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E. Papini(1), L. Franci(2)(3), S. Landi(1)(3), P. Hellinger(4), A. Verdini(1), and L. Matteini(5)

(1) *Dipartimento di Fisica e Astronomia, Università degli Studi di Firenze, via G. Sansone 1, Sesto Fiorentino 50019, Italy*
(2) *School of Physics and Astronomy, Queen Mary University of London, London E1 4NS, United Kingdom*
(3) *INAF, Osservatorio Astrofisico di Arcetri, Largo E. Fermi 5, Firenze 50125, Italy*
(4) *Astronomical Institute, CAS, Bocni II/1401, Prague CZ-14100, Czech Republic*
(5) *LESIA, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, Univ. Paris Diderot, Sorbonne Paris Cité, place Jules Janssen 5, 92195 Meudon, France*

Summary. — Properties of decaying Alfvenic plasma turbulence are investigated by means of two-dimensional Hall-magnetohydrodynamic and hybrid particle-in-cell numerical simulations. In most cases, spectral properties of turbulent fluctuations find good agreement in both the numerical models. The power spectra of the magnetic field exhibit a double power-law with spectral index -5/3 at large, fluid scales and -3 at sub-ion scales, while for velocity fluctuations the spectral index at fluid scales is -3/2. In both models, the development of a turbulent cascade is concurrently characterized by magnetic reconnection events that are fast, with inverse reconnection rates much smaller than the characteristic large-eddy turnover times. Moreover, these reconnection events trigger a direct energy transfer from large to sub-ion scales. This supports the existence of a reconnection-mediated turbulent regime at sub-ion scales. We conclude that the Hall-MHD fluid description captures to a large extent the transition of the turbulent cascade between the fluid and sub-ion scales.

1. – Introduction

Turbulence is ubiquitous in laboratory and space plasma. It is characterized by disordered and chaotic fluid motion and often shows violent and random changes in the kinetic and magnetic properties of the plasma. A key aspect of turbulence concerns understanding how energy is transferred from the large global scales across ion and sub-ion scales, down to the scales where it is dissipated. This is of particular interest in weakly collisional plasmas, since such mechanisms play a role in, *e.g.*, the formation of hot coronae, the heating and acceleration of the solar wind and its interaction with planetary atmospheres, solar flares and coronal mass ejections. Recently, Hybrid particle-in-cell (HPIC) numerical simulations [1, 2] performed using the CAMELIA code [3], which describe full kinetic ions and fluid massless isothermal electrons, successfully reproduced most of the turbulent properties observed in the solar wind and in the magnetosheath [3]. Such a model is rather complex, as it retains all ions' kinetic properties. It is therefore convenient to focus on a model that contains fewer physical ingredients, in order to isolate or rule out some of the candidate mechanisms taking place at kinetic scales. The Hall-MHD (HMHD) model is a suitable candidate for turbulence studies [4, 5, 6], as it contains the same induction equation of the HPIC model and is the only equation that changes with respect to MHD. Moreover, it describes the electron-ion velocity decoupling, whistler, ion-cyclotron, and kinetic Alfvén waves, and it contains the ion inertial length as a characteristic scale. In this work we present the results of a comparative study that highlights the similarities and differences between two simulations of decaying turbulence, one employing the HMHD model, the other the HPIC model. We complement the analysis with results from a MHD simulation, performed by using the HMHD code and setting the Hall term to zero.

