

Tailoring the Actuator Line Theory to the Simulation of Vertical-Axis Wind Turbines

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Abstract

Analysis tools with a fidelity higher than the ubiquitous Blade Element Momentum (BEM) method are needed by now in wind energy; in particular, different research groups have recently proposed the application of the Actuator Line Method (ALM) to wind turbines, to exploit the benefits of an accurate discretization of the wake through Computational Fluid Dynamics and the computational cost saving associated to the lumped parameter modeling of the blade. When applied to Vertical-Axis Darrieus rotors, however, several shortcomings of present models are known to the scientific community, especially regarding the spreading of aerodynamic forces in the domain and the implementation of robust aerodynamic polars and dynamic stall models. Moving from this background, an ALM method numerical model for the simulation of VAWTs has been here developed within the commercial solver ANSYS® FLUENT®. Then, in the effort of tailoring the ALM to this type of machines, different features have been implemented and discussed in the present study, including a novel strategy for the sampling of the angle of attack from the resolved flow field, a sensitivity analysis on the force spreading within the domain and numerous sub-models to account for secondary aerodynamics effects. Attention has been given at ensuring robustness to the implementation of the pivotal modeling of dynamic stall. To prove the effectiveness of proposed solutions, an extensive validation has been carried out on selected test cases, for which both high-fidelity CFD and experimental data were available: a real 2-blade H-Darrieus rotor and a fictitious 1-blade machine. The developed solutions have increased the accuracy of the predicted torque up to 16% with respect to the ALM standard formulation.

Keywords: Actuator Line Model (ALM), Darrieus, Vertical Axis Wind Turbine (VAWT), Computational Fluid Dynamics (CFD), Dynamic stall

1. Introduction

Renewed interest in research applied to Vertical Axis Wind Turbines (VAWTs) has been shown lately by academia and companies [1]. These machines, in spite of the lower efficiency with respect to their horizontal-axis counterparts (HAWTs) [2], present specific advantages in non-conventional installations, like floating platforms for deep-water, offshore or small applications in turbulent environments [3]. In the first case, the possibility of shifting the generation unit on the platform, thus lowering the center of gravity, together with the insensitivity to misaligned flows [4], make these machines suitable for floating applications, where the variation of performance with respect to the wave-induced oscillatory motion of the supporting platform is key. On the other hand, installations in inherently complex flow environments, such as the urban one, might benefit from VAWT

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performance insensitivity to wind direction and turbulent fluctuations [5], especially when small rotors are considered.

These machines are nonetheless characterized by an extremely complex rotor aerodynamics. In their cycloidal motion, VAWT blades experience a continuous variation of the angle of attack, often largely exceeding the static stall limit for a relevant portion of each rotor revolution [6]. At start-up, such variation range can extend up to $\pm 180^\circ$. The result is an inherently unsteady behavior, making the estimation of the actual airfoil performance extremely difficult. This aspect, combined to the lack of research in past years, has strongly limited the effectiveness of low-order models such as the Blade Element Momentum (BEM), commonly used for HAWTs, showing that Computational Fluid Dynamics (CFD) is probably the only viable option for the analysis of these machines. On one hand, CFD is indeed able to provide an accurate description of the flow field developing around the rotating blades, allowing the proper estimation of the turbine behavior [7–10]. On the other hand, however, the computational cost associated to this numeric technique is extremely high, due to the corresponding strict requirements in terms of spatial and temporal discretization, especially when three-dimensional simulations are considered [7]. Therefore, it cannot be used yet for applications in which a high number of configurations has to be tested in a relatively limited amount time, e.g. in industrial design routines.

In order to fill the gap in terms of both computational resources and accuracy between low-order models and CFD, several so-called “hybrid methods” for the simulation of VAWTs have been proposed over the years. Among them, the Actuator Line Method (ALM) [11], is one of the most promising. In this approach, the rotating blade forces are estimated from the local inflow conditions, based on input tabulated polar data, and then plugged into the computational grid as equivalent volume momentum sources. A CFD-based approach is used on the other hand to solve the corresponding flow field. Since the actual geometry of the turbine blade is not numerically resolved, but it is modelled via a lumped-parameter representation, the overall grid density of the simulation, together with the corresponding computational cost, is notably reduced.

In the analysis of HAWTs analysis, since the original work of Sørensen and Shen [11], the ALM has been largely developed and validated, reaching nowadays a satisfying level of maturity [12]. Different codes are available to researchers, which can be already used for advanced applications such as wind farm simulation [13] or aero-elastic analysis [14]. Considering VAWT rotors on the other hand, various problems still remain unsolved or only partially tackled in the few examples presented in the relevant technical literature.

