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BrickWORK Software-Aided Analysis of Masonry Structures

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Abstract

Among the various types of software described in the earlier works of the authors and subsequently published by an Italian software-house, BrickWORK is by far the most significant software tool of the next generation. BrickWORK is the result of algorithmic calculations developed by the authors to perform nonlinear analysis of existing masonry structures. Its potential was realized during the analysis of masonry structures of historical and cultural importance, which will be discussed in this article. Using an iterative procedure, the program allows one to analyse and compute, in 2D and 3D, the degree of safety and the vulnerability level of a masonry structure when it is subjected to vertical loads, foundations settlements and the horizontal action of an earthquake.

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Keywords: Computer program BrickWORK; Numeric algorithm; Nonlinear analysis; Masonry structures; Architectural heritage

1. Introduction

In the world of international scientific research regarding the structural analysis of masonry buildings, there are three principle schools of thought:

- Masonry Continuum [1];
- Discrete Model [2];

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- Intermediate school of thought, that proposes a model based on a macro-elements model [3].

The authors of this paper belong to the second school of thought, since they are convinced that by modelling a masonry apparatus "similar" to the actual structure, it is possible to simulate the actual behaviour of any structure made of inert elements of different shapes (bricks, stones), arranged in different ways in the masonry apparatus, and characterized by the presence of different materials (brick, stone, mortar). It was, indeed, ascertained that, due to different external actions, dislocations generally occur in correspondence to the linking joints and that, therefore, the vulnerability of the masonry construction is a function of the number of blocks.

The load-bearing masonry is a non-homogeneous and non-isotropic "material". While its compression strength is very high, its tensile strength is very low. However, its shear strength and, therefore, its capacity to preserve sliding blocks along the joints, is difficult to quantify since it always depends on various intrinsic factors such as cohesion, the peculiar characteristic of mortar joints, and the level of internal compression which improves the friction. All these factors are affected by the shape and arrangement of the blocks in the masonry apparatus.

For these reasons, the calculation algorithm is applied to the mechanical model of a masonry structure modelled by a finite number of inert elements (blocks) arranged so as to obtain a precise structural geometry. The various structural typologies that have been analyzed by the program BrickWORK include arches, barrel vaults, cross vaults, Catalan vaults, porticos, load-bearing walls with and without openings, domes with or without an oculus and/or lantern, Roman and Pompeian forum entablatures.

2. The Frictional and Unilateral Joint

In the BrickWORK model of a masonry apparatus, a choice of joints is possible. The choice, mortar joint or contact joint to connect the blocks, determines the type of analysis to be performed. Mortar joint ("elastic-cracking joint") is used to compute the internal stress of the joints, displacements of the blocks, geometric configuration caused by external actions and position and depth of the cracks in the joints. While contact joint ("brittle-rigid joint"), indeed, is used only to discover the existence of one equilibrated solution capable of assuring construction stability. For this reason, in this case, the analysis belongs to limit analysis procedures and the result is only one of the infinite number of equilibrated solutions. To interlink the blocks in correspondence to the joints, a discrete interface device (Fig 1) was designed and modeled by a set of links orthogonal to the joints and another set of links tangent to the joints [4].

