



UNIVERSITÀ
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POLITÉCNICA

UNIVERSIDAD
POLITÉCNICA
DE MADRID

DIPARTIMENTO DI INGEGNERIA CIVILE E
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Università degli Studi di Firenze

DEPARTAMENTO DE INGENIERÍA CIVIL,
HIDRÁULICA Y ENERGÉTICA

Universidad Politécnica de Madrid

**A METHODOLOGY FOR THE
BIVARIATE HYDROLOGICAL
CHARACTERIZATION OF FLOOD
WAVES FOR RIVER-RELATED FLOOD
RISKS ASSESSMENTS**

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Abstract

The Flood Directive 60/2007/EC requires the European countries to verify the effectiveness of existing flood defence infrastructure to mitigate flood risks. The current practice establishes that the river flood control structures must respect a basic requirement, usually consisting of a certain safe freeboard under a design peak flow rate corresponding to a certain probability of exceedance. This requirement has some critical issues. It is based on a univariate frequency analysis of only flood peak, and therefore it assumes a perfect correspondence between the probability of occurrence of the hydrological variable and the failure of the flood-control structure. The thesis aims to define a methodology to overcome these issues implementing a bivariate hydrological risk analysis for river-related flood risk. The methodology is mainly focused on the overtopping failure for river levees. River levees are the most common river flood-control structure. They are raised, predominantly earth, structures (also called dykes, digues or embankments). The overtopping failure of a river levee caused several flood disasters such as Elbe flood (2002), New Orleans flood (2005), Emilia Romagna Flood (2017) or Arkansan flood (2019).

The proposed procedure is carried out through two steps: (i) the evaluation of hydrological failure related to the overtopping risk for a levee; (ii) the estimation of the probability of occurrence of the hydrological failure introducing the concept of the Bivariate Failure Return Period.

The hydrological failure is determined considering the mutual interaction between a bivariate hydrological load of peak discharge (Q) and the volume of the hydrograph (V), the river conveyance, and the levee resistance with respect to overtopping. The bivariate hydrological load considers an approximation of the real bivariate distribution of Q and V , functional to determine the hydrological failure. The shape of the hydrographs is classified concerning the overtopping introducing the Overtopping Hydrograph Shape Index (OHSI). The hydrological failure condition is represented by a curve in Q - V space containing all the hydrographs causing the initiation of the damage. This curve demonstrates that not only the peak flow but also the volume of the hydrograph are essential variables to characterise the overtopping failure.

The risk of overtopping failure is expressed by the probability of occurrence of the hydrological failure within a new interpretation of the return period. Because of the hydrological failure curve is a function in Q - V space, the return period is estimated in the bivariate framework. Several definitions of the bivariate return period are available, each of which gives a different interpretation of it. This critical issue is overcome introducing the Bivariate Failure Return Period. The Bivariate Failure Return Period assesses the probability of the failure curve of the hydraulic structure generating possible scenarios through a Monte Carlo simulation. Two case studies are presented to demonstrate the applicability of the methodology and the advantages of using it respect to the current practice. In the thesis, the

methodology is also applied to the problem of flood damage estimation demonstrating the flexibility and the validity of the proposed procedure. In this case, the hydrological failure consists in equal-euro curve in Q-V space, which includes all the hydrographs causing the same euro flood damages in a site.

The procedure proposed needs Q-V data at the target site where the risk is to be assessed but most of the sites are ungauged. This issue is overcome by testing the bivariate regional frequency analysis. It is applied to the case study of the entire Tuscany Region (Italy). By the bivariate regional analysis, the Q-V series can be estimated at ungauged sites, and the uncertainty is reduced at gauged sites.

Resumen

La Directiva de inundaciones 60/2007 / CE requiere que los países europeos verifiquen la efectividad de la infraestructura de defensa contra inundaciones existente para mitigar los riesgos de inundación de manera eficiente. La práctica actual establece que las estructuras de control de inundaciones del río deben respetar un requisito básico, que generalmente consiste en un cierto francobordo seguro bajo un caudal máximo con una cierta probabilidad. Este requisito tiene algunos problemas críticos. Como ejemplo, se basa en un análisis de frecuencia univariante de la caudal máxima, o supone una correspondencia perfecta entre la probabilidad de ocurrencia de la variable hidrológica y la falla de la estructura de control de inundaciones. La tesis tiene como objetivo definir una metodología para superar estos problemas implementando un análisis de riesgo hidrológico bivariado para el riesgo de inundación relacionado con el río. La metodología se centra principalmente en la falla de desbordamiento de los diques de los ríos. Los diques de los ríos son la estructura de control de inundaciones más común. Son estructuras elevadas, predominantemente terrestres (también llamadas diques, excavaciones o terraplenes). La falla de desbordamiento del dique de un río causó varios desastres de inundación como la inundación de Elba (2002), la inundación de Nueva Orleans (2005), la inundación de Emilia Romagna (2017) o la inundación de Arkansas (2019).

El procedimiento propuesto se lleva a cabo a través de dos pasos: (i) la evaluación de la falla hidrológica relacionada con el riesgo de desbordamiento de un dique; (ii) estimación de la probabilidad de ocurrencia de la falla hidrológica introduciendo el concepto del Período de retorno de falla bivariada.

La falla hidrológica se determina considerando la interacción mutua entre una carga hidrológica bivariada de pico de caudal (Q) y el volumen del hidrograma (V), el transporte del río y la resistencia del dique con respecto al sobrecorte. La carga hidrológica bivariada considera una aproximación de la distribución bivariada real de Q y V , funcional para determinar la falla hidrológica. La forma de los hidrogramas se clasifica con respecto al sobrecorte que introduce el Índice de forma del hidrograma de sobrecorte (OHSI). La falla hidrológica está representada por una curva en el espacio Q - V que contiene todos los hidrogramas que causan el inicio del daño. Esta curva demuestra no solo el flujo máximo sino que también el volumen del hidrograma es una variable esencial para caracterizar la falla de sobrecorte.

El riesgo de fallas excesivas se expresa por la probabilidad de fallas hidrológicas en el plazo amplio del período de retorno. Debido a que la curva de falla hidrológica es una función en el espacio Q - V , el período de retorno se estima en el marco bivariado. Existen varias definiciones del período de retorno bivariado, cada una de las cuales ofrece una interpretación diferente del mismo. Este problema crítico se supera al presentar el Bivariate Failure Return Period. El Bivariate Failure Return Period evalúa la probabilidad de la curva de falla de la

estructura hidráulica generando posibles escenarios a través de una simulación de Monte Carlo. Se presentan dos estudios de caso para demostrar la aplicabilidad de la metodología y las ventajas de usarla con respecto a la práctica actual. En la tesis, la metodología también se aplica al problema de la estimación de daños por inundación que demuestra la flexibilidad y la validez del procedimiento propuesto. En este caso, la falla hidrológica consiste en una curva de euro igual en el espacio Q-V, en el que hay todos los hidrogramas que causan los mismos daños por inundación en euros en un sitio.

El procedimiento propuesto necesita datos de Q-V en el sitio de destino donde se debe evaluar el riesgo, pero la mayoría de los sitios no están calibrados. Este problema se supera probando el análisis bivariado de frecuencia regional. Se aplica al estudio de caso de toda la región de Toscana (Italia). Mediante el análisis regional bivariado, la serie Q-V se puede estimar en sitios no calibrados, y la incertidumbre se reduce en los sitios calibrados.

Index

Abstract.....	iii
Resumen.....	v
Index.....	vii
List of figures.....	x
List of tables.....	xiii
List of symbols	xiv
Introduction.....	1
<i>Motivation</i>	<i>1</i>
<i>Objectives.....</i>	<i>2</i>
<i>Thesis structure</i>	<i>4</i>
1 Current state of knowledge.....	6
1.1 Risk assessment for a river hydraulic structure	6
1.1.1 Standard approach: univariate hydrological load.....	6
1.1.2 Recent developments: the multivariate hydrological load	9
1.2 The return period in bivariate field	12
1.2.1 The hydrological return periods.....	12
1.2.2 The structural return periods.....	15
1.3 The overtopping mechanism.....	15
1.4 Discussions	17
2 Methodology.....	20
2.1 Characterisation of the bivariate hydrological failure curve focusing on overtopping mechanism for a river levee	21
2.1.1 Selection of the events: Peak and Volume series.....	23
2.1.2 The hydrological load: the forcing hydrographs	25
2.1.3 Generation of forcing hydrographs.....	27

2.1.4	Hydraulic Conveyance and Critical Overtopping Flood Hydrograph.....	28
2.2	<i>Probability of the bivariate hydrological failure line: the Bivariate Failure Return Period ...</i>	29
2.2.1	Definition of the bivariate distribution.	31
2.2.2	Failure probability and the Bivariate Failure Return Period	37
3	Case study	40
3.1	<i>Bivariate hydrological failure line</i>	40
3.1.1	Hydrological Load estimation	40
3.1.2	Shape Ranking concerning the OHSI	42
3.1.3	Generation of the forcing hydrographs	43
3.1.4	Hydraulic Conveyance.....	43
3.1.5	Results and discussion	44
3.2	<i>Estimation of the Bivariate Failure Return Period</i>	48
3.2.1	Definition of Bivariate distributions	49
3.2.2	Results and discussions.....	54
4	Bivariate hydrological flood damage estimation.....	58
4.1	<i>Methodology.....</i>	59
4.1.1	Damage modelling.....	59
4.1.2	Case study.....	60
4.2	<i>Results and Discussions</i>	65
5	Multivariate Regional Frequency Analysis: the case of the Tuscany region (Italy)	67
5.1	<i>Study area.....</i>	68
5.2	<i>Methodology and objectives</i>	68
5.3	<i>Screening of the data</i>	69
5.4	<i>Delineation of Homogeneous Regions.....</i>	69
5.5	<i>Selection of the Bivariate Regional Distribution.....</i>	70
5.6	<i>Index Flood.....</i>	77

5.7	<i>Quantile estimation and evaluation of the model</i>	80
5.8	<i>Results and discussions</i>	82
6	Conclusions	85
6.1	<i>Main conclusions</i>	85
6.2	<i>Limitation of the results</i>	87
6.3	<i>Outlooks</i>	88
	Appendix	91
	<i>A1-Evaluation of monthly parameter ϵ and b</i>	91
	<i>A2- Overtopping Hydrograph Shape Index - OHSI</i>	92
	<i>A3- Figures relative to the copula analysis for the chapter 3.2</i>	94
	<i>A4- Figures relative to the copula analysis for the chapter 5</i>	98
	References	104

List of figures

Fig. 1. Steady flow rating curve (grey) and unsteady flow rating curve (black) (adapted from Di Baldassarre and Montanari (2009)).....	1
Fig. 2. Bivariate illustration of the Failure Scenarios (shaded area) according to different approaches, in the copula domain: the (top-left) OR, (top-right), AND, (bottom-left) Kendall, and (bottom-right) the Survival Kendall. In the Kendall case, the lower bound of the hazard region corresponds to the critical isline L_t , while in the Survival Kendall case it corresponds L_t (adapted from Salvadori et al., 2016).	14
Fig. 3. The three main scenarios of levee overtopping (adapted from Hughes, 2008).	16
Fig. 4 Mechanism of erosion at the levee's toe due to overflow (adapted from Hewlett et al., 1987).....	17
Fig. 5. Flow chart of the overall procedure.	20
Fig. 6. An illustration of the difference between the univariate load (left) and the bivariate one (right). In the univariate case, there is only one load (blue line) and the resistance (red line) is raised by only one load, on the contrary, in the bivariate case, there are two loads (blue and green line) and a possible infinite combination of these raising the resistance.....	22
Fig. 7. Flow chart of the whole methodology to estimate hydrological failure line respect to the overtopping failure for a river levee.	23
Fig. 8. Same cross-section: on the left, different hydrographs with same peak and volume but different shape, from triangular to rectangular; on the right, the corresponding overtopping volume, and overtopping duration vs. corresponding values of Overtopping Hydrograph Shape Index (OHSI).	27
Fig. 9. Three triangular hydrographs with the same peak, and volume but different shape, at the same cross-section (a); the Overtopping Hydrograph Shape Index -OHSI of each hydrograph (b); detail of the rating curves (c) and the corresponding overtopping volumes (d) for each hydrograph.....	27
Fig. 10. The flow chart of evaluation of Bivariate Failure Return Period – BFRP.	31
Fig. 11. The two case studies analysed. On the left the case study of Poggio a Caiano, on the right the case study of Capannoli.	40
Fig. 12. Scatter-plot of the peak (Q) and volume (V) pairs in log-log space with recorded and synthetic Q-V pairs. On the left the case of Poggio a Caiano on the right the case of Capannoli.	42
Fig. 13. Overtopping failure events in Tuscany (1992-2014).	42
Fig. 14. Percentage of overtopping failure cases for the first 10 positions in the shape ranking based on the sensitivity analysis for the Poggio a Caiano and Capannoli case studies. The chosen threshold of 85% is also reported	43
Fig. 15. The curve represents the initiation of damage condition for a levee with average grass cover proposed by Hewlett et al. (1987). The area above to the curve is the damaged area; the area below represents the no-damage area.	44
Fig. 16. Example of two different situations at the same cross section: two hydrographs H-a, and H-b (a) with the same peak but different volume and realistic shape with the corresponding rating curves (b). Below the detail of the rating curves (c), and the corresponding overtopping hydrographs (d).	45

Fig. 17. The Critical Overtopping Flood Hydrograph at the cross-section of the analysed levee. On the left the site of Poggio a Caiano, on the right the site of Capannoli. The Q500 is the peak flow rate of 500 years return period, and 0.5m is the safe freeboard. The Q500_GPD are the values estimated from the inference analysis of the GPD distribution; the Q500_RBA are the values obtained by River Basin Authority	46
Fig. 18. Case study of Poggio a Caiano. The plots show the data (Weibull plotting position), and candidate distributions in lognormal probability plot for Q and V. The distributions selected by at least one criterion (AIC, AICc, BIC, ADC), are plotted in black, and in bold in the table, the others are in grey.	50
Fig. 19. Case study of Capannoli. The plots show the data (Weibull plotting position), and c, candidate distributions in lognormal probability plot for Q and V. The distributions selected by at least one criterion (AIC, AICc, BIC, ADC), are plotted in black, the others are in grey. In the table in bold the selected distributions are underlined.	51
Fig. 20. From left to right: the scatter plot of the data, and the corresponding χ -plot, and K-plot for both case studies.	52
Fig. 21. Scatter plots of the bivariate distribution model used in the case studies: on the left the case study of Poggio a Caiano modelled with Galambos copula; on the right the case study of Capannoli modelled with Gumbel copula. The grey dots are simulated data, the black ones are the observed data.	54
Fig. 22. For both case studies. The dot points represent the PFi evaluated through the Monte Carlo simulation. The blue dashed line is the failure probability curve obtained through interpolation of the PFi. The orange line represents the failure probability curve $RfBFRP, l$ evaluated through equation Eq.31.....	55
Fig. 23. On the top, the percentage breakdown of buildings, present in the study area, based on the intended use. At the bottom, the percentage breakdown of residential buildings based on their characteristics (number of floors, with cellar/without cellar).....	61
Fig. 24. Two flooded area corresponding to two synthetic hydrographs are shown. The two hydrographs have the same discharge peak but different volume (higher for the right flooded area).....	62
Fig. 25. Reference schemes for the vertical value distribution and structural stage–damage relationships for buildings.....	63
Fig. 26. Reference schemes for the vertical value distribution and structural stage–damage relationships for buildings.....	64
Fig. 27. In this plot, the line in red represents the curve in which all the hydrographs cause the same monetary damage for the building in the municipality area of Poggio a Caiano. The Bivariate Failure Return Period (chapt. 3.2) is evaluated for about 25 years.....	65
Fig. 28. The estimated damage curves corresponding to the return periods of 100 years and 200 years	66
Fig. 29. Location of Tuscany Region, the main rivers, and the gauged sites.....	68
Fig. 30. Flow chart of the overall procedure for the regional multivariate frequency analysis	69
Fig. 31. The multivariate homogenous regions in Tuscany. Each region is delineated taking into account the limits of each gauge site catchment and is not extend downstream the last its gauge site. The white parts belong to the basin that have drainage flow to other Italian Regions.....	70
Fig. 32. At-site behaviour for high return periods of the marginal distributions that have passed the goodness-of-fit test, i.e. the GEV, the Lognormal (LN) and the Gumbel (GUMB) distributions.....	72

<i>Fig. 33. The scatter plots of the rank of q, and v (left), the χ-plots in the middle, and the K-plots for all the homogeneous regions.</i>	<i>75</i>
<i>Fig. 34. Graphical comparison between quantile curves estimated by the proposed regional procedure at several target sites, and at-site quantile curves estimated by a local approach.</i>	<i>83</i>
<i>Fig. 35. Illustration of the uncertainty analysis for the local, and regional approach using 1,000 simulated quantile curves for $p = 0.99$ at gauged site TOS01004779 (in north region).</i>	<i>84</i>

List of tables

<i>Tab. 1. Copula families considered in this study. This copula functions are the most implemented in the hydrological applications.</i>	<i>35</i>
<i>Tab. 2. In each column of the table, the Time Frame Failure Matrix –TFFM for each step of a Monte Carlo procedure, is represented. The elements in each matrix are indicative.</i>	<i>38</i>
<i>Tab. 3. Monthly recession parameters ε, b.</i>	<i>41</i>
<i>Tab. 4. The Kendall (τ) and the Spearman (ρ) dependence measures for both case studies.</i>	<i>52</i>
<i>Tab. 5. For both the case studies of Poggio a Caiano and Capannoli. The results for the goodness of fit test (AIC_{cop}, BIC_{cop}, S_n and the corresponding p-values of S_n) for each copula families is shown. In bold the copula that passes at least one test. The Gaussian copula and the Galambos (Poggio a Caiano), and the Gumbel copula (Capannoli) are the best.</i>	<i>53</i>
<i>Tab. 6. Upper tail dependence coefficients of the copulas that pass the goodness of fit tests.</i>	<i>54</i>
<i>Tab. 7. For each case study, the number of failure scenarios for each failure matrix (FMI) are indicated.</i>	<i>55</i>
<i>Tab. 8. For both case studies, the failure probability for the corresponding i-time frame that composed the design life ($i=1,2,3...$, DL)</i>	<i>55</i>
<i>Tab. 9. Results of goodness of fit tests (AIC, AICc, BIC) for the regional marginal distributions. The GEV and Lognormal distributions are the best for the discharge for North and South regions; the Gumbel and the GEV distributions for Central region. The Gumbel and the GEV are the best for the volume for all the regions.</i>	<i>71</i>
<i>Tab. 10. The Spearman's rank correlation coefficient (ρ), and the Kendall's rank coefficient (τ) for all homogeneous regions.</i>	<i>76</i>
<i>Tab. 11. The assessed copula families</i>	<i>77</i>
<i>Tab. 12. Regional copula, and the corresponding parameters for each homogeneous region.</i>	<i>77</i>
<i>Tab. 13. Regression models using as index-flood.</i>	<i>80</i>
<i>Tab. 14. Results of the regional error measures for evaluating quantile estimates using the bivariate regional procedure based on the multivariate index-flood model.</i>	<i>84</i>

List of symboles

Δ_2	L-covariation matrix	[-]
Δh	Variation of the water stage in the river	[m]
Δt	Time interval	[s]
$\Delta \tau$	Units of time	Years
$\Delta \tau_m$	mean interval time of the sampled events	[Year]
A	Catchment area	[km ²]
A_i	Footprint of the <i>i-th</i> building	[m ²]
AIC	Akaike Information criterion	[-]
AIC _c	Corrected Akaike Information criterion	[-]
AIC _{cop}	Akaike Information criterion for a copula	[-]
Asp	Aspect	[deg]
b	Parameter of recession equation	[-]
BFRP	Bivariate Failure Return Period	[Year]
BIC	Bayesian Information Criterion for copula	[-]
BIC _{cop}	Bayesian Information Criterion for copula	[-]
$c(u_1, u_2, \dots, u_n)$	Density function of Copula	[-]
$C(u_1, u_2, \dots, u_n)$	Copula function	[-]
C_d	Weir coefficient	[-]
c_i	Regression coefficient	[-]
C_n	Empirical copula	[-]
$C_n(u, v)$	Empirical copula	[-]

COFH	Critical overtopping of the Hydrograph	[-]
COM	Center of the mass	[-]
C_{θ_n}	Parametric copula	[-]
D	Duration of the hydrograph	[s]
d	coefficient of recession equation	[-]
DDF	Depth-Density curve	[-]
Df	Duration of the falling limb of the hydrograph	[s]
$D_{i\%}$	Damage percentage for a building	[-]
$D_{i\epsilon}$	Economic damage for a building	[-]
DL	Design Life	[Year]
D_r	Duration of the rising limb of the hydrograph	[s]
E	Exposure	[-]
E_i	Market value of the i -th building	[€/m ²]
EL_{rat}	Elongation ratio of the catchment	[-]
EV2	Frechet Distribution	[-]
FB	Freeboard for a river levee	[m]
FC(Q,V)	Failure Curve	[-]
F_d	Design value of the load	[-]
FM_i	Failure Matrix for the i -time frame	[-]
$f_x(x)$	Probability mass function	[-]
$F_x(x_i)$	Marginal cumulative distribution	[-]
$F_{x_1, x_2, \dots, x_n}(x_1, x_2, \dots, x_n)$	Joint cumulative distribution function	[-]
G	Gravitational water	[mm]

GEV	Generalized Extreme Value distribution	[-]
GUMB	Gumbel Distribution	[-]
H	Multivariate Homogeneity test	[-]
H_a	Hazard	[-]
H_c	Head above a crest of a weir	[m]
h_i	Flood stage next to a building	[m]
HL	Hydrological Load	[-]
H_m	Annual maximum stage in the river	[m]
H_{max}	Maximum elevation	[m asl]
H_{mean}	Mean elevation	[m asl]
H_{min}	Minimum elevation	[m asl]
HR	Hydraulic resistance	[-]
h_u	Uniform water stage in the river	[m]
IDF	Inflow Design Flood	[m ³ /s]
i_l	Steepness of levee	[m/m]
k	coefficient of recession equation	[-]
K_c	Kendall's distribution function	[-]
K_{cs}	Kendall's survival function	[-]
K_{sat50}	Saturated Hydraulic conductivity in the first 50 cm of soil	[mm]
L	Length of the overflow section	[m]
L_{LDP}	length of the longest drainage path	[km]
L_{ms}	length of the longest series of streams that connects the basin outlet to the foremost source point	[km]

LN	LogNormal Distribution	[-]
LP3	Log-Pearson type III distribution	[-]
m	Number of buildings in a flooded area	[-]
$M(u_1, u_2)$	Copula of the Fréchet-Hoeffding upper bound	[-]
MAR	Mean annual rainfall	[mm]
MPL	the maximum pseudo-likelihood method	[-]
MS	Mean slope	[%]
n	Manning coefficient	[s/m ^{1/3}]
N	Number of days	[day]
n	Length of sample size	[-]
N_i	number of floors in the i -th building	[-]
o	Model error	[-]
O_d	Overtopping duration	[s]
OHSI	Overtopping hydrograph Shape Index	[-]
O_v	Overtopping volume	[m ³]
P	Perimeter of the catchment	[km]
P3	Pearson type III distribution	[-]
PDS	Peak Discharge Series	[-]
P_{LDP}	Slope of the longest drainage path	[%]
POT	Peak Over Threshold method	[-]
$P_z(z)$	Cumulative distribution	[-]
Q	Peak discharge	[m ³ /s]
Q_0	Lateral overflow discharge	[m ³ /s]

q_0	Lateral overflow discharge per unit of length	[m ³ /s/m]
Q500	Discharge with 500 years of return period	[m ³ /s]
Q_i	Initial discharge due to only the baseflow	[m ³ /s]
Q_{ind}	Index flood	[m ³ /s]
Q_{min}	Minimum discharge of the hydrograph	[m ³ /s]
Q_{ps}	Synthetic peak flow	[m ³ /s]
Q_r	Discharge in receding limb	[m ³ /s]
Q_t	T-year floods	[m ³ /s]
$Q_{x,y(p)}$	Bivariate quantile of (X,Y)	[m ³ /s]
R	Failure probability	[-]
R_d	Resistance	[-]
$R_f(l)$	Failure probability curve	[-]
R_i	Rank of random variable (e.g. Q)	[-]
R_i	Risk	[-]
RRP	Routed Return Period	[Year]
S	Energy slope of the river	
S_0	River slope	[m/m]
S_f	Structure function	[-]
SF	Shape factor	[-]
S_i	Rank of random variable (e.g. V)	[-]
S_n	Cramer Von-Mises test	[-]
SRP	Structure return period	[Year]
S_{XY}	Survival function of X and Y	[-]

t	Probability level	[-]
T	Return period	[Year]
$T_{(u1 u2)}$	Conditional return period	[Year]
$T^{\wedge}$	AND return period	[Year]
$T^{\vee}$	OR return period	[Year]
T_{Kc}	Kendall's return period	[Year]
T_{ks}	Survival Kendall's return period	[Year]
T_s	Structure failure return period	[Year]
U_i	Random variable	[-]
u_i	Vector [0,1]	[-]
V	Volume of the hydrograph	[m ³]
v_b	Overtopping velocity	[m/s]
V_k	Random variable	[-]
V_u	Vulnerability	[-]
$W(u_1, u_2)$	Copula of the Fréchet-Hoeffding lower bound	[-]
X_{centr}	X-coordinate of centroid catchment	[m]
$X_{fd,ad}, Y_{fd,ad}$	Dimensionless coordinate of the center of mass of the falling limb of the hydrograph	[-]
X_i	Random variable	[-]
$X_{rd,ad}, Y_{rd,ad}$	Dimensionless coordinate of the center of mass of the rising limb of the hydrograph	[-]
X_{step}	number of discrete time steps	[-]
X_t	Growth curve	[-]
X_u	Uniform random variable [0,1]	[-]

Y_{centr}	Y-coordinate of centroid catchment	[m]
Y_i	Random variable	[-]
$\hat{Y}(x_0)$	Flood sample moment	[m ³ /s]
Z	Design variable	[-]
α_i	ratio between recovery cost, and market value	[-]
δh_s	change in water surface elevation during the Δt time interval	[ft/s]
ε	Parameter of recession equation	[-]
ε	Parameter of recession equation	[-]
θ_i	Copula parameter	[-]
$\Theta_{\text{reg},i}$	Parameter of regional copula	[-]
λ_u	Tail dependence coefficient	[-]
λ_u^c	Uppertail dependence coefficient estimated through copula	[-]
$\widehat{\lambda_U^{CFG}}$	Estimator of tail dependence coefficient	[-]
μ	Location	[-]
μ_{fail}	Average number of failure scenarios	[-]
ξ	shape	[-]
ρ	Spearman rank correlation coefficient	[-]
σ	Scale	[-]
τ	Kendall rank correlation coefficient	[-]
χ^2	Measure of independence for Chi-square test	[-]
ω	Threshold $\in [0, 1]$	[-]
l	Specified period	[year]

$\Pi(u, v)$

Independence copula

[-]

Introduction

Motivation

Nowadays, floods cause each year million euros of damages and affect a large number of people worldwide. The Flood Directive 60/2007/EC requires the European countries to verify the effectiveness of existing flood defence infrastructure to mitigate flood risks efficiently and to implement also the structural, and non-structural measures. Flood and storm events around the world continue to lead to critical flood defence failures resulting in tragic losses of life, and the devastation of large areas.

The current practice establishes that the river flood control structures, e.g. the river levees, must respect a standard requirement. Generally, it requires a certain safe freeboard under a peak flow rate with a certain probability, i.e. a return period (e.g. Moser, 1991). A univariate frequency analysis evaluates the probability of the flood peaks with a correspondence between the probability of the peak flow rate and the risk of the failure of the structure (Volpi and Fiori 2014). This correspondence is a critical issue that is originated from the approximation of the river flood flow process. This approximation assumes that the river flow is steady with a perfect correspondence between the water stage in the river and discharge. On the contrary, the river flood flow is unsteady with a rating curve characterised by a hysteresis loop with a two-way water stage-discharge relationship (Fig. 1).

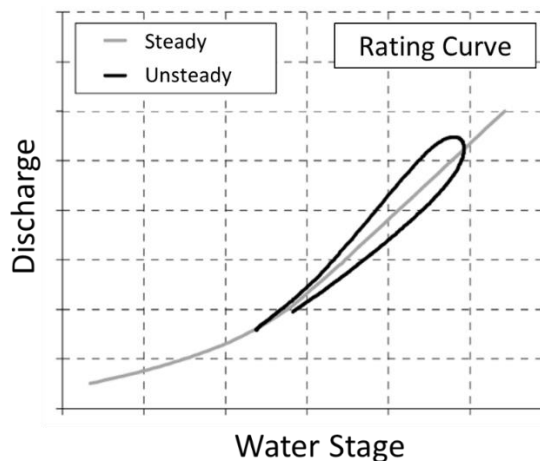


Fig. 1. Steady flow rating curve (grey) and unsteady flow rating curve (black) (adapted from Di Baldassarre and Montanari (2009))

The unsteady rating curve related to a specific cross-section of a river is not unique. It depends on the combination of all variables of the hydrographs and not only by the peak flow rate, as demonstrated by several authors (Fread 1975; Petersen-Øverleir 2006). To assess the water stage in the river and, thus, to assess the risk of failure or the design requirements for a flood control structure becomes necessary a multivariate frequency analysis on flood variable

(e.g. peak discharge, volume of the hydrograph, etc.) with an unsteady flow for modelling the critical floods. The multivariate frequency analysis is also useful for flood-control structure (e.g. storage areas) or river-related flood risk that need not only a peak flow rate with a certain probability but also a volume of the hydrograph as requirements. The current practice evaluates the volume of the hydrograph resulting from a design storm event with a duration equal the time of concentration of the analysed catchment or from several storms with same probability but different duration (Chen and Wong 1993). However, the probabilistic analysis refers only to the peak flow rate and not to the volume of the hydrograph, resulting in an under/overestimation of the risk of the value of design variable (Salvadori et al. 2016).

From this consideration, building a methodology, which concerns a bivariate hydrological load with the unsteady flow, improves river-related risks assessment. Several are the hydraulic model and tools that implement the unsteady flow analysis for the standard practice (Hec-Ras, Mike, Anuga, Telemac, Basement, Iber etc.). The bivariate frequency analysis for the hydrological load is prevalently used at a research-level with application in different hydraulic fields such as drought problems (Hao and Singh 2015; Li et al. 2017). Rare are research applications of the bivariate frequency analysis for river-related flood risk. Researching on this thematic could inspire more studies in this subject area that will help fill some of the identified gaps and address some practical concerns and limitations. The implementation of a bivariate hydrological load with unsteady flow requires many statistical and engineering skills that are scattered in the literature and needs a methodology that collects these together. Furthermore, it is necessary a procedure based on tools that are widely known to be replicable by the practitioners and public entities that are interesting in river flood risk.

All these considerations and issues motivate the necessity of creating a methodology that deals with estimating river-related flood risks with bivariate hydrological perspective.

Objectives

The thesis aims to define a methodology for the bivariate hydrological risk analysis for river-related flood risk. The methodology is focused on the river levees and tested on the flood damage estimation. River levees are the most common river flood-control structure. Their failure has caused several flood disasters such as Elbe flood (2002), New Orleans flood (2005), Emilia Romagna Flood (2017) or Arkansan flood (2019). They are raised, predominantly earth, structures (also called dykes, digues or embankments). In many instances, levees have been built up and extended over decades or sometimes centuries. Few of these were originally designed or constructed to modern standards and records of their construction, and historical performance may not exist (CIRIA, 2013). The current hydrological information may have modified the design standard initially estimated, or even may have changed. These considerations and further changes in time (e.g. the changes of river morphology, etc.) makes the focus on the risk analysis of river levees necessary. Vorogushyn et al. (2010) detect the overtopping as the leading cause of failure; thus, it is the type of levee failure analysed in this thesis.

Introduction

The further application on the flood damage estimation problem demonstrates the flexibility and the validity of the proposed procedure.

To improve the current practice of the overtopping risk for a river levee, and more in general of river-related flood risk, the aims of this research study are:

- Assess the hydrological failure condition for the specific risk developing a methodology to estimate the failure curve for the overtopping mechanism for a river levee. It demonstrates that not only the peak discharge (Q) but also the volume of the hydrograph (V) is a significant variable for failure. The hydrological failure curve represents the initiation of the failure condition allowing to evaluate the probability of failure of the structure.
- Assess the probability of hydrological failure condition, previously mentioned, developing a procedure to assess the return period in a bivariate hydrological environment.
- Demonstrate the flexibility of the proposed methodology, applying it to the flood damage estimation.

A typical problem for building a bivariate distribution is to have a sufficient amount of data, i.e. Q-V pairs. Classical approaches proposed a number of the data at least equal ten times the return period relative to the risk analysed (Chow et al. 1988), other use at least 15 years of data (e.g. Aron et al. 1987). A critical issue is the short recorded series of the target site or if it is ungauged. In these cases, continuous rainfall-runoff modelling are used to obtain the Q-V series (section 1.1.1). This type of model has several issues linked to the computational time, the necessity of a large number of required parameters, and the difficulties with modelling the high intermittence of short time step (Haan et al. 1976; Haberlandt et al. 2011; Wilks 1998). Regional frequency analysis techniques (Hosking and Wallis 2005) are possible to implement to overcome these issues. They based on the idea of transferring the information from gauged sites to the target ungauged sites.

- Further objective of the dissertation is tested the multivariate regional frequency analysis method of (Chebana and Ouarda 2009; Requena et al. 2016) Chebana and Ouardza (2009) and Requena et al. (2016). The technique is employed and tested in the case study of the Tuscany Region (Italy).

A more detailed specification of the objectives as drawn from the implications of the identified knowledge gaps is provided in the concluding section of chapter 1. Research questions Concerning the importance of improving the flood risk assessments the research questions are:

- **Is the design flood approach adopted in standard practice a useful method to assesses the risk for river-related flood problems?**

In standard practice, the design flood consists of one hydrograph with a certain probability of occurrence, i.e. return period. In the flood risk assessment, the practitioners evaluate the design flood and analysing if a certain type of failure is verified or not. This methodology is

based on the hypothesis of a perfect correspondence between the probability of design flood and the probability of the analysed failure. Understanding how this hypothesis is verified is a key aspect of flood risk assessment.

- **How assess a probability related to a certain type of failure or damage in a hydrological bivariate environment?**

In literature, several definitions of bivariate return period exist, and each of these gives a different interpretation on it. A different interpretation corresponds to a different assessment of flood risks. Analysing the concept of the bivariate return period and studying a solution that overcomes this critical issue is a challenge useful for improving the flood risk assessment.

- **Can the regional frequency analysis be useful in a bivariate hydrological environment?**

The bivariate frequency analysis of peak discharge (Q) and volume (V) requires a big quantity of data. If the site is ungauged, the necessary Q-V data are detected by rainfall-runoff models, but they are complicated for the spatial and the temporal heterogeneity of the influencing parameters (such as soil or rainfall) (Bardossy 2009), and they could work for a long computational time for the entire region. Studying other techniques to overcome this issue represents a contribution for applying in practice the bivariate frequency analysis.

Thesis structure

The thesis consists of six chapters.

In the first chapter, the current state of knowledge on the characterization of flood waves for river-related flood risk assessment is highlighted. The standard approach used in the river flood risk assessment is analysed; in particular, the limitations in implementing a univariate frequency analysis for evaluating the hydrological forcing are shown respect a bivariate approach. The bivariate return period definitions are also exposed.

The second chapter refers to the methodology, in particular to the overtopping risk for river levees. The first part of the methodology regards the estimation of the bivariate hydrological failure curve in Q-V space (par. 2.1). This aim is achieved using an approximation of the bivariate distribution with the introduction of Overtopping Hydrograph Shape Index for hydrograph shapes characterisation. The second part (par.2.2) introduces a new approach to evaluate the return period of the bivariate hydrological failure line: the Bivariate Failure Return Period.

The third chapter refers to the application of the methodology on two case studies. The study sites are in Tuscany (Italy) with different locations and upstream catchments. In both case studies, the volume of the hydrograph represents a significant variable for overtopping failure. The benchmark with standard design requirements is also implemented.

Introduction

The fourth chapter refers to the topic of flood damage estimation. The methodology for the evaluation of the overtopping is tested to estimate flood building damages in the municipality of Poggio a Caiano (Tuscany, Italy). The similarity of the trends of the curves for the flood damage estimation and the overtopping failure demonstrates the validity and the flexibility of the procedure.

The fifth chapter refers to the application of the multivariate regional frequency analysis method (Requena et al. 2016) tested to the case study of Tuscany Region.

In the end, the conclusions in which a summary of the thesis, key results, implications, limitations and future development are highlighted.

1 Current state of knowledge

In this chapter, the current knowledge on the hydrological risk analysis for a river levee respect to the overtopping failure is reviewed and analysed. Firstly, the standard approach for risk assessment of the hydraulic structure is presented. In particular, the methodologies for the estimation of the hydrological load underlying the issues of the traditional approach, and the importance of considering the hydrological load as multivariate (section 1.1). The return period is a concept that usually represents the measure of the risk. Section 1.2 shows current approaches for evaluating the return period in the bivariate environment. Since the dissertation mainly focusing on the overtopping mechanisms, the factors affecting the overtopping failure are described (section 1.3). Section 1.4 discusses current knowledge briefly.

1.1 Risk assessment for a river hydraulic structure

The product of three components (2007/60/EC) defines the flood risk (R):

$$R = H_a \cdot E \cdot V_u \quad \text{Eq.1}$$

Where H_a is the hazard: the probability of occurrence the event; E is the exposure: the population, and the value of the asset subjected to the event; V_u is the vulnerability that is the propensity to the damage of E component subjected to the H_a event (Muis et al. 2015). Several scientists have questioned on the quantification of hazards (such as Klein et al. 2009; Tsakiris 2014; Volpi and Fiori 2012). The hazard component for river hydraulic structure consists of the hydrological load. The estimation of the hydrological load for a hydraulic structure is the first, and one of the most crucial steps to be considered in the risk assessment, and in the design process (Smith 1992). Traditionally, the probability of occurrence of the hydrological load expresses the measure of the risk. The potential damages occurring as a consequence of the failure of the hydraulic structure set a priori this probability (Volpi and Fiori 2014). From this consideration, the estimation of a hydrological load in probabilistic terms becomes fundamental in the assessment of the hydraulic risk for the hydraulic structure.

1.1.1 Standard approach: univariate hydrological load

Traditionally, the hydrological load is associated with a return period expressing with the concept of the design flood (Beard 1975). Design Flood is also known as the Inflow Design Flood (IDF) (FEMA, 2004; USACE and USBR 2015). In common practice, the IDF can be the entire flood hydrograph or the peak discharge of the flood hydrograph with a certain probability or return period. There are numerous methods available to estimate the magnitude and the probability of the hydrological load. Swain *et al.* (2004), and USACE and USBR (2015) present a list of currently available methods along with inputs, and assumptions with some of their strengths, and limitations. Since each target site is a particular mixing of climatic, hydrologic, and watershed variables with different levels of data availability, and required levels of

confidence, no approaches address all hydrologic issues (USACE and USBR 2015). Improvements to standard methods may be added as project needs, research, and experience dictate.

Methods can generally either be classified as streamflow-based statistical analysis, rainfall-based with statistical analysis on the generated runoff (Maidment 1993).