A first issue concerns the *proper extraction of the blade angle of attack* from the resolved flow field, especially when dealing with severely unsteady operating conditions of the machine. Different studies have tried to overcome this problem over the years, by either sampling in one point in front of the rotating airfoil [15–17], in correspondence of the actuator line [18,19], or in multiple locations such as the Effective Velocity Method (EVM) [20,21]. All these methodologies nonetheless lack a proper validation, due to the chronic shortage of reliable performance data on small VAWTs, which has been partially filled only in recent years. Furthermore, none of these works, except for the one of Shamsoddin and Porté-Agel [22], includes a strategy dedicated to the low-pass filtering of the sampled AoA signal, which might be biased by the disturbances coming from the solved flow field, (e.g. numeric noise, vortices, local errors, etc...), especially in the downwind region of the rotor. Although this might seem a minor problem, it is the reason why implementing a stable dynamic stall model into an ALM code has been so far a challenging task. In fact, the numerical derivation over time of the AoA signal, as required by this class of models, may amplify flow field disturbances up to the point that severe oscillations and, in extreme cases, divergence of the solution occur [20].

A second issue is associated to the VAWT blades peculiar *cycloidal motion*. As a matter of fact, since blade-wind interaction and energy extraction occur on the same plane, the number of aerodynamic effects contributing to torque production is definitely higher than in HAWTs. This, combined with the inherently unsteady operation of these machines, makes the accuracy of hybrid models such as the ALM extremely sensitive to the combination of the various *sub-models* used to

account for such secondary phenomena [23]. Although the implementation of some of these effects, such as *flow curvature* [18–20] and *dynamic stall* [18,19,22,24], into an ALM framework has already been investigated by some researchers, very few studies conducted a systematic analysis of their separated and combined influence on the predicted turbine performance. Other VAWT features, such as the contribution to torque induced by the airfoil *pitching moment* and the influence of the position of the *Blade-Spoke Connection (BSC)* point on the blade loads, have never been considered so far to the best of authors' knowledge.

The third issue is the *insertion of the computed blade forces into the computational domain*. As a matter of fact, the majority of studies on ALM applied to VAWTs tend to marginalize this aspect, adjusting the parameters of the insertion function (*Regularization Kernel*) using as only criterion the stability of the simulation. Various works in the HAWT field nonetheless [25–27] demonstrated how relating the kernel distribution width to a characteristic dimension of the rotor blade, e.g. the chord, can increase the approach robustness and provide a more physically sound description of the flow pattern around the blades, in particular of the tip vortex.

1.1 Outline and contributions of the study

Starting from this background, this study presents the implementation of an Actuator Line tool for vertical-axis turbines in the widely diffused commercial CFD software ANSYS® FLUENT® (version 18.2). To this end, the User Defined Function (UDF) feature available in the software has been exploited, allowing the creation of an independent module, written in C language, which can be loaded into a generic CFD case without additional set-up by the user. Beyond the specific solver used in the study, it is worth remarking, however, that the proposed methods and models are of general validity and easily replicable in different software environments.

The proposed tool has been specifically designed to address the three issues outlined in the previous section, in an effort to fill the development gap of the ALM approach between horizontal- and vertical-axis machines.

To tackle the problem of *velocity sampling*, a novel strategy for obtaining a high-quality angle of attack signal from the resolved flow field [28] has been introduced. The latter combines a *LineAverage* [29] method for the AoA sampling, with a Cubic Spline Smoothing (CSS) algorithm for the low-pass filtering of the extracted signal. This approach, which shares many features with the latest generation of integral methods available for HAWTs [30,31], such as the low-pass filtering of spatial flow disturbances, has been adapted by the authors to VAWTs and validated on high-fidelity CFD and experimental data of a test Darrieus rotor [32].

The main contribution of this work to the modelling of the effects associated to the blade *cycloidal motion* comes instead from the proposed modification of the ALM base formulation. The latter in fact has been here adapted to rotor configurations, in which the blade is connected to the shaft at an arbitrary position along the chord, i.e. at a custom Blade-Spoke Connection (BSC) point [33]. Combined with another element of novelty of this work, which is the correction of the blade torque for the airfoil *pitching moment* [33], such new formulation notably increases the variety of Darrieus rotor designs than can be successfully simulated with this kind of tools. Other *sub-models* implemented in proposed ALM, such as *flow curvature effects* [34], and *dynamic stall* [35], although already available in some existing pieces of software, have nonetheless benefitted from the experience acquired by the authors over years of research in the field. In particular, a more advanced strategy for the modelling of flow curvature effects, based on the conformal mapping of the blade airfoil, has been adopted, supposedly improving the accuracy of the algebraic corrections currently employed in the field.