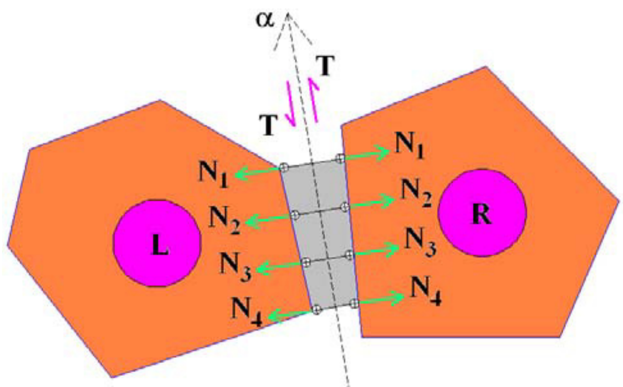

Fig. 1. discrete model of the joint between two blocks

The orthogonal links are designed in such a way as to transmit the normal forces from one block to another. In the case of mortar joints, the only forces which can be transferred are the compression forces which are lower than the value which causes the masonry to crush and the tensile forces which, as a result of the low mortar tensile strength, do not cause the blocks to detach. On the other hand, in the case of contact joints, the limit tensile strength is equal to zero and, therefore, the material is assumed to be no-tension. Therefore, in both cases, a limit value for compression and another for tension must be defined by the user.

The function of the tangential links, however, is to transmit shear forces and manage the phenomena of reciprocal sliding blocks. Therefore, also in this case, a limit value for the transmittable shear force must be defined; when this value is exceeded, the sliding or incipient sliding phenomenon occurs. The algorithm considers both the case of the *"sliding friction"*-like behaviour proposed by Coulomb (when the limit shear strength is reached, the block slides and the interface shear force becomes zero), and the case of the *"plasting shearing"*-like behaviour proposed by Drucker (the shear force can never exceed the limit value which is a function of the interface compression force N and the friction factor f: $T = f{\cdot}N$. Once the limit value is reached, the block does not move, unlike in the previous case, but the section of the joint undergoes a plastic deformation and the transmittable shear force assumes the constant value of its limit value) [5]. As stated above, the joints of the model can be defined as "unilateral and frictional joints" (Fig 2).

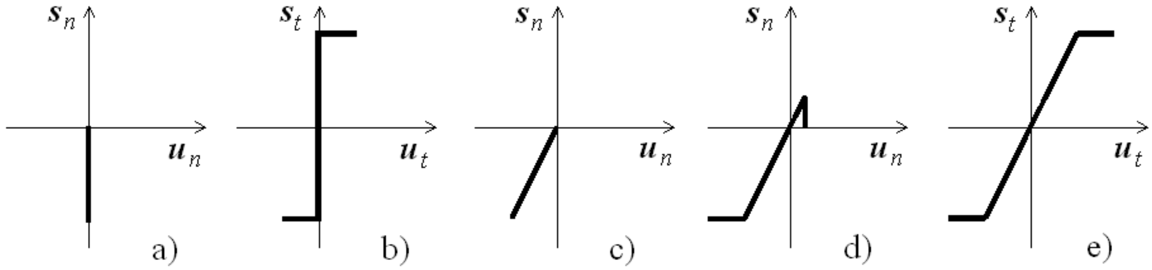

Fig. 2. behaviour of the interface joints: brittle-rigid joint in normal direction (a) and tangential (b) – elastic-cracking joint in normal direction (c) or (d) and tangential (e)

3. Brief Introduction to the BrickWORK Algorithm

The choice of the masonry discrete structure model, computed by the numerical procedure illustrated below [6], was derived from testing and observing the differences between the properties of mortar (when present), bricks and stones. It is not necessary to explain in detail the deformability differences between mortar and blocks. Since mortar is much more deformable than blocks, the model (called "concentrated elasticity model") was assumed to be composed of rigid blocks and deformable mortar joints. Contact joint deformability, an abstract concept, is instead imagined to be infinite.

Two equations govern the discrete model made of rigid blocks, unilateral and frictional joints and elasticity concentrated in the joints: the *equilibrium equation* (first equation in system (1) and (2)) and the *elastic-kinematic equation* (second equation in system (1) and (2)). The equations are written in algebraic and matrix form; so they can be immediately translated into program language for software development.

In system of equations (1) and (2), vector X constitutes the static unknown of the problem: normal forces X_n and shear forces X_t transmitted by interface links. Matrix A is populated by the coefficients of the unknowns. Vectors F (loads) and Δ (foundation settlements) are, however, known vectors and constitutes external actions on the construction. Coefficients of vector x, the kinematic unknown of the problem, are the block displacements. Finally, matrix K, used only in the case of elastic-cracking joint, is a matrix that contains the deformability coefficients of the joints in correspondence to each link.