2. – Hall-MHD pseudospectral simulations of Alfvénic turbulence

We integrate the nonlinear fully-compressible and viscous-resistive HMHD equations in a 2D periodic domain, by means of a pseudospectral code we developed [7, 8]. The numerical setup of the HMHD simulation is the same of the CAMELIA simulation employed in [9]. For further details on the initialization and numerical implementation, see [10]. In Hall-MHD, the presence of the Hall term in the induction equation introduces the ion inertial length $d_i = c/\omega_{pi}$ (where ω_{pi} is the ion plasma frequency) as new characteristic scale, and $\tau_A = \Omega_i^{-1}$ (*i.e.* the inverse of the ion-cyclotron frequency $\Omega_i = eB_0/m_i c$) as characteristic time scale. We evolve the plasma in a 2D periodic box of size $L_x \times L_y = 256\ d_i \times 256\ d_i$ using a square grid of 2048^2 points. An out-of-plane mean magnetic field $\boldsymbol{B}_0 = B_0\hat{\mathbf{z}}$ is set. We populate the initial state with freely-decaying large-amplitude Alfvénic-like fluctuations in the xy-plane and up to the injection scale $\ell_{\text{inj}} = 2\pi d_i/k_\perp^{\text{inj}}$, with $k_\perp^{\text{inj}} d_i \simeq 0.2$, where $k_\perp^2 = k_x^2 + k_y^2$. The rms amplitude relative to B_0 of the fluctuations is $\simeq 0.24$. We set the plasma beta to $\beta = 2$, and $S = 10^3$, corresponding to a global Reynolds number of $\sim 4 \cdot 10^4$. We note that for the MHD simulation the inertial length d_i has no physical meaning. However, we use d_i (only in a numerical sense) as a normalization length also in the MHD case.

In all simulations, the initial Alfvénic fluctuations quickly evolve to form large vortices with current sheets in between. These sheets then shrink down to a critical width (of the order of d_i in the HMHD and the HPIC run, smaller in the MHD run) and disrupt due to the triggering of magnetic reconnection events. Then new reconnection events take place in newly formed current sheets, until turbulence fully develops. Bottom panels of Fig. 1 show the distribution of the reconnection rates as a function of time. Reconnection is very fast, the average reconnection time being 9.6 τ_A, 12.2 τ_A, and 4.3 τ_A for the MHD, the HMHD, and the HPIC case respectively. Top panels of of Fig. 1 show the characteristic turbulent nonlinear time, $\tau_{nl}(k_\perp) = (k_\perp \mathrm{v}_e(k_\perp))^{-1}$, at a given scale $\ell = 2\pi/k_\perp$. $v_e(k_\perp)$ is the electron velocity amplitude at that scale (in the MHD case, $v_e(k_\perp)$ is replaced by the fluid velocity). These plots are much informative, since $\tau_{nl}(k_\perp)$ is also a proxy for the amount of energy that is being deposited at the scale ℓ. In all simulations, when the first reconnection events are triggered (see Fig. 1), energy is directly transfered to the smallest scales accessible in the HPIC and in the HMHD run and at $k_\perp d_i \simeq 4$ in

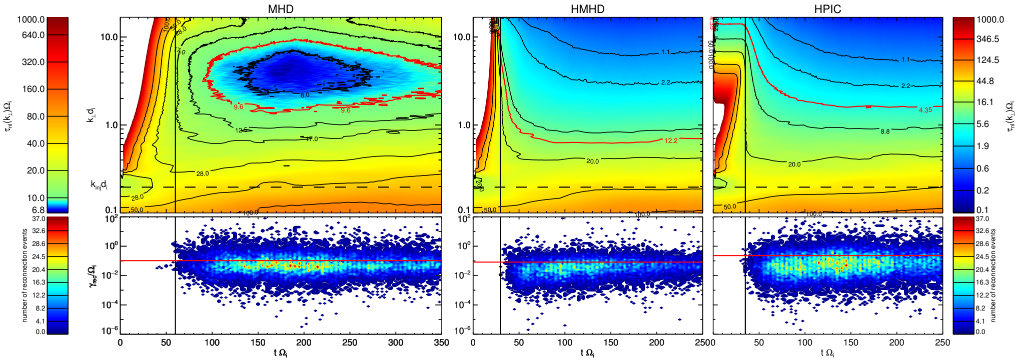

Fig. 1. – Coloured contours of the characteristic nonlinear time $\tau_{nl}(k_\perp)$ (top panels) and distribution of reconnection rates (bottom panels) for a MHD (left), a HMHD (center) and for a HPIC (right) simulation with same numerical setup. Horizontal red lines denote the average reconnection rates of 0.1, 0.08, and 0.22 respectively. For each simulation, black vertical lines indicate the time of the first reconnection events.

the MHD run, since at those scales $\tau_{nl}(k_\perp)$ suddenly decreases. Later, in a transient phase that last 20 to 30 Alfvén times (characterized by almost vertical isocontour lines of $\tau_{nl}(k_\perp)$), the energy is fed to larger and larger scales. We interpret this behavior as the signature of coalescence of plasmoids. The number of reconnection events increases very rapidly and reaches a statistically stationary value. After this transient, the nonlinear time $\tau_{nl}(k_\perp)$ changes only slightly.