1.1.1.1 Statistical analysis of the flood data

The statistical analysis of observed flows is a classical approach for estimation of the flood-frequency relationship when consistent numbers of observed flows are available (e.g., Singh and Singh, 1991; Vogel and Wilson, 1996; Yue *et al.*, 1999). This technique is also employed for testing the non-stationarity behaviours of extreme flood flow (Ishak et al. 2013; Machado et al. 2015; Šraj et al. 2016; Xiong et al. 2015). The main limitation of the statistical approach remains the availability and amount of flow data at the site of interest (Renard et al. 2013). If only a short record of streamflow is available, the data set have to increase with another source of information. As an example, the cultural evidence or flood marks (Caporali et al. 2016; Pekárová et al. 2013) find in the memories of the long-time residents can be used for upgrading the dataset (Stedinger and Cohn 1986). If a short time series at the target site is disposal or the target site is ungauged, the regional frequency analysis is possible to employ as statistical analysis technique (see, as example, Ahn and Palmer, 2016; Cunnane, 1988; Dalrymple, 1960; Hosking and Wallis, 2005; Smith et al., 2015; Stedinger, 1983). The regionalisation is based on the idea of transferring the flood information from hydrologically similar catchments to the target catchment is at the base of the regionalisation (Hosking and Wallis 2005). This method follows three main approaches: (i) Pooling schemes; (ii) Functional relationship to catchment attribute; (iii) Geostastical methods. The pooling schemes method transposes the flood information from catchments with similar hydrological behaviour to the site of interest (Merz 2011). The pooling schemes commonly follow the index flood procedure (Bocchiola et al. 2003; Caporali et al. 2008; Dalrymple 1960; Fiorentino et al. 1987; Javelle et al. 2002; Kjeldsen et al. 2002; Stedinger 1983). The index flood approach identifies homogeneous groups of catchments for which floods with a return period of T years (T-year floods) is:

$$Q_t = Q_{ind} \cdot x_t \quad \text{Eq.2}$$

Where Q_t is the T-year floods, Q_{ind} is the index flood, and x_t is the growth curve, which describes the relationship between the dimensionless flood and the return period T (Merz 2011).

The second approach for regionalization is the functional relationship to catchment attributes. This method assumes an existing of a relationship between flood statistics and catchment attributes in gauged catchments. This relationship determines the flood statistics in ungauged catchments (Acreman and Sinclair 1986). The most common method of developing

functional relationships is multiple regressions (see, among the others, Brath et al., 1997). In multiple regression, the following equation regulates the flood statistics at the site of interest:

$$\hat{Y}(x_0) = c_0 + c_1 \cdot Y_1(x_0) + c_2 \cdot Y_2(x_0) + \dots + c_n \cdot Y_n(x_0) + o \quad \text{Eq.3}$$

Where $\hat{Y}(x_0)$ is the flood sample moment; $Y_1(x_0) \dots, Y_n(x_0)$ are the catchment attributes at locations x_0 ; $c_0 \dots c_n$ are the regression coefficients, and o is the model error. Recently, Ahn and Palmer (2016) propose a ‘spatial proximity’ approach in which they use two types of regression: the first is the quantile regression technique, and the second parameter regression technique.

The geostatistical methods are the third main approaches to flood regional frequency analysis. They assume that flood statistics are spatially correlated, and hence use the spatial correlation for the spatial interpolation of flood statistics (Merz 2011). Several approaches can be found in hydrological literature based on the Ordinary Kriging technique, and its developments, such as Top-Kriging (Archfield et al. 2013; Chokmani and Ouarda 2004; Daviau et al. 2000; Merz and Blöschl 2005; Nezhad et al. 2010).

1.1.1.2 Rainfall runoff-modelling.

If flow data are not available, rainfall-runoff modelling (Beven 2011; Chu and Steinman 2009; Goñi et al. 2013; Yang et al. 2014) is a good alternative, either using event-based or continuous simulation. The result of these models is the flood hydrograph from a rainfall input. The design storm approach with event-based rainfall-runoff models is one of the most popular methods for design flood estimation (Pathiraja et al. 2012). Indeed, it is characterized by simplicity, and low computational cost (Berk et al. 2017), as well as the availability of input rainfall data in the form of Intensity Duration Frequency curve (Chen 1983) or Depth Duration Frequency curve, which are provided by meteorological services in most countries (see among the others (Caporali et al. 2014)). By the event-based approach, a rainfall input with the design return period is transformed into the design flood (Brocca et al. 2011). A disadvantage of the design storm approach is the assumption of the equality of the return periods for the storm, and the resulting design flood (Haberlandt et al. 2011). This assumption is a reliable approximation because the design flood is a function of other random parameters such as the antecedent soil moisture condition (Shaw 2005; Verhoest et al. 2010). Assuming to fix the duration of the storm, equal to the time of concentration of the analysed catchment (Chen and Wong 1993), represents another issue of the Design Storm approach. This last hypothesis neglects the possible contribution of storms with different durations on the flood return period. These critical issues of the design storm approach affect the estimation of the probability of design flood (Adams and Howard 1986; Gaume et al. 2009; Grimaldi and Petroselli 2015; Li et al. 2014). Using the rainfall-runoff continuous model avoids these critical issues (Camici et al. 2011). The

input of the model is the complete rainfall time series. The statistical analysis on the modelled flow time series allow to describe the design flood (see, among the others, Blazkova and Beven, 2002; Boughton and Droop, 2003; Calver and Lamb, 1995; Cameron et al., 1999; Haberlandt et al., 2008; Lamb and Kay, 2004; Pathiraja et al., 2012). This kind of hydrological modelling needs sufficient continuous rainfall time series with high temporal, and sufficient spatial resolution (Haberl et al. 2011). Given the limited availability of data, synthetic precipitation is used more commonly for this purpose (Aronica and Candela 2007; Cowpertwait et al. 2006; Fatichi et al. 2011; Moretti and Montanari 2008; Rodriguez-Iturbe et al. 1987). However, the rainfall-runoff continuous model is more suitable for daily or longer time step precipitation due to difficulties with modelling the high intermittence of short time step data, and a large number of required parameters (Haan et al. 1976; Haberlandt et al. 2011; Wilks 1998).

1.1.2 Recent developments: the multivariate hydrological load

As shown in the previous section, the statistical analysis techniques, and rainfall-runoff modelling are used to define the hydrological load for river hydraulic structure. In standard approach, the hydrological load is univariate, based on frequency analysis on flood peaks series (statistical analysis and continuous rainfall-runoff modelling) or the equality of the probability of occurrence between the design storm and design flood (event-based rainfall-runoff modelling) (Kite 1978; Naghettini and Potter 1996; England Jr et al. 2019; Stedinger 1983). This method is not entirely correct for the risk analysis of river hydraulic structures because using univariate frequency analysis may cause an over/underestimation of the risk associated with a given flood (Salvadori and De Michele 2004; Serinaldi 2016). Indeed, the primary variable, affecting the type, and the size of river hydraulic structures for flood control, is the river stage raised during the routing process that depends not only on the flood peak but it is influenced by the entire hydrograph (Chebana and Ouarda 2011; Mediero et al. 2010; Requena et al. 2013). The water stage is affected by different random variables such as the flood peak, the volume, the shape of the hydrograph, and duration (Klein et al. 2011). Therefore, to define the risk for a river hydraulic structure is useful employing a joint probability of the flood variables. The multivariate approach is also recommended to use by the European Flood Directive (2007/60/EC). In literature, several distributions are tested for the multivariate frequency analysis to assess the joint probability of flood peak, and volume of the hydrograph. Hiemstra et al. (1976)) derived the bivariate lognormal probability distribution for bivariate frequency analysis. Singh and Singh (1991) developed the bivariate exponential distribution. Several authors employ the bivariate normal distribution (e.g. Goel et al., 1998), the bivariate gamma distribution (Yue et al. 2001), and the bivariate extreme value distribution (Shiau 2003; S Yue, Ouarda, and Bobée 2001). Sandoval and Raynal-Villasenor (2008) applied a trivariate extreme value distribution for frequency analysis. These models show the issue in their necessary condition. Indeed, the marginal distributions of the random variables belong to the same family of the multivariate one (Klein et al. 2011). Frequently, the marginal distributions

fitting best the flood variables (flood peak, and volume) can be different (e.g. Duan et al., 2016; Mitková and Halmová, 2014).

1.1.2.1 Copula model

Recently the application of copulas to estimate the joint probabilities of random variables overcomes the critical issues of using the traditional multivariate distributions (Klein et al. 2011). Since the first application in hydrology by De Michele and Salvadori (2003), several papers in hydrology have shown the theoretical, and practical advantages of using a copula approach. Indeed, Copulas enable one to model the dependency structure independently from the marginal distributions (Salvadori et al. 2007). Therefore, it is possible to build multivariate distributions that differ in their marginal distributions. Several applications in the hydrological literature related to different risk areas (flood, drought, etc.) support their usage (e.g. Chen et al., 2011; Goodarzi et al., 2013a; Michailidi and Bacchi, 2017; A. Requena et al., 2013).

In the following, the main concepts of the Copula model are explained; more details are possible to find in Joe (1997); Salvadori *et al.* (2007); Nelsen (2007).

A copula (C) is a function defined as $C: [0,1]^n \rightarrow [0,1]$, such that for all $u_1, u_2, \dots, u_n \in [0,1]$ the copula function $C(u_1, u_2, \dots, u_n) = 0$ if at least one of the arguments is equal to zero, and $C(u_1, u_2, \dots, u_k, u_n) = u_k$ if all the arguments are 1 except u_k .

The theorem of Sklar (1959) establishes the link between a copula and a multivariate distribution:

$$F_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n) = C[F_{X_1}(x_1), F_{X_2}(x_2), \dots, F_{X_n}(x_n)] \quad \text{Eq.4}$$

where $F_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n)$ is the joint cumulative distribution function with the continuous marginals $F_{X_1}(x_1), F_{X_2}(x_2), \dots, F_{X_n}(x_n)$ of the random variables. In hydrological applications, the multivariate frequency analysis usually reduced to a bivariate frequency analysis of flood peak, and volume of the hydrograph (Klein et al. 2011; Nelsen 2007; Salvadori et al. 2007), thus the content of this chapter is concentrated on bivariate analysis of the random variables X and Y with the marginal distributions $F_X(x)$, and $F_Y(y)$ with a joint cumulative distribution function:

$$F_{XY}(x, y) = C[F_X(x), F_Y(y)] \quad \text{Eq.5}$$

Because copulas are invariant for strictly increasing transformations of X , and Y it is possible to use two uniformly distributed random variables U_1 and U_2 on $[0,1]$, defined as $U_1 = F_X(x)$, and $U_2 = F_Y(y)$.

Assuming continuous marginal distributions with the probability density functions $f_X(x)$ and $f_Y(y)$ the joint probability density function (pdf) becomes:

$$f_{X,Y}(x,y) = c[F_X(x), F_Y(y)]f_X(x)f_Y(y) \quad \text{Eq.6}$$

defined as $U_1 = F_X(x)$, and $U_2 = F_Y(y)$.

Where c is the density function of the copula C :

$$c = \frac{\partial^2 C(u_1, u_2)}{\partial u_1 \cdot \partial u_2} \quad \text{Eq.7}$$

Three special cases of copulas are of particular importance:

- The copula of the Fréchet-Hoeffding lower bound W :

$$W(u_1, u_2) = \max(u_1 + u_2 - 1, 0) \quad \text{Eq.8}$$

- The copula of the Fréchet-Hoeffding upper bound M :

$$M(u_1, u_2) = \min(u_1, u_2) \quad \text{Eq.9}$$

The independence copula Π :

$$\Pi(u_1, u_2) = u_1 \cdot u_2. \quad \text{Eq.10}$$

The copulas W and M are the general bounds for any copula C , and any pair $(u, v) \in [0,1]^2$

$$(u_1, u_2) = \max(u_1 + u_2 - 1, 0) \leq C(u_1, u_2) \leq M(u_1, u_2) = \min(u_1, u_2). \quad \text{Eq.11}$$

The empirical copula (C_n) (Deheuvels 1979) derived from a random sample $(X_1, Y_1), \dots, (X_n, Y_n)$ of size n is formally defined by:

$$C_n(u_1, u_2) = \frac{1}{n} \sum_{i=1}^n 1\left(\frac{R_i}{n+1} \leq u, \frac{S_i}{n+1} \leq v\right) \quad \text{Eq.12}$$

where R_i is the rank of $X_i \in X_1, \dots, X_n$, and S_i is the rank of $Y_i \in Y_1, \dots, Y_n$ of the sample and $1(A)$ the indicator function of the set A . Hydrological applications generally use three families of copulas (Joe 1997): the Archimedean copulas, the Elliptical copulas, and the Extreme-value copulas. In hydrologic applications, the bivariate distribution of flood peak (Q) and volume of the hydrograph (V), based on the copula model, is usually implemented (e.g. Requena, Mediero, and Garrote, 2013). The use of trivariate distribution implemented with the use of the Vine Copula is also used (Bevacqua et al. 2017; Liu et al. 2018; Shafaei et al. 2017). How to build the Vine Copula for model trivariate or multivariate distribution is an open

research theme (Czado 2019; Schepsmeier 2019) that it is outside the goal of this thesis, more robust are the bivariate model that, in the following, are taken into account.

1.2 The return period in bivariate field

In the previous paragraph, the employment of a multivariate distribution, modelled by the copula, is underlined, improving the assessment of the risk for the river hydraulic structures. In hydrologic applications, bivariate distributions of Q and V are usually implemented (e.g. Chowdhary et al., 2011). The risk for a river structure represents a loss related to failure probability, in turn, expressed in terms of probability of the hydrological load. Traditionally, the probability of the hydrological load is expressed with the widespread concept of return period (Fernández and Salas 1999; Vogel and Castellarin 2017; Volpi 2019a). Fuller (1913) introduced the return period concept firstly. It is one of the most used concepts in hydrological and geophysical risk analysis. The classical definition of the return period (T) is that an event “A” will occur on average once every T years, which is the time unit commonly used for return periods in hydrology. Mathematically, following the symbology of Volpi (2019), it is:

$$\frac{T}{\Delta\tau} = E[X_{step}] = \sum_{x=1}^{\infty} x f_x(x) \quad \text{Eq.13}$$

Where X_{step} is the number of discrete time steps to an occurrence of the event A , and $f_x(x) = P_r(X_{step} = x)$ is its probability mass function. The equation refers to discrete-time processes, T is measured in units of time, i.e. $\Delta\tau$; if $\Delta\tau = 1$ year, the return period is in years. When the flood risk analysis deals with a bivariate problem, several approaches exist for estimating bivariate design events for a given design return period (Gräler et al. 2016; Salvadori et al. 2016; Serinaldi 2015). The first primary difference respect to the univariate case is that in bivariate frequency analysis, the design flood that corresponds to a return period is not unique (Yue 1999) but infinite, as shown in several papers (e.g. Mediero et al., 2010; Requena et al., 2013; Salvadori et al., 2016). In hydrological applications, the multivariate frequency analysis usually reduced to a bivariate frequency analysis of flood peak and volume of the hydrograph, and in the following, we report the types of bivariate return periods present in literature. The copula model facilitates not only the construction of the bivariate distributions but also the evaluation of the bivariate return period (Brunner et al. 2016). Two main categories group the bivariate return periods: (i) Hydrological return period; (ii) Structural return period.

1.2.1 The hydrological return periods

The hydrological return period takes into account only the occurrence of the hydrological load. Gräler *et al.* (2013) and Brunner *et al.* (2016) explain the types and each difference between the hydrological return periods. The first type is the conditional return period (Yue et al. 1999b, 2001). The use of copula models in the construction of the bivariate distributions has

helped the application of the conditional distributions (e.g. Renard and Lang, 2007; Salvadori and De Michele, 2010; Verhoest *et al.*, 2010; Gräler *et al.*, 2016). Considering the random variables U_1 and U_2 with uniform distributions, and the two conditionals probability $Pr[U_1 > u_1|U_2 > u_2]$, and $Pr[U_2 > u_2|U_1 > u_1]$ associated to the two events $E^>_{U_1|U_2} = \{U_1 > u_1|U_2 > u_2\}$, and $E^>_{U_2|U_1} = \{U_2 > u_2|U_1 > u_1\}$ the conditional return period is defined, respectively, as:

$$T_{u_1|u_2} = \Delta\tau_m \frac{1-u_2}{1-u_1-u_2+C(u_1,u_2)}, \text{ and } T_{u_2|u_1} = \Delta\tau_m \frac{1-u_1}{1-u_1-u_2+C(u_1,u_2)} \quad \text{Eq.14}$$

Where $\Delta\tau_m$ is the mean interval time of the sampled events (one year), $C(u_1, u_2)$ is the copula model. This approach does not take into account the joint distribution of the two variables. The conditioned distribution is used to obtain the other quantile, again based on the principles of a univariate return period (Gräler *et al.* 2013).

The joint return periods are the second types of hydrological return period. In Hydrology, the joint return period of flood peak (Q) and volume of the hydrograph (V) deal with 3 cases: (i) both Q and V exceeding their predefined threshold; (ii) only Q exceeds its predefined threshold; (iii) only V exceeds the predefined threshold. The (i) case is known as “AND” return period, the (ii), and (iii) cases are known as the “OR” return period. Considering the previous variable (U and V) the events referring to, respectively, the “AND” and the “OR” cases are:

$$\{U_1 > u_1\} \wedge \{U_2 > u_2\}; U_1\{U_1 > u_1\} \vee \{U_2 > u_2\} \quad \text{Eq.15}$$

The “AND” return period and the “OR” return period expressed in copula terms are respectively (Vandenberghe *et al.* 2011):

$$T^\wedge = \frac{\Delta\tau_m}{1-u_1-u_2-C(u_1,u_2)}; T^\vee = \frac{\Delta\tau_m}{1-C(u_1,u_2)}; \quad \text{Eq.16}$$

The “OR”, and the “AND” return period depends only by the Copula model, and not by the marginal distributions. Regardless infinite critical regions are present with “OR” (“AND”) return period characterised by the same joint probability. It is impossible to choose among these critical regions (Salvadori and De Michele 2010). This situation is a dichotomy that it was tried to avoid with the introduction of Kendall’s distribution that measures the chance to observe an event in one of the two unique subregions defined by a level curve characterised by a unique value of joint probability (Serinaldi 2015). The Kendall’s distribution function K_c is the cumulative distribution of the copula’s isolines with the same likelihood in U_1 - U_2 space and is given in a bivariate case by:

$$K_c = P[W \leq t] = P[W = C(U_1, U_2) \leq t] \quad \text{Eq.17}$$

Where $W = C(U_1, U_2)$ is a univariate random variable. In literature for bivariate cases, relationships for the estimation of K_c exists for Archimedean, and extreme value copula families

(Ghoudi et al. 1998; Salvadori et al. 2007), in another way, K_c can be evaluated with a numerical algorithm (Salvadori and De Michele 2010). Any critical probability level t , detected through the Kendall distribution function, divide the space into a supercritical, and a noncritical region. Therefore, The Kendall's return period (T_{K_c}) corresponds to the mean inter-arrival time of critical events lying on the probability level t (Brunner et al. 2016) which is given by:

$$T_{K_c} = \frac{\Delta\tau_m}{1 - K_c(t)} \quad \text{Eq.18}$$

Using the Kendall return period approach, some noncritical events might have larger marginal values than any selected design event (Gräler et al. 2013). Salvadori et al. (2013) avoid this issue, introducing the concept of the survival Kendall's return period – T_{K_s} . It creates a bounded safe region, where all the variables of interest are finite and limited. The survival Kendall's distribution function is at the base on the survival of Kendall's return period, this last defined as:

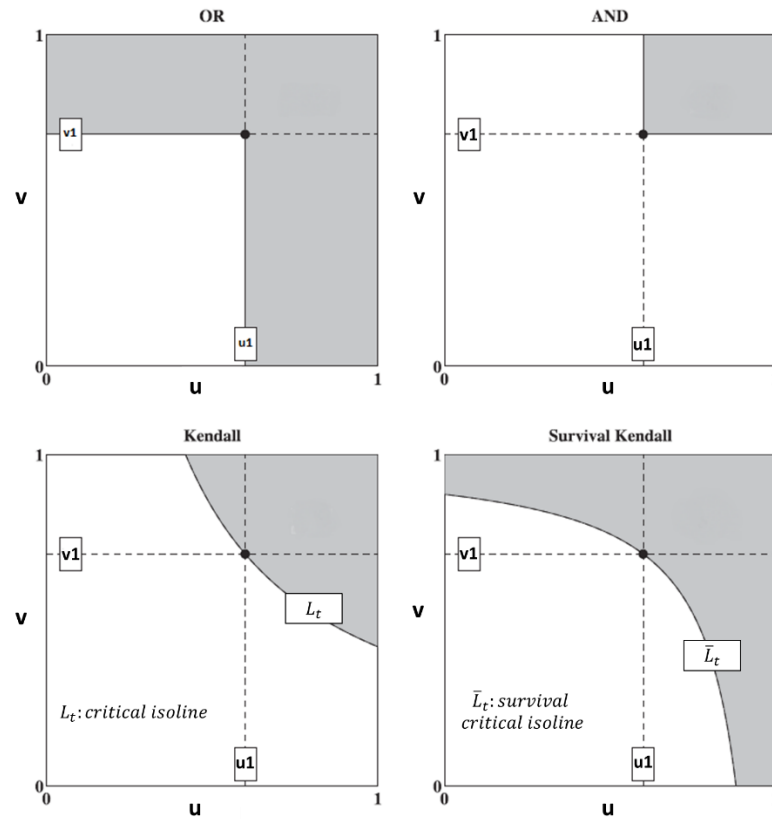


Fig. 2. Bivariate illustration of the Failure Scenarios (shaded area) according to different approaches, in the copula domain: the (top-left) OR, (top-right), AND, (bottom-left) Kendall, and (bottom-right) the Survival Kendall. In the Kendall case, the lower bound of the hazard region corresponds to the critical isoline L_t , while in the Survival Kendall case it corresponds $\bar{L}_t$ (adapted from Salvadori et al., 2016).

1.2.2 The structural return periods

The hydrological return period assesses the probability only of the hydrological load, and do not consider the interaction with the hydraulic structure, and its mechanism of failure. In the univariate case, the return period of the unique variable involved, e.g. flood peak, is equal to the return period of failure of the structure. On the contrary, in the bivariate case, the return period is the result of the interactions between the hydrological load acting and structure. Indeed, the return period of structure failure usually does not correspond that of the hydrological loads (Requena et al. 2016; Volpi and Fiori 2014). The bivariate flood risk assessment must take into account this interaction. In this case, the set of failure occurrences is dictated by a structure-function S_f accounting for the interactions between the hydraulic structure and the hydrological loads acting on it (De Michele et al. 2005; Requena et al. 2013; Volpi and Fiori 2014). Volpi and Fiori (2014) proposed the “Structure Failure Return Period” – T_s . The idea is to move from the bivariate distribution of the hydrologic loads (e.g. peak and volume of the input hydrograph in a reservoir) to that univariate of the design variable Z (e.g. the height upon the spillway). This approach is known as the transformation of two random variables (e.g. Papoulis and Pillai, 1976), the applications in a univariate context has been used in several applications (e.g. (Ashkar and Aucoin 2011; Kunstmann and Kastens 2006; Serinaldi 2013). Unfortunately, this approach fails if the failure mechanism depends on two or more variables.

1.3 The overtopping mechanism

The methodology is tested firstly for the overtopping risk for a river levee. In this paragraph, a description of the overtopping mechanism and the critical variables that conditionate the failure are exposed.

The overtopping has been identified as the leading cause of failure of a river levee (Vorogushyn et al. 2010). It typically occurs if the river discharge exceeds the design discharge of the levee during a flood event or, broadly, if the water level exceeds the levee crest or the flow overtops a weak levee segment. In Fig. 3, the three main overtopping scenarios at an earthen levee are shown (CIRIA 2013; Hughes 2008a): (i) the wave-only overtopping (Fig. 3a) can occur when the surge level is below the levee crest elevation; (ii) the surge only overtopping (Fig. 3b) occurs when the surge level exceeds the levee crest elevation without accompanying wave action; (iii) the wave and surge overtopping (Fig. 3c) occurs as a combination of the above scenarios.

Current state of knowledge

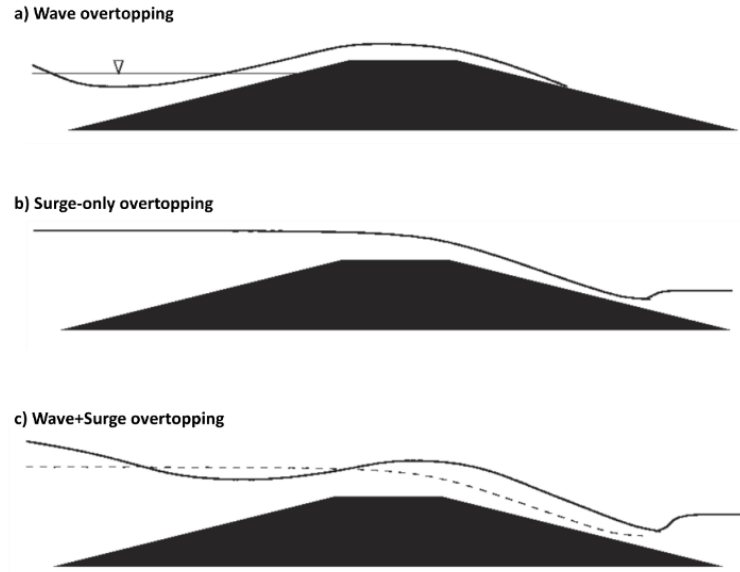


Fig. 3. The three main scenarios of levee overtopping (adapted from Hughes, 2008).

The river flood waves are long-period wave, classified as shallow-water waves (Ferrick 1985). The motion of the long-period wave is substantially horizontal with slight accelerations in the vertical direction whose effect can be neglected compared with the effects of the horizontal forces (Valken and Van Heemskerck 1963). Then, river flood waves are normally perceived as a change in the water level rather than short-period waves. For river dykes, the surge-only overtopping is, therefore, the primary detected failure mode, and, thus, it is the one considered in this thesis. In the surge-only overtopping scenario, the overtopping flow (Q_0) can be modelled as flow over a broad-crested weir as described for open channel flow (Chow 1958; Henderson 1966a):

$$Q_0 = C_d L H H_c^{1.5} \text{ or } q_0 = C_d H_c^{1.5} \quad \text{Eq.19}$$

Where Q_0 (m^3/s) is the total discharge, q_0 ($\text{m}^3/\text{s}/\text{m}$) is the unit discharge per length of overtopping section, C_d is the weir discharge coefficient, L (m) is the length of the overflow section, and H_c (m) is the head above the crest of the weir. Literature presents several approaches for C_d values; the most appropriate depends on the characteristic of the site under investigation. C_d values are proposed by Brunner (2002), depending on the type of levee coverage, and its roughness. Hager (1987) developed a methodology for estimating the weir coefficient C_d takes into the effects of flow depth, approaching velocity, lateral outflow direction, and side weir channel shape. Francis (CIRIA 2013) proposed a C_d of 0.623. Rehbock proposed a formula for assessing C_d , depending on the head above the crest, and the crest height (Rao and Muralidhar 1963).

The integral of overtopping discharge in the time results the overtopping volume (O_v) (Eq.20):

$$O_v = \int_0^{O_d} Q_0 \quad \text{Eq.20}$$

Where O_d is the overtopping duration (s). In the overtopping failure risk analysis, the critical overtopping velocity is usually located at the toe of the landward slope because it is the most common location for the initiation of erosion (CIRIA 2013). Indeed, the flow accelerates to reach supercritical and proceeds downslope until it reaches the base of the slope where a hydraulic jump develops (Fig. 4). The erosion occurs due to the high velocities and turbulence under the hydraulic jump.

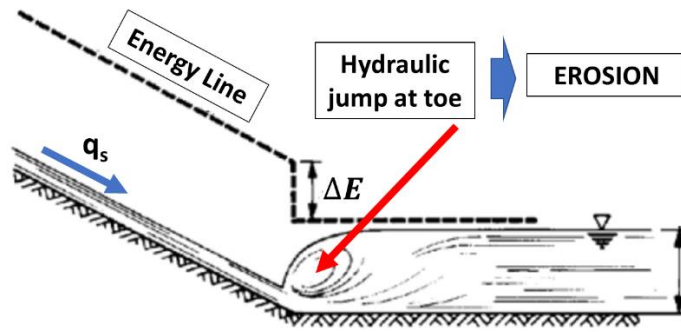


Fig. 4 Mechanism of erosion at the levee's toe due to overflow (adapted from Hewlett et al., 1987)

Considering the typical overtopping mechanism for the river, the surge-only overtopping (Hughes 2008a), the water momentum and the frictional resistance of the slope material reach the balance condition; thus the flow becomes steady (CIRIA 2013). This assumption is valid if the landward slope is long enough to reach equilibrium. The steady flow velocity v_b equation (CIRIA 2013) is:

$$v_b = \left(\frac{\sqrt{\sin(i_l)}}{n} \right)^{0.6} Q_o^{0.4} \quad \text{Eq.21}$$

Where n is the roughness of the Levee (Manning), i_l is the steepness of the levee (m/m). Hewlett et al. (1987) recommended $n = 0.03$ for slopes of 1:10, decreasing linearly to $n = 0.02$ for 1:3 slopes, and steeper.

1.4 Discussions

This dissertation aims to define a methodology for the characterisation of the hydrological risk for river-related flood risk. This methodology focused on river levee for different reason:

(i) they are the most common flood control structure in rivers; (ii) they built up and extended over decades or sometimes centuries; thus, the current hydrological information may have modified the design standard initially estimated, or even may have changed; (iii) Their failure caused several flood disasters. The overtopping failure is the primary failure mechanism for a river levee (Vorogushyn et al. 2010). The study of risk of overtopping for a river levee, require an integrated approach that considers the hydrological load, and the levee resistance criteria. Many studies are present in the literature for each branch. Indeed, in the previous paragraphs, the attention is focused on the evaluation of multivariate hydrological load. The multivariate approach is usually applied to dams for the overtopping analysis, but they don't address to find the hydrological failure line, rather than the risk in terms of the hydrological return period. Indeed, Mediero et al. (2010) proposed a design flood hydrograph investigating the relationship between the flood peak and hydrograph volume, implementing a Monte Carlo approach. Michailidi and Bacchi (2017) considered the overtopping water level as a function of flood peak, volume and hydrograph shape, and, implementing multivariate modelling with the use of copula distributions. The characteristics of the dam-reservoir system (reservoir volume, length of spillway crest, etc.) influence which hydrograph features (peak, volume, shape) are significant in the determination of the risk of dam overtopping, as is shown in the changes of the shapes of the equal-risk curves (Requena et al. 2013).

Considering only the levee resistance, many studies have been conducted on the initiation of the process, and causes of erosion, and breach formation on the levee due to piping or overtopping (Aguilar-López et al. 2016; Briaud et al. 2008; Dean et al. 2010; Dou et al. 2014; Hughes 2008b; Schmocker et al. 2014; Wei et al. 2016).

Rarely integrated methodology that considers the combination of the hydrological load, and levee resistance criteria, can be found. Mazzoleni et al. (2017) analysed the effects of levee cover strength on levee failure probability, flood propagation, and consequent impacts to population, and assets, but consider the hydrological load as univariate.

The risk is usually expressed in terms of the return period of hydrological load. Several approaches for the estimation of the bivariate return period are shown (par. 1.2). Regardless of the approach used, there are too many definitions of the multivariate return period that induce a subjective and arbitrary choice (Serinaldi 2015). To overcome this issue Vogel and Castellarin (2017) and Salvadori et al. (2016) suggest using the failure probability in the design life period of the structure because it can fit in every environment (univariate, bivariate, etc.). The failure probability may provide a consistent way of assessing the hydrological load, and, thus, the hydrological risk (Read and Vogel 2015).

2 Methodology

This PhD work deals with an in-depth analysis of the hydrological load to define a methodology for the assessment of river-related flood risks. In particular, the research focuses on assessing the risk of failure for a river levee respect the overtopping, but it also is useful for a different type of failures and risks. The methodology represents a development in the risk analysis because it considers an integrated approach in the bivariate hydrologic environment, which assesses and combines all the main components: the bivariate hydrological load, the river conveyance, and the levee resistance.

Firstly, the bivariate hydrological failure curve is estimated. The hydrological failure curve is the curve in Q - V space (Q is the peak discharge; V is the volume of the hydrograph) in which all the hydrographs cause the initiation of the damage due by overtopping. From this consideration, the hydrological failure curve is called as Critical Overtopping Flood Hydrograph (COFH). The COFH considers the mutual interaction between a simplified bivariate hydrological load of Q and V , the river conveyance, and the levee resistance. Indeed, this bivariate hydrological load does not account for the real bivariate distribution of Q and V , but it is an approximation, functional to determine the hydrological failure line. The novelty factor “Overtopping Hydrograph Shape Index” (OHSI) classifies the shape of hydrographs respect to the overtopping.

Secondly, the probability of the failure line is determined in terms of the return period. Because of the hydrological failure line, i.e. the COFH, is a function in Q - V space, the return period is estimated in the bivariate framework. As shown in section 1.2, several definitions of the bivariate return period are available, each of which gives a different interpretation on it (Salvadori et al. 2016). Therefore, the Bivariate Failure Return Period is introduced. The Bivariate Failure Return Period is the return period that corresponds the failure probability of the hydraulic structure in a horizon period of the “design life” of the structure. In Fig. 5, the flow chart of the main steps of the procedure is shown.

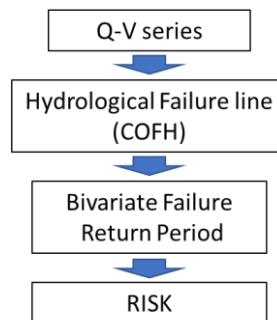


Fig. 5. Flow chart of the overall procedure.

2.1 Characterisation of the bivariate hydrological failure curve focusing on overtopping mechanism for a river levee

The overtopping has been identified as the leading cause of failure for a river levee (Vorogushyn et al. 2010). It typically occurs if the river discharge exceeds the design value of the levee during a flood event or, broadly, if the water level exceeds the levee crest or the flow overtops a weak levee segment (Hughes 2008b). In chapter 1, the importance of considering the hydrological load as bivariate is discussed. In particular, for estimation of the risk, the failure probability may provide a consistent way of assessing the hydrological load, and thus the hydrological risk for the hydraulic structure (Read and Vogel 2015). The first step is to define the hydrological failure curve.

In structural engineering, the traditional design approach for a structure, e.g. the limit state design (Gulvanessian et al. 2002; Mordà and Mancini 2018), is based on the following relationship:

$$F_d \leq R_d \quad \text{Eq.22}$$

Where the F_d is the design value for an action, and the R_d is the design value of a material or product property. Therefore, the structural engineer supposes that the critical status of a structure occurs when the $F_d = R_d$. In hydraulic engineering, the traditional approach of the risk analysis verifies if a hydraulic structure do not fail against a hydrological load of certain probability (Maidment and others 1993). This method is different respect to the structural design approach, and it is acceptable for the hydraulic engineering traditional approach that considers the load or action as univariate. Indeed, in this case, there is the equality between the probability of occurrence the hydrological variable, and the failure probability of the structure (Volpi and Fiori 2014).

In bivariate environment, the probability of hydrological load is usually different from the probability of failure of the structure because of the interaction between them (Volpi and Fiori 2014; Serinaldi 2015; G Salvadori et al. 2016). An ideal case is built to explain the difference between the traditional approach, and the bivariate one, considering an ideal rectangular structure in Fig. 6. The structure's failure is supposed to fail under a horizontal load only. In the univariate case, only one load (blue line) raises the resistance of the structure (red line and causes the critical status explained in Eq.22. The bivariate case is represented, adding another load (green line). There are infinite combinations of the two loads that produce the resistance of the structure, and thus the critical situation (Fig. 6). Therefore, transferring from this simple case to more complicated fluvial problems, infinite combinations of Q and V cause critical condition.

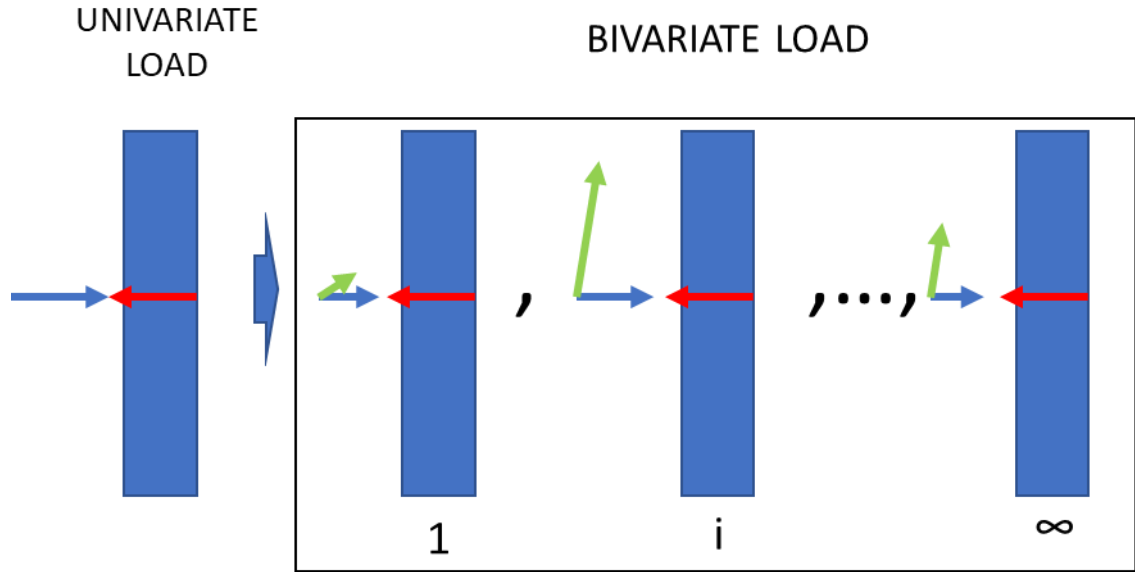


Fig. 6. An illustration of the difference between the univariate load (left) and the bivariate one (right). In the univariate case, there is only one load (blue line) and the resistance (red line) is raised by only one load, on the contrary, in the bivariate case, there are two loads (blue and green line) and a possible infinite combination of these raising the resistance.

From this consideration, a structural engineering approach for the estimation of hydrological failure curve becomes more performing. The Failure Curve - FC in the peak flow (Q) and Volume of the hydrographs (V) space is:

$$FC(Q, V) = HL - HR = 0 \quad \text{Eq.23}$$

Where HL is the hydrological load and HR is the Hydraulic Resistance. The solution of Eq.23 must take into account the complex interaction between bivariate hydrological load, river conveyance, and levee resistance. Rarely integrated methodology that combines all the factors above can be found, Mazzoleni et al. (2017) analysed the effects of levee cover strength on levee failure probability, flood propagation, and consequent impacts to population, and assets, but consider the hydrological load as univariate.

To define the hydrological failure curve of overtopping for a river levee, taking into account the combination of a bivariate hydrological load, the river conveyance, and the resistance of the levee. The bivariate hydrological load takes into account for a classification of the hydrograph shape concerning to the overtopping failure through the introduction of the Overtopping Hydrograph Shape Index (OHSI). The hydraulic resistance is determined by the river morphology, establishing a water level that may exceed the levee crest and the levee resistance. The hydrological failure consists of a curve Q-V called the Critical Overtopping Flood Hydrograph (COFH). The COFH contains all the hydrographs that reach the threshold for the overtopping failure condition in the analysed segment of the river.