Joint strengths, expressed by specific tensile, compression and shear limits, are described in the mathematical form by the inequalities in (1) and (2). This material nonlinearity requires the use of an iterative algorithm. The algorithm begins by computing the initial static solution, represented by vector X_0. Next all its coefficients are controlled and compared with the corresponding limits (ultimate strengths). If their values do not exceed any of the limits, the initial solution is assumed to be the final solution of the problem ($X=X_0$) and no iteration will be carried out. In this case, the resulting solution is equilibrated and compatible.

$$\begin{cases} AX+F=0 \\ A^T x = \Delta + \delta \end{cases} \quad sub \begin{cases} X_n \le 0 \\ X_t \le f \cdot \sum_{j=1}^{k} X_{nj} \\ \overline{\delta} \ge 0 \end{cases} \qquad \textit{brittle-rigid contact joint (1)}$$

$$\begin{cases} AX+F=0 \\ A^T x + KX = \Delta + \overline{\delta} \end{cases} \quad sub \begin{cases} X_n \le 0 \\ X_t \le f \cdot \sum_{j=1}^{k} X_{nj} \\ \overline{\delta} \ge 0 \end{cases} \qquad \textit{elastic-cracking mortar joint (2)}$$

Instead, even in the case where only one coefficient of X_0 exceeds the predetermined limit value, the solution is equilibrated but not compatible. Therefore, the algorithm performs at least one iteration to modify the value of such a coefficient so that the compatibility condition is respected. If that coefficient is a tensile force, the iteration nullifies the force and consequently a crack in a joint can occur (block detachment and separation). Instead, if that coefficient is a shear force, depending on the choice of the behaviour suggested by Drucker or Coulomb, the iteration will nullify the force or lower it so as to assume the limit value which activates the plasticization of the section.

In any case, the nullifying of a value of one of the forces in a link corresponds to its removal from the mechanical model and it, therefore, provokes the loss of one degree of static indeterminacy of the structure. As a fundamental consequence, the number of possible iterations, that is the number of changeable link values, is equal to the degree of static indeterminacy [7]. The removal of a further link would make the structure unstable and a precise collapse mechanism will develop.

To change the value of the force in an interface connecting link, the algorithm uses impressed distortions. These distortions are compulsory changes in the geometry of the structure and they vary the inner stress. When the inner stress due to distortions (coactions) is overlapped with the stress due to loads, the whole inner stress is modified. If those distortions are introduced in the system in the correct position(s), it will be possible to modify the solution X_0 so as to satisfy the compatibility requirements. The inner distortion vector δ is an additional unknown of the problem which can be computed thanks to the constraint conditions imposed by the inequalities in (1) and (2). At the *i-th* step of the procedure, the distortion vector assumes the form as follows:

$$\delta_i = L - C_i^{-1} X_{0i} \tag{3}$$

where L constitutes the limit strength in terms of a force and, therefore, it is the result of the product between an ultimate strength and the cross section of the link affected by distortion.

Applying the distortion vector to the system, at the end of the iterations the final static solution assumes the binominal relationship $X = X_0 + X_N$, where X_N is the force vector in the interface links as a consequence of the only distortions.