Finally, regarding the *insertion of the computed aerodynamic forces into the computational domain*, a more rigorous criterion, based on the turbine blade chord, has been here adapted to VAWT simulation.

An extensive validation has been carried out on selected test cases: a theoretical 1-blade H-Darrieus rotor, derived from an existing turbine prototype, and a real 2-blade machine. An element

of novelty of this work with respect to available literature is that, in addition to the wake velocity data usually employed for the validation of this kind of tools, high-fidelity CFD and experimental blade loading data was used to assess the code reliability. More in detail, the availability of the blade forces profiles in the spanwise direction and along one rotor revolution allowed to systematically investigate the effects of the different code features and their combination on the predicted turbine performance. Eventually, this data has been used to gain a deeper insight into the capability of the proposed ALM approach to reconstruct the different features characterizing the three-dimensional flow field developing across a VAWT rotor, e.g. the tip vortexes.

2. Test cases

Two different test cases have been selected for the simulations carried out to develop and validate the proposed models, namely the *R515* and *R500* H-type Darrieus rotors. The corresponding geometric and operational characteristics are reported in Table 1 and Table 2, respectively.

- *R515*: prototype three-blade machine, extensively tested in the experimental facility of the Politecnico di Milano by Dossena et al. [36]. Recently, it has been used also as the validation test case for the Actuator Line Model developed in [20]. For the present study, however, it was considered as test case its fictitious one-blade variant, simulated with a high-fidelity, three-dimensional blade-resolved CFD approach by Balduzzi et al. [7]. As a matter of fact, the corresponding availability of detailed wake velocity and blade spanwise loading data, combined with the absence of the solidity effects commonly encountered in the real multi-blade rotors, allowed to gain a deeper insight into the capability of the present ALM tool to reproduce the blade three-dimensional behavior, e.g. the tip vortexes. Furthermore, the peculiar position of the blade-spoke connection ($x/c=0.5$), makes this rotor the ideal candidate for the testing of the corresponding model introduced in this study (please refer to Section 3.1).
- *R500*: real two-blade, H-shaped Darrieus turbine (see Figure 1), tested in different studies [37,38] in the Open Jet Facility (OJF) at TU Delft. For this machine, both three-dimensional flow fields [37] and azimuthal blade forces profiles [38] are available from *ad hoc* post-processing of measured PIV (Particle Image Velocimetry) data, allowing to extend the analysis carried out for the *R515* single-blade turbine to a higher solidity rotor. To further support the validation process, two-dimensional blade-resolved CFD simulations from Melani et al. [39] are also employed as benchmark next to experimental measurements. A schematic depiction of the machine is shown in Figure 1. In this case, the availability of reference blade loading data for both a turbine *unstable* (TSR=2), i.e. characterized by dynamic stall over a major portion of one rotor revolution, and *stable* (TSR=4.5) operation point, respectively, allowed to investigate the capability of the methodologies (please refer to Section 3.5) implemented in the ALM tool to describe the effect of the dynamic stall phenomenon on turbine behavior.

As apparent, the choice of these rotors for the investigation is justified not only by the quantity and quality of the corresponding available benchmark data, but also by the possibility of testing the ALM prediction capabilities over a wider range of turbine architectures, solidities and airfoils. Furthermore, both test cases fall in the low-to-medium solidity range, which is particularly indicated for hybrid simulation models, as their accuracy in fact strongly decreases going towards high-solidity configurations [40].