Methodology

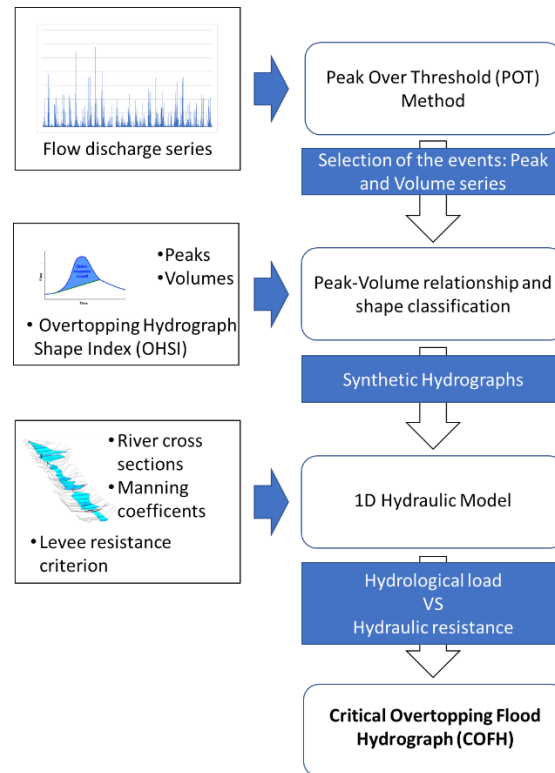


Fig. 7. Flow chart of the whole methodology to estimate hydrological failure line respect to the overtopping failure for a river levee.

2.1.1 Selection of the events: Peak and Volume series

2.1.1.1 Peak discharge series

The observed series of peak discharges at the levee site allows the identification of the flood peak frequency curve. If the gauging data are available at a nearby location in the river, the values can be transferred to the levee site through scaling analysis, using drainage area as similarity criterion (Leopold and Dunne 1978; Pazzaglia et al. 1998), provided that there are no significant tributaries to the main river in the differential reach. The available recorded data series ascertain the marginal distribution of peak flows. The annual maximum series approach is the most common method used in probabilistic hydrology (Cunnane 1973). However, the length of the available sample of discharge values is generally too short of estimating the return periods employed in design practice for the overtopping of a levee (200-500 years). This problem is solved using the Peak Over Threshold method (POT) (Lang et al. 1999), also called “partial duration series” approach. The POT method considers all the peak discharges exceeding a predefined threshold (Bayiliss 1999; Rhao and Hamed 2000). The main drawbacks of the partial duration series method are the threshold selection and the choice of independence criteria of the peak discharges. The Cunnane independence criterion (1979) is applied, and the

threshold by a graphical approach is selected: the mean residual life plot (Lang et al. 1999). In the following, the Peak Discharge Series is called PDS. The peak flow data are fitted to a 3-parameter Generalized Pareto Distribution - GPD (Pickands 1975), known to be a stable distribution for exceedance beyond a threshold (Coles 2001; Lang et al. 1999).

Many authors have used the model, such as (Davison and Smith 1990; Hosking and Wallis 1987; Smith 1985). The Eq.24 defined the GPD:

$$F(x) = 1 - \left[1 - \frac{\xi(x - \mu)}{\sigma} \right]^{\frac{1}{\xi}} \quad \text{Eq.24}$$

Where μ is a location parameter, σ is a scale parameter, and ξ is a shape parameter. The parameter estimation is based on the L-moments method.

2.1.1.2 Volume series

A valuable aspect of the methodology is to identify a series of appropriate hydrograph volumes corresponding to peak values. High volumes are probably associated with hydrographs with long duration but low peaks that are unlikely to damage a levee. The volume series linked to the PDS is chosen to advance a consistent methodology. Hydrograph volume is evaluated as the time integral from the start time to the end time of the quick flow component of the hydrograph. The constant slope method (Dingman 2015) is used for the baseflow separation to determine the quick flow component. The overlapped events case is analysed as one event because the selected events meet the independence criteria, which is a prerequisite to any statistical frequency analysis. Several criteria have been proposed in the literature to verify this hypothesis on the separation of successive flood events (Lang et al. 1999). The Cunnane criterion (Cunnane 1979) is employed as an independence criterion. Besides, the author avoided the scenario in which two independent events overlap each other by considering the start and end of an event during the phase in which the total discharge is equal to the baseflow component only. For the evaluation of the start time, and the end time of the quick flow component of the hydrograph, widely applied criteria are used. Notably, the start of the event coincides with the first abrupt rise of 20% of the discharge (Mediero et al. 2010), after a period of receding discharge of length N days ($N = 0.83 A^{0.2}$, where A is the catchment area in km^2) and at least 5 days, in which the total discharge is equal to the baseflow component (Halford and Mayer 2000).

The end time is defined when the baseflow becomes dominant, and is identified by the point from which the receding limb is described with a relative error of 15% by the following non-linear function (Coutagne 1948; Srinivasulu and Jain 2009; Thiesen et al. 2019):

$$Q_r = Q_0 \left(1 + \frac{(1-b)Q_0^{\frac{1}{1-b}}}{\varepsilon \cdot b} \right) \quad \text{Eq.25}$$

Where Q_0 is the initial discharge due to only the baseflow, while ε and b are the parameters of the recession equation. The two parameters ε and b of the recession equation (Eq.25) are estimated for each month. The procedure consists of two main steps: the first one is the definition of the Hourly recession Limb Discharge (HLD) series, the second one consists of the calibration of the parameter using an iterative method. The two steps are described in appendix A1-Evaluation of monthly parameter ε and b .

2.1.2 The hydrological load: the forcing hydrographs

2.1.2.1 Generation of synthetic peak and volume pairs

The dependence of the volume (V) on the peak discharge (Q) is analysed to approximate the joint distribution. This approximation allows generating random synthetic plausible Q-V pairs for the analysed site. The chapter aims to find the critical failure condition and not its probability. The use of a rigorous bivariate distribution may complicate the process for the generation of the Q-V sample uselessly, due to the difficulties that arise from the need to obtain a satisfying fit of the available data to a theoretical model.

A regression equation over the observed Q-V pairs is applied. The volume V for a given peak discharge Q is estimated with Eq.26:

$$V = c_1 \cdot Q^{c_2} \quad \text{Eq.26}$$

Where c_1 and c_2 are the coefficients of the regression equation, fitted from the observed pairs compiled in data analysis. The synthetic peaks are generated from the fitted GPD according to the inverse sampling method (Seber and Salehi 2012). A random sample of 5000 probability values from a uniform distribution in the interval $[0,1]$ is generated. The random sample is transformed into a set of synthetic peak flows (Q_{ps}) by the inverse GPD (Eq.27):

$$Q_{ps} = \frac{\sigma}{\xi} \left(1 - (1 - X_u)^\xi \right) + \mu \quad \text{Eq.27}$$

Where X_u is a uniform random variable in the interval $[0, 1]$. The synthetic volumes are generated from synthetic peak flows through the regression equation to obtain the synthetic pairs. According to the Shapiro-Wilk test (Shapiro and Wilk 1965), the residuals of the regression equation are generally distributed in the log-log space of variables. The authors select the volumes from a normal randomisation for each synthetic peak flow, with mean equal to the

result of the regression equation (Eq.27), and standard deviation equal to the residual variance of the regression. This model approximates the correct bivariate probability distribution, but it is acceptable because the aim is to generate a large sample of realistic Q-V pairs covering the expected range of Q-V at the levee location. A similar procedure was applied, for instance, by Mediero et al. (2010).

2.1.2.2 Hydrograph shape

To describe the hydrological load, the shape of the hydrograph corresponding to each peak - volume pair should be defined. Hydrograph shapes are selected among those that maximise the duration of overtopping conditions and thus produce larger overtopping volumes. There are many empirical formulae in the literature, modelling the stage-discharge relationship such as Jones (1915); Henderson (1966); Fread (1975); Chow (1958); Marchi (1976); Fenton (1999). These equations guide to characterise the levee overtopping risk. In every formula, considering a time interval length Δt , there is a non-linear direct proportionality between the variation of discharge (Q) and the rate of change of the river stage (ΔH). Thus, at the generic time t , $H(t)$ depends on $Q(t)$ but is also correlated with the variation of discharge during the previous periods. This factor strongly influences the amplitude and the magnitude of the hysteresis loop. For the same hydraulic conditions (roughness, cross-section, etc.), the hysteresis loop depends on the variation of the ratio between discharge and time, which is represented by the slope of the hydrograph.

An index that expresses their risk of levee overtopping classifies the hydrograph shapes of the events included in the PDS series are first. For a given peak, and volume of the hydrograph, the worst shape for the overtopping risk is the rectangular one because it maximises the duration of overtopping. This research proposes the Overtopping Hydrograph Shape Index (OHSI), which takes into account how much the hydrograph approaches the rectangular shape. The OHSI is defined as:

$$OHSI = \left[\frac{X_{rl,ad}D_r + X_{fl,ad}D_f}{D} \right] \left[\frac{Y_{rl,ad}(Q_p - Q_{min}) + Y_{fl,ad}(Q_p - Q_{min})}{(Q_p - Q_{min})} \right] \quad \text{Eq.28}$$

All the terms and the procedure to obtain the OHSI are reported in appendix II. The OHSI links the Centre Of Mass (COM) of the discharge change rate in time. For a given pair of peak and volume, the larger the value of OHSI, the farther the COM is located from the peak, thus producing a higher hysteresis loop. This fact leads to a higher probability of having more overtopping volume if the overtopping threshold is reached. Therefore, large values of OHSI correspond to the more dangerous hydrograph shapes concerning to the overtopping failure. Different OHSI, associated with different hydrographs with the same peak and volume but different shapes (from triangular to rectangular), were tested on a rectangular channel (Fig. 8). The overtopping volume and overtopping duration are evaluated, demonstrating that there is a direct proportionality among the

Methodology

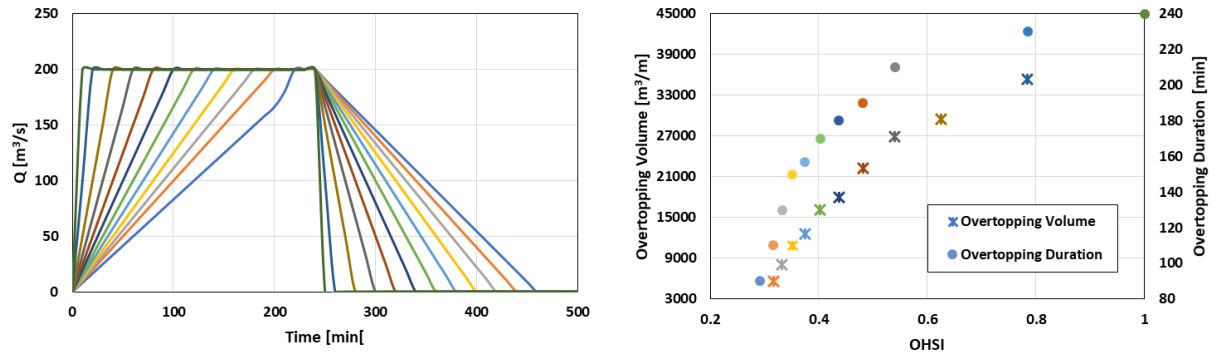


Fig. 8. Same cross-section: on the left, different hydrographs with same peak and volume but different shape, from triangular to rectangular; on the right, the corresponding overtopping volume, and overtopping duration vs. corresponding values of Overtopping Hydrograph Shape Index (OHSI).

Several triangular hydrographs with the same peak, volume, and duration were also tested. The position of the peak establishes a different value of the OHSI for each hydrograph. The results for three hydrographs are shown in Fig. 9. The results are impressive because as the OHSI increases, the amplitude of the hysteresis cycle enlarges, thus increasing the overtopping volume.

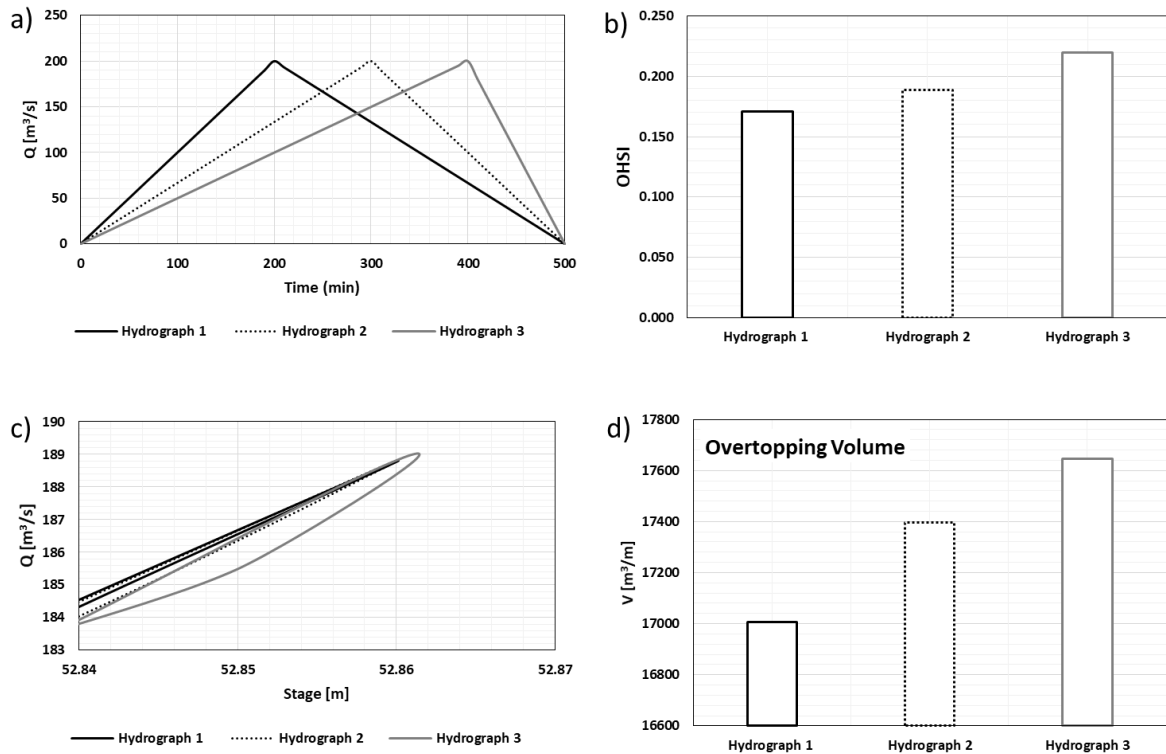


Fig. 9. Three triangular hydrographs with the same peak, and volume but different shape, at the same cross-section (a); the Overtopping Hydrograph Shape Index -OHSI of each hydrograph (b); detail of the rating curves (c) and the corresponding overtopping volumes (d) for each hydrograph.

2.1.3 Generation of forcing hydrographs

Each Q-V pair is transformed in a hydrograph to which a shape is associated. The hydrograph shapes are taken from the observed events, considering the shapes that are most

likely to cause overtopping. Since the most dangerous shapes concerning the overtopping failure are not known a priori, the OHSI index is used to rank hydrograph shapes. The shapes are classified in decreasing order according to the OHSI. A sensitivity analysis is carried out to find the most dangerous shapes to be applied randomly to the Q-V pairs. In the sensitivity analysis, hydrographs with peak values close to the overtopping discharge, and different shapes are tested. The percentage of the hydrograph that exceeds the overtopping threshold condition is analysed for each shape. The overtopping threshold condition is approximated to $10000 \text{ m}^3/\text{m}$ for 2 hours (Damme et al. 2016), which is the most used reference value in the Tuscany Region. The shapes that exceed the threshold condition in 85% of the cases are selected and are applied randomly to the Q-V pairs to generate the hydrograph sample.

2.1.4 Hydraulic Conveyance and Critical Overtopping Flood Hydrograph

The hydraulic conveyance is evaluated with the help of a one-dimensional hydraulic model. The levee failure condition depends on water depth, duration of the overflow, overtopping volume, overtopping velocity or a combination of the factors mentioned above. A hydraulic model is required to evaluate these factors for a given forcing hydrograph. For the given hydrograph, the hydraulic model allows estimating the water level corresponding to the discharge, properly accounting for the hysteresis loop of the stage-discharge relationship. As the river flow in systems with levees is subcritical (CIRIA 2013), the hydraulic model should cover the river reach downstream of the levee under analysis. The hydraulic model accounts for the backwater effects that determine the water depth at the levee location. The river reach under analysis should be extended downstream of the levee site to a cross-section of known boundary condition or a location in which the effect of the chosen boundary condition is negligible at the levee site. Site-specific conditions should determine the adequate length of the river to be analysed. The levee resistance criterion, characterised by the overtopping threshold condition, should be evaluated at the levee site using the output from the hydraulic model. The methodology depends on adopting a failure criterion that involves variables that can be derived from the hydrodynamic model of the river reach. The general guideline is to use as levee resistance criterion the one that is most suitable for the area analysed (CIRIA 2013). There are several overtopping resistance criteria in the literature (e.g. Hewlett et al. 1987), and the most appropriate for the case studies after a preliminary analysis of the area is selected.

The existing levee resistance criteria require overtopping volume or overtopping duration or the overtopping velocity or a combination of these factors. The evaluation of these quantities can be approximately carried out with a simple 1D hydraulic model and depends on the overtopping failure mechanism. The overtopping failure mechanism can be divided into three main groups (Hughes 2008a): surge-only overtopping, wave-only overtopping, and a combination of surge, and wave overtopping. The river flood waves are long-period waves and are classified as shallow-water waves (Ferrick 1985). The motion of the long-period wave is substantially horizontal with slight accelerations in the vertical direction whose effect can be neglected compared with the effects of the horizontal forces (Van Douwen 1963). Then, the

river flood wave is normally perceived as a change in the water level rather than a short-period wave. For river dykes, the surge-only overtopping is, therefore, the primary detected failure mode, and it is the one considered in this research. The overtopping hydraulic quantities are estimated with the formula described in section 1.1. The application of the chosen levee resistance criterion allows the classification of the forcing hydrographs in two groups, depending on whether they cause overtopping failure or not. The boundary between those groups is formed by the hydrographs that strictly satisfy the overtopping threshold conditions and determines a curve in the $Q - V$ space that is the Critical Overtopping Flood Hydrograph (COFH).

2.2 Probability of the bivariate hydrological failure line: the Bivariate Failure Return Period

Previously, the critical hydrological failure line is estimated, i.e. the COFH. It consists of a curve in $Q-V$ space in which all the possible hydrographs cause the overtopping failure. The next step is to assess the probability of this failure condition in terms of the return period. In state of the art, different approaches for evaluating the bivariate return period are discussed. In particular, the importance to consider a structural approach (Requena et al. 2016; Volpi and Fiori 2014) taking into account the interaction between the hydrological load and the structure. The existing methodologies show some issues (Serinaldi 2015; Vogel and Castellarin 2017), avoiding them using the failure probability concept. Although the return period is generally adopted in the hydraulic structure design, the interest of practitioners dealing with risk analysis should focus on quantifying the failure probability of the system at h , and over a specified period, for instance, the design life (Volpi et al. 2019).

The classical equation of the return period T (Kottegoda and Rosso 1997):

$$T(z) = \frac{\Delta\tau}{1 - P_z(z)} \quad \text{Eq.29}$$

Where $P_z(z) = P(Z \leq z)$ is a marginal cumulative distribution. The return period definition, $T(z)$ quantifies the expected time to the occurrence of a dangerous event $A = \{Z > z\}$, which implies that the probability of occurrence of A in one-time unit $\Delta\tau$ (year) is $1/T$. Differently, the failure probability of the system measures the probability that the dangerous event A occurs at least once over the specified period (l). It is expressed by the following relationship (Volpi, 2019):

$$R = P\left\{X \leq \frac{l}{\Delta\tau}\right\} = \sum_{t=1}^{l/\Delta\tau} f_x(t) \quad \text{Eq.30}$$

Where R is the failure probability, and $l/\Delta\tau$ is the number of discrete time steps corresponding to l . Under stationarity, and independence conditions, the failure probability depends on the return period:

$$R(z, l) = 1 - \left(1 - \frac{\Delta\tau}{T(z)}\right)^{\frac{l}{\Delta\tau}} \quad \text{Eq.31}$$

Usually, the $\Delta\tau$ is one year.

From the Eq.31 results a failure probability of 63% for an event with 100 years of return period, considering a time horizon of 100 years, and demonstrates the importance to find the failure probability in risk analysis of hydraulic structure (e.g. Benjamin and Cornell 1970; Serinaldi 2015).

In this paragraph, the goal is to define the bivariate return period taking into account the failure probability of the hydraulic structure. In this sense, the procedure proposed is called: Bivariate Failure Return Period (BFRP). To assess the failure probability, the bivariate hydrological failure curve, e.g. the COFH, becomes necessary. It detects which events are in the safe or failure area of the Q-V space. The COFH procedure simplifies the bivariate distribution construction to have a shareable methodology for the practitioners, and the administrations. The critical failure line can be estimated with other methods (e.g. Volpi and Fiori 2014) that complicates its evaluation.

The BFRP is based on: i) definition of Q-V bivariate distribution; ii) Using the bivariate distribution, failure scenarios are generated through Monte Carlo Simulation; iii) the failure probability curve $(R_f(l))$ is estimated by Poisson distribution; iv) The BFRP is the value of the return period replaced in Eq.31 approximates with 5% tolerance the $R_f(l)$.

$$R_f(BFRP, l) \simeq 1 - \left(1 - \frac{\Delta\tau}{BFRP}\right)^{\frac{l}{\Delta\tau}} \quad \text{Eq.32}$$

In Fig. 10, the overall procedure is shown.

Methodology

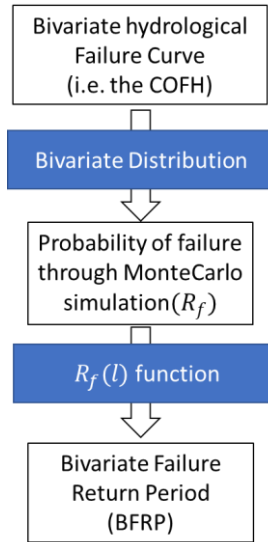


Fig. 10. The flow chart of evaluation of Bivariate Failure Return Period – BFRP.

2.2.1 Definition of the bivariate distribution.

The copula method (par. 1.1.2) is used to model the bivariate distribution of Q and V . Traditional multivariate techniques, assume that the marginal distributions belong to the same family of distributions, and the dependence between variables follows a linear relationship (Tong 2012). However, the flood variables may well fit by different marginals distributions (e.g. Zhang and Singh 2006). Copula models can avoid these difficulties following the theorem of Sklar (1959) (Eq.1) (Salvadori et al. 2007). As shown in section 1.1.2, a copula allows for modelling the dependence structure of random variables regardless of the family that the marginal distributions belong.

2.2.1.1 Definition of annual maxima series

In the hydrological extreme value theory, the distribution derives from the annual maxima series (Cunnane 1973) of the hydrologic variable, i.e. Q . In the univariate case, the magnitude is sortable. Detecting of the annual maximum value becomes clear (as for example $Q=100$ is higher than $Q=90$). In a bivariate environment, assessing an annual maximum is not straightforward. Establish a priori that the interaction between Q_1 and V_1 produces an effect most hazardous than that one generated by Q_2 and V_2 (where $Q_1 > Q_2$). Indeed, as shown in figure Fig. 8, different hydrographs with the same Q , V generate different hysteresis loops, and thus, different water stages in the river. In literature, most of the authors propose to select the bivariate annual maxima series by the annual maxima series of Q with the corresponding V (e.g. Goodarzi et al. 2013b; Mediero et al. 2010). This method is useful if the aim is to estimate the critical hydrological failure line, not a probability analysis of the failure (chapt.2). The annual maxima series of Q - V pairs should belong to the hydrographs causing the maximum effect, i.e. water stage, in the year. Considering the maximum peak in the year and the

corresponding volume is correct when a significant difference between peak values in the same year is evident, and the slope of the river is above to the 0.01%. If there are these conditions, the magnitude of the hysteresis loop belonging to the unsteady rating curve (Δh , the variation of the water stage respect the steady rating curve) is insignificant (Fread 1975). If these conditions are not verified, a further analysis of the discharge series is proposed. The detecting of the annual maxima series of hydrographs, and consequently, the annual maxima of Q and V is at the base of the methodology. The annual maxima series of hydrographs is chosen respect to the resulting water stage in the river in the cross-section where the Q and V are analysed.

Firstly, a simplified rating curve is estimated considering an equivalent cross-section, with the same perimeter, and area of the actual one, and the Chezy formula for steady flow. The shape of the equivalent cross-section is detected from which best approximate the real cross-section. The result of this step is the maximum steady water stage ($h_u[ft]$) corresponding to the peak of discharge.

Secondly, the Δh (m) is evaluated with the Fread formula (1975):

$$\Delta h \simeq 3,28084 \cdot h_u \left[1 - \left(\frac{S_0}{S} \right)^{0.3} \right] \quad \text{Eq.33}$$

S_0 (m/m) is the slope of the river, and S is the energy slope approximated by:

$$S \simeq S_0 + \delta h_s \left[\frac{0.52}{\frac{h_u^{2/3}}{n} \sqrt{S_0}} + \frac{0.01 h_u^{2/3} \sqrt{S_0}}{h_u \cdot n} \right] \quad \text{Eq.34}$$

n is the Manning roughness coefficient ($s/ft^{1/3}$), δh_s is the change in water surface elevation during the Δt time interval, in ft/sec, and $\delta h_s = (h - h')/\Delta t$. Usually, the daily discharge series is available. The Cunnane independence criterion (1979) is used to select a peak discharge belonging to separate events. The Fuller's formula (Fill and Steiner 2003) transforms the mean daily peak discharge into the peak of the event. For the calculation of h_s , Δt is accounted for one day. The peak of daily discharge and the following one value are referred to compute the corresponding h and h' with the Chezy formula. The maximum water stage generated by the hydrograph is evaluated as:

$$H_a = h_u + \Delta h \quad \text{Eq.35}$$

The annual maximum H_a detects the annual maximum hydrograph. The annual maximum values of Q and V are extracted from the selected hydrograph.

2.2.1.2 Selection of the marginal distributions

The bivariate distribution is modelled through the copula. The basics of copula theory is described in paragraph 1.1.2. Following a copula-based approach, to define the bivariate distribution of Q and V, it is necessary to define the marginal distributions of Q and V and the function Copula.

The selection of the marginal distribution is carried out by the following steps:

- Selection of a list of candidate distributions;
- estimation of the marginal distribution parameters;
- application of model selection criteria.

The marginal distributions tested are the traditionally employed in hydrologic extreme value analysis: Normal, Log-Normal-LN, Gumbel-GUMB, Frechet (EV2), Generalized Extreme Value (GEV), Pearson type III (P3), and Log-Pearson type III (LP3).

In literature, different methods are proposed for estimating parameters of univariate distribution (see, among the others, Elamir and Seheult 2003; Greenwood et al. 1979; Hosking 1990; Kottegoda and Rosso 1997). In the present study, the maximum likelihood method is chosen. The main advantage of the maximum likelihood method is related to the small variance of parameters, and quantiles that the method provides (Dombry et al. 2019).

The choice of the marginal distribution relative to each random variable (Q and V) is supported by the application of the following selection criteria:

- the Akaike information criterion (AIC) (Sakamoto, Ishiguro, and Kitagawa 1986);
- the Bayesian information criterion (BIC) (Marshall, Nott, and Sharma 2005);
- the Anderson-Darling criterion (ADC) (Laio 2004).

A detailed description of these model selection criteria and their hydrologic applications can be found in Laio et al. (2009).

2.2.1.3 Copula selection

The steps involved in selecting the appropriate copula model are:

- dependence evaluation between Q and V;
- parameters estimation;
- goodness-of-fit tests;
- tail dependence

A dependence analysis among Q and V is conducted to assess if dependence can be deduced from the data. It can be carried out by graphical analyses and dependence measures. The scatter plot of pairs $(R_i/(n+1); S_i/(n+1))$ can display a graphical analysis of

dependence. R_i is the rank of Q_i among $Q_1 \dots Q_n$, S_i is the rank of V_i among $V_1 \dots V_n$, and n is the length of the sample size. Besides, the scatter plot of ranks, two graphical tools for detecting dependence have been proposed in the literature: χ -plot (Fisher and Switzer 1985) and K-plot (Genest and Boies 2003). The χ -plot measures the location of an observation regarding the whole of the observations (λ_i) against a measure of the χ^2 test statistic for independence (χ_i). If the random variables are independent, it is expected that $\chi_i = 0$. Therefore, the values of χ_i that fall far from zero indicate a certain degree of association between two variables. The chi-plot also includes control limits drawn at $\chi_i = \pm C_p / \sqrt{n_s}$, where n_s is the sample size, and C_p values are suggested by Fisher and Switzer (2001). The C_p values proposed are 1.54, 1.78, and 2.18 with the p-values of 0.9, 0.95, and 0.99, respectively. Positive dependence is established when scatter points fall mainly on the upper side of the control limits; otherwise, negative dependence is found. The K-plot relates the order statistics ($H_{(i)}$) estimated from the observed data to the expected values of these statistics ($W_{(i:n)}$) generated under the null hypothesis of independence between the marginal distributions. A straight line across the main diagonal is superimposed on the graph to the case of independence, and a smooth curve $K(w)$ located above the straight is associated with perfect positive dependence. In case of perfect negative dependence, it is shown as a string of data points aligned on the x-axis. Besides, dependence measures are needed to procure a quantitative value of the dependence relation between variables. Two well-known nonparametric dependence measures are used: the Spearman rank correlation coefficient (ρ) (Spearman 1961) and Kendall rank correlation coefficient (τ) (Kendall 1938). These two dependence measures are associated with their p-value (independence is rejected if p-value < 0.05, Genest and Favre 2007). The copula functions tested are the ones usually analysed in the hydrologic applications (Chowdhary et al. 2011; Favre et al. 2004; Reddy and Ganguli 2012; Renard and Lang 2007; Zhang and Singh 2006), reported in Tab. 1.

Methodology

<i>Copula</i>	$C_\theta(u_1, u_2)$	θ space
Clayton	$\{u_1^{-\theta} + u_2^{-\theta} - 1\}^{-1/\theta}$	$[-1, \infty) \setminus \{0\}$
Gumbel	$\exp\left\{-\left((-\ln u_1)^\theta + (-\ln u_2)^\theta\right)^{1/\theta}\right\}$	$[-1, \infty)$
Frank	$-\frac{1}{\theta} \ln\left\{1 + \frac{(e^{-\theta \cdot u_1} - 1)(e^{-\theta \cdot u_2} - 1)}{e^{-\theta} - 1}\right\}$	$(-\infty, \infty) \setminus \{0\}$
Galambos	$u_1 u_2 \exp\left[\left((-\ln u_1)^{-\theta} + (-\ln u_2)^{-\theta}\right)^{-1/\theta}\right]$	$[0, \infty]$
Husler-Reiss	$\exp\left[\ln u_1 \Phi\left\{\frac{1}{\theta} + \frac{\theta}{2} \ln \frac{-\ln u_1}{-\ln u_2}\right\} + \ln u_2 \Phi\left\{\frac{1}{\theta} + \frac{\theta}{2} \ln \frac{-\ln u_2}{-\ln u_1}\right\}\right]$	$[0, \infty]$
Gaussian	$\int_{-\infty}^{\phi^{-1}(u_1)} \int_{-\infty}^{\phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{(1-\theta^2)}} \exp\left\{-\frac{x^2 - 2\theta xy + y^2}{2(1-\theta^2)}\right\} dx dy$ ¹	$[-1, 1]$
Student-t	$\int_{-\infty}^{t_{\theta_2}^{-1}(u_1)} \int_{-\infty}^{t_{\theta_2}^{-1}(u_2)} \frac{\Gamma((\theta_2 + 2)/2)}{\Gamma(\theta_2/2) \pi \theta_2 \sqrt{1 - \theta_1^2}} \left(1 + \frac{x^2 - 2\theta_1 xy + y^2}{\theta_2}\right)^{\frac{\theta_2 + 2}{2}} dx dy$ ²	$\theta_1 \in [-1, 1]$ $\theta_2 \in (0, \infty)$

¹ ϕ is the univariate standard Normal distribution; ² Γ is the gamma distribution

Tab. 1. Copula families considered in this study. This copula functions are the most implemented in the hydrological applications.

The set of copulas is filtered after testing the acceptable range of dependence in terms of Kendall's τ value. In Michiels and De Schepper (2008) provides ranges of acceptable Kendall's τ to different copulas for the bivariate case.

The next step consists of the estimation of the copula parameters. In literature, different methods are proposed for copula parameters estimation (e.g. Song and Singh 2010; Weiß 2011). In the present work, the maximum pseudo-likelihood method (MPL) is used (Genest and Favre 2007). The MPL estimation method does not require any prior assumptions on the marginal distributions of hydrologic variables. The MPL method transforms the flood variables into uniformly distributed vectors using its empirical distribution function. Then, the copula parameters are estimated using the maximization of pseudo log-likelihood function (Salvadori et al. 2007). The evaluation of the adaptation of the copula functions to the pseudo-observations is carried out through graphic comparisons, and suitable statistical tests. The behaviour of the copulas can be drawn via a scatter plot, where a synthetic sample of pairs generated from each copula of study $(u_{1,i}, u_{2,i})$, being $i = 1, \dots, n$, and n the sample size, is compared to the pairs related to the observations $(R_i/(n+1); S_i/(n+1))$. Another useful plot consists of fitting the marginal distributions of the random variables to transform the pairs generated from the copula into their original units $(Q_i; V_i)$, by the equation:

$$(Q_i; V_i) = (P_Q^{-1}(u_{1,i}); P_V^{-1}(u_{2,i})) \quad \text{Eq.36}$$

where P_Q^{-1} , and P_V^{-1} are the inverses of the marginal distributions. While the first plot provides a graphical test of the goodness-of-fit of the dependence structure taken in isolation, the latter one makes it possible to judge the viability of the complete model for frequency analysis (Joe 1997; Nelsen 2007; Salvadori et al. 2007, 2015).

Although graphical tools provide a general view of the goodness-of-fit, numerical tests are needed to quantify it. Copulas can be selected according to the Akaike Information Criteria, Bayesian Information Criteria (Grønneberg and Hjort 2014), and Cramer Von Mises Test (Salvadori et al. 2007). For observations $u_{i,j}$, $i = 1, \dots, N$, $j = 1, 2$, the Akaike Information criterion for a copula – AIC_{cop} with parameter(s) θ is defined as:

$$AIC_{cop} = -2 \sum_{i=1}^N \ln[c(u_{i,1}, u_{i,2} | \theta)] + 2k \quad \text{Eq.37}$$

Where $k=1$ with one parameter copula, $k=2$ with two parameters copula.

The Bayesian Information Criterion for copula – BIC_{cop} is defined as:

$$BIC_{cop} = -2 \sum_{i=1}^N \ln[c(u_{i,1}, u_{i,2} | \theta)] + \ln(N)k \quad \text{Eq.38}$$

The selected copula should have a lower value both for the AIC_{cop} , and for the BIC_{cop} .

The Cramer-von Mises test (S_n) (Serinaldi et al. 2007) is adopted. The S_n is an error measure between the values of the empirical copula and those obtained by the fitted copula.

$$S_n = \sum_{i=1}^n \left\{ C_n \left(\frac{R_i}{n+1}; \frac{S_i}{n+1} \right) - C_{\theta_n} \left(\frac{R_i}{n+1}; \frac{S_i}{n+1} \right) \right\}^2 \quad \text{Eq.39}$$

where:

$$C_n(u, v) = \frac{1}{n} \sum_{i=1}^n \mathbf{1} \left(\frac{R_i}{n+1} < u; \frac{S_i}{n+1} < v \right) \quad u, v \in [0, 1] \quad \text{Eq.40}$$

C_n is the empirical copula, C_{θ_n} is the parametric copula with the parameter previously estimated from the observed data (Q_i, V_i). The p -value is obtained through a parametric bootstrap-based procedure (Genest et al. 2009). The statistic value is used to classify the copula models as the p -value can only be utilised to accept or reject each copula model. Consequently, the selected copula should have the lower value of the statistic with an admissible p -value (i.e. larger than 0.05). The chosen copula is not only the one that best represents the dependence

between peak, and volume but also the one that allows making better evaluations on the extreme events. Thus, the analysis of upper tail dependence is developed. The coefficient λ_u (Serinaldi, 2008) measures the tail dependence:

$$\lambda_u = \lim_{\omega \rightarrow 1^-} (F_{X_1}(x_1) > \omega | F_{X_2}(x_2) > \omega) \quad \text{Eq.41}$$

The Eq.41 demonstrate that the upper tail dependence is related to the probability of the marginal distribution of x_i being more significant than the threshold $\omega \in [0, 1]$, when the marginal distribution of y being also higher than that threshold. The Eq.41 can also be estimated using copulas through the following equation:

$$\lambda_u^c = \lim_{\omega \rightarrow 1^-} \frac{1 - 2\omega + C(\omega, \omega)}{1 - \omega} \quad \text{Eq.42}$$

To understand which copula model is suitable for the analysis of extreme, the λ_u^c obtained from each copula should be compared to the coefficient estimated from the observed data represented by the estimator $\widehat{\lambda_U^{CFG}}$ (Reddy and Ganguli 2012):

$$\widehat{\lambda_U^{CFG}} = 2 - 2 \exp \left[\frac{1}{n} \sum_{i=1}^n \log \left(\frac{\sqrt{\log \frac{1}{u_{i,1}} \log \frac{1}{u_{i,2}}}}{\log \frac{1}{\max(u_{i,1}, u_{i,2})^2}} \right) \right] \quad \text{Eq.43}$$

2.2.2 Failure probability and the Bivariate Failure Return Period

The second step is identified as the failure probability. The failure probability measures the likelihood that the dangerous event A occurs at least once over the specified period, i.e. the Design Life - DL (Salvadori et al. 2016; Volpi et al. 2019). The Design Life (DL) of the hydraulic structure is fixed according to the country law where hydraulic structure is. Through the bivariate distribution previous evaluated, failure scenarios are built. Each failure scenario assumes that the bivariate distribution is based on the annual maxima series of Q and V. Therefore, synthetic Q-V pair, randomly generated from bivariate distribution, represents an annual maximum of a hypothetical year. To clarify, if a random sample of ten pairs Q-V is generated, it would represent a hypothetical scenario of the next ten years, and each pair would be the annual maxima of Q-V. This series does not probably represent the real scenario for the next ten years, and, therefore, it is necessary to create a Monte Carlo methodology for this purpose. A Monte Carlo simulation is performed for every time frame that the design life of the structure is composed $(1, 2, \dots, i, \dots, DL)$. The main result of the Monte Carlo simulation is the Failure Matrices (FM). The Failure Matrices are in number equal to the design life (DL) of the hydraulic structure. To have a sufficient number of scenarios (Chow 1958), each Failure Matrix

– FM_i is built with a number of rows equal ten times the design life. The number of columns of each FM_i balance the length of each time frame (l) of design life

Tab. 2). As an example, if a $DL=50$ years and the time frame of $i = 10$ years, the FM_{10} has 500 rows and 10 columns. Each row represents a possible scenario (failure or not failure) for the i -time frame. The elements of the FM_i are the generated synthetic Q-V, expressed as ‘0’ if the synthetic Q-V falls in the safe zone and ‘1’ if it falls in the failure zone. The safe zone and failure zone are previously identified in Q-V space by the COFH (par 3.1.5) or in any case, a hydrological failure line obtained with other methodologies (Volpi and Fiori 2014). A row represents a failure scenario if there is at least one element equal ‘1’ otherwise, it represents a safe scenario. An example of the Failure Matrices are shown in

Tab. 2 with symbolic elements.