In the case of elastic-cracking joint, the two vectors, which added together provide the final solution, assume the relationships (4) and (5):

$$X_0 = K^{-1}A^T(AK^{-1}A^T)^{-1} \cdot F + K^{-1}(I - A^T(AK^{-1}A^T)^{-1}AK^{-1}) \cdot \Delta \qquad (4)$$

$$X_N = (I - K^{-1}A^T(AK^{-1}A^T)^{-1}A) \cdot \overline{\delta_1} \qquad (5)$$

Instead, in the case of brittle-rigid joint, the two vectors assume the simplified relationships (6) and (7):

$$X_0 = A^T(AA^T)^{-1} \cdot F + (I - A^T(AA^T)^{-1}A) \cdot \Delta \qquad (6)$$

$$X_N = (I - A^T(AA^T)^{-1}A) \cdot \delta \qquad (7)$$

If the user adopts the elastic-cracking mortar joint, the kinematic solution can also be obtained. The block displacement vector can be computed considering that the final static solution X is the consequence of a nonlinear calculation procedure, in which cracks in the joints may have occurred. Therefore, this vector can be computed numerically by implementing a mathematical procedure that involves the partition of the matrices A^T and K respectively in the sub-matrices $[A^T_T \mid A^T_C]$ and $[K_T \mid K_C]$, where subscript "c" and subscript "t" refer to coefficients of the two matrices as follows: subscript "c" is relative to forces in the interface links which are not affected directly by distortions and subscript "t" is relative to forces whose values are purposely modified because, in the previous step of the algorithm, the forces exceeded the user-defined ultimate strengths. The solution of system (2), re-written by partitioning the two matrices, provides both the displacement vector x (8) and the vector which quantifies the crack depth as well as their position (9):

$$\overline{x} = -(A_c A_c^T)^{-1} A_c K_c X_c + (A_c A_c^T)^{-1} \cdot \Delta_c \qquad (8)$$

$$\overline{\delta_2} = A_i^T \overline{x} + \Delta_i \qquad (9)$$

4. Relevant Case Studies

The potential of the algorithm and the versatility of the software have been ascertained several times in the analysis of relevant masonry structures of historical interest. Previous releases of the algorithm were implemented in software devoted to the analysis of specific structural typologies, such as the barrel vault [8] and the cross vault [9]; some of these were also published and marketed by an Italian software-house [10]. BrickWORK, is, however, the most advanced computer program in this category developed by the authors, and implements the most updated release of the calculation algorithm. In addition to the above mentioned typologies, it allows one to study other more generic types of masonry structures as well.


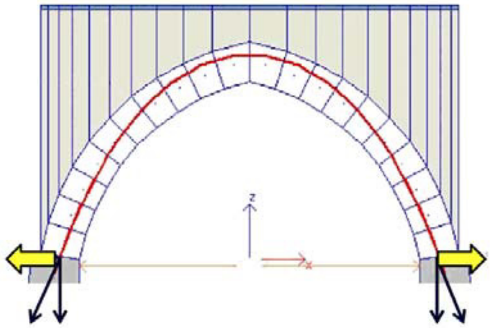

Fig. 3. (a) an aerial photo of the Columbian Fort San Fernando; (b) stability analysis of one of its barrel vaults and line of thrust output

The most recent case study, Fort San Fernando de Bocachica in Colombia (Fig. 3a), is significant because BrickWORK computer simulations allowed the authors to diagnose the causes of the damages of the outer circular wall on which a sequence of barrel vaults is set. An analysis of barrel vaults was carried out (Fig.3b). Their stability condition was ascertained and graphically shown by the line of thrust, automatically drawn by the software inside the section's profile of the vault at the end of the analysis. Actions transferred to the circular wall by the vaults were computed and allowed the authors to ascertain the incipient mechanism condition of the wall overturning. This case study is going to be discussed, in detail, at the Spanish conference *Rehabend 2014* which will be held in April 2014 in Santander [11].