We now focus on the spectral properties of fully developed turbulence. This state is reached at $t \simeq 180\ \tau_A$, $150\ \tau_A$, and at $200\ \tau_A$ in the MHD, HMHD, and HPIC run respectively. Figure 2 shows the isotropized power spectra $\mathrm{P}(k_\perp)$ of magnetic, velocity, and density fluctuations for all the three models at those times. The HPIC and HMHD power spectra of the total magnetic field fluctuations nicely overlap, from the large fluid scales, where they produce a cascade with slope $-5/3$, through the kinetic range, where

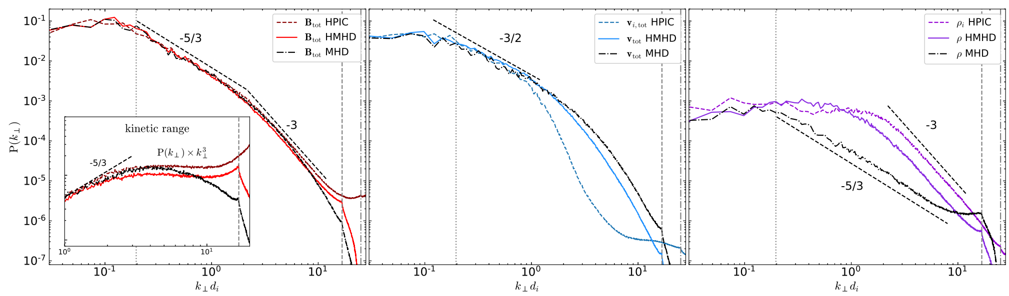

Fig. 2. – Isotropized power spectra $\mathrm{P}(k_\perp)$ as a function of $k_\perp = \sqrt{k_x^2 + k_y^2}$ from the MHD, the HMHD, and the HPIC run, of total magnetic field (left), perpendicular ion velocity (middle), and density (right) fluctuations. The vertical dotted, dashed, and dot-dashed lines denote the injection scale $k_\perp^{\mathrm{inj}} d_i \simeq 0.2$, the 2/3 filter's cutoff of the HMHD model, and the Nyquist wavenumber respectively. Black dashed lines denote reference slopes. The inset panel shows a zoom of the magnetic spectra in the sub-ion range, compensated with $k_\perp^{-3}$.

they both show a power-law of slope −3, and down to the 2/3 filter's cutoff. The MHD power spectrum also matches at large scales, and differs at scales smaller than d_i, as expected (see the kinetic range inset). Spectra of the velocity fluctuations of all models match at fluid scales and diverge at kinetic scales. Interestingly, the MHD and the HMHD velocity spectrum are not the same at small scales, as the Hall term indirectly affects the fluid velocity through the Lorentz force. Right panel of Fig. 2 shows the spectrum of the ion plasma density. Both the HPIC and the HMHD spectrum have a power law with the same index of −3, but shifted. This is due to the HMHD adiabatic prescription, which affects the sound speed, thus changing the properties of kinetic Alfvén waves [11] with respect to the HPIC run. Finally, the spectrum of the density of the MHD run shows an extended −5/3 power law, from the injection scale down to the dissipation scale.

3. – Discussion

We have shown that Hall-MHD simulations of Alfvénic decaying turbulence are able to recover most of the properties of Hybrid-PIC simulations. In particular, the ability of HMHD to reproduce the spectral properties of magnetic field fluctuations is striking. The dynamics of magnetic reconnection, especially in relation to the turbulent evolution, is also similar in both cases, unlike in MHD where there is no characteristic scale as the ion inertial length. Results of this work confirm the potential use of Hall-fluid models for studying turbulence at kinetic scales.

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