Tab. 2. In each column of the table, the Time Frame Failure Matrix –TFFM for each step of a Monte Carlo procedure, is represented. The elements in each matrix are indicative.

	Failure Matrix					
	YEAR					
	$l = 1$	$l = 2$	$l = 3$	$l = 4$	...	$l = DL$
Row number	FM_1	FM_2	FM_3	FM_4		FM_{DL}
1	0	0,0	0,0,0	1,0,0,1		0,0,0,1...0
2	0	1,0	0,1,0	0,0,0,0		0,1,0,1...0
...		...	...	1,1,0,1		
$10 \cdot DL$	1	0,0	0,0,0	0,0,0,0		1,0,0,0...1

The collection of the FM_i ’s rows can be assimilated to a statistical Poisson experiment because it has the following properties:

- The experiment results in outcomes (the FM_i ’s rows) that can be classified as no failures or failures.
- The average number of failures is specified, counting the number of failure scenarios ($\mu_{fail,i}$) in FM_i .
- The probability that failure scenarios will occur is proportional to the size of the region ($10 \cdot DL$).
- The probability that success will occur in a tiny region is virtually zero.

The Poisson probability for each i-time frame can be expressed:

$$P_i(x; \mu) = \frac{e^{-\mu} \cdot \mu^x}{x!} \quad \text{Eq.44}$$

Where x is the actual number of successes that result from the experiment, and μ represents the average number of failure scenarios. The objective is to detect the number of failure scenarios with high probability of occurrence for each i-time frame. The failure probability (PF_i) for each i-time frame (year) that composed the DL ($i=1,2,3,\dots,DL$) can be estimated as:

$$PF_i = \frac{F_i(0.95; \mu_{fail,i})}{10 \cdot DL} \quad \text{Eq.45}$$

Where $F_i(0.95; \mu_{fail,i})$ is the quantile of the Poisson Distribution corresponding to a probability of 95% related to a given number of failure scenarios occurring in the region of $10 \cdot DL$.

The probability failure curve is obtained ($R_f(l)$) plotting and interpolating the PF_i values respect the corresponding time frame.

The Bivariate Failure Return Period is the return period that, replaced in the Eq.32, approximates the probability failure curve $R_f(l)$ with a 5% relative error. Therefore, the BFRP take into account both the multivariate nature of the hydrological load for river hydraulic structures and the risk of failure in its design life.

3 Case study

The methodology is illustrated through its application in two case studies: The levee of Ombrone Pistoiese River located in the town of Poggio a Caiano and the levee of Era River located in the town of Capannoli. The Ombrone Pistoiese River and the Era River are tributaries of the Arno River in Tuscany, Central Italy. The catchment areas are respectively 450 km² at Poggio a Caiano and 335 km² at Capannoli.

The two reaches are completely banked. In the two case studies, the levees are earth embankments about 11 m high, and grass-covered. The cross-section of the dyke has a trapezoidal shape. The width at the crest is about 3 m, and the width at the bottom is about 40 m. The slope of the riverside is 1.25:1.5, while the slope of the landside is 2:1. The landside area has mostly urban land use. Both the towns of Poggio a Caiano and Capannoli have a hydrometric gage with a consistent number of years (>20 years) of hourly observations. The study aims to determine the COFH curve for overtopping failure. As boundary conditions and the model parameters are used that of previous studies of (Minatti et al. 2016) and by the River Basin District Authority of Northern Apennine (www.adbarno.it/opendata).

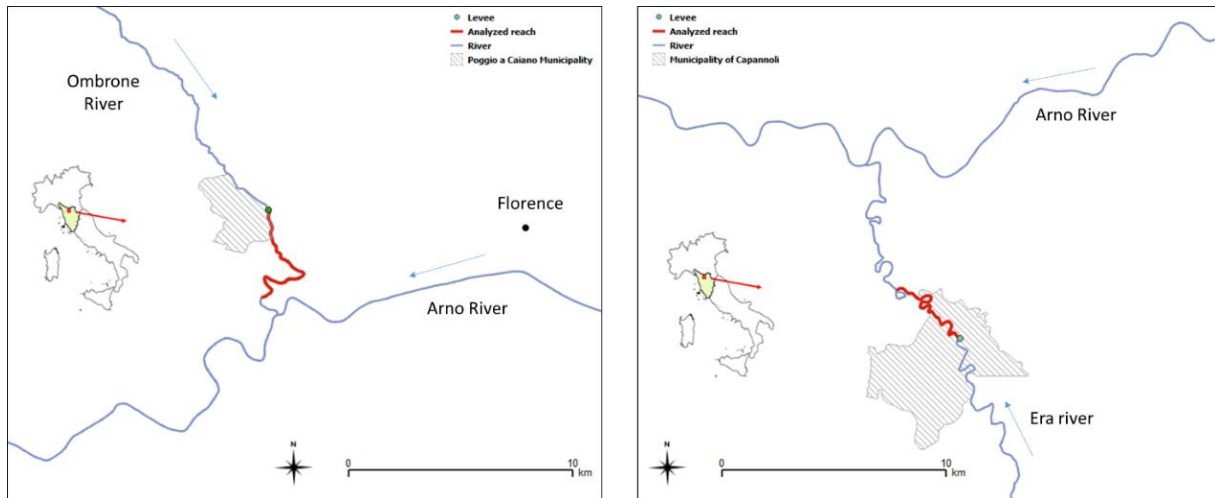


Fig. 11. The two case studies analysed. On the left the case study of Poggio a Caiano, on the right the case study of Capannoli.

3.1 Bivariate hydrological failure line

3.1.1 Hydrological Load estimation

Generation of peak-volume synthetic pairs

The Peak Discharge Series (PDS) are selected fixing a threshold respectively of 98 m³/s for Poggio a Caiano, and 40 m³/s for Capannoli, by the mean residual life plot (Lang et al. 1999). The PDS consist of 78 peak discharges for 26 years (1992-2017) for Poggio a Caiano and 40 peaks for 30 years (1988-2017) for Capannoli. The univariate frequency distribution of the PDS is modelled with the Generalized Pareto Distribution. L-moments determine the

Case study

parameters of the Generalized Pareto Distribution (GPD). For Poggio a Caiano the parameters are $\mu = 98.431$, $\sigma = 62.116$ and $\xi = 0.182$. For Capannoli the parameters are $\mu = 47.708 \text{ m}^3/\text{s}$, $\sigma = 37.297 \text{ m}^3/\text{s}$ $\xi = -0.182 \text{ m}$.

The volume (V) series is linked to the PDS to have consistent methodology. The volume is the time integral of the quick flow component between the start and the end time of the event. The start and the end time are defined as established by the methodology. For each case study, Tab. 3 reports the monthly parameters ε , b of the non-linear recession curve for estimating the non-linear recession curve.

Tab. 3. Monthly recession parameters ε , b .

	b	ε
January	0.53	48.52
Febraury	0.52	49.71
March	0.48	44.55
April	0.5	36.12
May	0.47	31.12
June	0.46	29.41
July	0.46	23.27
August	0.46	23.65
September	0.52	31.56
October	0.55	37.84
November	0.53	46.51
December	0.54	48.21

The dependence of the volume on the peak discharge is analysed for estimating the plausible synthetic Q-V pairs. Figure Fig. 12 shows the scatter-plot of the peak and volume pairs in log-log space for the two case studies. For the Poggio a Caiano the relationship $\log(V) = 1.1821\log(Q) + 2.57$ is identified with R^2 equal to 0.9333. For Capannoli the relationship is $\log(V) = 1.707\log(Q) + 5.16$ with R^2 equal to 0.70.

Five thousand synthetic peaks are generated according to Eq.27, and the corresponding synthetic volume values are generated with a normal randomisation for each synthetic peak value, with a mean equal to the result of the regression equation (Eq.26) and standard deviation equal to the residual variance of the regression. The synthetic pairs are represented in log-log space in Fig. 12.

Case study

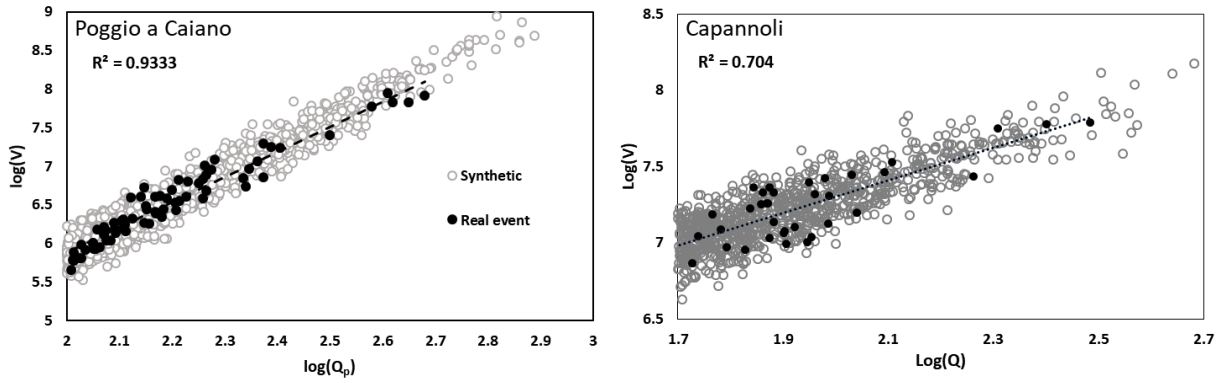


Fig. 12. Scatter-plot of the peak (Q) and volume (V) pairs in log-log space with recorded and synthetic Q - V pairs. On the left the case of Poggio a Caiano on the right the case of Capannoli.

3.1.2 Shape Ranking concerning the OHSI

The OHSI classifies the shapes related to the event of the PDS series in decreasing order, so the first position is the most dangerous for overtopping failure. The number of shapes of the ranking that are used in the randomisation is determined through the sensitivity analysis. A database of river levee failures for North Italy is compiled for this purpose. The database contains the river levee failure events for the period 1992-2014 of the records of the EEA-European Environmental Agency, of the Italian Civil Protection, and the Italian National Institute for Environmental Protection and Research (ISPRA - Istituto Superiore per la Protezione e la Ricerca Ambientale). The reports of Tuscany Hydrological Service are also considered. In the database, the river levee failures are divided into three categories: i) overtopping; ii) piping; iii) erosion. In Fig. 13 the overtopping failure events in Tuscany provided by the database are reported.

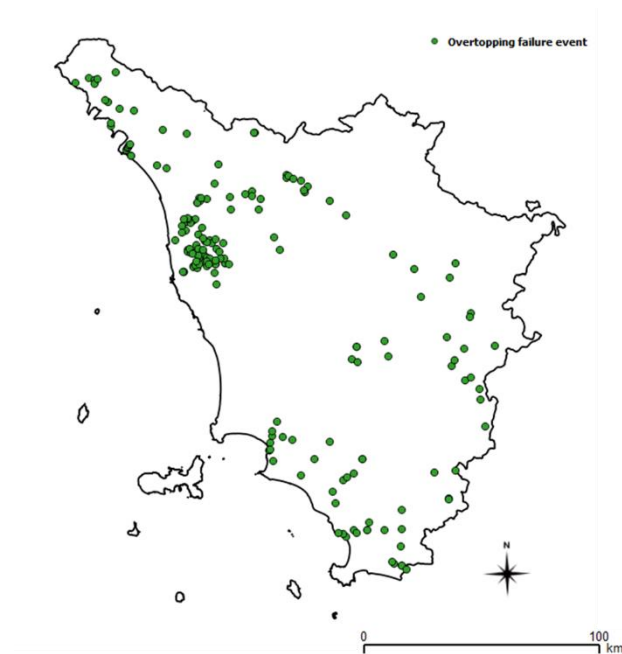


Fig. 13. Overtopping failure events in Tuscany (1992-2014).

In the database, there are four overtopping failure events in the period 1992-2014 associated to the levee of Poggio a Caiano. These four events allow us to define the threshold event for the sensitivity analysis. Among these, the event with a minimum peak of 428 m³/s fixes the threshold event. The five overtopping failure events for the levee of Capannoli (1992-2014), deduced from the database, identify the threshold event of 195 m³/s. The methodology for the generation of synthetic flow peaks and volumes is applied to produce a sample of synthetic Q-V pairs with a flow peak values at least equal or higher to the threshold event. Figure Fig. 14 shows the results of the sensitivity analysis. It represents the percentage of cases from the constrained sample that produced levee failure for the ten shapes with highest OHSI score. The first six shapes from the shape ranking for Poggio a Caiano and the first seven shapes for Capannoli were selected for shape randomisation because they exceed the threshold condition in 85% of the cases.

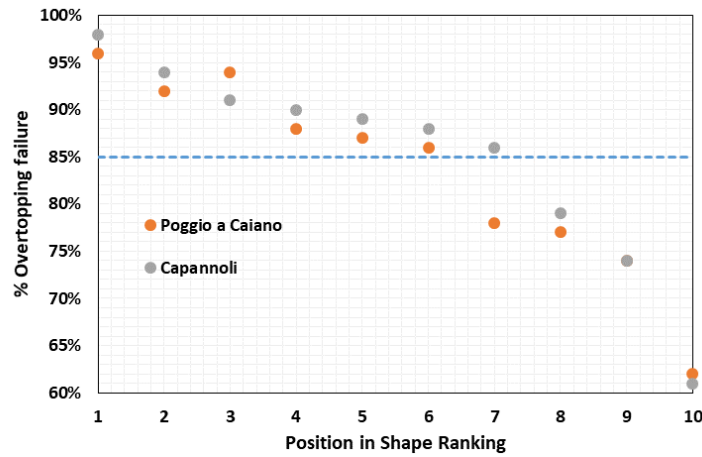


Fig. 14. Percentage of overtopping failure cases for the first 10 positions in the shape ranking based on the sensitivity analysis for the Poggio a Caiano and Capannoli case studies. The chosen threshold of 85% is also reported

3.1.3 Generation of the forcing hydrographs

The shapes selected are standardised, as suggested by Yue et al. (2002). The hydrograph abscissa is divided by its duration (D), and the ordinates of the hydrograph are divided by D/V. The generation of the 5000 forcing hydrographs follows these steps: firstly, take a Q – V pair, previously calculated, then randomly choose a shape from the selected ones. The Q and V resize the standardised shapes to build the forcing hydrograph.

3.1.4 Hydraulic Conveyance

Each set of 5000 forcing hydrographs is routed in the corresponding river reach, modelled with 1D HEC-RAS. Considering the average slope of the rivers in Tuscany Region, and the boundary conditions of the two sites, a river reach length about of 5 km have been modelled. The geometry of the HEC-RAS model is defined by the cross-sections of the River Basin District Authority of Northern Apennine (www.adbarno.it/opendata), and the Manning roughness coefficient is taken from the previous study (Minatti et al. 2016).

The levees are modelled as a lateral structure of the river, and the overtopping quantities are evaluated through Eq.19 and Eq.21. The weir coefficient (Eq.19) is fixed as 1.45 to account for the angle between the outflow concerning the main flow of the river (USACE 2009). The analysis consists of 5000 scenarios; each of these has one of the forcing hydrographs as upstream boundary condition. The downstream boundary condition for the Capannoli case is the uniform flow depth. For the case study of Poggio a Caiano, the reach of Ombrone Pistoiese is affected by a backwater effect caused by the presence of the Arno River. Given the consistent number of forcing hydrographs, the downstream boundary condition takes into account of the mean value of the water stage observed in the Arno River during the flood events registered in Poggio a Caiano gauge station. The limit on the overtopping volume of $10000 \text{ m}^3/\text{m}$ for 2 hours (Damme et al. 2016) is adopted as levee resistance criterion because it is well suited to the case study of Poggio a Caiano, confirmed by the technicians of the Land Reclamation Authorities of Tuscany Region. The overtopping volume and overtopping duration are estimated through the Eq.20. In the case study of Capannoli, the levee resistance criterion of Hewlett et al. (1987) is selected. This criterion establishes the initiation of damage on grass-covered levees, exposed to steady overflow, basing on curves in the space of overtopping velocity - overtopping duration respect to the degree grass cover. The levee of Capannoli is average grass cover, and the initiation of damage curve for this degree of grass cover is shown in Fig. 15. The overtopping duration is evaluated as above and the overtopping velocity is estimated through Eq.21.

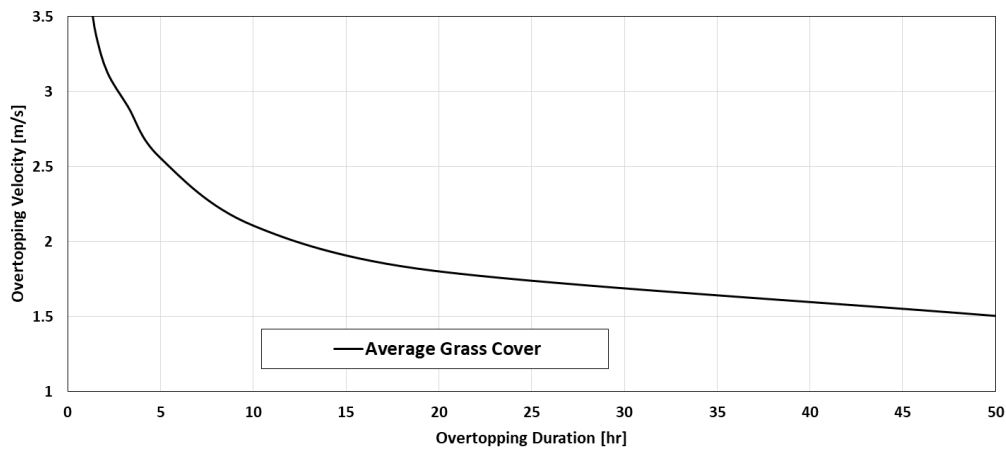


Fig. 15. The curve represents the initiation of damage condition for a levee with average grass cover proposed by Hewlett et al. (1987). The area above to the curve is the damaged area; the area below represents the no-damage area.

3.1.5 Results and discussion

The hydrological overtopping risk for a river levee is analysed in this work. Three factors are considered: the hydrological load, represented by the ensemble of Q-V pairs, the river morphology, represented by the hydraulic model, and the levee resistance, represented by the overtopping threshold condition.

The two hydrographs (Fig. 16), Hydrograph-a (H-a) and Hydrograph-b (H-b), are selected from the case study of Poggio a Caiano to illustrate the interaction between the factors that

determine the overtopping condition. The two hydrographs have the same peak, of about 450 m^3/s , but different volumes and shapes. The shapes are selected from the six critical realistic shapes established through the OHSI index (Fig. 14).

Fig. 16 also shows the hysteresis loop and the two overtopping hydrographs. The two hydrographs exceed the embankment crest differently because they cause a different hysteresis loop in the overtopped cross-section. Particularly, evaluating the overtopping hydrographs, only H-b exceeds the levee resistance criterion and causes overtopping failure.

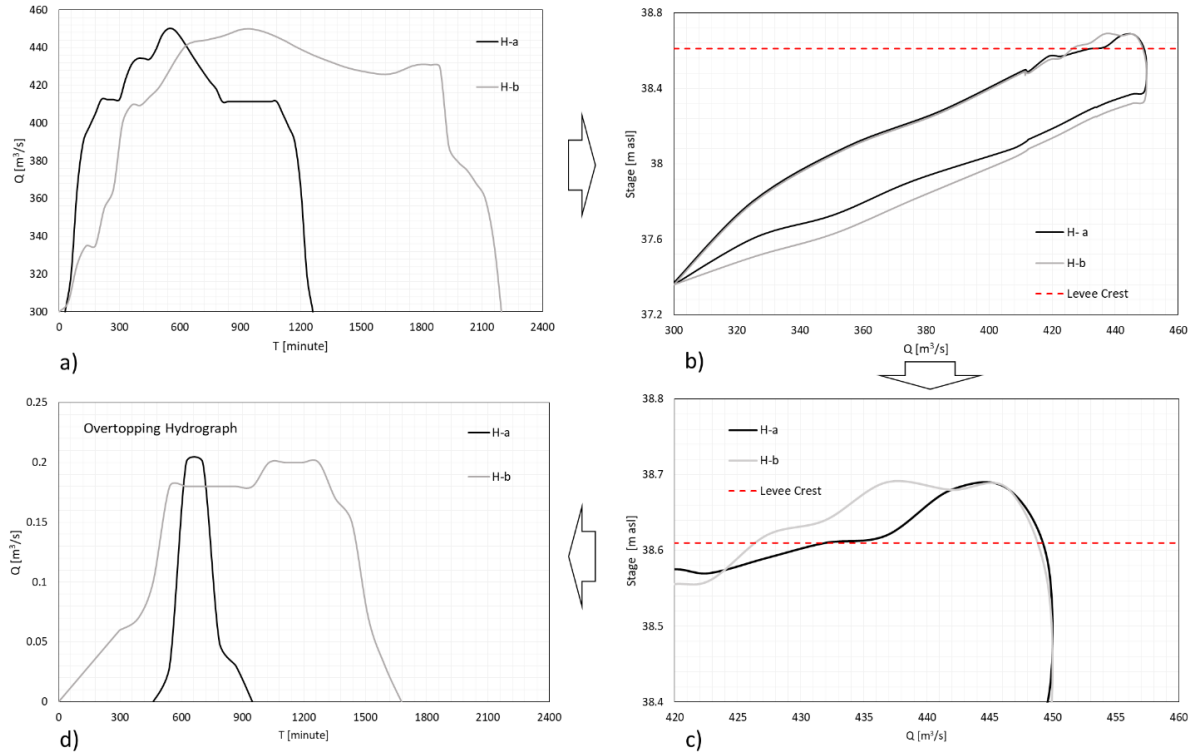


Fig. 16. Example of two different situations at the same cross section: two hydrographs H-a, and H-b (a) with the same peak but different volume and realistic shape with the corresponding rating curves (b). Below the detail of the rating curves (c), and the corresponding overtopping hydrographs (d).

The main results of the analysis are shown in Fig. 17. The whole ensemble of 5000 forcing hydrographs was analysed. A critical curve in the Q - V space is identified for each case study, defined by the Q - V pairs that strictly satisfy the levee resistance criterion: the Critical Overtopping Flood Hydrograph-COFH, displayed in Fig. 17. The COFH for a levee is not a unique hydrograph, but rather a curve in the Q - V space that represents the limiting failure condition. The critical failure line divides the Q - V space in a safe zone and a failure zone. The safe zone is located below, and to the left of the critical failure line and includes the hydrographs that do not cause levee failure. The area above and to the right of the COFH is the failure zone and includes the hydrographs that cause levee failure.

Case study

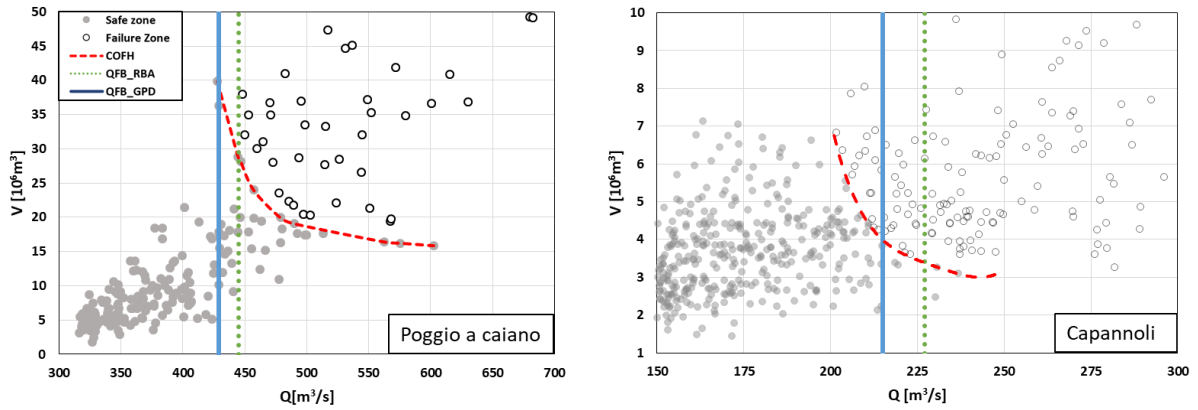


Fig. 17. The Critical Overtopping Flood Hydrograph at the cross-section of the analysed levee. On the left the site of Poggio a Caiano, on the right the site of Capannoli. The Q500 is the peak flow rate of 500 years return period, and 0.5m is the safe freeboard. The Q500_GPD are the values estimated from the inference analysis of the GPD distribution; the Q500_RBA are the values obtained by River Basin Authority

This critical failure line is the basis for risk assessment for the levee. It allows the estimation of the annual failure probability or the corresponding return period. Further natural development of this work is to apply a methodology that would generate equally likely Q-V points following the bivariate distribution and to determine the fraction of points that fall on either side of the critical failure line to estimate the failure probability, i.e. the return period. In this sense, the procedure to find the COFH is a significant contribution to flood risk estimation for a levee. The classic definitions of the bivariate return period described in the literature account only for the probability of occurrence of the hydrological load. For instance, unconditional, and conditional return periods (Salvadori and De Michele 2004) for hydrological analyses focus on the primary return period (the so-called “OR” return period, where either Q or V are exceeded, and the “AND” return period, where both Q and V are exceeded), and the secondary return period, aided by the definition of sub-, super-, and critical events. They defined sample isolines of return period in the Q-V space for each definition assuming certain copula distributions. In practice, the isolines of the return period of structure failure do not usually correspond those of the hydrological loads (Volpi and Fiori 2014). To overcome this issue, Serinaldi (2015) suggests using the failure probability of the structure. The failure probability is not addressed explicitly in this work because the methodology proposed here does not consider a full bivariate distribution of Q and V; it is instead an approximation. The approximation is obtained by fitting the univariate probability distribution of discharges to a GPD, and estimating the corresponding volume through linear regression (Eq. 4). It is not a rigorous bivariate distribution, but it is useful to estimate the critical failure line. The evaluation of the critical failure line in Q-V space is based on a large sample of realistic hydrographs covering the expected range of hydrographs at the levee location. The aim of the simplified approach of the bivariate distribution is to generate this sample. Using a rigorous bivariate

distribution (see, among others, Hawkes and Svensson 2006; Salvadori and Michele 2010) may complicate the process for the generation of the sample of synthetic hydrographs. Focusing the aim on evaluating the critical failure line rather than on the estimation of the failure probability, the approximate bivariate distribution allows avoiding these complications.

The results also show significant deviations from the standard levee design procedures. A typical river dyke design requires the determination of a design flowrate, usually specified in terms of a return period, plus a certain freeboard (FB) to be added as an additional safety margin. The theoretical failure line may be obtained by assuming a value for freeboard. For the two case studies analysed in this work, design discharges are determined by the River Basin Authority (RBA) as corresponding to a return period of 500 years (Q500) and a freeboard of 0.5 m. The discharge corresponding to the water stage reached adding the freeboard (QFB) is estimated through a steady hydraulic model. These discharges (QFB_RBA), represented in Figure 8 as vertical dotted lines are 445 m³/s for Poggio a Caiano and 215 m³/s for Capannoli. When the Q500 is evaluated from the univariate GPD distribution the corresponding design discharges (QFB_GPD), represented as vertical blue lines in Fig. 17, are 429 m³/s for Poggio a Caiano, and 227 m³/s for Capannoli.

Both theoretical estimates correspond to the same order of magnitude as the critical failure line obtained by the procedure presented in this work. However, there are striking differences that emphasise the utility of the bivariate approach. In both cases, but particularly in the case of Capannoli, discharges smaller than QFB may produce the levee failure if the corresponding hydrograph has enough volume and an adverse shape. Similarly, the levee may be safe if forced with hydrographs with peak discharge above QFB, but with small volume. The COFH demonstrates how the volume is a relevant variable to evaluate the risk of overtopping because the overtopping failure can occur for lower or higher discharges than the traditional design condition. Moreover, the COFH demonstrates that overtopping failure is dependent on volume. For the same discharge, different volumes can cause different scenarios: some Q-V pairs fall into the failure zone, and others fall into the safe one.

The proposed methodology is a useful approach to determine the critical failure line within the bivariate space defined by peak discharge and hydrograph volume and may be generalised to a large number of similar cases. However, the methodology is also subject to some limitations, derived from the aim, the data analysis and the modelling choices.

The approach is limited to bivariate analysis. It may be enhanced to include other single-value variables, like hydrograph duration. However, the next natural step would be to include hydrograph shape, which is not readily amenable to multivariate analysis. Hydrograph shape has been included in a simplified way, assuming adverse shapes selected through the OHSI ranking. This assumption leaves the results on the safe side, but the corresponding critical line may be overestimating the failure probability since not all hydrographs could have unfavourable shapes.

Regarding data analysis, some simplifications have also been made. In addition to the simplified procedure adopted for the generation of the sample of forcing hydrographs, other aspects of the data analysis deserve discussion. The probabilistic analysis of the Peaks of Discharge Series-PDS is based on the assumption of stationarity. There is concern that the assumption of stationarity may not hold due to several factors, like seasonality or global change. However, many similar studies are based on the same assumption (see, among others, G Salvadori and Michele 2004). Future developments of this work may include the assessment of the validity of the stationarity assumption and the development of multivariate non-stationary distributions to characterise the hydrological forcing. In this case, other external factors should be taken into account (for example, the land-use change) to have a complete definition of the non-stationary distribution (Huang et al. 2017). Some recent studies focused on this field (see, for example, Bender et al. 2014; Kang et al. 2019), and can be taken as a starting point.

The hydraulic model chosen for this work is a simple one-dimensional hydrodynamic code. This model was chosen for its simplicity and ease of use. It has, however, some limitations. The HEC-RAS model is not able to model diverted flow over the levee crest. The overtopped volume was estimated from water stage using the weir equation, but the overtopped volume was not removed from the flow. This approximation is acceptable because the volume diverted is minimal compared to the hydrograph volume. For future developments, 2D hydraulic models may be used, to fully account for backwater effects in meandering rivers, and to improve the representation of the overtopped volume. The hydraulic model did not include the possibility of sediment transport during flood episodes. The dynamics of sediment erosion and deposition may further alter the stage-discharge relationship and the hysteresis behaviour that determine the overtopped volume. Some cases may require the use of a 2D model with sediment transport to characterise the resistance factor produced by the river each accurately. However, a large number of the required simulations (5000 hydrographs) would imply long computational times for the complete analysis. The failure criterion used in this analysis was chosen for its simplicity. The actual implementation is also based on simplifications: the use of weir equation to estimate laterally overtopped flow, and the assumption of steady-state of it, the surge only overtopping mechanism, neglecting short-period wave effects. In future developments, other types of overtopping failure criteria may also be assessed to take into account other types of events, like hurricanes or coastal flooding, starting from the empirical approach proposed by Hughes (2008a) and CIRIA (2013).

3.2 Estimation of the Bivariate Failure Return Period

The methodology is applied to both case studies previous presented (Poggio a Caiano and Capannoli). The hydrological failure lines were determined in the previous paragraph, i.e. the COFH, and it useful for implementing the Bivariate Failure Return Period methodology.

3.2.1 Definition of Bivariate distributions

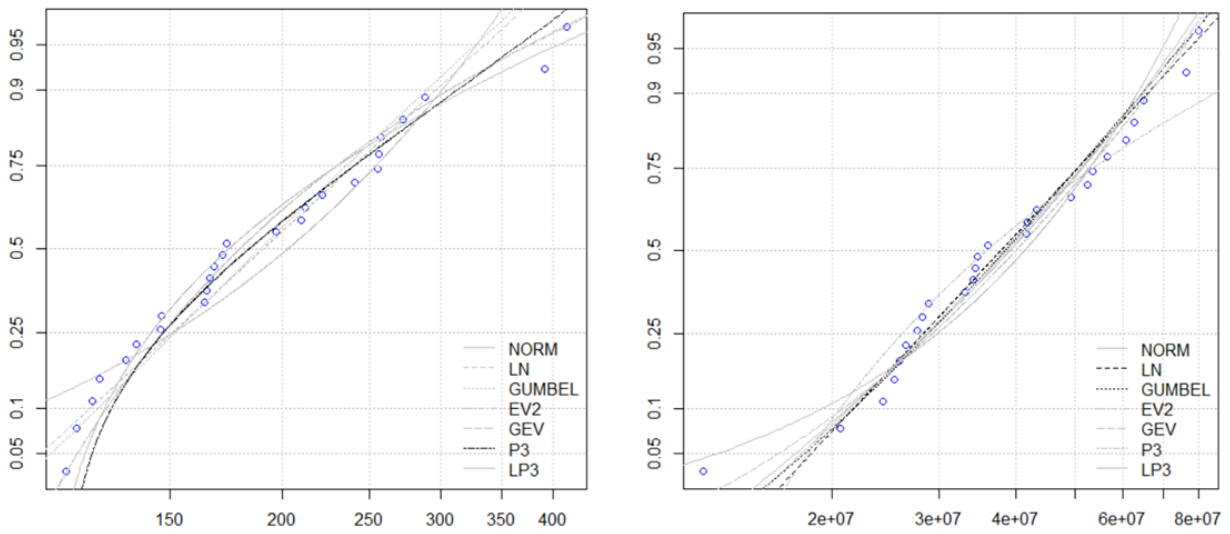
3.2.1.1 Definition of annual maxima series

Tuscany Hydrological Services provide the mean daily discharge series for Poggio a Caiano (1992-2017), and Capannoli (1988-2017). The events are separated using Cunnane separation criterion (1973), and the maximum daily discharge can be transformed into the maximum peaks by Fuller's Formula (Fillard and Steiner 2003). To obtain the annual maxima series of Q and V, firstly the methodology for the estimation of annual maximum hydrographs (par 2.2.1) is implemented. As an illustration, two events recorded at Poggio a Caiano of the year 2014 are selected. One event has a peak of 257 m³/s, the highest in the year, and the second has a peak of 235 m³/s. The traditional method (e.g. Mediero et al., 2010; De Michele et al., 2003) would be taken into account of the first event (257 m³/s) for the annual maxima series of Q, and V. Analysing the corresponding water stage (Eq.35) of the two events with the proposed approach, it results that $H_{235} > H_{257}$. Therefore, for the construction of the annual maxima series of Q and V, the event with a peak of 235 m³/s is chosen because it causes a higher water stage than the event with 257 m³/s. This result comes out from a different distribution of the volume in the time that causes different Δh (Eq.33) of the hysteresis loop. The resulting annual maxima series consists of 26 pairs of Q and V for Poggio a Caiano, and 30 pairs for Capannoli.

3.2.1.2 Selection of marginals distribution

The first step is to determine the univariate marginal distributions for Q and V. Different probability models are taken into account because they are the most used in hydrological applications: Normal, Log-Normal, Gumbel, Fréchet (EV2), Generalized Extreme Values (GEV), Pearson type III (P3), and Log-Pearson type III (LP3). Four different model selection criteria are considered (Laio et al. 2009): the Akaike Information Criterion (AIC), the Akaike Information Criterion with a correction for small sample sizes (AICc), the Bayesian Information Criterion (BIC), and the Anderson-Darling Criterion (ADC). The best-fitting distribution function is selected by the minimum values of AIC, AICc, BIC, and ADC. The results are shown in Fig. 18 and Fig. 19: (i) for the case study of Poggio a Caiano the P3 distribution for Q ($\mu = 115.14$; $\sigma = 72.12$; $\xi = 1.26$) and two distributions show best performance for volume. The Gumbel distribution ($\mu = 33276618$; $\sigma = 13907763$) is chosen because it presents the minimum values for AIC, AICc, BIC. (ii) The EV2 ($\mu = 4.212$; $\sigma = 0.385$) for Q, and the Lognormal distribution ($\mu = 16.590$; $\sigma = 0.618$) for V are detected for Capannoli.

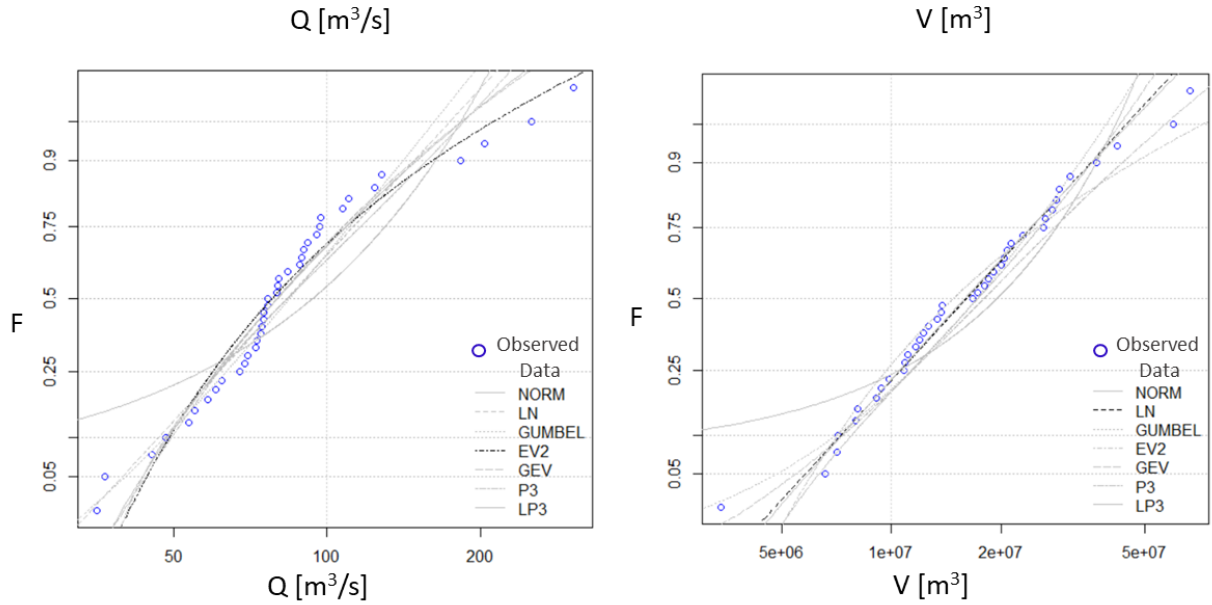
Poggio a Caiano



Poggio a Caiano									
Distribution of Q					Distribution of V				
Probability Distribution	AIC	AICc	BIC	ADC	Probability Distribution	AIC	AICc	BIC	ADC
Normal	303.4	303.9	305.9	0.684	Normal	944.2	944.8	946.7	0.318
LogNormal	295.4	295.9	297.9	0.155	LogNormal	941.7	942.2	944.2	0.0757
Gumbel	295.3	295.9	297.9	0.182	Gumbel	941.4	941.9	943.9	0.088
EV2	293.4	293.9	295.9	0.088	EV2	948.1	948.6	950.6	0.359
GEV	295.4	296.5	299.2	0.133	GEV	943.8	944.9	947.6	0.173
P3	291.8	292.8	295.5	0.05	P3	943.1	944.2	946.9	0.109
LP3	294.1	295.2	297.9	0.125	LP3	943.1	944.2	946.9	0.139

Fig. 18. Case study of Poggio a Caiano. The plots show the data (Weibull plotting position), and candidate distributions in lognormal probability plot for Q and V. The distributions selected by at least one criterion (AIC, AICc, BIC, ADC), are plotted in black, and in bold in the table, the others are in grey.