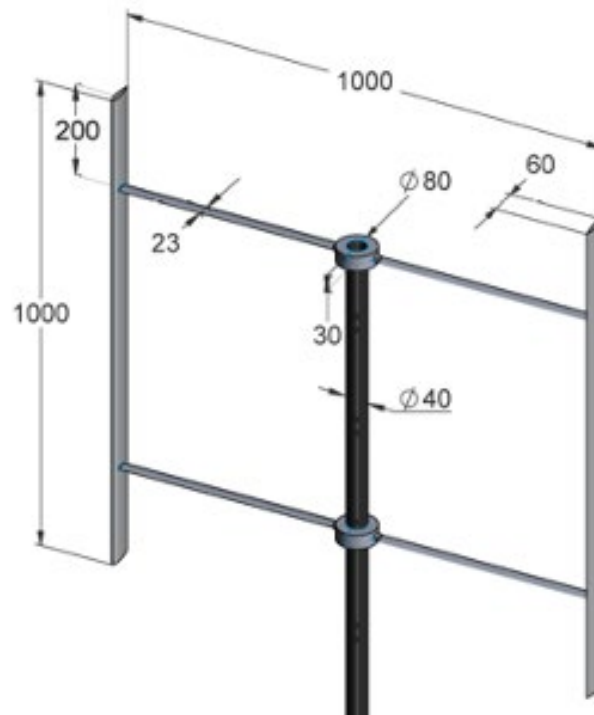


Figure 1 - R500 test case

Table 1 - Details of test rotors selected for the investigation.

Dimension	R515	R500
Number of blades [-]	1	2
Radius [m]	0.515	0.500
Chord [m]	0.086	0.060
Airfoil	NACA0021	NACA0018
Blade-spoke connection [x/c]	0.5	0.25

Table 2 – Test operating conditions.

Inlet value	R515	R500	
Tip-speed ratio [-]	3.3	2	4.5
Wind speed [m/s]	9.0	10.2	9.1
Air pressure [Pa]	1.1e5	1.1e5	
Air temperature [°C]	27	20	
Turbulence intensity [%]	0.05	0.24	

3. Actuator Line Model

In this section, the different components of the proposed ALM model are presented in detail. First, the ALM base formulation is briefly illustrated, together with the modifications to the rotor modelling introduced in this study to account for the *pitching moment* and the *blade-spoke connection* effects. The novel *LineAverage* method for sampling the angle of attack from the CFD flow field is then presented, followed by the strategies adopted for the insertion of the blade forces into the computational domain (*regularization kernel*) and the compilation of the tabulated polar data to be used as an input for the code. Eventually, the strategy recently devised by the authors to tackle the implementation of dynamic stall models into ALM codes is explained; as a demonstrative case, the Berg model is considered, even though the proposed strategy is of use for any other method.

3.1 Actuator Line algorithm

In its most general formulation, the ALM algorithm can be synthetically described as follows. Please refer to Figure 2 for the adopted conventions. At each timestep, the Aerodynamic Center (AC) of each blade element, i.e. the corresponding airfoil quarter-chord point, is located on the grid based on the turbine characteristics and operating condition.

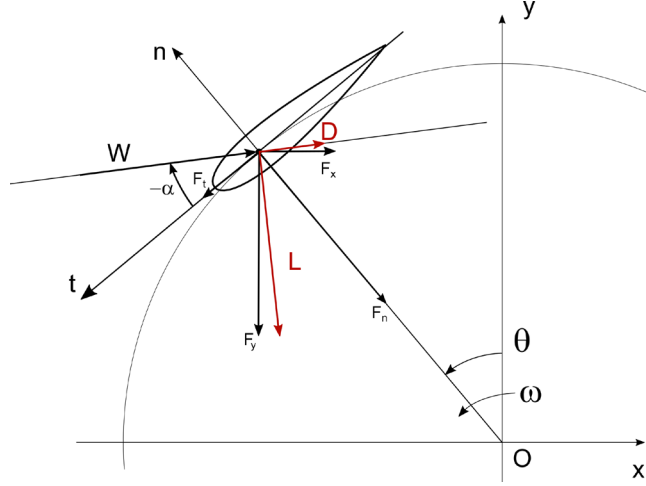


Figure 2 - Conventions adopted for the present study.

The local undisturbed velocity V is then extracted from the previous timestep resolved CFD flow field, according to selected velocity sampling strategy. By combining it with the blade peripheral speed ωR , the blade relative velocity W is obtained and used to evaluate the blade angle of attack α and chord-based Reynolds number Re_c , as in Eqs. 1 and 2:

$$\alpha = \arctan\left(-\frac{W_n}{W_t}\right) \quad (1)$$

$$Re_c = \frac{\rho W c}{\mu} \quad (2)$$

$$L = 0.5 \rho c W^2 \cdot C_L(\alpha, Re) \quad (3)$$

$$D = 0.5 \rho c W^2 \cdot C_D(\alpha, Re) \quad (4)$$

where W_n and W_t are the blade relative velocity components in the normal and tangential directions with respect to the blade trajectory, respectively, c its chord, ρ the freestream flow density and μ the freestream dynamic viscosity. Such information is used to compute the airfoil lift (C_L) and drag (C_D) coefficients via linear interpolation of the tabulated aerodynamic data given as input to the code. In order to take into account second-order aerodynamic effects, these coefficients are corrected via *ad hoc* sub-models. Eventually, the blade element lift (L) and drag (D) forces are computed according to Eqs. 3 and 4 and inserted into the computational domain as volume momentum sources f_b , after having been smeared out by means of a proper function (*Regularization Kernel*). As a matter of fact, their direct insertion into the mesh cell intersected by the actuator line would cause a singularity in the solution, with the corresponding instability and numerical oscillations issues [41].