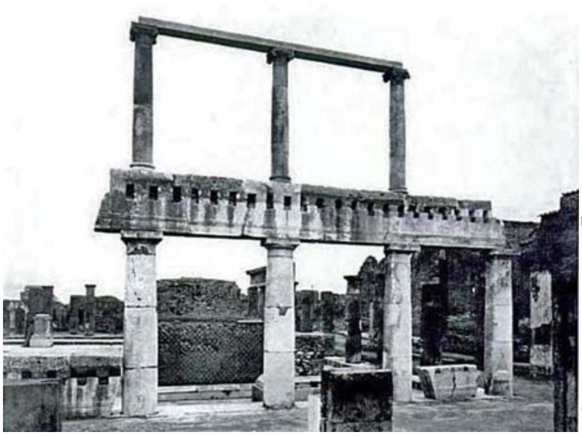
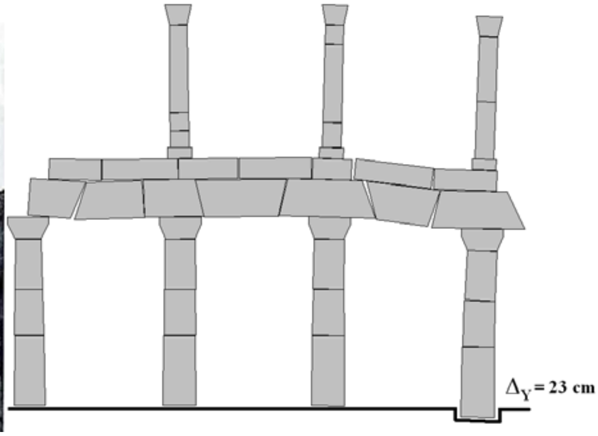

Fig. 4. (a) a photo of the "Trabs" in the Pompeian Forum; (b) vulnerability analysis of a portion due to a vertical displacement in correspondence to the right pillar

Our next case study which analyses a portion of the Pompeian Forum (Italy), illustrated in Fig 4a, is unique. In order to compute the vulnerability level of this construction relative to probable displacements due to foundation settlements, BrickWORK was used to perform the nonlinear analysis both materically and geometrically. To numerically quantify the vulnerability level, the peak value of the vertical displacement of the right pillar was computed. It was necessary to perform a step by step analysis, augmenting the vertical displacement value at each level and continually update the geometric configuration of the structure (Fig 4b).

During the analysis, the changes in the graphic configuration was performed by the software in real-time. For each of the updated geometric configurations, BrickWORK performed a materic nonlinear analysis. Therefore, numerous materic nonlinear analyses were performed [12].

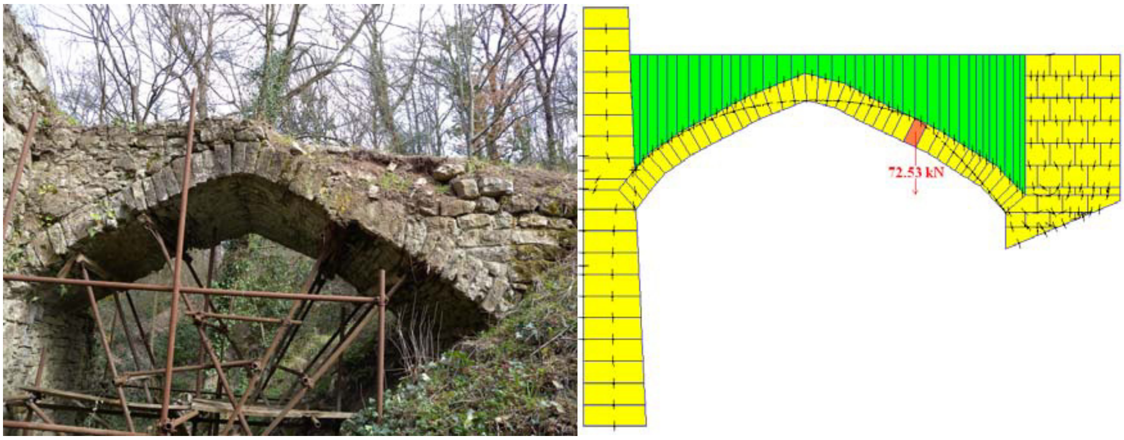

Fig. 5. (a) a side photo of the Rignalla bridge (FI) from uphill; (b) safety degree analysis in case of a moving load due to pedestrian traffic on the bridge