Capannoli



CAPANNOLI									
Distribution of Q					Distribution of V				
Probability Distribution	AIC	AICc	BIC	ADC	Probability Distribution	AIC	AICc	BIC	ADC
Normal	426.5	426.8	429.8	5.1012	Normal	1394	1395	1398	1.96272
LogNormal	397.2	397.5	400.5	0.8364	LogNormal	1371	1372	1375	0.045
Gumbel	400	400.3	403.3	1.0043	Gumbel	1376	1376	1379	0.28268
EV2	394.5	394.8	397.8	0.508	EV2	1377	1377	1380	0.23619
GEV	395.1	395.7	400	0.562	GEV	1376	1376	1381	0.9077
P3	397.7	398.4	402.7	1.0462	P3	1371	1372	1376	0.17583
LP3	395.8	396.5	400.8	0.6357	LP3	1373	1374	1378	0.05081

Fig. 19. Case study of Capannoli. The plots show the data (Weibull plotting position), and c , candidate distributions in lognormal probability plot for Q and V . The distributions selected by at least one criterion (AIC, AICc, BIC, ADC), are plotted in black, the others are in grey. In the table in bold the selected distributions are underlined.

3.2.1.3 Copula model selection

The next step consists of studying the dependence between Q and V . The scatter plot of the observed data is displayed in Fig. 20 on the left. In both case studies, the scatter plot $(R_i/(n+1); S_i/(n+1))$ leads to a positive dependence between variables. The χ -plot (Fig. 20, in the middle) and the K-plot (Fig. 20, on the right) confirm the dependence between data for both case studies. In the former, most values are located above the upper limit indicating positive dependence ($\chi_i > 0$). In the latter, the points are plotted over the diagonal line, so positive interaction is also drawn. Further analysis of the χ -plot of the case study of Capannoli leads to an upper tail dependence. The majority of the points are located on the right edge of the plot and tend to be far from the zero value of the y-axis, which is the independence hypothesis. Further tests must be implemented to confirm these preliminary statements.

Case study

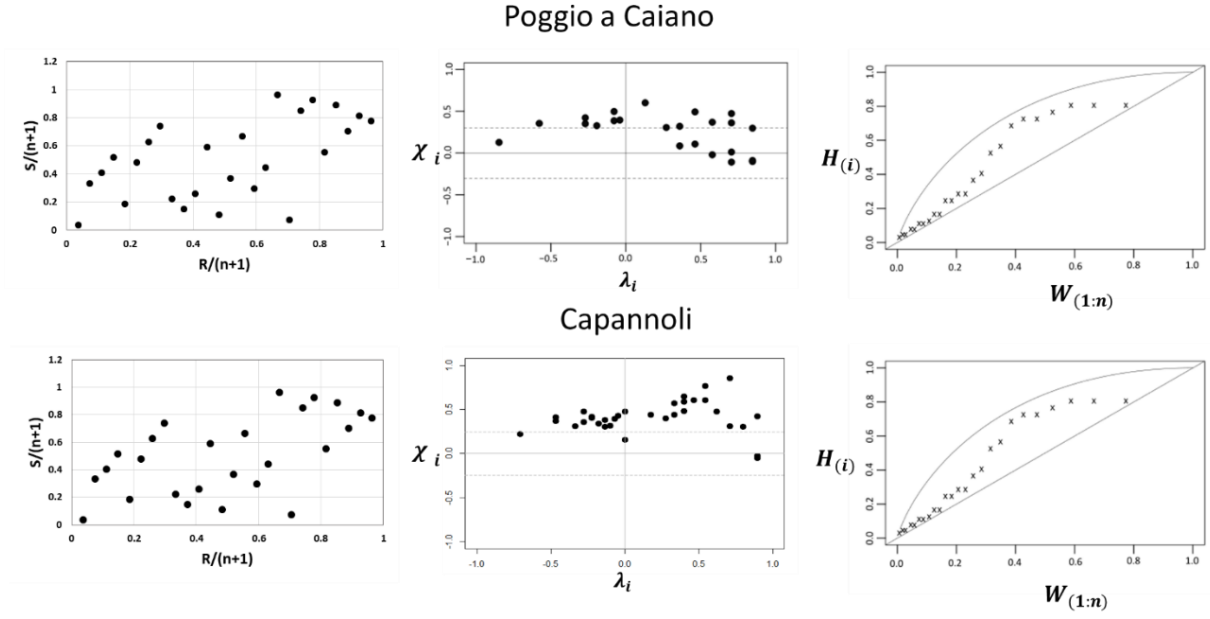


Fig. 20. From left to right: the scatter plot of the data, and the corresponding χ -plot, and K-plot for both case studies.

The values of the Spearman's rho (ρ) and Kendall's tau (τ) rank-based nonparametric measures of dependence corroborate the results provided by the graphical information. The values of each dependence measures, as well as its linked p -value, are summarised in Tab. 4.

Poggio a Caiano		Capannoli	
Spearman		Spearman	
ρ	p-value	ρ	p-value
0.561	0.0033	0.655	0.0024
Kendall		Kendall	
τ	p-value	τ	p-value
0.3785	0.0062	0.4899	0.0041

Tab. 4. The Kendall (τ) and the Spearman (ρ) dependence measures for both case studies.

Since a preliminary measure of the ordinal association between Q and V is verified for both case studies, next steps focus on the detection of the best copula functions among the copula families used in this study (Tab. 1). The parameters of copulas (θ) are estimated using the maximum pseudo-likelihood method (Tab. 5), see for more details Serinaldi et al. (2007). When dealing with bivariate data, the most natural way of a preliminary checking the adequacy of a copula model is to compare a scatter plot of the pairs $(R_i/(n+1)-S_i/(n+1))$ derived from the observed data (i.e., the support of the empirical copula C_n) with a synthetic dataset generated from parametric family of copulas (C_θ). Considering the selected copulas (Gaussian, Student's t, Frank, Clayton, Gumbel, Galambos and Husler-Reis), a synthetic sample of pairs (u_1, u_2) is generated from each ones, and it is compared to the pairs related to the observations $(R_i/(n+1); S_i/(n+1))$ (in the appendix). Other plot consists of benchmarking the Q_i , and V_i , deduced by $u_{1,i}, u_{2,i}$ (Eq.36), with the observed pairs of Q , and V (Fig. A1, Fig. A2, Fig. A3, Fig. A4 in the

Case study

appendix A3). The information provided by the plots is not sufficient for filtering on the list of copula families. It should be highlighted that in the case study of Capannoli the Normal, the Frank, and the Clayton copulas seem not to be suitable for the data because the synthetic sample pairs are too sparse. The same consideration can be carried out for the Clayton, Frank, and Gumbel copulas for the Poggio a Caiano case study. However, the goodness of fit tests are implemented to assess these preliminary results.

The performance of various copula families is made by comparing the outcomes of the goodness of fit tests of AIC_{cop} (), BIC_{cop} (Eq.38), S_n (Eq.39), and the corresponding p-value of the Cramer Von Mises test is evaluated.

In Tab. 5 the results of the tests are shown, in bold the best copula are indicated: the Gaussian, and the Galambos copulas for the case study of Poggio a Caiano; the Gumbel and the Galambos copulas for Capannoli. Therefore, these tests do not provide obviousness for choosing the best copula function. Anyway, the aim of the study is the assessment of the risk for river hydraulic structures that correspond to high return period for the hydrological load. The best copula is not only that represents better the whole dataset but also that model better the behaviour at high return period. A tail dependence assessment is developed for the copulas resulting in the best by the goodness of fit tests. The upper tail coefficient for each copulas (Eq.42) is compared with its estimator (Eq.43). In the Tab. 6 the resulted are reported. Only the extreme value copulas show upper tail dependence. Indeed, the Gaussian copula for the case study of Poggio a Caiano has a null result, as it has by definition an upper tail dependence of zero. For the evaluation of the Bivariate Failure Return Period, the Galambos copula is chosen because it takes into account the upper tail dependence, and, at the same time, model the dependence structure between Q and V. By same consideration, the Gumbel copula is chosen respect the Galambos copula for the case study of Capannoli. Indeed, both Gumbel and Galambos are extreme values copulas but the difference between λ_u^c , and $\widehat{\lambda_u^{cvg}}$ provided a better upper tail dependence in Gumbel copula (Tab. 6).

Tab. 5. For both the case studies of Poggio a Caiano and Capannoli. The results for the goodness of fit test (AIC_{cop} , BIC_{cop} , S_n and the corresponding p-values of S_n) for each copula families is shown. In bold the copula that passes at least one test. The Gaussian copula and the Galambos (Poggio a Caiano), and the Gumbel copula (Capannoli) are the best

Poggio a Caiano						Capannoli					
Copula	θ	AIC	BIC	S_n	p-value	Copula	θ	AIC	BIC	S_n	p-value
Gaussian	0.630	-8.09	-6.83	0.0257	0.614	Gaussian	0.7550	-26.52	-24.86	0.0286	0.2458
Student's t	0.620 ; 30	-5.91	-3.39	0.0265	0.5793	Student's t	0.754 ; 30	-24.31	-20.990	0.0286	0.249
Clayton	1.054	-5.79	-4.54	0.0554	0.1085	Clayton	0.464	-19.55	-17.88	0.0672	0.2849
Gumbel	1.623	-6.59	-5.33	0.0291	0.5181	Gumbel	2.045	-27.62	-24.96	0.023	0.3421
Frank	3.981	-6.95	-5.7	0.0321	0.5665	Frank	5.289	-19.88	-18.210	0.0446	0.0445
Galambos	0.927	-7.81	-6.91	0.0255	0.6160	Galambos	1.353	-28.610	-23.970	0.019	0.287
Husler-Reis	1.436	-6.95	-5.7	0.0260	0.6283	Husler-Reis	1.962	-16.699	-15.296	0.037	0.037

Case study

Tab. 6. Upper tail dependence coefficients of the copulas that pass the goodness of fit tests.

Poggio a Caiano				Capannoli			
Copula	θ	λ_u^c	λ_u^{CFG}	Copula	θ	λ_u^c	λ_u^{CFG}
Gaussian	0.63	0	0	Gumbel	2.045	0.94	0.95
Galambos	0.927	1.180	1.22	Galambos	1.35	1.02	0.95

In summary, the Galambos copula with $\theta=0.927$ is implemented for model bivariate distribution for the case study of Poggio a Caiano, the Gumbel copula with $\theta=2.045$ is detected for the case study of Capannoli. In Fig. 21 the bivariate distributions for both case studies are shown.

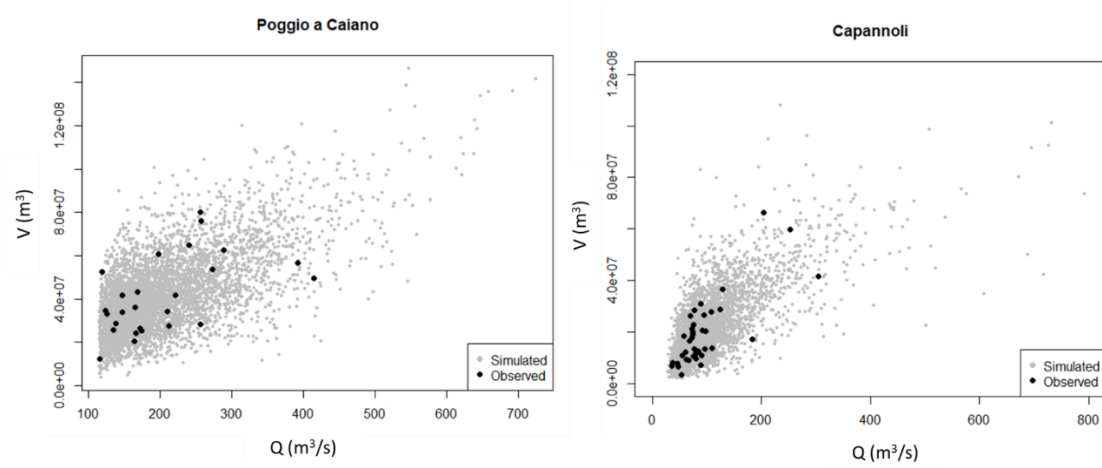


Fig. 21. Scatter plots of the bivariate distribution model used in the case studies: on the left the case study of Poggio a Caiano modelled with Galambos copula; on the right the case study of Capannoli modelled with Gumbel copula. The grey dots are simulated data, the black ones are the observed data.

3.2.2 Results and discussions

For both case studies, the design Life – DL for the river levee is fixed ad 50 years according to the Italian National Design Codes. The Monte Carlo procedure (par. 2.2.2) is implemented. By the copula functions, the synthetic pairs of Q-V are generated verifying in which side respect the hydrological failure line fall (safe zone or failure zone). The Failure Matrices for each time frame are determined (FM_i , $i = 1, 2, \dots, DL$), each of which has number of rows equal to $10 \cdot DL = 500$, and i columns. The corresponding amount of failure scenarios ($\mu_{fail,i}$), represented by the number of rows with at least one Q-V pair falling in the failure zone, are counted (Tab. 7).

Case study

Tab. 7. For each case study, the number of failure scenarios for each failure matrix (FM_i) are indicated.

Number of failure scenarios H_{fail}														
Failure Matrix	Poggio a Caiano	Capannoli	Failure Matrix	Poggio a Caiano	Capannoli	Failure Matrix	Poggio a Caiano	Capannoli	Failure Matrix	Poggio a Caiano	Capannoli	Failure Matrix	Poggio a Caiano	Capannoli
FM ₁	11	15	FM ₁₁	103	198	FM ₂₁	177	300	FM ₃₁	234	382	FM ₄₁	287	423
FM ₂	21	36	FM ₁₂	113	202	FM ₂₂	187	317	FM ₃₂	240	383	FM ₄₂	287	425
FM ₃	32	71	FM ₁₃	117	214	FM ₂₃	195	308	FM ₃₃	243	379	FM ₄₃	294	432
FM ₄	42	81	FM ₁₄	126	215	FM ₂₄	195	314	FM ₃₄	252	363	FM ₄₄	299	424
FM ₅	51	90	FM ₁₅	135	232	FM ₂₅	199	333	FM ₃₅	263	399	FM ₄₅	300	433
FM ₆	58	95	FM ₁₆	143	249	FM ₂₆	203	333	FM ₃₆	261	396	FM ₄₆	310	434
FM ₇	69	127	FM ₁₇	151	280	FM ₂₇	213	365	FM ₃₇	263	397	FM ₄₇	312	435
FM ₈	79	158	FM ₁₈	155	276	FM ₂₈	217	355	FM ₃₈	276	418	FM ₄₈	318	435
FM ₉	81	158	FM ₁₉	167	270	FM ₂₉	230	352	FM ₃₉	279	422	FM ₄₉	323	443
FM ₁₀	93	187	FM ₂₀	166	302	FM ₃₀	233	375	FM ₄₀	282	406	FM ₅₀	322	433

The failure probability (PF_i) for each FM_i is estimate using Eq.45 (Tab. 8):

Tab. 8. For both case studies, the failure probability for the corresponding i-time frame that composed the design life ($i=1,2,3\dots, DL$)

Probability of failure	Poggio a Caiano	Capannoli	Probability of failure	Poggio a Caiano	Capannoli	Probability of failure	Poggio a Caiano	Capannoli	Probability of failure	Poggio a Caiano	Capannoli	Probability of failure	Poggio a Caiano	Capannoli
PF ₁	0.034	0.05	PF ₁₁	0.240	0.462	PF ₂₁	0.398	0.682	PF ₃₁	0.518	0.856	PF ₄₁	0.630	0.944
PF ₂	0.058	0.102	PF ₁₂	0.262	0.472	PF ₂₂	0.420	0.718	PF ₃₂	0.532	0.858	PF ₄₂	0.630	0.948
PF ₃	0.084	0.182	PF ₁₃	0.270	0.498	PF ₂₃	0.436	0.7	PF ₃₃	0.538	0.85	PF ₄₃	0.644	0.962
PF ₄	0.106	0.206	PF ₁₄	0.290	0.5	PF ₂₄	0.436	0.712	PF ₃₄	0.556	0.816	PF ₄₄	0.656	0.946
PF ₅	0.126	0.226	PF ₁₅	0.308	0.536	PF ₂₅	0.444	0.752	PF ₃₅	0.580	0.892	PF ₄₅	0.658	0.964
PF ₆	0.142	0.236	PF ₁₆	0.326	0.572	PF ₂₆	0.454	0.752	PF ₃₆	0.576	0.886	PF ₄₆	0.678	0.966
PF ₇	0.166	0.308	PF ₁₇	0.342	0.64	PF ₂₇	0.474	0.82	PF ₃₇	0.580	0.888	PF ₄₇	0.682	0.968
PF ₈	0.188	0.376	PF ₁₈	0.352	0.63	PF ₂₈	0.484	0.8	PF ₃₈	0.608	0.932	PF ₄₈	0.696	0.968
PF ₉	0.192	0.376	PF ₁₉	0.378	0.618	PF ₂₉	0.510	0.792	PF ₃₉	0.614	0.942	PF ₄₉	0.706	0.986
PF ₁₀	0.218	0.44	PF ₂₀	0.374	0.686	PF ₃₀	0.516	0.842	PF ₄₀	0.620	0.908	PF ₅₀	0.704	0.964

The probability failure curve $R_f(l)$ is obtained plotting the PF_i values respect the corresponding time frame and interpolating these values. The Bivariate Failure Return Period is the return period that, replace in the equation Eq.32, approximates the probability failure curve $R_f(l)$ for both case studies, the failure probability curves obtained through interpolation of the PF_i values (Fig. 22). The BFRP respect the overtopping failure estimated through the estimated COFH is (i) a return period of forty years for the case study of Poggio a Caiano, and (ii) a return period of fifteen years for the case study of Capannoli.

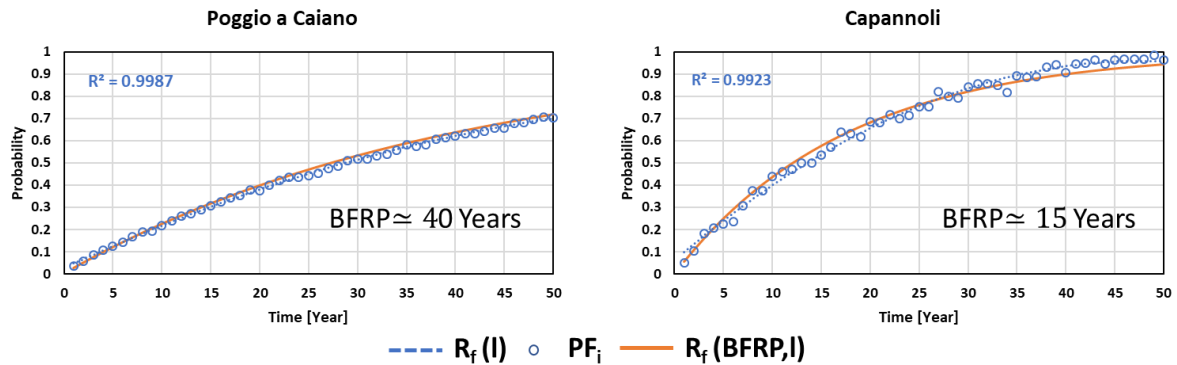


Fig. 22. For both case studies. The dot points represent the PF_i evaluated through the Monte Carlo simulation. The blue dashed line is the failure probability curve obtained through interpolation of the PF_i . The orange line represents the failure probability curve $R_f(BFRP, l)$ evaluated through equation Eq.31

In both cases, BFRP allows an estimate of the return period in a bivariate field starting from the definition of the failure curve. The evaluation of BFRP is based on concept, and empirical formula widely known and applied in the standard practice of risk assessment, and design. To prove the efficacy, the BFRP is compared with bivariate return periods existing in the literature. The Hydrological-based return period (par. 1.2.1), such as the “OR”, “AND”, considers the probability of occurrence of the hydrological load regardless of the hydraulic structure with which it interacts. As underlined by many authors (e.g. Salvadori et al. 2016; Serinaldi 2015; Volpi and Fiori 2014), the probability of occurrence of the hydrological load is normally different from the failure probability of the hydraulic structure, and the corresponding return period can underestimate or overestimate the return period of failure of the structure. The failure risk for a hydraulic structure (Requena et al. 2016; Volpi 2019b) must take into account the particular failure mechanism associating to the particular hydraulic structure, as BFRP does. According to these considerations, the BFRP is compared with the structural return period (par. 1.2.2): the Structure Return Period -SRP (Volpi and Fiori 2014), and Routed Return Period- RRP (Requena et al. 2013). The SRP estimates the critical variable for the structure by the interaction of the bivariate hydrological load and a structure-function. The return period is determined by a univariate probabilistic analysis of the determined of the critical variable. The procedure consists in moving from the multivariate distribution of the Q-V to that of the univariate of the critical variable Z for the specific mechanism of failure (e.g. water stage in the reservoir). This approach is known as the transformation of two random variables (Papoulis and Pillai 2002). In river hydraulic structures, this approach can be ineffective for several issues. Indeed, in the case study of Poggio a Caiano not all the hydrographs cause the critical overtopping variable (i.e. overtopping volume). Many hydrographs, during the routing process, do not raise the limit water stage in the river, and if also raised they could not cause the overtopping damages because in Poggio a Caiano the failure mechanism depends by exceeding the threshold for the overtopping volume (Damme et al. 2016). This circumstance can condition the probability analysis of the critical variable of failure mechanism because many hydrographs cause a null value of the critical variable, so it altered the evaluation of the Structural Return Period. If we consider the overtopping mechanism of the case study of Capannoli the SRP approach raises another issue. The dimensionality reduction (from bivariate to univariate) at the base of the SRP approach does not verify. In the case of Capannoli, the analysis starts from a bivariate problem (hydrological load) transferring to another bivariate problem (the resistance criterion). The resistance criteria depends on a function of overtopping velocity and overtopping duration (Hewlett et al. 1987). The BFRP avoid this issue because the estimation of the hydrological failure curve (i.e. the COFH) and the estimation the probability of this failure are each other independent. Indeed, the hydrological failure curve takes into account the possible multivariate nature of the failure mechanism.

Otherwise, the Routed Return Period – RRP starts from the failure curve as the curve in Q-V space in which all the Q-V pairs cause the initiation of the failure mechanism. The RRP depends on the probability evaluated as the ratio between the number of Q-V pairs above the

failure curve respect the total number. Although RRP and BFRP assume a failure curve, the return periods estimated with the two methods are different. The BFRP for Poggio a Caiano case is about 40 years while the RRP is 60 years. The BFRP for Capannoli case is about 15 years while the RRP is 30 years. This difference is due because the RRP approach assesses the probability in its entirety without taking into account the possible succession of events that may occur during the design life of the structure. On the contrary, they are taken into account by the BFRP approach building of the failure matrices by the Monte Carlo simulation.

Broadening the benchmark on multivariate return period existing (OR, AND, SRP, etc.), Serinaldi (2015) highlights that they are not interchangeable because the underlying probabilities are not interchangeable. The risk of failure in the design life period has a unique and general definition that can fit every situation. The BFRP approach represents an improvement respect this issue because assessing the failure probability in the design life of the structure

4 Bivariate hydrological flood damage estimation.

The flood damage estimation is a crucial step for flood risk management purposes, answering the requirements of the European Flood Directive 60/2007/EC and UNISDR (2015). Models and methodologies to estimate economic damages are vital for evaluating and comparing flood mitigation measures, and for defining flood risk management plans (Kreibich et al. 2016). The literature mainly focuses on the relevant issues in the application of flood damage models, following explained. Most flood damage models are based on data resulting from a small number of events (Cammerer et al. 2013; Jongman et al. 2012; Smith 1994). Because flooding can be characterized by different factors (water depths, contamination, flood duration, etc.), and in many different types of areas (building types, historical monuments, crops, and, etc.) any model might miss considerable parts of the spectrum of possible options (Wagenaar et al. 2016). Indeed, damage data from one event are often not transferrable to other areas or events (Ballio et al. 2015). Since only damage data representative of the event under consideration can be used, few data are available, and hence significant uncertainties are introduced in flood damage modelling (Molinari et al. 2014).

Furthermore, the models often assume that the potential damage is constant throughout the year, while this does not hold for agricultural land, where crop value varies depending on crop maturity (Amadio et al. 2016). Also, most of the damage models are not considered social factors like people's gender. For example, women, elders, and children, worldwide are more exposed than men to disaster risk, rates of mortality, significant damage to their livelihoods (Sohrabizadeh et al. 2014).

The recent literature, previously mentioned, proves the interest and the importance of this topic. Although, as well as the literature for the overtopping risk for a river levee, the flood damage analysis of the literature mainly focuses on the variables that conditionate the damage models (Dottori et al. 2016) without considering the hydrological load. The flood damage estimation usually consists in (e.g. Arrighi et al. 2013; Scorzini and Frank 2015): (i) considering a univariate hydrological load with a certain probability, e.g. hydrograph $Tr=200$; (ii) routing the hydrological load into 2d hydraulic model; (iii) estimation of the flood hydraulic variable, i.e. water stage; (iv) estimation of flood damage by a stage-damage curve for the exposed part. This standard approach provides a perfect correspondence between the probability of occurrence of the hydrological load is the same as the flood damage.

In previous chapters, it is highlighted how to consider the hydrological load related to river flood risk as univariate is not the proper way because the risk depends on the water stage that is influenced by all the variable of the hydrograph. In literature rarely are the studies on the use of a bivariate hydrological load in flood damage estimation.

The study aims to demonstrate the flexibility and the validity of the methodology proposed and tested for the overtopping risk. Hydrological load has to be considered as bivariate in the flood damage estimation, not only the peak of discharge but also the volume of the

hydrograph is a crucial variable. The methodology proposed for the assessment of the overtopping risk for a river levee is used, adapting it to the flood damage estimation. Testing to other types of risk, i.e. the flood damage, the flexibility of the proposed procedure is demonstrated.

4.1 Methodology

The methodology proposed (chapt. 2) is adapted to this type of risk. The damage models replace the levee resistance criterion, and the hydraulic conveyance consists of a 1d-2d hydraulic model, 1d for the channel 2d for the flood area.

Firstly, the probability of occurrence of a monetary damage threshold of 20% respect to the total real estate value of the studied area is established. The defined threshold is not completely rigorous, but it is useful to achieve the defined aims. Indeed, one of the aims is to demonstrate the flexibility and the validity of the methodology proposed (chapt. 2) applying to other type of river-related flood risk. At this moment, it is not necessary to stress in the damage models (stage-damage curve). In the overtopping cases (chapt. 3) the critical variables are the overtopping velocity etc., here the critical variable for the damage is the flood stage because stage-damage curves are the core of the flood damage model (Smith 1994).

Similarly to the COFH (2.1.4), the result of this study is the critical curve in Q-V space in which all the hydrographs cause the same monetary damage for the study area. The probability of this failure in terms of BFRP is also assessed.

The damage model process is explained in the following paragraph.

4.1.1 Damage modelling

The traditional approach for assessing economic damage is the use of stage-damage curves (Merz et al. 2004), which represent the vulnerability of exposed buildings for a given flood stage. In other words, the damage curves establish the link between the damage induced to a building, expressed as a percentage of the exposed value, and the flood stage.

The steps that allow determining the economic value of damage are:

- Definition of the flood depth at each building (h_i).
- Definition of the damage percentage for each building ($D_{i\%}$).

$$D_{i\%} = d_i(h_i) \quad \text{Eq.46}$$

where:

d_i : stage-damage curve for i-th building [%].

- Definition of the economic value of damage for each building ($D_{i\epsilon}$).

$$D_{i\epsilon} = D_{i\%} \cdot N_i \cdot A_i \cdot E_i \cdot \alpha_i \quad \text{Eq.47}$$

where:

N_i : number of floors in the i -th building [-];

A_i : footprint of the i -th building [m^2];

E_i : market value of the i -th building [$\text{€}/\text{m}^2$];

α_i : ratio between recovery cost, and market value [-].

- Definition of economic damage in the flooded area ($D_{tot \text{ €}}$)

$$D_{tot \text{ €}} = \sum_{i=1}^m D_{i\epsilon} \quad \text{Eq.48}$$

Where m is the number of buildings in the flooded area.

4.1.2 Case study

The flood damage estimation considering a bivariate hydrological load is applied to the case study of the municipality of Poggio a Caiano previous presented.

In the Poggio a Caiano municipality there are 365 buildings for residential and industrial use; 92% of buildings have a residential use while 8% are single-floor industrial buildings (Fig. 23, on top). For the residential structures the following subcategories are present (Fig. 23, at the bottom): (i) two-floor buildings without cellar; (ii) two-floor buildings with cellar; (iii) three-floor buildings without cellar; (iv) three-floor buildings with cellar; (v) four-floor buildings without a cellar. The subcategories are detected by field campaigns and data provided by the Tuscany Region.

Bivariate hydrological flood damage estimation.

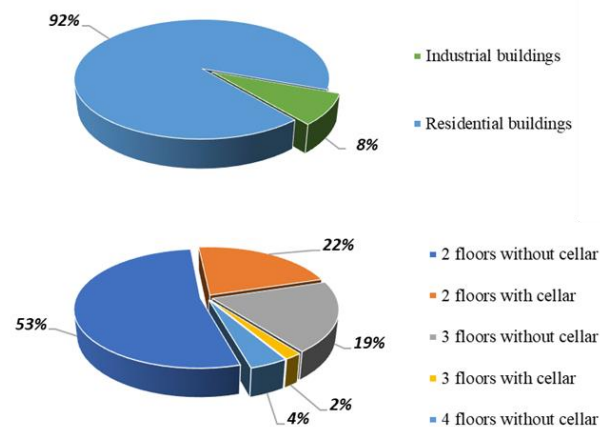


Fig. 23. On the top, the percentage breakdown of buildings, present in the study area, based on the intended use. At the bottom, the percentage breakdown of residential buildings based on their characteristics (number of floors, with cellar/without cellar).

Five thousand of synthetic hydrographs are generated, considering the linear relationship between Q and V , and the OHSI index for the shape selection (para. 2.2). Each hydrograph is routed into a combined 1D / 2D hydrodynamic model:

- one-dimensional (1D) for the Ombrone Channel;
- two-dimensional (2D), in the urban area with cells 1x1m.

The connection between channel and flooded area is modelled with a lateral structure, as advised by USACE (2015). The levees in the area are assumed stable for each scenario avoiding eventual piping or overtopping failure. This simplification is used because the aim is to demonstrate that the volume of the hydrograph is also an essential variable in the flood damage estimation, and the failure of overtopping would complicate the model process. Furthermore, the traditional flood damage estimation process considers the levee stable (USACE 1989), and the only chance of the traditional approach in this study is to implement the bivariate hydrological load. For each synthetic hydrograph, one flood map is obtained. As an illustration, two flood maps corresponding to two different hydrographs with the same peak, and different volumes are shown (Fig. 24). From this figure, it is possible to assess that the volume is an important variable. Indeed, the hydrograph with higher volume (Fig. 24, on the left) causes a larger flood area.

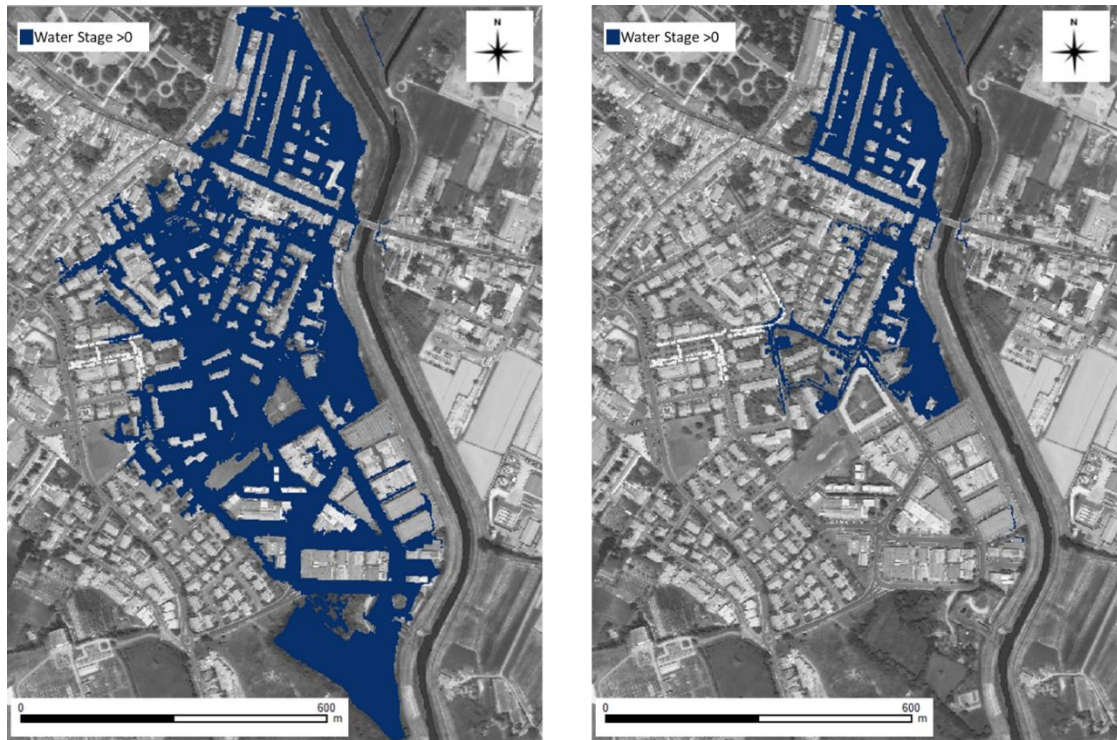


Fig. 24. Two flooded area corresponding to two synthetic hydrographs are shown. The two hydrographs have the same discharge peak but different volume (higher for the left flooded area).

A stage-damage curve must be implemented to evaluate the damage. In this area, and more in general in Tuscany, no stage-damage curves exist; thus a simplify stage-damage curve is created for each type of residential buildings.

The following hypotheses were adopted to define the damage curves for residential structures:

- It is assumed that there is no damage before the water reaches a depth of 0.25 m (Messner et al. 2007); this threshold represents the average elevation of the building entrance door upon the road level. This threshold a simplifying assumption because most buildings are not waterproof, and could be severely damaged even with a lower flood depth, depending on the efficiency of the local sewage system.
- As done previous work (e.g. Elmer et al. 2010), the damage grows linearly with water depth. The curves have higher slopes at the height of the floors. This choice is due to the hypothesis that floodwater against building causes severe damage to the floor up to a height threshold after which it is, however, necessary to proceed with a total renovation of the flooded floor.

As regards the damage curve relating to industrial buildings, the curve proposed by Huizinga et al. (2017) is adopted.

Bivariate hydrological flood damage estimation.

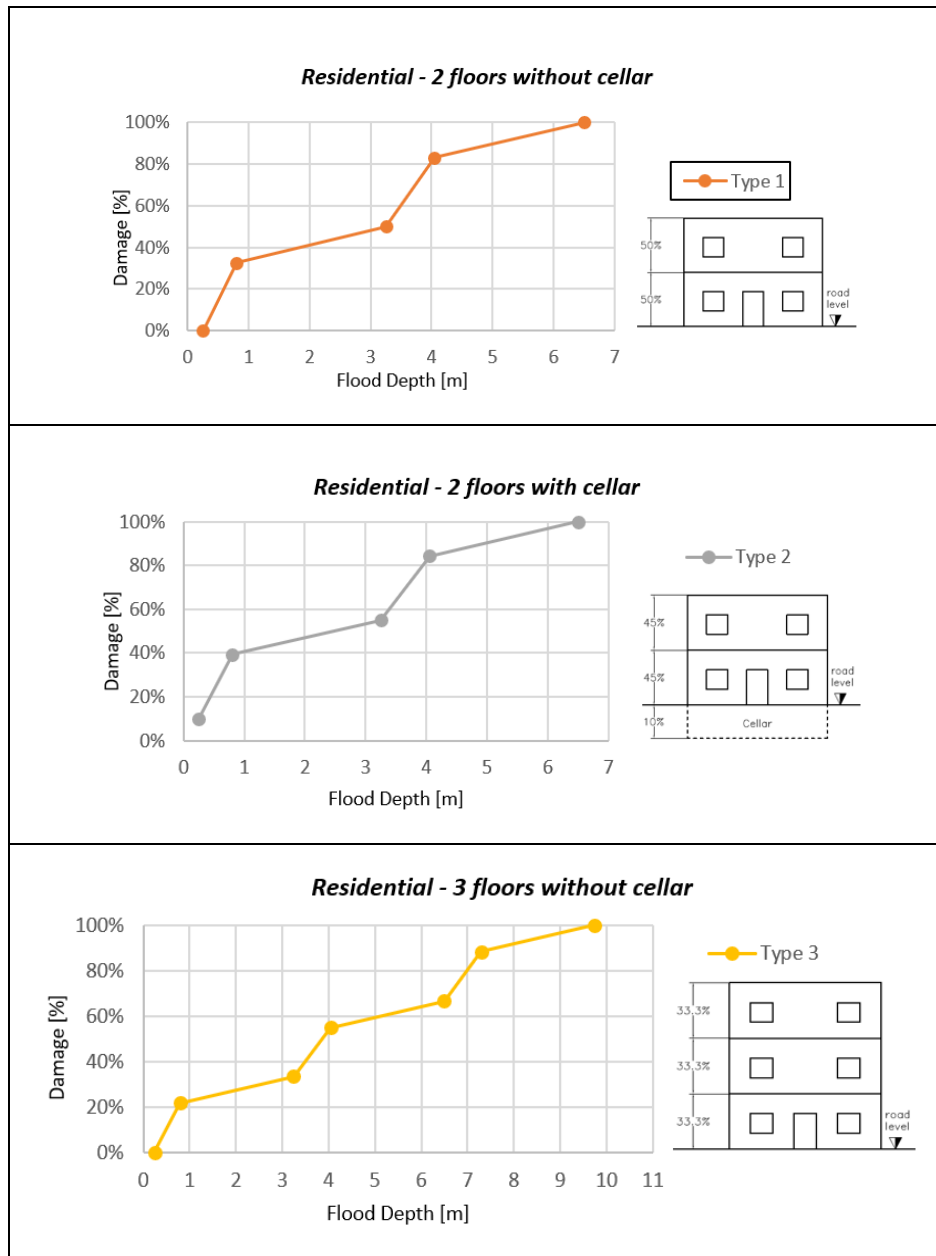


Fig. 25. Reference schemes for the vertical value distribution and structural stage–damage relationships for buildings.

Bivariate hydrological flood damage estimation.