In 2010, the Reclamation Consortium of Central Tuscany, the authority for the protection and maintenance of soil and waterways, entrusted the authors with a research project aimed at a cognitive analysis of the Rignalla bridge (Fig 5a) and a rehabilitation and functional recovery project for a pedestrian public use. The Rignalla bridge is a small masonry arch bridge that was discovered in the countryside near Florence (Italy). It was necessary to determine its degree of safety relative its ability to support occasional pedestrians. For this purpose, BrickWORK allowed us to numerically ascertain the safety degree by simulating pedestrian movement by applying an increasing vertical load to each of the stone blocks in the bridge (moving load analysis). This analysis allowed the authors to determine that the structure was secure and, after necessary consolidation and restoration interventions, the bridge was finally reopened to the public [13].

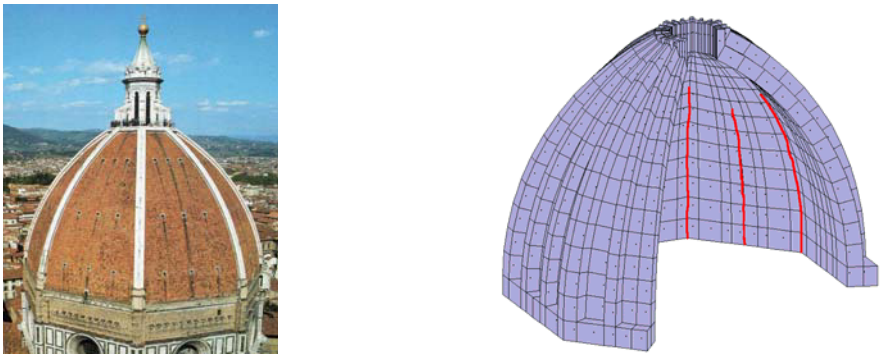

Fig. 6. (a) a photo of the dome of S. Maria del Fiore in Florence; (b) BrickWORK structural model and crack lines

The next case is a very interesting three-dimensional study of the Dome of Santa Maria del Fiore, the internationally well-known basilica in Florence, Italy (Fig 6a). Using BrickWORK, the authors were able to reconstruct the dome three-dimensionally and numerically demonstrate the advantages of the construction technique adopted by Brunelleschi. His technique consisted of arranging the blocks along inclined surfaces mathematically described by fourth degree equations called *quartics*. Thanks to this technique, Brunelleschi was able to construct an octagonal dome using circular dome construction rules. Furthermore, the numerical results highlighted the presence of cracks along the diagonal ribs and in the middle of the lunes. The cracks ascertained by BrickWORK are in the exact same position as the actual cracks that developed immediately after construction of the dome in the 1400s. The historical and constructive details and the results of the analysis carried out by BrickWORK were presented in 2012 at the international conference *Domes in the World* held in Florence [14].

5. Summaries

The topic of conserving architectural, historical an cultural heritage is a very real and important topic in many nations, especially in Europe and South America. In particular, Italy has many historical masonry constructions and archaeological sites.

The first step in the conservation of historical constructions is to analyse their degree of safety and vulnerability level as they relate to vertical loads, foundation settlements, seismic actions or simply an increase of the service loads as a consequence of the change in building use.

The computer program BrickWORK, even if still unpublished, was designed to be an operative tool for academics and professionals alike to numerically quantify the degree of safety and vulnerability level of masonry constructions. It can be used to carry out local verifications. Therefore, by utilizing BrickWORK, the user can achieve a higher level of knowledge about an architectural construction, in particular to forecast its structural behaviour simulating different events, thereby identifying specific types and locations of weak points. Thus, the numerical results and graphs furnished by the program can be used to assist the user in adequately designing strengthening and consolidation interventions.

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