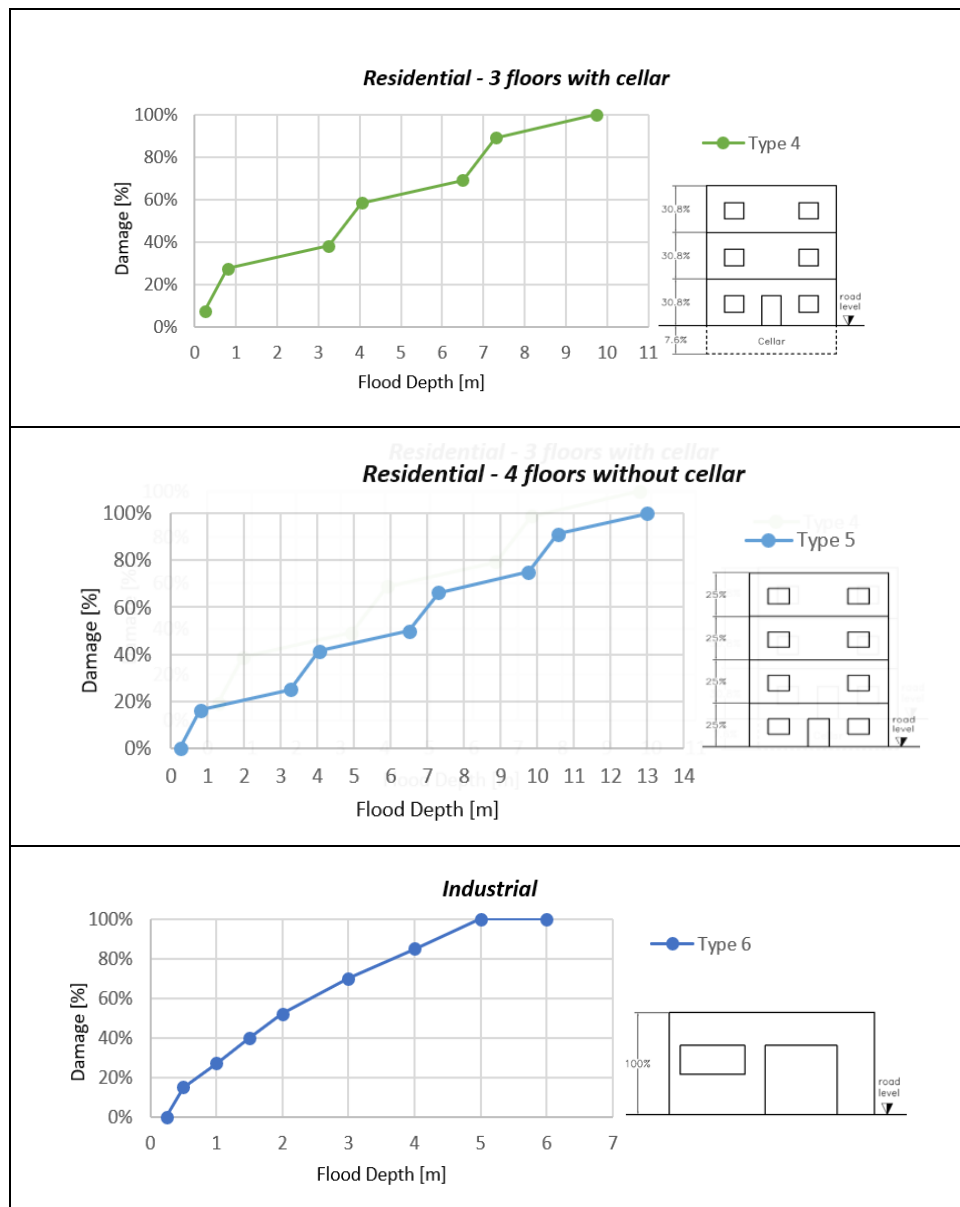


Fig. 26. Reference schemes for the vertical value distribution and structural stage–damage relationships for buildings.

The flood depth is obtained from the hydraulic simulation; the average flood depth at the perimeter for each building is computed through GIS tools. This value represents the independent variable in the stage–damage relationship (Fig. 25Fig. 26). After stage–damage function application, per cent losses can be transformed into economic losses using monetary value for each class. The recovery cost is used to estimate the cost of damages to buildings (Messner et al. 2007). Depending on the maintenance status of buildings, the damage caused by the floods can, in theory, vary from minimal destruction to a complete one. In the considered case study, where most of the real estate are well-maintained masonry buildings, we assume that after a flood a typical building will require a prominent renovation (e.g. restoration of electrical, and heating plants, building frontage, floors). From real estate appraisals, the ratio between the market value of a flat that needs a deep renovation, and a perfect condition flat is

supposed about 0.2 (personal communication with constructions companies); therefore, the value of the alpha parameter (Eq.47) is assumed to be 0.2. The market value of the buildings is estimated to be 2075 €/m² for residential buildings, and 625 €/m² for industrial buildings (OMI, 2019). The other parameters present in equation Eq.47 (N_i , A_i) are obtained through on-site inspections. The economic damage to buildings for each synthetic event is evaluated using the stage-damage curve.

4.2 Results and Discussions

Each hydrograph is routed in the hydraulic model, and the total damage (Eq.48) in the study area is evaluated for each flood map. The total real estate value of the area is assessed as about 70 millions of euros. The 20% of these values is about 14 millions of euros. As done in paragraph 3.1.5, the hydrograph causing this threshold of damage value is detected, and the critical curve in Q-V space is developed (red line, Fig. 27). The BFRP (chapt. 3.2) is about 25 years. The curve shows that the same damages can be raised by a peak of discharge varying from about 380 m³/s to 490 m³/s, thus a non-negligible range of discharge.

As well the overtopping risk, the critical failure line demonstrates that in the flood damage estimation not only peak but also volume is a relevant variable because the critical line in Q-V space has a slope. Indeed, the more the tendency of the line is horizontal, the more the flood damage is conditioned by the volume.

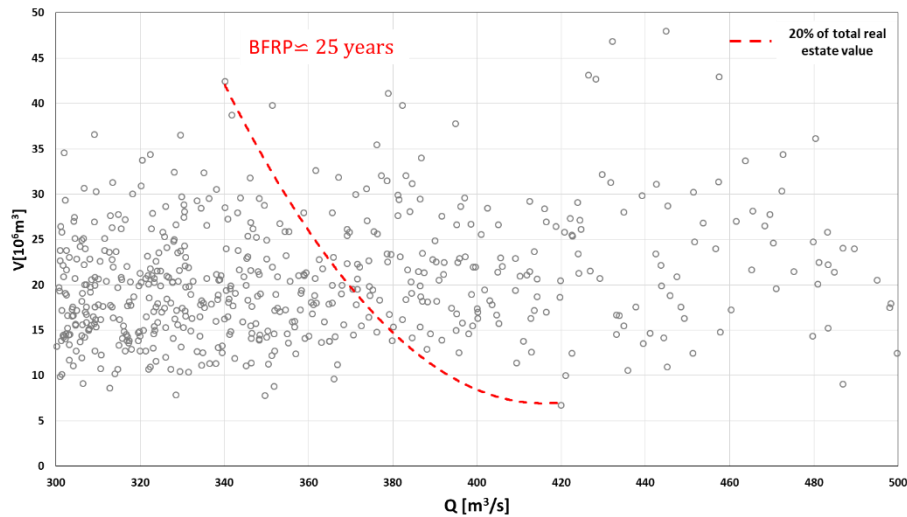


Fig. 27. In this plot, the line in red represents the curve in which all the hydrographs cause the same monetary damage for the building in the municipality area of Poggio a Caiano. The Bivariate Failure Return Period (chapt. 3.2) is evaluated for about 25 years.

Further analysis can be implemented following a traditional approach with the evaluation of the probability of occurrence of the damage corresponding to the return period for 100 years, and 200 years, analysing the damaging series obtaining by each synthetic hydrograph. In this case, to apply the BFRP approach some preprocess needing. Indeed, the BFRP starts from a threshold preliminary defined, e.g. the overtopping failure condition or 20% of the total real estate value. In this case, no preliminary threshold in damage is fixed but only the return period. This issue is avoided with the root-finding algorithm basing on iterations. This algorithm produces a sequence of threshold of damage to which correspond a critical curve in Q-V space and a BFRP. The algorithm is based on the Newton method stopping when the differences between the BFRP belonging to two consecutive iterations is less than 5%.

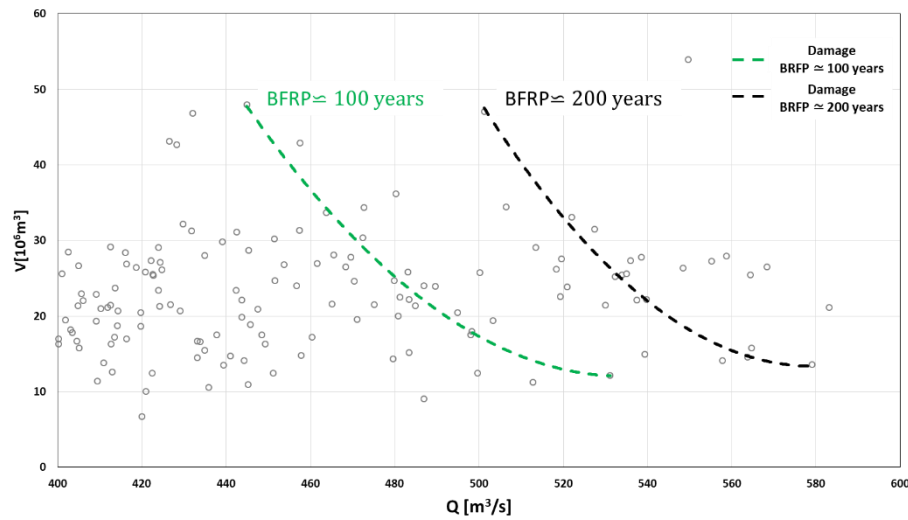


Fig. 28. The estimated damage curves corresponding to the return periods of 100 years and 200 years

In the plot of figure Fig. 28, the damage curves corresponding to the return periods of 100 years, and 200 years are shown. These curves have a similar trend of the damage curve of figure Fig. 17, confirming the importance of the volume of the hydrograph in flood damage estimation. In this chapter, the evaluation of damage in the bivariate environment is estimated. In the literature rarely are the studies that use a bivariate hydrological load for the flood damage estimation. The methodology creates for the overtopping risk analysis for a river levee is apply in this other type of flood risk. In the case study, the damage model is approximated, and more data would need to a more consistent one. This issue is outside the aim of chapter referring to other literature topic. The hydrological damage curves detect in this case study shows how necessary is considering the volume in the evaluation of the flood damage.

5 Multivariate Regional Frequency Analysis: the case of the Tuscany region (Italy)

In chapter 2, the importance to consider a bivariate hydrological load for the risk of failure for hydraulic structures, or more general for flood risk, is underlined.

Several methodologies and applications of the multivariate frequency analysis are applied in different fields of hydrologic engineering, and risk assessment. Among the others, for coastal, and offshore engineering (Salvadori et al. 2015; Wahl et al. 2012), for irrigation, and water allocation design (Li et al. 2016), for drought problems (Li et al. 2017), for hydraulic risk analysis (Goodarzi et al. 2013; Klein et al. 2009).

All these approaches take into account a bivariate hydrological load that consists of a large number of synthetic hydrographs. These hydrographs are generally based on the bivariate distribution of peak of discharge (Q) and volume (V) associated with a shape. In particular, the estimation of the bivariate distribution depends on where the investigation area is. If it is in gauged site, with a sufficient length of recorded data, the Q-V bivariate distribution can be defined by the suitable copula model based on data (Grimaldi and Serinaldi 2006). If the site is ungauged, the necessary Q-V data are detected by rainfall-runoff models, but they are complicated for the spatial, and the temporal heterogeneity of the influencing parameters (such as soil or rainfall) (Bardossy 2009), and could work for a long computational time for the entire region. For more details of the issues of rainfall-runoff models refers to paragraph 1.1.1. In the future perspective of using the hydrological load as bivariate, to improve the flood design and risk analysis, the practitioners can easily, and quickly obtain the synthetic hydrographs. According to these considerations, the regional frequency analysis (Hosking and Wallis 2005) is tested in this chapter for the prediction of the bivariate distribution of Q-V.

Regional frequency analysis uses data from several measuring sites. A "region" is a group of sites, each of which is assumed to have data drawn from the same frequency distribution. More details for regional frequency analysis are in paragraph 1.1.1.

To use bivariate hydrological load, Chebana and Ouarda (2007), and recently Requena et al. (2016) propose a multivariate extension for the index flood method (Hosking et al. 2003) to develop Multivariate Regional Frequency Analysis (MR-HFAs) and apply this procedure to the case study of the region in the Ebro catchment in Spain. In the present chapter, the potentials of MR-HFAs approach (Requena et al. 2016) are verified in the Tuscany region (Italy). The MR-HFAs is analysed at ungauged sites (regional), and when a reduction in uncertainty at gauged sites is required (especially when a short data length is available). In Tuscany, univariate regional frequency analysis is developed for extreme precipitation, and discharge (Caporali et al. 2008). The Tuscany Hydrological Service uses the extreme precipitation regional frequency analysis as a reference in the design, and flood risk analysis. In the future perspective in which the multivariate frequency analysis is increasing in the application by the practitioners, the MR-

HFAs of the Q-V can be useful for the professional activities, and the Tuscany Hydrological Service activities.

5.1 Study area

The Tuscany has an area of about 23000 km²; it is characterised by a climate that ranges from temperate to Mediterranean maritime, and by a complex topography. It presents plain areas near the sea, and around the main river valleys, hilly internal zones, and the mountainous area of the Apennines. The total annual precipitation ranges from 530 to 2600 mm. The main rivers are the Arno River with a catchment area of 8228 km², the Serchio River with a catchment area of 1565 km², and the Ombrone Grossetano River with a catchment area of 3494 km².

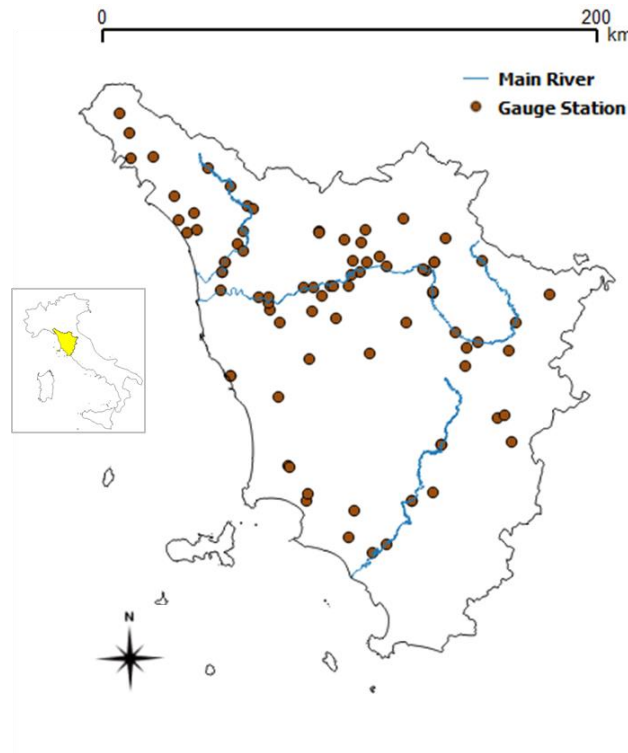


Fig. 29. Location of Tuscany Region, the main rivers, and the gauged sites.

5.2 Methodology and objectives

The study assumes the application of the bivariate index flood method (Chebana et al. 2007) of the flood peak (Q) and the volume of the hydrograph (V).

In bivariate environment, the full index method is defined as follows:

$$Q_{x,y}(p) = \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} \cdot q_{x,y}(p) \quad \text{Eq.49}$$

$Q_{x,y}(p)$ is the bivariate regional quantile for (X,Y) for a given probability p . μ_x and μ_y are the index flood respectively for X and Y. $q_{x,y}(p)$ is the bivariate dimensionless quantile curve for a given probability p . It should be noted that in the univariate approach, the quantile is a single value, whereas, in the bivariate approach, it is a curve.

The methodology is finalized to define each term of Eq.45, and it is based on five main steps: (i) screening of the dataset; (ii) delineation of homogeneous regions based on Multivariate Homogeneity Test; (iii) selection of the bivariate regional quantile curve; (iv) estimation of the index flood for X, and Y; (v) estimation of quantile curves.

The overall procedure is shown in Fig. 30.

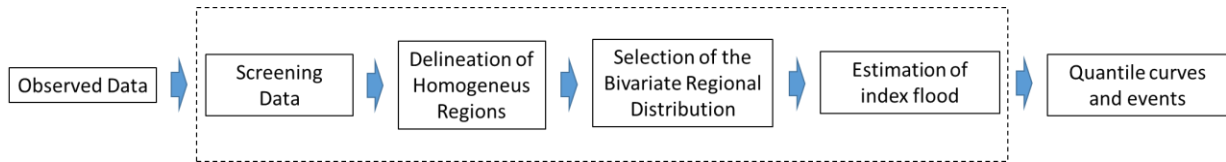


Fig. 30. Flow chart of the overall procedure for the regional multivariate frequency analysis

5.3 Screening of the data

Screening of the data is the first step in the multivariate regional frequency analysis. The data screening consists of detecting the multivariate outliers.

The Hydrological Service has to validate the recorded data at gauge sites. The incorrect events and the anomalous registrations that can affect the datasets are removed. The gauged sites are 177, but only 62 are chosen considering those that at least have 20 years of data (Fig. 29). This limit is applied in other studies that concern regional frequency analysis (Hosking and Wallis 2005).

5.4 Delineation of Homogeneous Regions

In a homogeneous region, the multivariate frequency distribution is considered the same for the whole region, apart from a scale factor. The homogeneity is verified by the Multivariate Homogeneity Test - H (Chebana and Ouarda, 2007). The H test is an extension of the Hosking and Wallis (1993) homogeneity test for the univariate case and it is based on the Multivariate L-moments (see among others, Serling et al. 2007, Chebana and Ouarda 2007), focusing on L-covariation matrix (Δ_2). The test compares the variability in Δ_2 for the analysed region with

the variability estimated from a set of synthetic homogeneous regions with the same number of sites, and same data length at each site. For more details on the Multivariate homogeneity test - H , see Chebana and Ouarda (2007). The region is declared to be homogeneous, if $H < 1$, acceptably homogenous, if $1 < H < 2$, and definitely heterogonous, if $H > 2$. Firstly, the homogeneity is verified for the entire region, but a H value of 4.157 indicates that the whole Tuscany Region is statistically definitely heterogeneous. Then, to detect the homogeneity the Tuscany is divided into subregion. Three Subregions (Fig. 31) are considered: North region, Central region and South region. The H values for each region are: North $H=0.76$; Central $H=0.267$; South= 1.274 . All the sub-region are statistically homogeneous. The delineation of these region considers the limits of each gauge site catchment and previous regional studies related to other hydrologic variable (Caporali et al. 2014). Some Tuscany's areas aren't taken into account in this analysis (white parts in Fig. 31) because belonging to rivers flowing to other Italian regions.

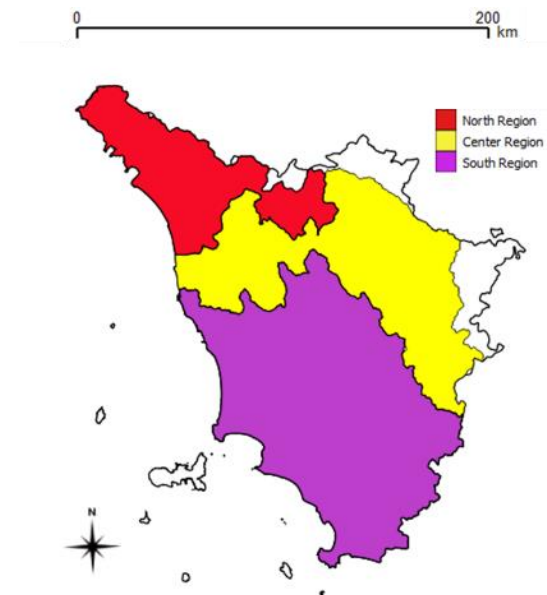


Fig. 31. The multivariate homogenous regions in Tuscany. Each region is delineated taking into account the limits of each gauge site catchment and is not extend downstream the last its gauge site. The white parts belong to the basin that have drainage flow to other Italian Regions.

5.5 Selection of the Bivariate Regional Distribution

5.5.1.1 Definition of the regional marginal distribution

The Bivariate Regional Distribution - BRD is obtained for each homogenous region. It is modelled by the suitable copula model based on the best fit univariate distribution of the marginal variable Q and V . As the BRD is dimensionless, for each site the mean value usually standardises the observed data series (e.g. Dalrymple 1960) or the median one (e.g. Institute of Hydrology 1999). The data for each gauged site are standardised using the mean value of the

Multivariate Regional Frequency Analysis: the case of the Tuscany region (Italy)

sample. Each region has the series of the standardised (q_i, v_i) where: $q_i = Q_j / Q_{mean,j}$, and $v_i = V_j / V_{mean,j}$ where Q_j , and V_j are, respectively, the annual maxima of the discharge, and the volume for the site; $Q_{smean,j}$, and $V_{smean,j}$ are, respectively, the mean discharge, and the mean volume for the site. The steps to build the bivariate regional distribution for a homogeneous region are the same previously used for the detection of the bivariate distribution for a site, see paragraph 2.2.1 for more details.

In the table are reported the results for the goodness of fit test (AIC, AICc, BIC) for the marginal distribution for q and v .

Tab. 9. Results of goodness of fit tests (AIC, AICc, BIC) for the regional marginal distributions. The GEV and Lognormal distributions are the best for the discharge for North and South regions; the Gumbel and the GEV distributions for Central region. The Gumbel and the GEV are the best for the volume for all the regions.

Distribution	Standardized Discharge (q)							Standardized Volume (v)						
	Normal	LogNormal	Gumbel	EV2	GEV	P3	LP3	NORM	LN	Gumbel	EV2	GEV	P3	LP3
North Region	Akaike Information Criterion (AIC):							Akaike Information Criterion (AIC):						
	714.9	515.4	529.7	628.6	514.3	532.9	532.9	815.1	645.3	616.3	812.3	612.6	624	624
	Corrected Akaike Information Criterion (AICc):							Corrected Akaike Information Criterion (AICc):						
	714.9	515.4	529.8	628.7	514.3	532.9	532.9	815.2	645.4	616.3	812.3	612.6	624	624
	Bayesian Information Criterion (BIC):							Bayesian Information Criterion (BIC):						
	722.7	529.5	537.6	636.5	526	544.6	544.6	823	653.2	634.1	820.1	624.3	635.7	635.7
Central Region	Akaike Information Criterion (AIC):							Akaike Information Criterion (AIC):						
	761.7	648	584.9	1107.2	582.7	604	604	776.3	738.2	677.1	1051.1	678.2	681.4	681.4
	Corrected Akaike Information Criterion (AICc):							Corrected Akaike Information Criterion (AICc):						
	761.7	648	584.9	1107.2	582.7	604	604	776.3	738.3	677.1	1051.1	678.2	681.5	681.5
	Bayesian Information Criterion (BIC):							Bayesian Information Criterion (BIC):						
	770.1	656.4	593.3	1115.5	595.2	616.5	616.5	784.6	746.6	685.4	1059.5	688.7	694	694
South Region	Akaike Information Criterion (AIC):							Akaike Information Criterion (AIC):						
	727.1	537	552.5	675.3	534.3	559.1	559.1	886.7	617.9	569.1	726.8	563.5	584.9	584.9
	Corrected Akaike Information Criterion (AICc):							Corrected Akaike Information Criterion (AICc):						
	727.1	537.1	552.5	675.4	534.3	559.1	559.1	886.8	617.9	569.1	726.9	563.6	585	585
	Bayesian Information Criterion (BIC):							Bayesian Information Criterion (BIC):						
	734.8	549.7	564	683	545.8	566.7	566.7	894.4	605.6	587.7	734.5	575	596.4	596.4

In the North, and South Regions, the GEV, and the Lognormal distributions for the standardised variable q show similar best results; the Gumbel and the GEV distributions have similar best outcomes for the standardised variable v . Because the tests do not give a unique indication, the best univariate distribution is chosen through at-site behaviour for high return periods. The variability of q and v are calculated for $T=500$ years. Both for q and v , The variability is evaluated benchmarking the 500 years quantile from the univariate regional distribution with the corresponding quantile of at site univariate distribution.

In Fig. 32 at site-behaviour for high return periods of the standardised distributions, and the calculation of their variability ($\sigma(q)$ or $\sigma(v)$) for $Tr=500$ years are shown. The GEV distribution is chosen for modelling the marginal distribution for q and v in the North region. The Gumbel distribution is chosen for modelling the marginal distribution for q and v in the Central homogeneous region. The GEV distribution for q , and the Gumbel distribution for v are chosen to model the marginals in the South region.

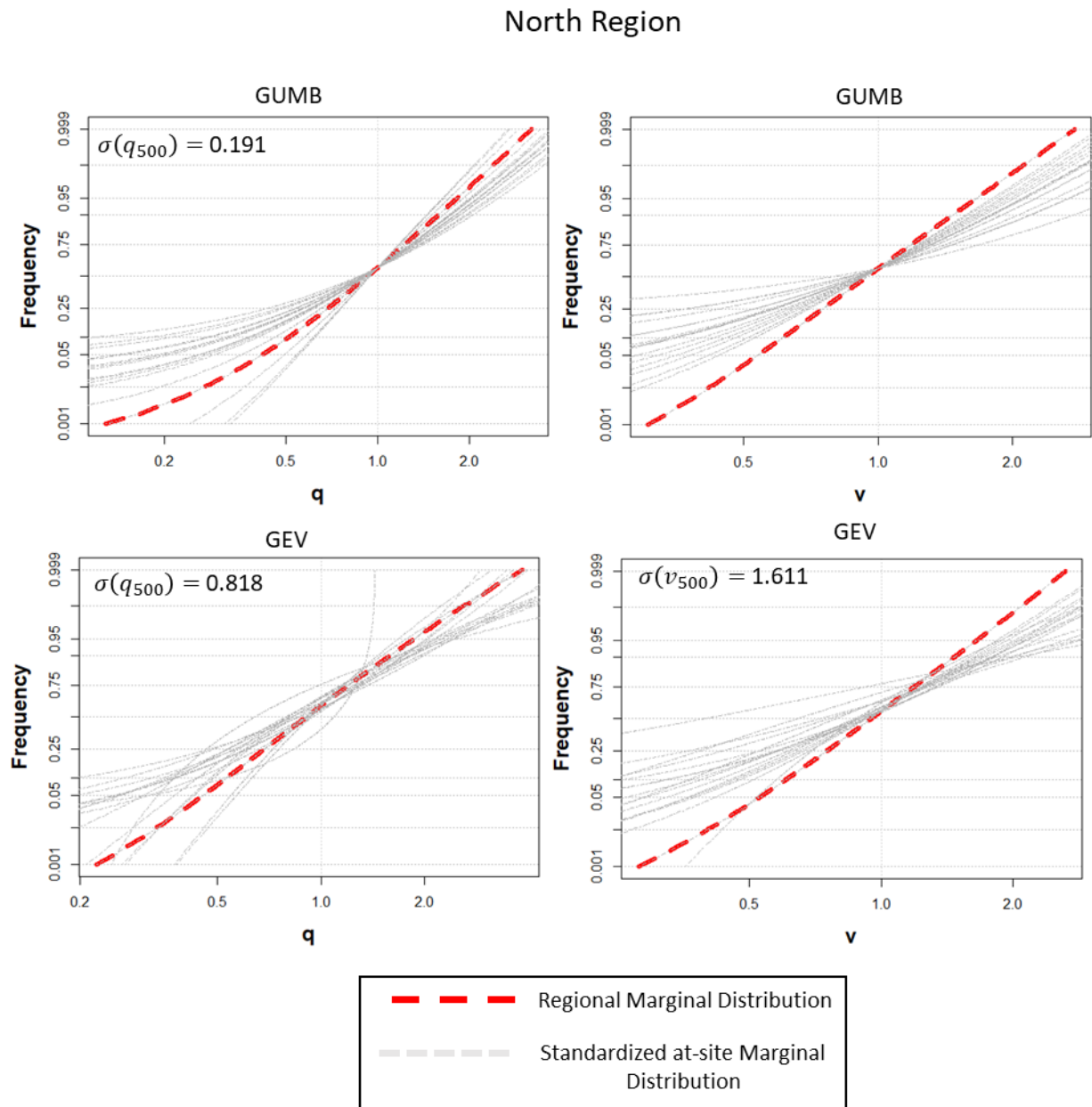


Fig. 32. At-site behaviour for the North Region for high return periods of the marginal distributions that have passed the goodness-of-fit test, i.e. the GEV, the Lognormal (LN) and the Gumbel (GUMB) distributions.

Central Region

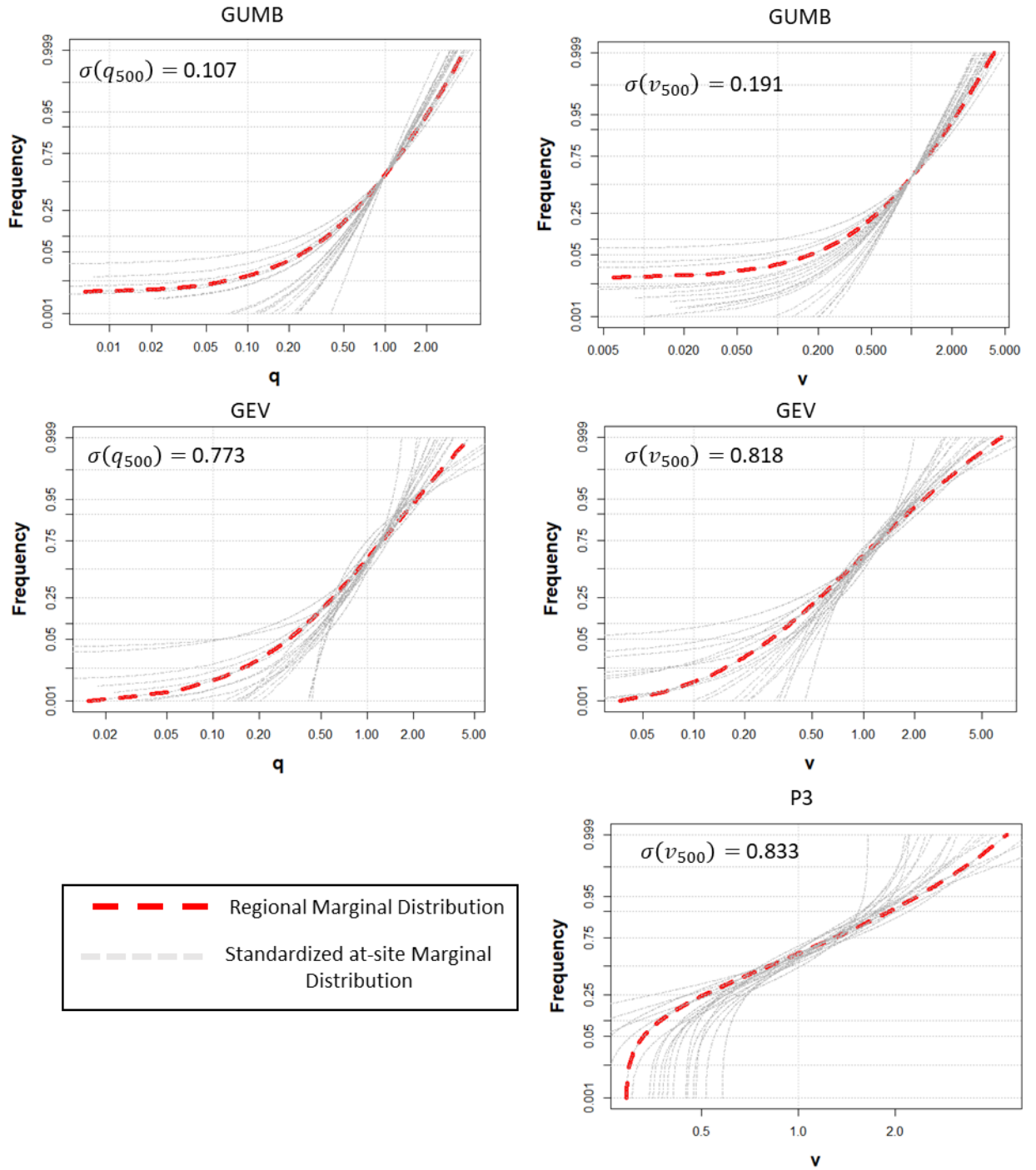


Fig. 33. At-site behaviour for the Central Region for high return periods of the marginal distributions that have passed the goodness-of-fit test, i.e. the GEV, three parameter Pareto distribution (P3) and the Gumbel (GUMB) distributions.

South Region

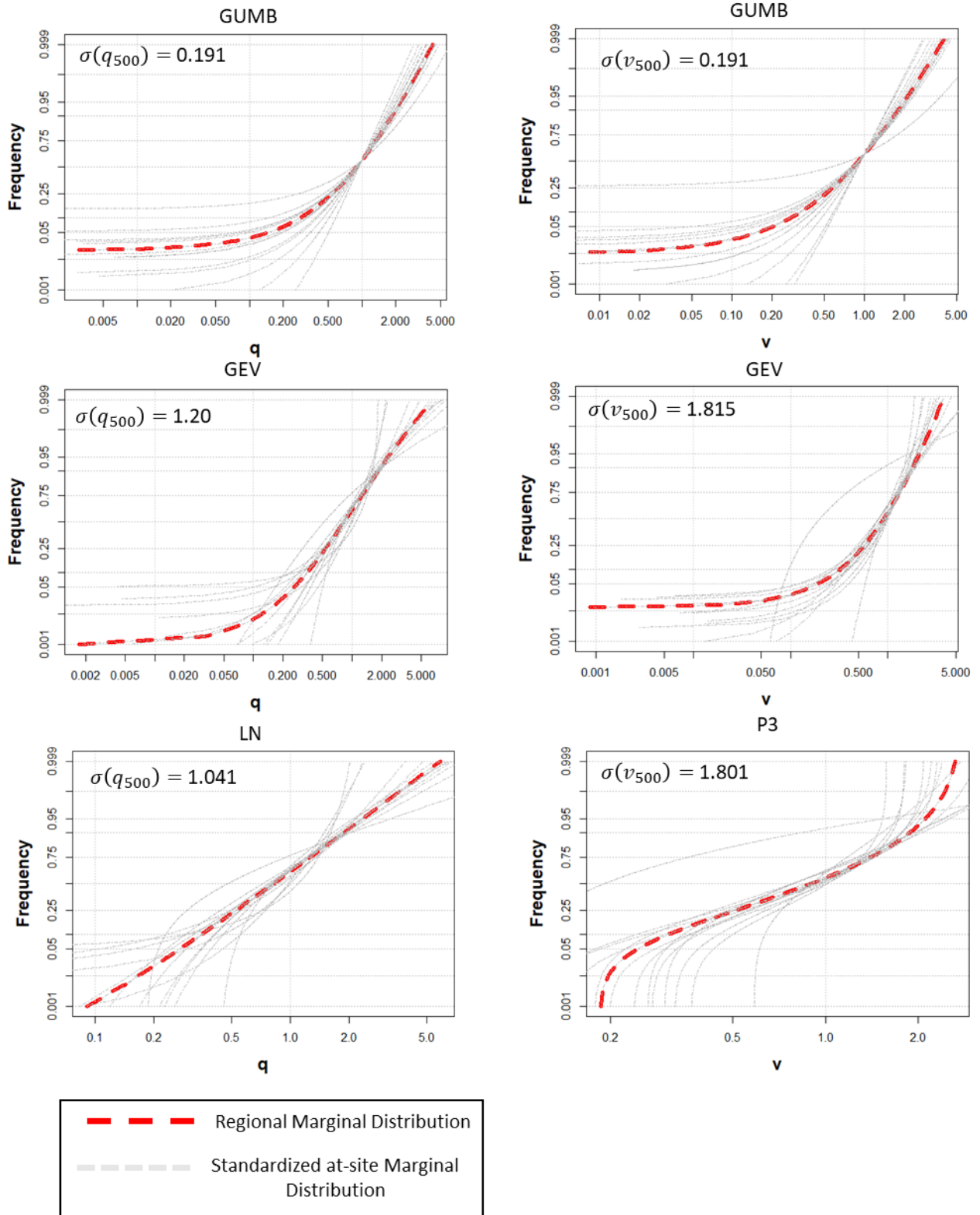


Fig. 34. At-site behaviour for the South Region for high return periods of the marginal distributions that have passed the goodness-of-fit test, i.e. the GEV, three parameter Pareto distribution (P3), LogNormal (LN) and the Gumbel (GUMB) distributions.

5.5.1.2 Selection of regional copula

The selection of regional copula starts with the analysis of the dependence between q and v . It carries out by graphical analyses and dependence measures. As shown in paragraph 2.2.1, the graphical analysis consists of three plots: the scatter plot of the ranks of q and v ($R_i/(n+1); S_i/(n+1)$), the χ -plot, and the K-plot. In Fig. 35 the plots are reported for each homogeneous region.

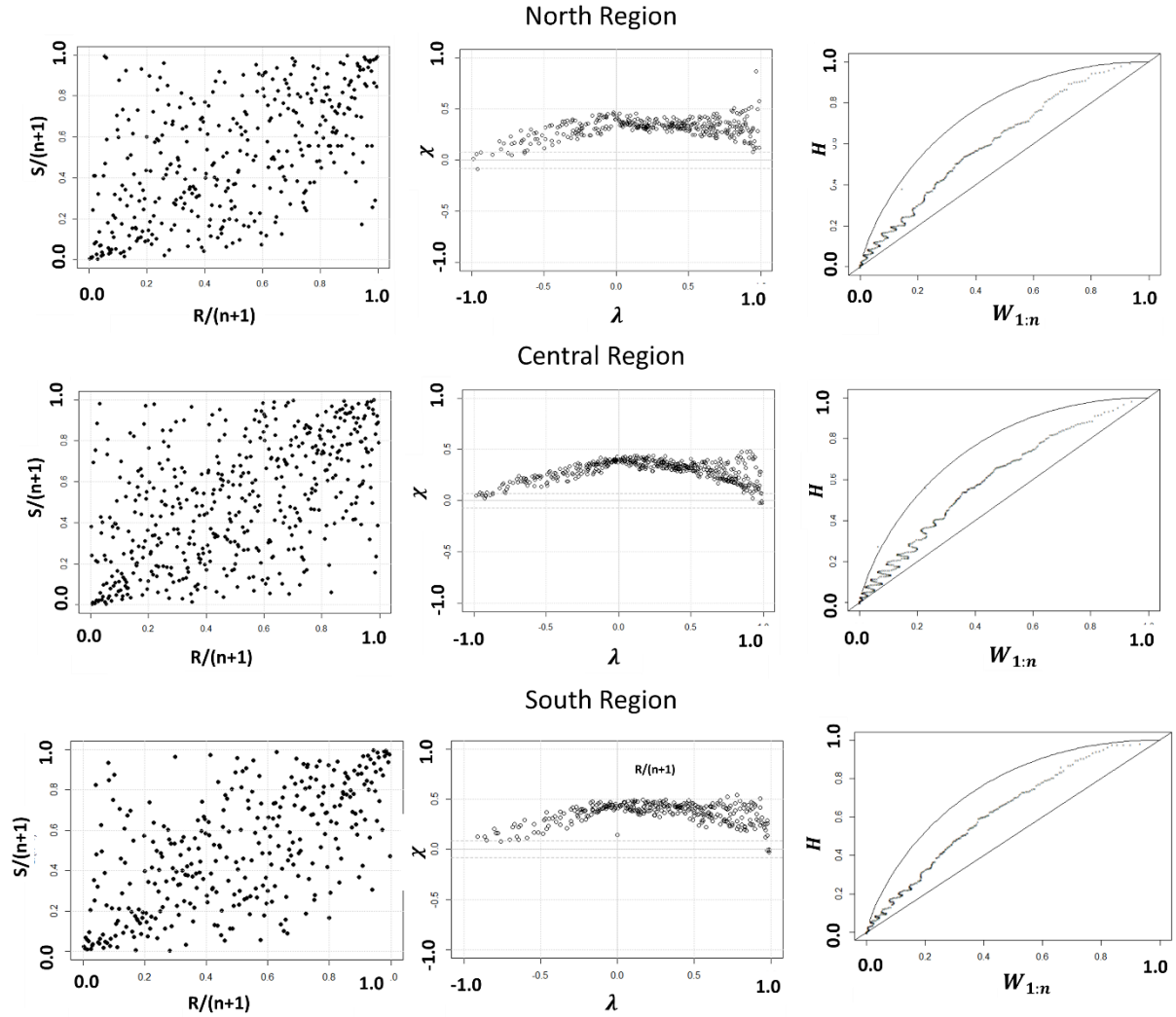


Fig. 35. The scatter plots of the rank of q , and v (left), the χ -plots in the middle, and the K-plots for all the homogeneous regions.

The scatters plot ($R_i/(n+1); S_i/(n+1)$) leads to a positive dependence between q and v for all the homogeneous regions. The χ -plots (Fig. 35, in the middle), and the K-plots (Fig. 35, on the right) confirms the dependence between data for all regions. In the former, most values fall above the upper limit indicating positive dependence ($\chi_i > 0$). In the K-plot, the values are collocated above the diagonal line indicating a positive dependence.

Multivariate Regional Frequency Analysis: the case of the Tuscany region (Italy)

Tab. 10. The Spearman's rank correlation coefficient (ρ), and the Kendall's rank coefficient (τ) for all homogeneous regions.

North Region		Central Region		South Region	
Spearman		Spearman		Spearman	
ρ	p-value	ρ	p-value	ρ	p-value
0.544	0.002	0.38	0.002	0.616	0.002
Kendall		Kendall		Kendall	
τ	p-value	τ	p-value	τ	p-value
0.389	0.002	0.535	0.002	0.44	0.002

The parameters of copulas (θ) are estimated using the maximum pseudo-likelihood method (Tab. 5). As shown in paragraph 2.2.1, two plots are used to check preliminary the best fitting copula: (i) a synthetic sample of pairs (u_1, u_2) are generated from the analysed copula families, and they are compared to the pairs related to the observations $\left(\frac{R_i}{n+1}; \frac{S_i}{n+1}\right)$ (Fig. A5, Fig. A7, Fig. A9 in appendix A4); (ii) the second plot consists of the benchmark between the Q_i and V_i (Eq.36) with the observed pairs of Q , and V (Fig. A6, Fig. A8, Fig. A10 in the appendix A4). The copula family assessed are the previous analysed (Tab. 3). The information provided by the plots is not sufficient for filtering the list of copula families. It should be highlighted that the Galambos copula seems suitable for North and South homogeneous regions, the Gumbel copula for the Central one.

The goodness of fit tests of AIC_{cop} (Eq.37), BIC_{cop} (Eq.38), S_n (Eq.39), and the corresponding p-value of the Cramer Von Mises confirms the preliminary graphical result (Tab. 11). The Student's t copula is chosen for modelling the regional bivariate distributions for all the homogeneous region. At each copula the weighted regional parameters ($\theta_{reg,1}; \theta_{reg,2}$) are associated to each copula. The regional copula parameters are obtained from at site parameters ($\theta_{1i}; \theta_{2i}$) evaluated, starting from standardized data (q_i, v_i) by maximum likelihood for marginal distributions, and maximum pseudo-likelihood for the copula:

$$\theta_{reg,i} = \frac{\sum_{j=1}^N n_j \cdot \theta_{ki}}{\sum_{j=1}^N n_j}, k = 1, 2. \quad \text{Eq.50}$$

Where N is the total number of the gauge site for each homogeneous region, n is the number of pairs for j-site of the homogeneous region. The parameters for each regional copula are reported in Tab. 12.

Tab. 11. The assessed copula families

Copula	North Region			
	AIC _{cop}	BIC _{cop}	S _n	p-value
Gaussian	-146.848	-142.92	0.0266	0.1028
Student's t	-158.96	-135.116	0.267	0.0988
Clayton	-142.87	-138.94	0.107	0.0009
Gumbel	-143.437	-137.65	0.046	0.002
Frank	-133.437	-129.513	0.043	0.0089
Galambos	-167.39	-159.54	0.267	0.0988
Husler-Reis	-163.54	-139.009	0.0630	0.0040
Copula	Centre Region			
	AIC _{cop}	BIC _{cop}	S _n	p-value
Gaussian	-161.56	-157.38	0.02577	0.1487
Student's t	-147.32	-143.14	0.05783	0.0009
Clayton	-149.8	-145.62	0.1622	0.0009
Gumbel	-174.09	-165.72	0.55269	0.2904
Frank	-164.47	-160.29	0.0249	0.1108
Galambos	-149.52	-142.044	0.06767	0.0029
Husler-Reis	-156.996	-149.146	0.104	0.0009
Copula	South Region			
	AIC _{cop}	BIC _{cop}	S _n	p-value
Gaussian	-161.56	-157.38	0.02577	0.1148
Student's t	-159.536	-155.481	0.067678	0.002994
Clayton	-149.8	-145.62	0.1622	0.000998
Gumbel	-147.32	-143.14	0.05783	0.000998
Frank	-164.47	-160.29	0.024982	0.1008
Galambos	-174.09	-165.72	0.0208	0.2565
Husler-Reis	-168.867	-160.748	0.10463	0.000998

Tab. 12. Regional copula, and the corresponding parameters for each homogeneous region.

Homogeneous Region	Copula	θ_{reg}
North	Galambos	0.885
Central	Gumbel	1.53
South	Galambos	1.053

5.6 Index Flood

The “index-flow” parameter can be either the sample mean or the sample median (e.g. (Hosking and Wallis 2005)). Viglione (2007) shows that the estimation of the mean is less biased than that of the median because low skewness coefficients generally characterise the

hydrological variables. For this reason, the sample mean is used as the index-flow. The most frequently used method to estimate the index-flow is the multiregressive that relates the index-flow to catchment characteristics, such as climatic indices, geologic, and morphologic parameters, land cover type, etc., through linear (used here) or non-linear equations (Kottegoda and Rosso 1997).

Eight multiregression models are tested to represent index flood $\mu = (\mu_Q, \mu_Q)$ (Viglione 2007):

$$\begin{aligned}
 \mu &= \beta_0 + \beta_1 M_1 \dots + \beta_m M_m \\
 \mu^{0.5} &= \beta_0 + \beta_1 M_1 \dots + \beta_m M_m \\
 \mu^{1/3} &= \beta_0 + \beta_1 M_1 \dots + \beta_m M_m \\
 \log(\mu) &= \beta_0 + \beta_1 M_1 \dots + \beta_m M_m \\
 \mu &= \log(\beta_0) + \log(\beta_1) M_1 \dots + \log(\beta_m) M_m \\
 \mu^{0.5} &= \log(\beta_0) + \log(\beta_1) M_1 \dots + \log(\beta_m) M_m \\
 \mu^{1/3} &= \log(\beta_0) + \log(\beta_1) M_1 \dots + \log(\beta_m) M_m \\
 \log(\mu) &= \log(\beta_0) + \log(\beta_1) M_1 \dots + \log(\beta_m) M_m
 \end{aligned}
 \tag{Eq.51}$$

Where β_i are the vectors of regression parameters, and the vectors M_i represent the catchment and morphoclimatic descriptors.

For each catchments relative the gauged sites (Viglione, 2007) are considered:

- i. The catchment area (A [km^2]);
- ii. The position of the catchment Centroid (X_{centr}, Y_{centr}) [m];
- iii. The catchment perimeter (P [km]);
- iv. the maximum, the minimum, and the mean elevation ($H_{min}, H_{max}, H_{mean}$ [m a.s.l.]) of the catchment;
- v. The Aspect (Asp [deg]), i.e. the direction towards which a slope faces that is the angle of exposure of the cell computed from the north;
- vi. The Circularity ratio that is the ratio between the perimeter of catchment, and the circle with the same area of catchment (R_c);
- vii. The Mean Slope defined as: $MS[\%] = \tan^{-1} \left(\frac{H_{max} - H_{min}}{\sqrt{A}} \right)$;
- viii. The length of the longest series of streams that connects the basin outlet to the foremost source point (L_{ms} [km]);
- ix. The main stream mean slope defined as $MSMS [\%] = \frac{H_{max} - H_{min}}{L_{ms}}$;

- x. The length of the longest drainage path (L_{LDP} [km]): the longest drainage path is the longest path between the basin outlet, and the most distant point on the basin border, following drainage directions;
- xi. The slope of the longest drainage path (P_{LDP} [%]);
- xii. The ratio between the diameter of a circle with an area equivalent to the basin area, and the length of the longest drainage path (EL_{rat});
- xiii. The Shape Factor defines as $SF = A/L_{LDP}^2$;
- xiv. The number of links that constitute the main stream, or number of confluences to the main stream (d_t);
- xv. The Drainage density (D_d [km/km²]) that is the total network length divided by the area of the drainage basin;
- xvi. The Concentration time evaluated with Giandotti formula (Giandotti 1933);
- xvii. The Compactness coefficient C_c : the ratio between the perimeter of the basin, and the diameter of the equivalent circle.

Other parameters are climatic parameters defined as the average climatic features can be considered attributes or “descriptors”, similarly to its morphologic parameters (Viglione 2007).

The climatic parameters available:

- i. The Mean Annual Rainfall (MAR[mm]) averaged over the catchment over the catchment,
- ii. The parameters ‘ a ’, and ‘ n ’ of the DDF curve for extreme precipitation (average of each catchment) provided by Regional Frequency Analysis available in Tuscany Region (Caporali et al. 2012),
- iii. The Saturated Hydraulic conductivity in the first 50 cm of soil average for each catchment (K_{sat50} [cm/s]) (Tuscany Region),
- iv. The Gravitational Water (G [mm]) (Busoni et al. 1986),
- v. The Curve Number (CN).

The best regressive model is chosen to respect the R_{adj}^2 (Viglione 2007):

$$R_{adj}^2 = 1 - \frac{\frac{SS_{res}}{n-p}}{\frac{SS_T}{n-1}} \quad \text{Eq.52}$$

Where SS_T is the quadratic mean of the observations, and $\frac{SS_{res}}{n-p}$ is the quadratic mean of the residuals. The coefficient of determination R_{adj}^2 can be used to choose the best model. It is also recommended (Claps and Laio 2008; Viglione 2007) to apply a cross-validation method, calculating the root of the mean square error ($RMSE$) of the residuals $\hat{\mu}_i - \mu_i$, where $\hat{\mu}_i$ is the

estimated value of the index flood μ_i , based on the regression obtained using all the observations except the i -th. The $RMSE$ is defined as:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{\mu}_i - \mu_i)^2} \quad \text{Eq.53}$$

The models chosen for each region are shown in Tab. 13:

Tab. 13. Regression models using as index-flood.

Homogeneous Region	Index flood relationship	R_{adj}^2	$RMSE$
North	$\mu_Q = 13754.84 + 0.1250 \cdot A + 0.4008 \cdot (H_{max} - H_{min}) - 0.0029 \cdot Y_{centr}$	0.975	716.1258
	$\mu_V^{0.5} = -42573 + 5650 \cdot \ln(H_{mean}) + 5544 \cdot \ln(P) - \ln(H_{min})$	0.963	1.48E+08
Central	$\mu_Q^{0.5} = 30304 + 6.845 \cdot \ln(A) + 6.462 \cdot \ln(H_{mean}) - 234.85 \cdot \ln(Y_{centr})$	0.963	715
	$\mu_V = -47910159 + 54571 \cdot A + 212179 \cdot P - 2373118 \cdot MS$	0.975	28299833
South	$\mu_Q^{1/3} = 657.714 + 1.719 \ln(A) + 1.432 \cdot \ln(H_{mean}) - 46.753 \cdot \ln(X_{centr})$	0.973	690
	$\mu_V^{0.5} = -47910159 + 54571 \cdot P + 212179 \cdot (H_{max} - H_{min}) - 2373118 \cdot C_c$	0.999	1.68E+08

5.7 Quantile estimation and evaluation of the model

The (dimensionless) regional quantile curve for a given probability is obtained from the identified bivariate regional distribution. In the univariate case, the estimate of the analogous regional growth curve is straight. However, some issues arise in the bivariate approach. Indeed, several return period definitions existing for the bivariate return period, conduct to different curves in Q-V space, thus different flood events, and different interpretation of the probability (Serinaldi 2015). In previous paragraphs, this issue is highlighted and avoided proposing the Bivariate Failure Return Period. A comparison between bivariate quantile curves estimated by the regional and local procedure is developed to evaluate the performance of the regional approach. The benchmark is conducted using the same copula, and the same marginal distribution with regional parameters in one case, and at-site parameters in the local one. The quantile curve for a target site is obtained by rescaling the regional quantile curve via the estimated index floods.

The quantile curve obtained by regional, and at-site approach is benchmarked for each homogeneous region. The quantiles curves (Salvadori et al. 2013) corresponding to a low probability of occurrence ($p=0.9; 0.99, 0.998$) are analysed. The quantile curves are associated with the “AND” return period (par. 1.2.1). The distance between both curves (local and regional) in the bivariate space is quantified through the measures presented by Chebana, and Ouada (2009). The regional bias $RB^r(p)$ assesses the performances of the regional approach, the absolute regional bias, $ARB^r(p)$ and the quadratic error $RRMSE^r(p)$.

The $RB^r(p)$ is defined as:

$$RB^r(p) = 100 \frac{1}{N} \sum_{i=1}^N RIE_i^*(p) \quad \text{Eq.54}$$

Where N is the number of the sites of the homogeneous region, and $RIE_i^*(p)$ is the relative integrated error defined as:

$$RIE_i^*(p) = \frac{1}{L_p^i} \int_{Q_{cp}^i} RE_p^i(x) dx \quad \text{Eq.55}$$

where L_p^i is the length of the proper part of the quantile curve defined by Chebana, and Ouarda (2011), and $RE_i(p)$ is the relative error that is a measure of the vertical distance between quantile curves (local and regional), defined as:

$$RE_i(p) = \frac{\widehat{G}_p^i(x) - G_p^i(x)}{G_p^i(x)}, 0 < p < 1 \quad \text{Eq.56}$$

Where the $\widehat{G}_p^i(x)$, and $G_p^i(x)$ are the quantile curves at the same site evaluates respectively by regional, and at-site approach.

The $ARB^r(p)$ is defined as:

$$ARB^r(p) = 100 \frac{1}{N} \sum_{i=1}^N |RIE_i^*(p)| \quad \text{Eq.57}$$

The $RRMSE^r(p)$ is defined as:

$$RRMSE^r(p) = 100 \sqrt{\frac{1}{N} \sum_{i=1}^N (RIE_i(p))^2} \quad \text{Eq.58}$$

Where the $RIE_i(p)$ is the relative integrated errors:

$$Q_0 = \frac{1}{L_p^i} \int_{Q_{cp}^i} |RE_p^i(x)| dx \quad \text{Eq.59}$$

The uncertainty analysis proposed by Mediero (2014) is performed to assess the uncertainty reduction provided by a regional approach. The procedure is the following: (i) One

thousand of synthetic pairs from the selected copula family generates synthetic series of length n fitted to the data, either at-site or regional; (ii) the same copula family is fitted to each synthetic series, obtaining its parameter(s); (iii) a new series of (u_1, u_2) is generated by each simulated copula parameter(s); (iv) 1000 Q-V series are generated by the selected marginal distribution fitted to the data; (v) the same marginal distributions are fitted to each synthetic Q-V series, obtaining its parameters; (vi) a given (u_1, u_2) series from (iii), and a given set of marginal distribution parameters from (v) are used to generate a quantile curve for a given probability p (resulting a total of 1000 quantile curves for such a p -value); (vii) the standard deviation σ of the asymptotic values of the simulated quantile curves is individually calculated for V and Q. In the procedure, at-site number of data (N) is considered for the local estimate, while pooled data are considered for the regional estimate.

Further benchmark is conducted for the overtopping failures analysed in previous case studies with the bivariate regional distribution.

For these purposes, the synthetic standardised Q-V pairs can be obtained via the multivariate regional distribution, destandardising them by the index flood (Eq.48) of each variable at the target site.

5.8 Results and discussions

The quantile curves for every gauged site are evaluated via the regional approach and the local one. As an illustration, the quantile curves obtained by the regional approach are plotted together with the local quantile curves for several sites (Fig. 36). To estimate local quantile, the regional shape parameter curve is used in the GEV distribution to reduce the uncertainty in local estimates for high return periods due to the variability of the at-site coefficient of L-skewness (Jimenez et al. 2012). The plots in Fig. 36 show a similar result by regional approach and the local one for the quantile curve estimation. As expected in the regional approach, the distance between the corresponding quantile curve becomes the more significant, the lower is the probability of occurrence. Another aspect that influences the distance between the quantile curve is the accuracy of the estimation of the index-flood. Indeed, site TOS01005191 is in the downstream part of the Arno basin, with an upstream catchment area of 8031 km², with many different types of land use, with many types of morphologies etc., influencing the evaluation of the index flood.

The assessment of the model is also carried out by the evaluation of $RB^r(p)$ (Eq.54), $ARB^r(p)$ (Eq.57), and $RRMSE^r(p)$ (Eq.58). The values of each homogeneous region for the probability of 0.90, 0.99, and 0.998 are shown in the Tab. 14. The results display that in general errors increase as p increases, due to the higher uncertainty in the estimate. The values for the probability of 0.99 for the probability of 0.90, 0.99, and 0.998 are benchmarked to the corresponding values of the similar regional studies developed by Requena et al. (2016) and Chebana and Ouada (2009). In this case study, the value of (0.99) $RB^r(p)$ is positive, and

larger. It entails a trend to obtain uniformly high quantile estimates for the whole sub-region. The value of $ARB^r(p)$ is also larger, involving a higher tendency to obtain consistently high or low estimates at given sites. The smaller values of $RRMSE^r(p)$ values could be due to the use of the multivariate local quantile curve as a proxy of the real multivariate quantile curve, which could be closer to that estimated by the multivariate regional approach. Similar results are obtained when other probability values are compared.

As an illustration of the benefits of the regional approach, the uncertainty results for TOS01004779 (belonging to the North Region) are shown for $p=0.99$, considering it as gauged (Fig. 37). In this case, it decreases from 0.274 to 0.165 for peak, and from 0.332 to 0.220 for volume. Hence, and as expected, the uncertainty is reduced by the regional approach, for which 356 pooled data were considered, in comparison with the local approach where 22 at-site data were available.

However, the uncertainty in the regional approach is not negligible. Regardless of such uncertainty, the (in our case, regional) bivariate approach is essential in obtaining a better hydrological characterisation than that provided by the univariate context (Dung et al., 2015).

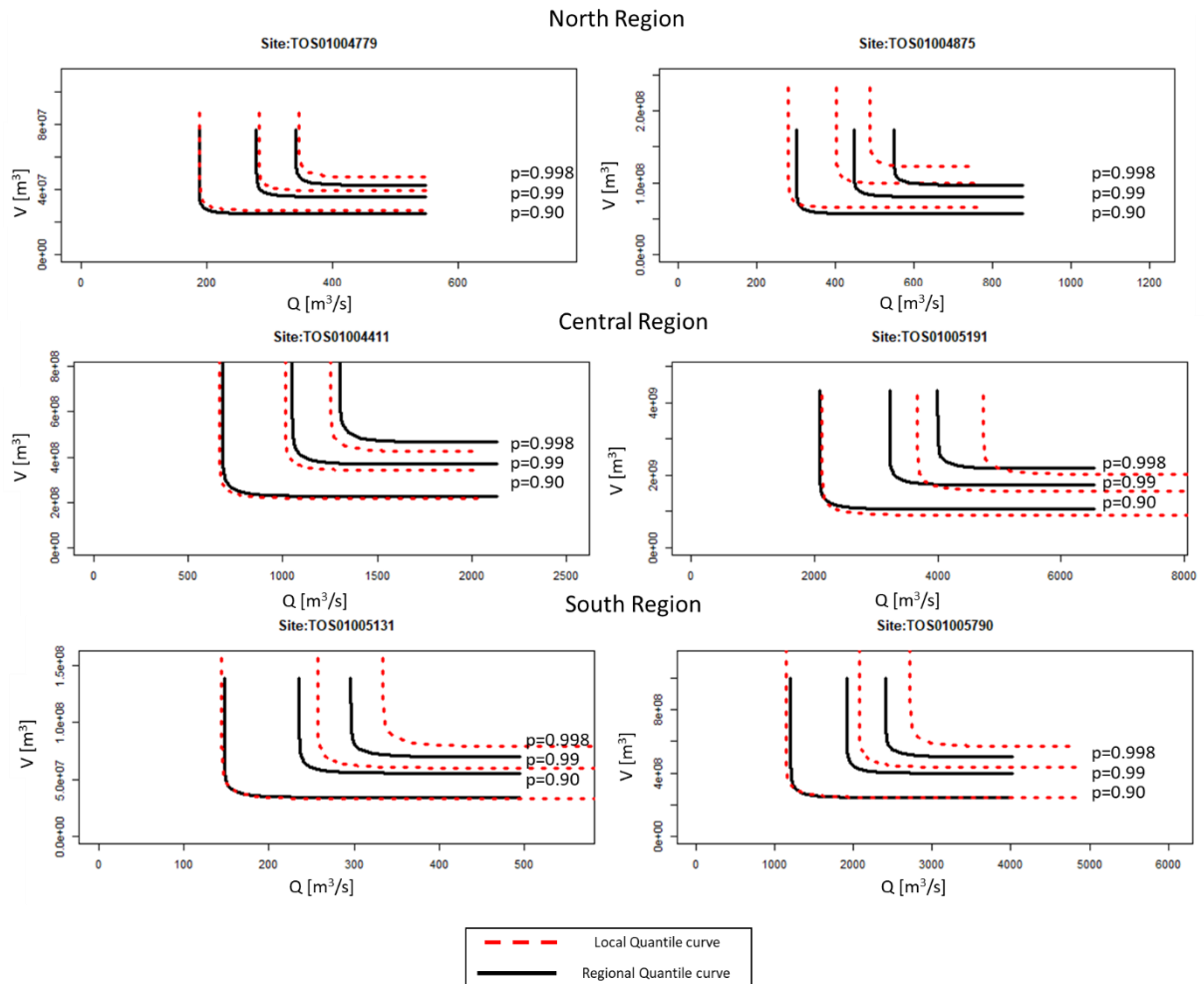


Fig. 36. Graphical comparison between quantile curves estimated by the proposed regional procedure at several target sites, and at-site quantile curves estimated by a local approach.

Multivariate Regional Frequency Analysis: the case of the Tuscany region (Italy)

Tab. 14. Results of the regional error measures for evaluating quantile estimates using the bivariate regional procedure based on the multivariate index-flood model.

Homogeneous Region	Probability (p)	Error measure (%)		
		$RB^r(p)$	$ARB^r(p)$	$RRMSE^r(p)$
North	0.9	1.854	7.963	10.897
	0.99	1.816	8.801	12.471
	0.998	1.778	9.690	14.135
Central	0.9	1.523	3.591	4.174
	0.99	1.344	3.920	4.484
	0.998	1.015	3.957	4.691
South	0.9	2.236	5.272	6.127
	0.99	1.973	5.755	6.583
	0.998	1.490	5.810	6.886

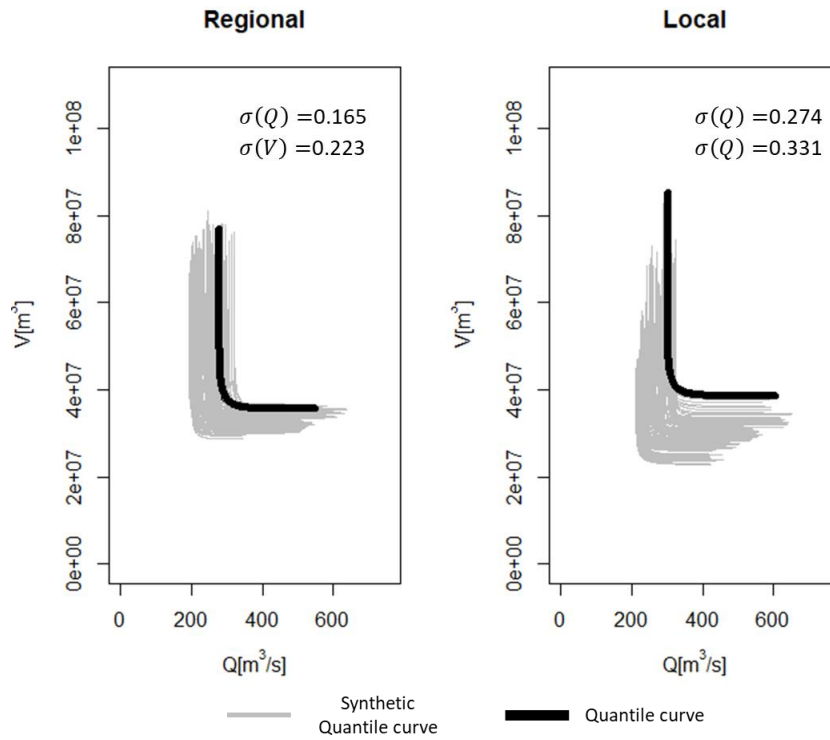


Fig. 37. Illustration of the uncertainty analysis for the local, and regional approach using 1,000 simulated quantile curves for $p = 0.99$ at gauged site TOS01004779 (in north region).

6 Conclusions

The conclusions that have been reached as a result of the present research, the original contributions to the proposed methodology, as well as possible further research are presented in this final chapter.

The present research provides methodologies for guiding practitioners to perform the analyses for river-related flood risk considering a bivariate hydrological load.

6.1 Main conclusions

The results of this PhD research are here presented as answers to the research questions listed in the introduction

- **Is the design flood approach adopted in standard practice a correct method to assesses the risk for river-related flood problems?**

In standard practice, the design flood approach is used in flood risk analysis (e.g. Okoli et al. 2019). In this approach, the hydrological load consists of one hydrograph with a probability of occurrence. This approach is justified on the approximation of a perfect correspondence between peak discharge and the water stage in the river. The methodology proposed in this thesis clarifies if this approximation conditions the flood risk assessment. The hydrological failure curves (Fig. 17 and Fig. 27) represents the hydrological condition that causes the initiation of the type of failure assessed. Two types of failure are evaluated: (i) the overtopping for a river levee (ii) in the flood damage estimation. In both cases, the hydrological failure curves (Fig. 17 and Fig. 27) have a similar trend demonstrates that not only the peak flow but also the volume of the hydrograph are essential variables to characterise the type of failure. The milder the slope of the hydrological failure curve in Q-V space, the higher the dependence on volume, and the steeper the slope, the greater the dependence on peak flow, as demonstrated by Mediero et al. (2010); Requena et al. (2013) for dam safety analyses. From these considerations, the design flood approach of the standard practice is not completely correct. Indeed, the design flood is not a unique hydrograph, but it is composed of infinitives different hydrographs laying on the hydrological failure curve.

Another critical issue of the standard design flood approach is not only that the hydrograph is unique, but it is the same independently of the type of considered failure. The methodology proposed to overcome this issue demonstrating that each failure type corresponds to different hydrographs. Indeed, the graphical outcomes in Figs 17 and 27 of the Poggio a Caiano case study demonstrate that the overtopping for a river levee and the flood damage estimation are represented by different hydrological failure curves.

The hydrological failure curve suggests two main practical consequences. Firstly, it is possible to benchmark it with the standard design value (Q500 or Q200) to assess the

potential risk and to detect which are the levees that are weakest, needing further evaluation. Secondly, the curve can be used to improve the flood alert system to establish early warning threshold conditions, and to help in the emergency management during the flood event.

- **How to assess a probability related to a certain type of failure or damage in a hydrological bivariate environment?**

The risk analysis demands to an estimation of a return period for a certain type of risk. In bivariate hydrological environment, several definitions of return periods exist in literature. Considering the same probability of occurrence, to each definition of the bivariate return period (“OR”, “AND”; Kendall etc.) corresponds a different curve in the Q-V space. For this reason, the existing bivariate return period definition gives a different interpretation of the same probability of occurrence, thus a different risk estimation (Salvadori et al. 2016). To overcome this issue, it is established to evaluate the probability starting from the condition that triggers the failure considered proposing the new approach of the Bivariate Failure Return Period-BFRP. The proposed approach is based on well-established theories which give it efficiencies in its applications. The flexibility of the BFRP is demonstrated because this return period is applied to two different case studies with different modes of the same failure mechanisms. To prove the efficacy, the BFRP is benchmarked with similar approaches existing in literature: the Structure Return Period – SRP (Volpi and Fiori 2014) and the Routed Return Period - RRP approach (Requena et al. 2013). Briefly, the SRP approach is based on the idea to transfer the evaluation of the probability from a bivariate problem (Q-V distribution) to a univariate one related to the critical damage variable (e.g. the water stager above a weir of the reservoir). Thus, the main issue of the SRP approach is that it is not applicable if the failure mechanism depends by two or more variable (e.g. the overtopping failure). Respect to the SRP, the BFRP is also applicable to failure mechanism depend on multiple critical variables because the failure mechanism estimation and the return period are separately estimated. The BFRP is also benchmarked with the Routed Return Period – RRP (Requena et al. 2013). The BFRP approach considers the failure probability during the design life of the hydraulic structure while the RRP estimates the return period as the frequency of the failure hydrographs without evaluating the possible failure scenarios. In this sense, the BFRP represents an improvement in the bivariate risk assessment. The different return periods present in literature lead to compare the probabilities describing different sets of events corresponding to different mechanisms of failure, only one of which describes the response of the system at hand (Serinaldi 2015). The BFRP tries to overcome this issue because referring the return period to the risk of failure of the design life for the structure (Serinaldi 2015).

A limit of the BFRP approach is that it needs a priori the hydrological failure curve in Q-V space related to the failure mechanism. The first part of the methodology proposed in chapter 2 allows estimating the hydrological failure line and can be useful to implement the BFRP approach.

- **Can the regional frequency analysis be useful in a bivariate hydrological environment?**

The bivariate regional frequency analysis is applied to the Tuscany Region. The procedure was proposed by Chebana and Ouarda 2007 and was developed by (Requena et al. 2016). The aim is to test an alternative for rainfall-runoff modelling to supply the Q-V pair series to develop bivariate distributions at the ungauged sites. The bivariate regional frequency analysis could be useful for providing a sufficient number of plausible hydrographs at ungauged sites. The multivariate regional frequency analysis is based on an extension of the index-flood method (Hosking and Wallis 2005). The Tuscany is divided into three homogeneous regions verified with the Multivariate Homogeneity Test (Chebana and Ouarda 2007). The proposed procedure is useful for practitioners to estimate quantiles and select design events by a multivariate regional hydrological frequency analysis. Quantile curves can be obtained at an ungauged target site located in a homogenous region through the information collected at a set of gauged sites, supplying an estimate at a location where no flood data exist. Also, the uncertainty of multivariate quantile estimates at a given gauged site can be reduced by considering the multivariate regional approach.

Furthermore, a large set of synthetic flood hydrographs necessary for extending the observations can be obtained via the regional approach which can be useful in performing dam design or safety analyses based on the routed return period approach. Hence, the multivariate regional approach could be used for estimating multivariate quantiles when the focus is on the characterisation of the hydrological phenomenon or on obtaining the multivariate distribution of the hydrological variables when the aim is the return period of the structural failure. Further research could be performed regarding the inclusion of cross-correlation between at-site observations in the model, to reduce possible uncertainty or bias in the results.

It should be noted that although the methodology is focused on analysing floods, it could be adapted to studying other kinds of hydrological events, such as droughts.

6.2 Limitation of the results

The methodology is subjected to some significant limitations, derived from the type of the approaches, the data analysis, and the modelling choices. In the estimation of the critical failure lines for the overtopping, and flood damages, the approach is limited to bivariate analysis. It may be enhanced to include other single-valued variables, like hydrograph duration. Hydrograph shape has been included in a simplified way, supposing worst shapes selected through the OHSI ranking. This assumption leaves the results on the safe side, but the corresponding critical line may be overestimating the failure curve since not all hydrographs will have unfavourable shapes. The probabilistic analysis is based on the assumption of stationarity. There is concern that the assumption of stationarity may not hold true due to several

Conclusions

factors, like seasonality or global change. However, many similar studies are based on the same assumption (see, among others, Salvadori and De Michele, 2004).

Several assumptions of the hydraulic part of the thesis are more or less pragmatic (e.g. the 2D- simulations for urban areas), but the focus of the dissertation is building a methodology to estimate the hydrological load. The hydraulic model for the overtopping analysis is a simple one-dimensional hydrodynamic code. This model was chosen for its simplicity, and ease of use. It has, however, some limitations. The overtopped volume was estimated from water stage using the weir equation, but the overtopped volume was not removed from the flow. This approximation is acceptable because the volume diverted is tiny compared to the hydrograph volume. The hydraulic model did not include the possibility of sediment transport during flood episodes. The dynamics of sediment erosion and deposition may further alter the stage-discharge relationship and the hysteresis behaviour that determine the overtopped volume. Some cases may require the use of a 2D model with sediment transport to properly characterise the resistance factor produced by the river each. However, a large number of the required simulations (5000 hydrographs) would imply long computational times for the complete analysis. The failure criterion used in this analysis was chosen for its simplicity. The actual implementation is also based on simplifications: the use of weir equation to estimate laterally overtopped flow, and the assumption of steady-state of it, the surge only overtopping mechanism, neglecting short-period wave effects.

In general, some small deficits in the hydraulic part are more than balanced by the opportunities of bivariate statistics for design purposes. In this thesis, the theoretical approach is transposed to be applied to solve real-world problems. It also requires some simplifications and assumptions, which could be modified according to the experience of the user and which should be combined with assessments of the resulting uncertainties. Further developments are involved in improving the hydraulic modelling of the process.

In the Bivariate Failure Return Period, the main limitation is that is based on the critical hydrological failure curve, thus it is influenced by the limitation above mentioned if the failure line is determined with the proposed methodology. In flood damage estimation, beside the limitation on the estimation of the hydrological load previous discussed, the main limitation is in the estimation on the factor that conditioned stage-damage curves that it is an open research topic, but it is outside the aim of the thesis.

6.3 Outlooks

Future studies could deepen and extend the bivariate frequency analysis including the hydrograph shape in probability analysis implementing a trivariate frequency analysis with the use of Vine Copula (Joe and Kurowicka 2011). The hydrograph shape is not readily usable because the worst shape may be associated not to hazard flood event, and more, in general, it is difficult to sortable. In this sense, more in-depth analysis in the OHSI should be conducted. Future developments of this work may include the assessment of the validity of the stationarity

Conclusions

assumption, and the development of multivariate non-stationary distributions to characterise the hydrological forcing. In this case, other external factors should be taken into account (for example, the land-use change) to have a complete definition of the non-stationary distribution (Huang et al. 2017). Some recent studies are focused on this field (see, for example, Bender et al. 2014; Kang et al. 2019), and can be taken as a starting point. In future developments, other types of overtopping failure criteria may also be assessed to take into account other types of events, like hurricanes or coastal flooding, starting from the empirical approach proposed by Hughes and Nadal (2008) and CIRIA (2013). In the overtopping analysis, the 2D hydraulic models may be used, to fully account for backwater effects in meandering rivers and to improve the representation of the overtopped volume. Other types of failure should be taken into account, such as the piping. In this sense, the scenarios should include the interaction between the hydrological load, and the burrowing activity of the animals into the levee (Haubrock et al. 2016).

Appendix

A1-Evaluation of monthly parameter ε and b

We start from the entire daily discharge series available and extract the daily recession discharge series in a matrix Y_d in which a single daily recession represents each column discharge series with these characteristics:

- The series is descending.
- The minimum length of the series is 5 days (Aksoy and Wittenberg 2011).
- The series starts from N days from the previous maximum of discharge, according to the equation of (Halford and Mayer 2000):

$$N = 0.83A^{0.2} \quad A.1$$

Where A is the catchment area in km^2 .

The hourly discharge series corresponding to the periods that are recognized in the previous steps is taken, to have as many Hourly recession Limb Discharge (HLD) series (long at least 5 days, which is, 120 hours) as many periods that are identified. The matrix Y_h is created in which the single HLD series represent each column. Initially, the parameters ε , and b are evaluated for every HLD, and after they are grouped for a month by the average value. The procedure shown below is applied for each HLD. For simplicity, in the next step the single column of Y_h is called Q . The parameter b , usually ranging from 0 to 1 (Aksoy, Bayazit, and Wittenberg 2001). Wittenberg (1999) assumes that $b = 0.5$ would provide a better approximation of the observed data than the linear reservoir approach. The parameter b is first fixed equal to 0.4 and incremented of 0.01 until a maximum value $b = 0.6$ in iterative process.

- For every value of b , the parameter ε is determined according to:

$$\varepsilon = \frac{\Delta t \cdot \sum_{i=1}^n (Q_{i-1} + Q_i)}{2 \cdot \sum_{i=1}^n (Q_{i-1}^b + Q_i^b)} \quad A.2$$

- By a backward computation along the time axis, the baseflow (Q_{calc}) at the time step $t - \Delta t$ is estimated by the equation (Wittenberg 1999)

$$Q_{calc,t-\Delta t} = \left(Q_{calc,t}^{b-1} + \frac{b-1}{\varepsilon \cdot b} \Delta t \right)^{b-1} \quad A.3$$

A2- Overtopping Hydrograph Shape Index - OHSI

The hydrograph is divided into the rising, and falling limb. For each limb, the increments of discharge is evaluated, and the position of the centre of mass of the discharge increment is calculated.

$$Rising\ Limb = \begin{cases} X_{rl} = \sum_{i=0}^{D_r} \frac{D_{r(i)}^2}{2} Q_i \\ Y_{rl} = \sum_{i=0}^{D_r} \frac{Q_i^2}{2} D_{r(i)} \end{cases}, Falling\ Limb = \begin{cases} X_{fl} = \sum_{i=0}^{D_f} \frac{D_{f(i)}^2}{2} Q_i \\ Y_{fl} = \sum_{i=0}^{D_f} \frac{Q_i^2}{2} D_{f(i)} \end{cases} \quad A.4$$

Where X_{rl} , Y_{rl} are the coordinate of the centre of mass of the rising limb respect the start time of the hydrograph; X_{fl} , Y_{fl} are the coordinate of the centre of mass of the falling limb respect the start time of the hydrograph. These coordinates depend on the duration of the entire hydrograph, and the difference between peak, and the limbs smaller value of discharge, therefore a standardization of the coordinates is performed dividing X by the entire duration of the hydrograph, and Y by the difference between the peak discharge, and the smaller value of discharge of the hydrograph:

$$Rising\ Limb = \begin{cases} X_{rl,ad} = \frac{X_{rl}}{D} \\ Y_{rl,ad} = \frac{Y_{rl}}{Q - Q_{min}} \end{cases}, Falling\ Limb = \begin{cases} X_{fl,ad} = \frac{X_{fl}}{D} \\ Y_{fl,ad} = \frac{Y_{fl}}{Q - Q_{min}} \end{cases} \quad A.5$$

Where Q , Q_{min} are respectively the peak, and the minimum of discharge of the selected event. $X_{rl,ad}$, $Y_{rl,ad}$ are the adimensional coordinate of the centre of mass of the rising limb respect the start time of the hydrograph; $X_{fl,ad}$, $Y_{fl,ad}$ are the coordinate of the centre of mass of the falling limb respect the start time of the hydrograph. By multiplication the weighted mean of X coordinate, and the weighted mean of Y coordinate, the authors define the Overtopping Hydrograph Shape Index:

$$OHSI = \left[\frac{X_{rl,ad} D_r + X_{fl,ad} D_f}{D} \right] \left[\frac{Y_{rl,ad} (Q_p - Q_{min}) + Y_{fl,ad} (Q_p - Q_{min})}{(Q_p - Q_{min})} \right] \quad A.6$$

Appendix

A3- Figures relative to the copula analysis for the chapter 3.2

Poggio a Caiano

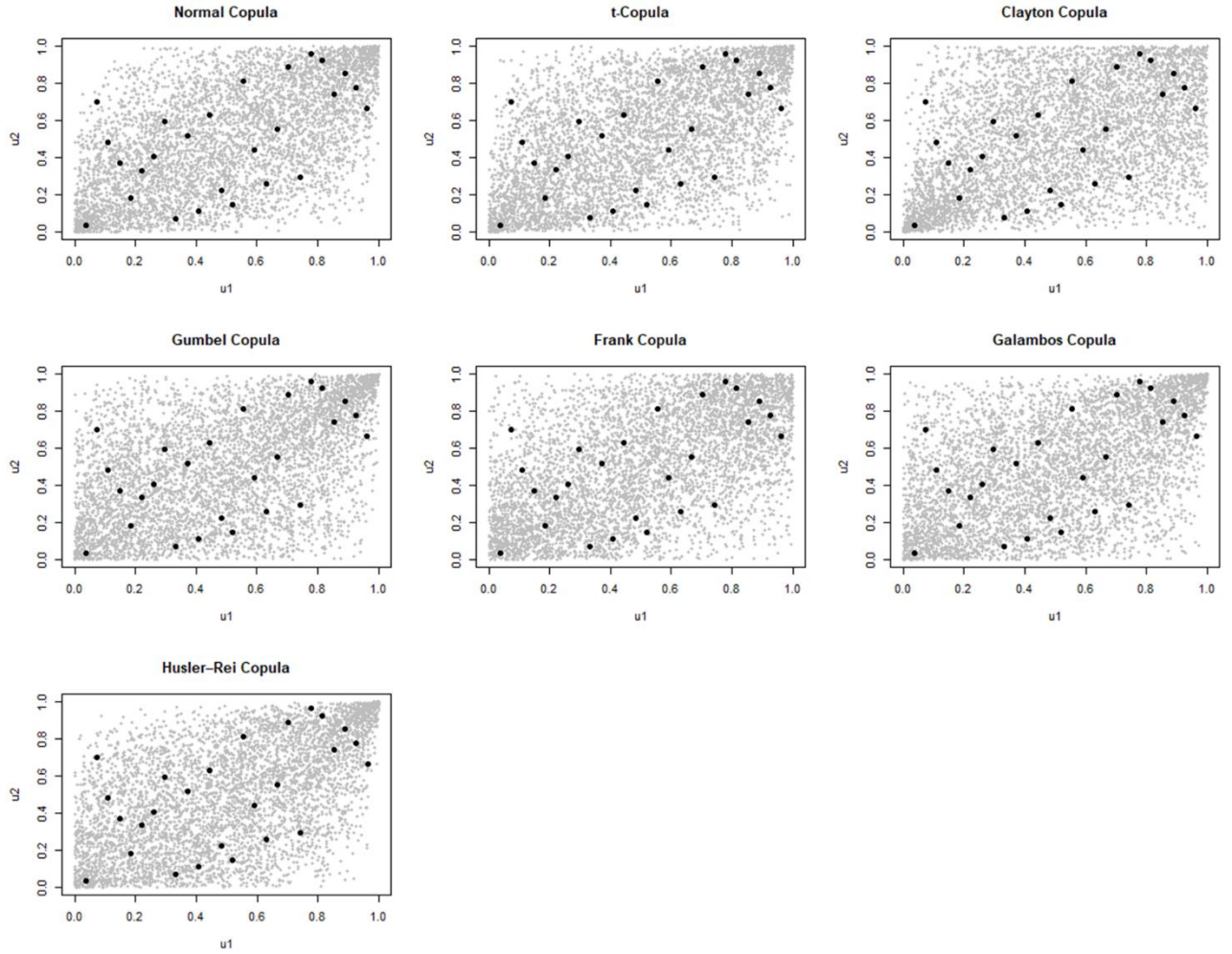


Fig. A1. Case study of Poggio a Caiano. Comparison between the sample generated from the chosen copula, and the observed data.

Appendix

Capannoli

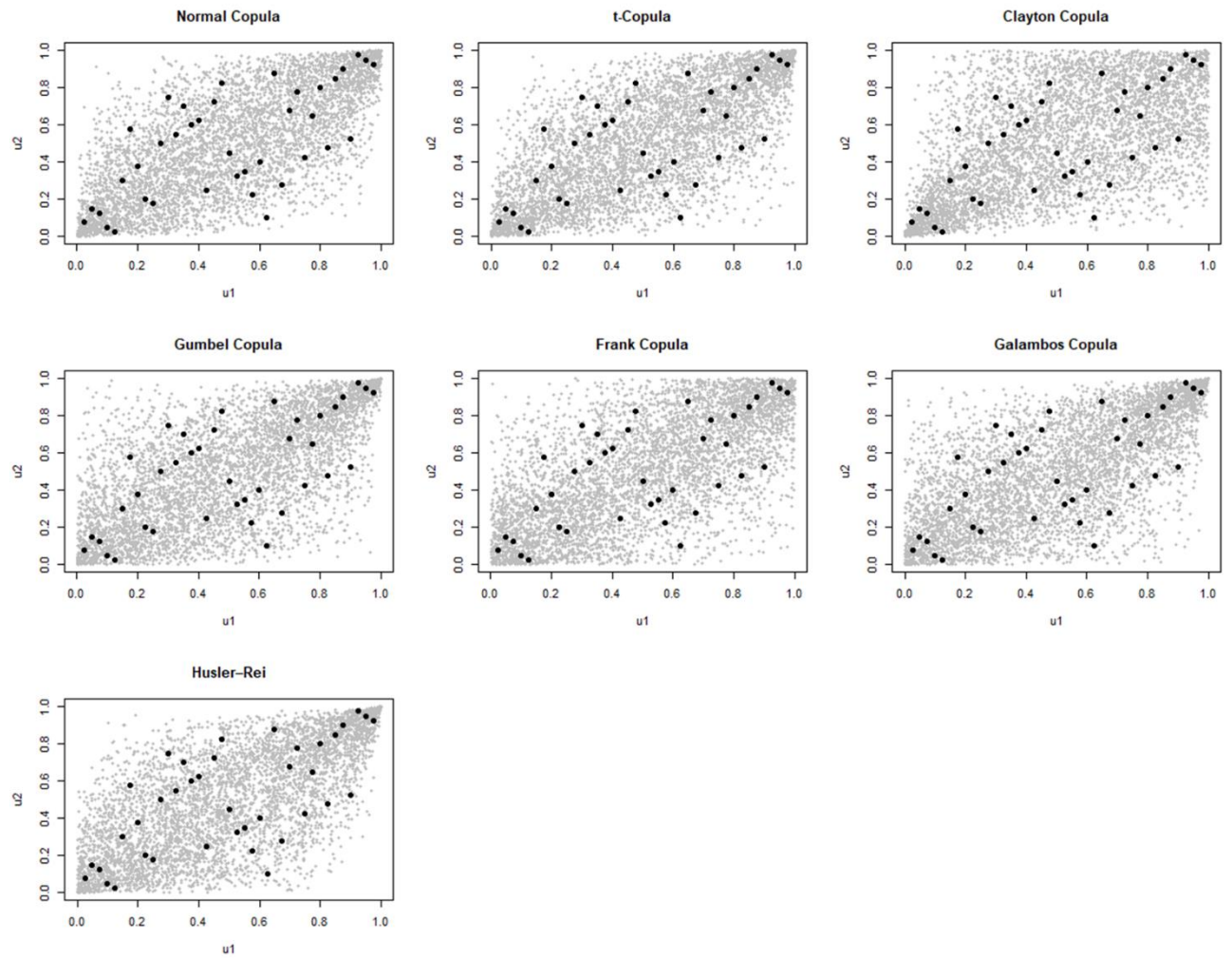


Fig. A2. Case study of Capannoli. Comparison between the sample generated from the chosen copula, and the observed data.

Poggio a Caiano

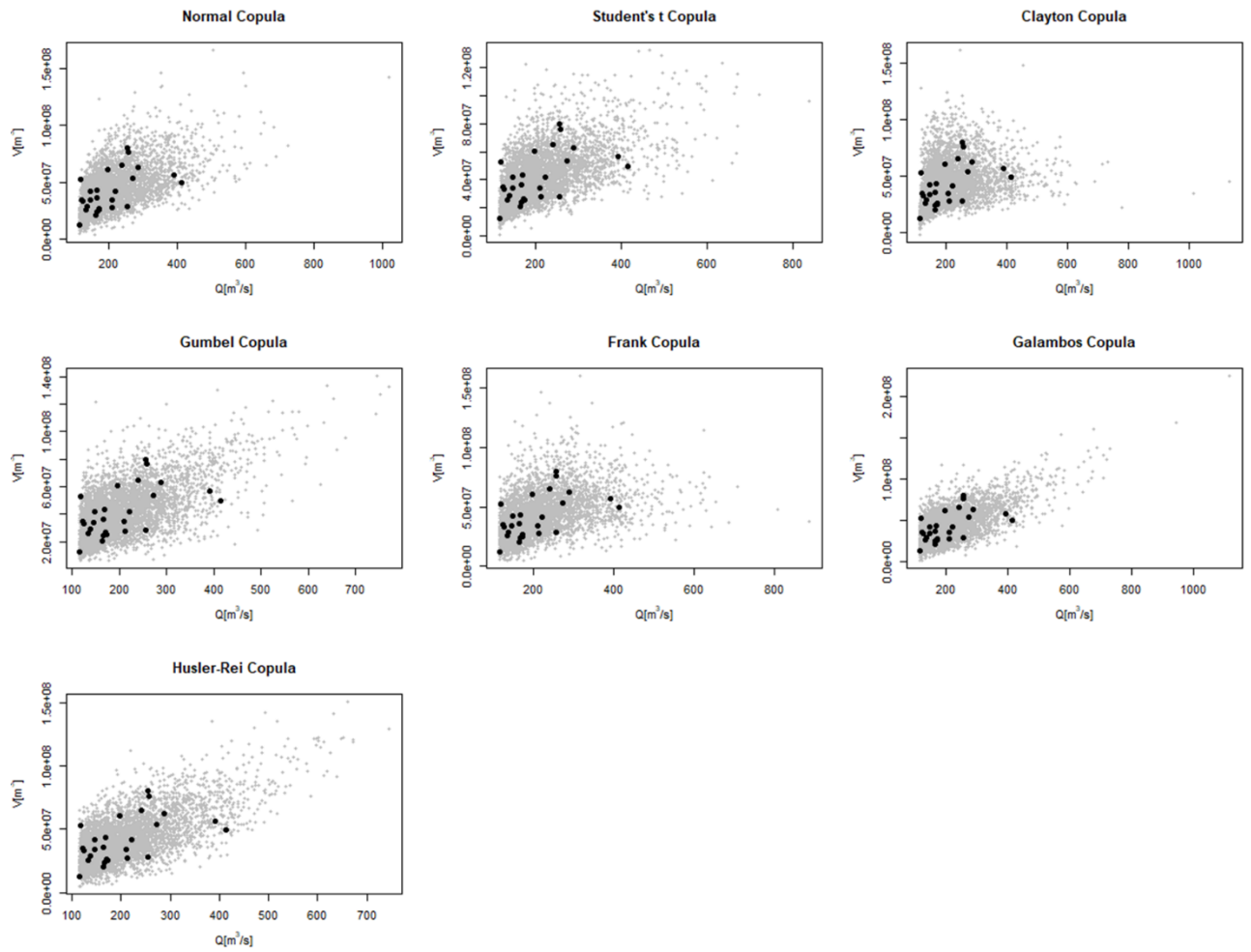


Fig. A3. Case study of Poggio a Caiano. Comparison between the samples (grey dots) generated from the chosen copula, and transformed in the quantile from the corresponding marginal distribution. The black dots represent the observed data.

Capannoli

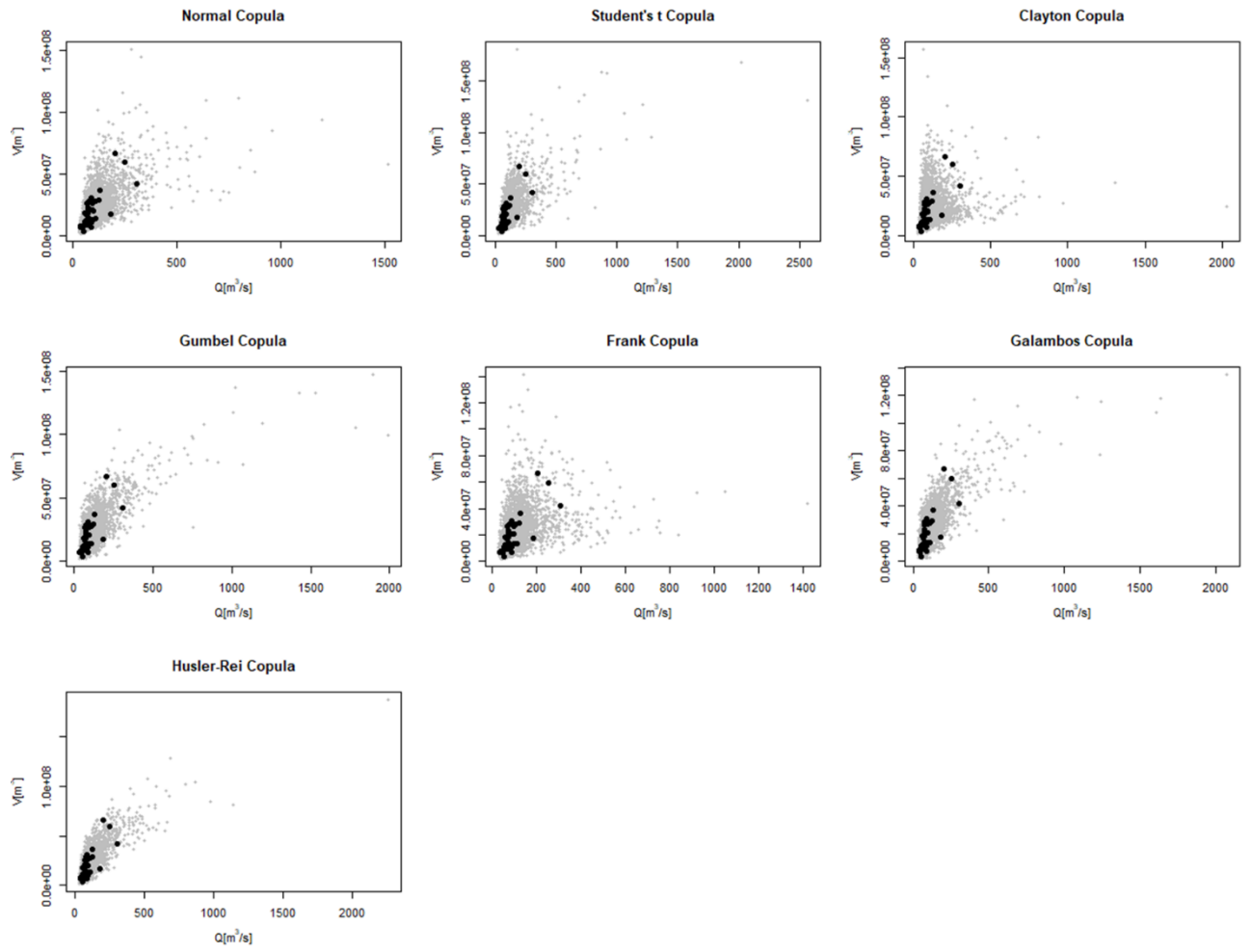


Fig. A4. Case study of Capannoli. Comparison between the samples (grey dots) generated from the chosen copula, and transformed in the quantile of corresponding marginal distribution. The black dots represent the observed data.

A4- Figures relative to the copula analysis for the chapter 5

North Region

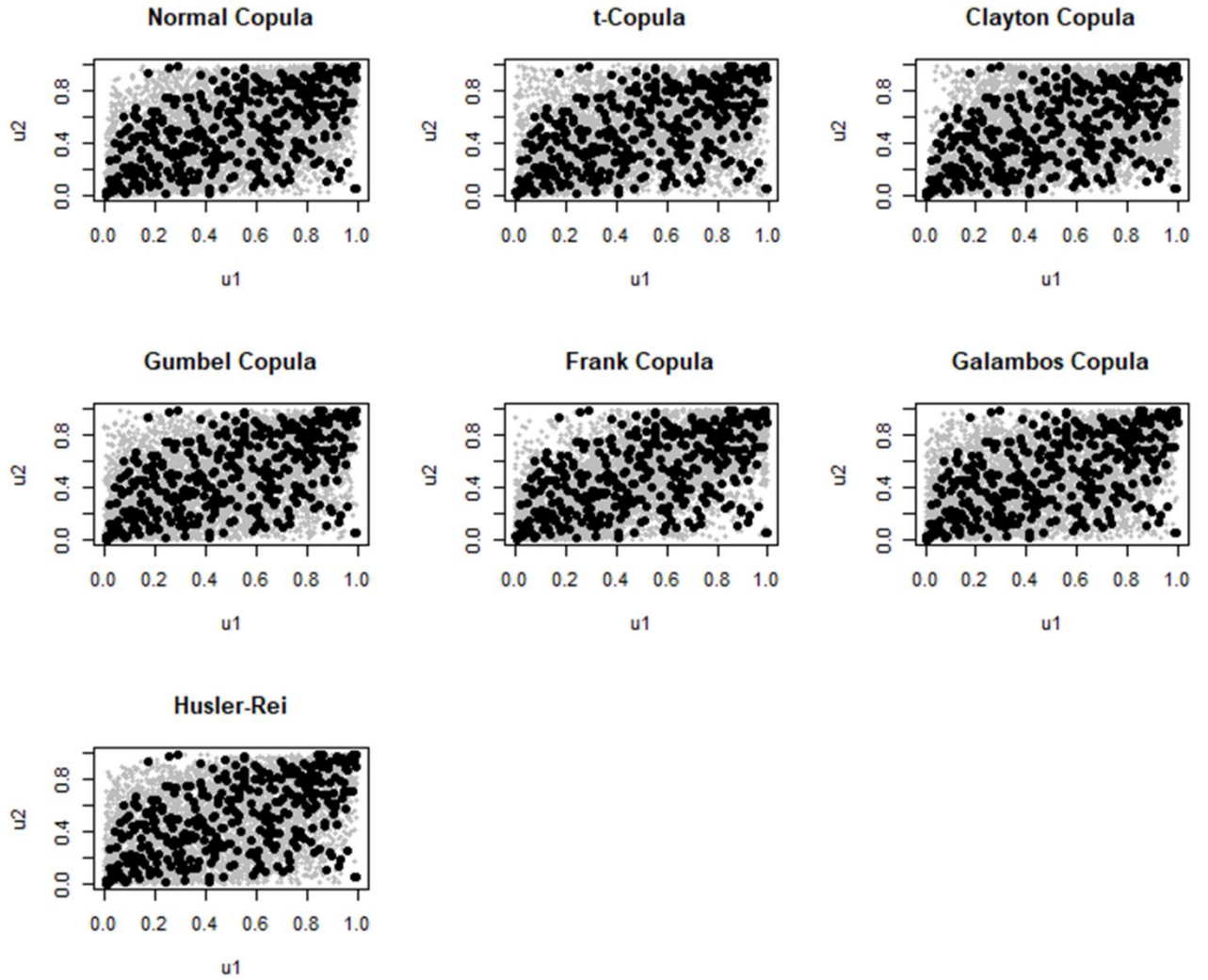


Fig. A5. Comparison between the sample generated from the chosen copula, and the observed data for the North region.

North Region

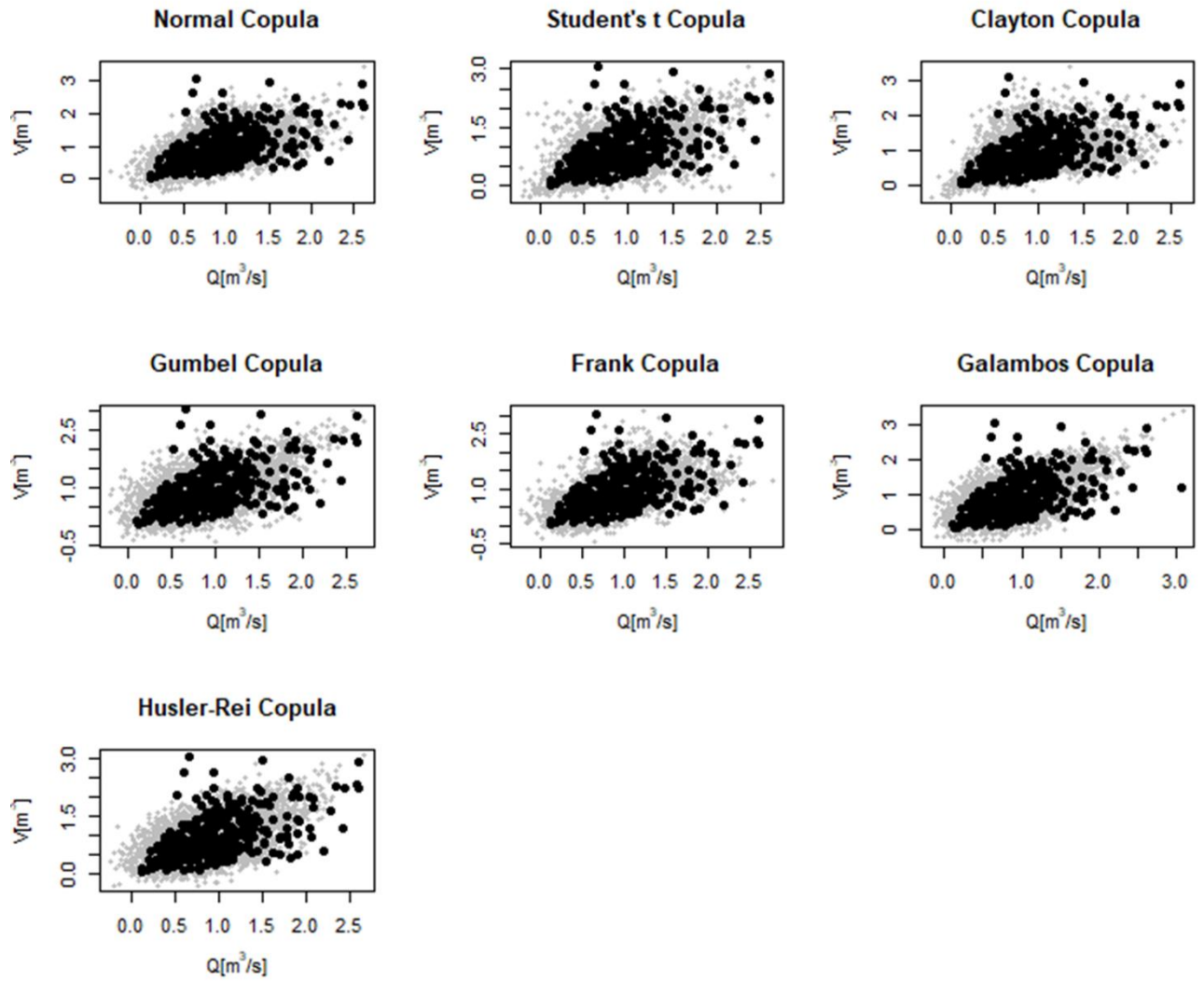


Fig. A6. North region: comparison between the samples (grey dots) generated from the chosen copula, and transformed in the quantile from the corresponding marginal distribution. The black dots represent the observed data.

Central Region

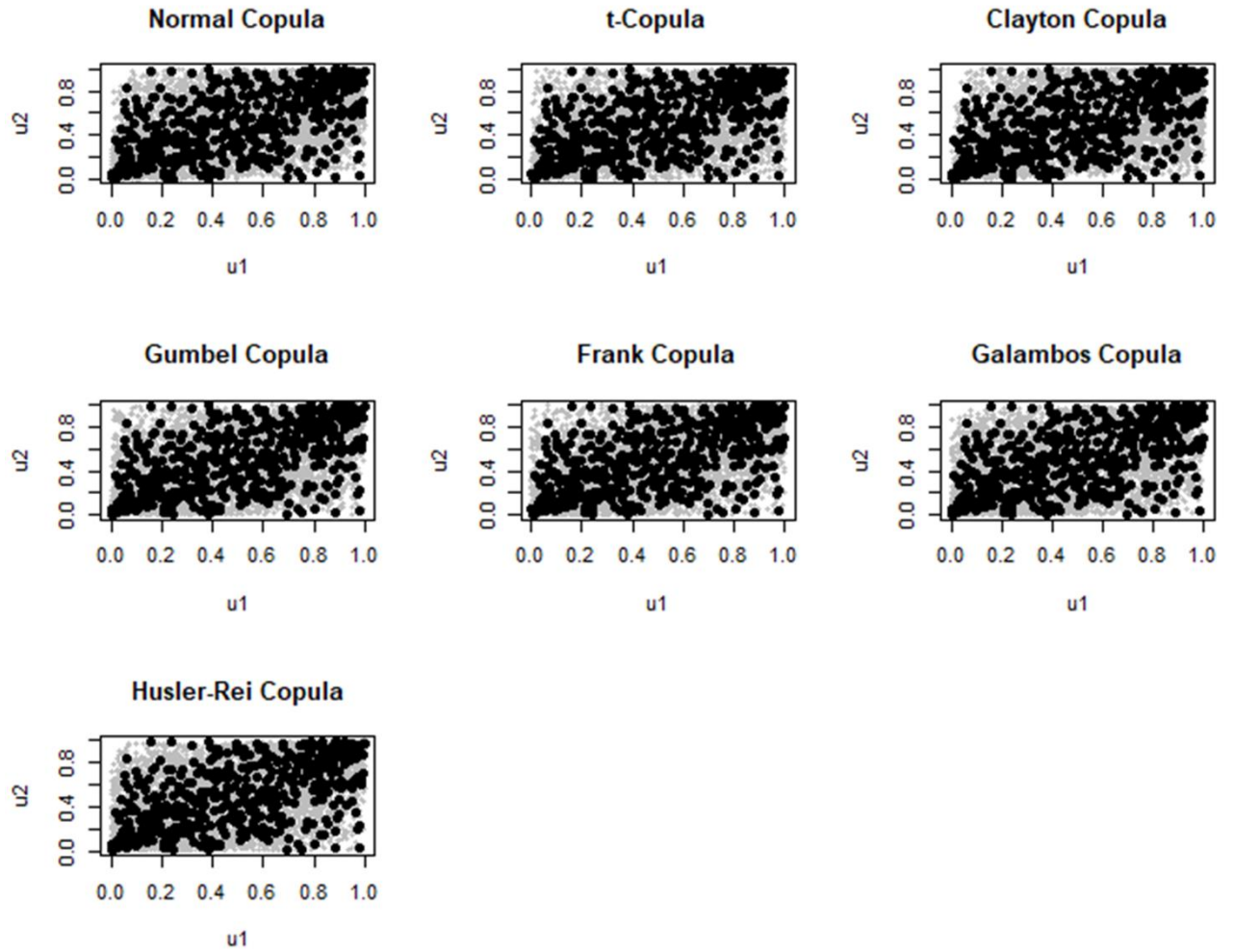


Fig. A7. Comparison between the sample generated from the chosen copula, and the observed data for the Central region.

Central Region

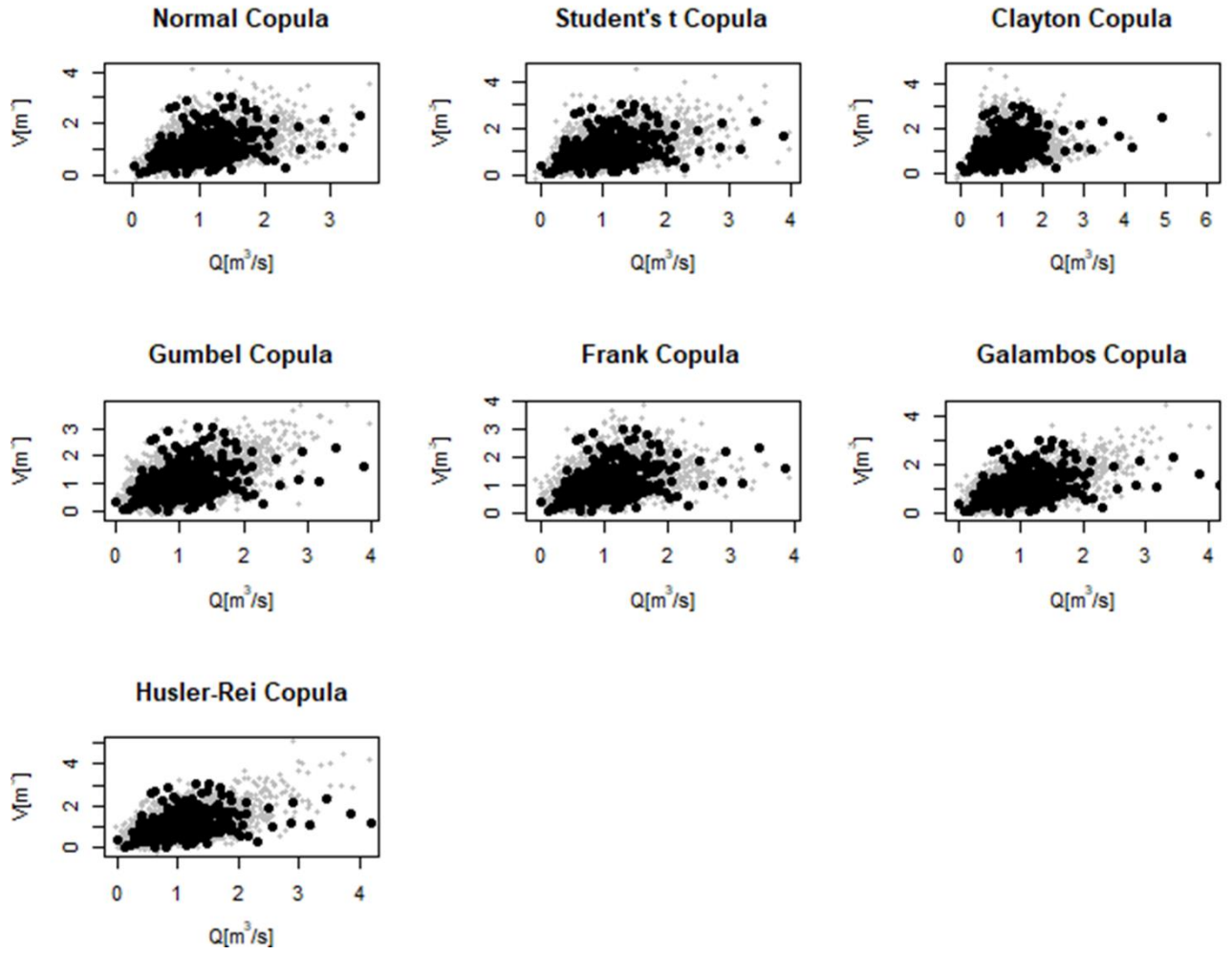


Fig. A8. Central region: comparison between the samples (grey dots) generated from the chosen copula, and transformed in the quantile from the corresponding marginal distribution. The black dots represent the observed data.

South Region

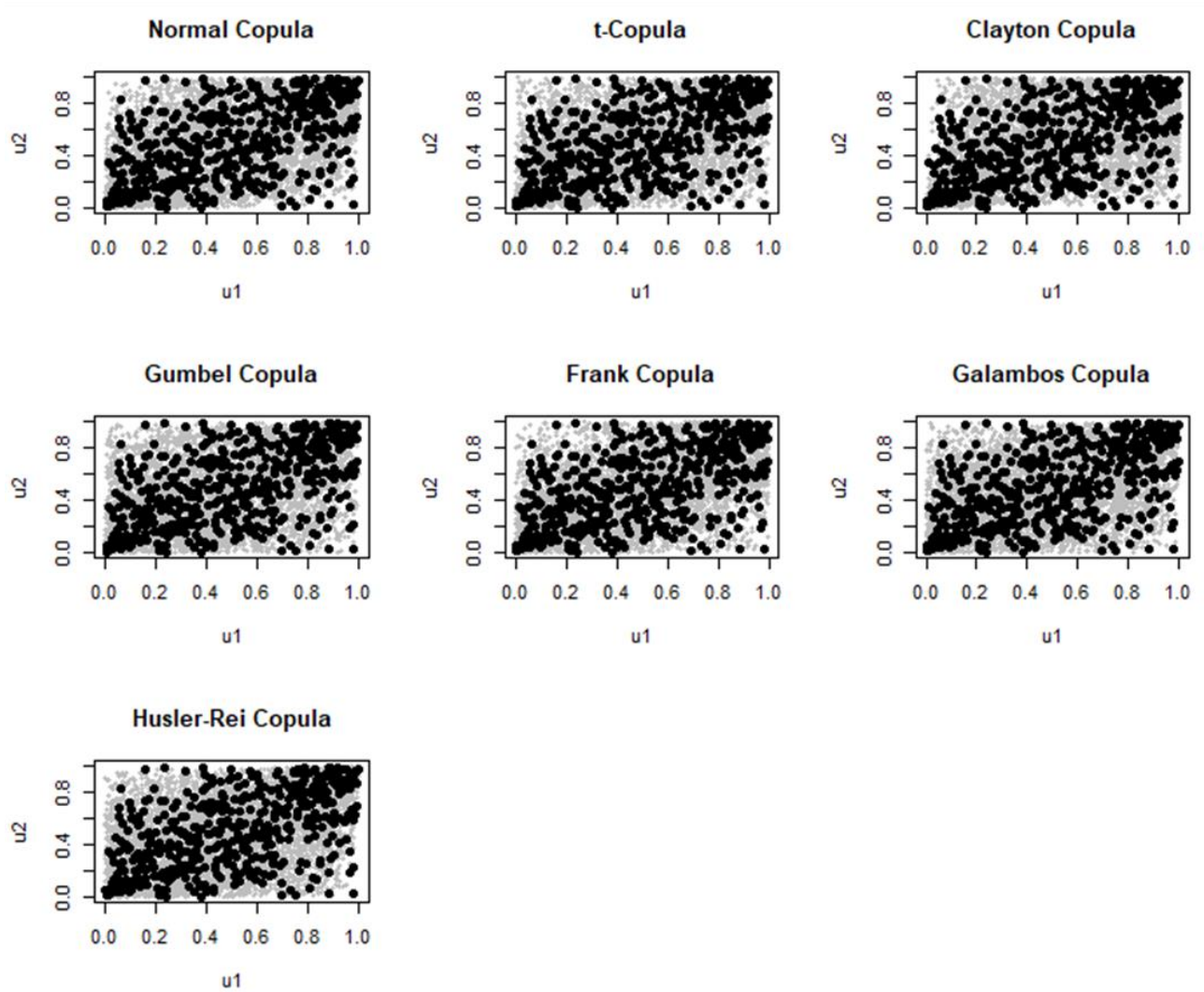


Fig. A9. Comparison between the sample generated from the chosen copula, and the observed data for the South region.

South Region

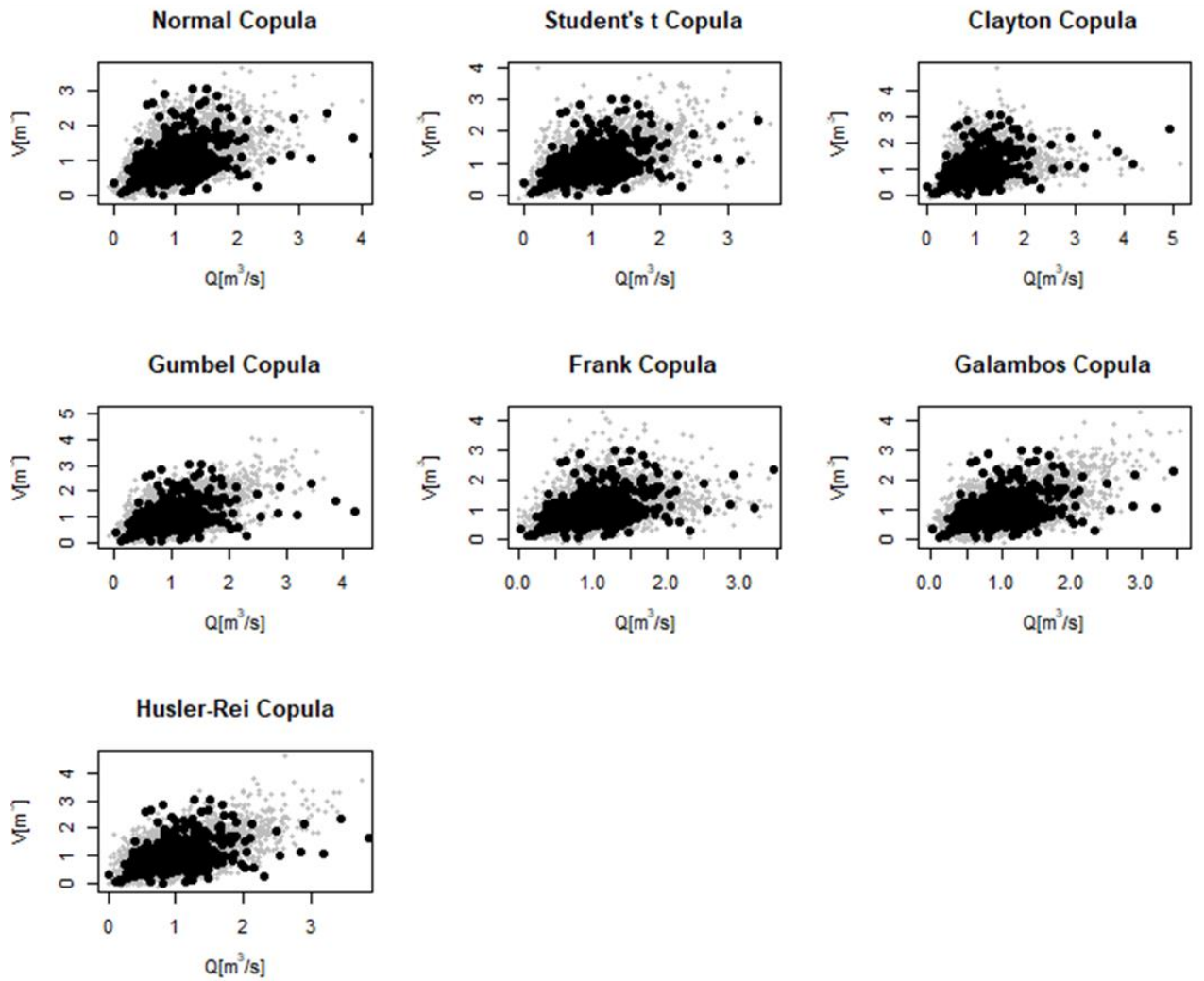


Fig. A10. South region: comparison between the samples (grey dots) generated from the chosen copula, and transformed in the quantile from the corresponding marginal distribution. The black dots represent the observed data.

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