After the above procedure has been performed for all the elements along the blade span, the modified governing equations for an incompressible flow, i.e. the continuity (Eq. 5) and momentum (Eq. 6), are solved:

$$\nabla \cdot \mathbf{v} = 0 \quad (5)$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) \right) = -\nabla p + \nabla \cdot (\mu \nabla \mathbf{v}) + \mathbf{f}_b \quad (6)$$

where $\mathbf{v}$ is the resolved flow velocity vector.

When dealing with vertical-axis machines, however, the formulation described so far, which is the standard for the majority of ALM codes, needs to be partially revised, due to the specific behaviour presented by the turbine blades when moving in a cycloidal fashion. More in detail, two modifications, coming from the experience of the authors in the field of VAWT dynamics, are here introduced for the first time: the *pitching moment* and the *blade-spoke connection (BSC)* corrections.

Pitching moment correction

In contrast with HAWTs, in Darrieus rotors the airfoil pitching moment C_M operates in the same plane of the turbine rotation [42]. Thus, it must be taken into account as an additional contribution in the shaft torque computation, as in Eq. 7:

$$\text{torque} = F_t \cdot R + M = F_t \cdot R - C_M \cdot 0.5 \cdot \rho \cdot c^2 \cdot W^2 \quad (7)$$

where the pitching moment C_M comes from input tabulated data.

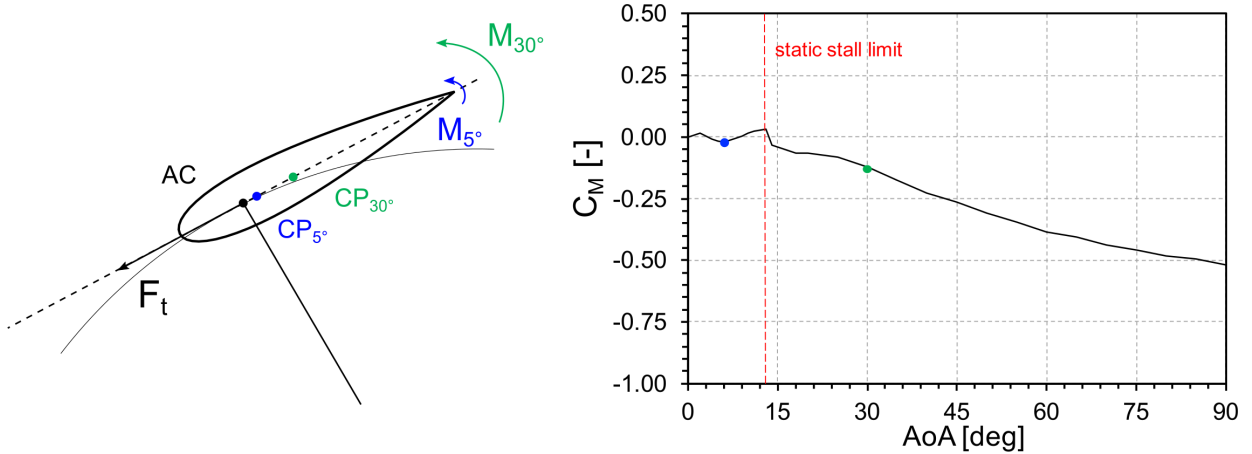


Figure 3 - Schematic representation of the pitching moment effect.

As shown in Figure 3, The magnitude of the pitching moment grows considerably as the static stall limit is exceeded, since after that point the position of the airfoil *Center of Pressure (CP)*, where the blade forces are actually applied, starts deviating significantly from the one of the aerodynamic center [43]. As a consequence, the weight of this correction becomes increasingly important as the operation AoA range is increased, i.e. as the TSR is reduced.

Blade-spoke connection (BSC) correction

The majority of aerodynamic codes are based on the assumption that the blade aerodynamic center (AC) and the Blade-Spoke Connection point (BSC), i.e. its point of connection to the corresponding support strut, always coincide. As demonstrated by Bianchini et al. [33], however, in the general case of $BSC \neq AC$, this hypothesis introduces a notable error in the blade angle of attack and forces computation.

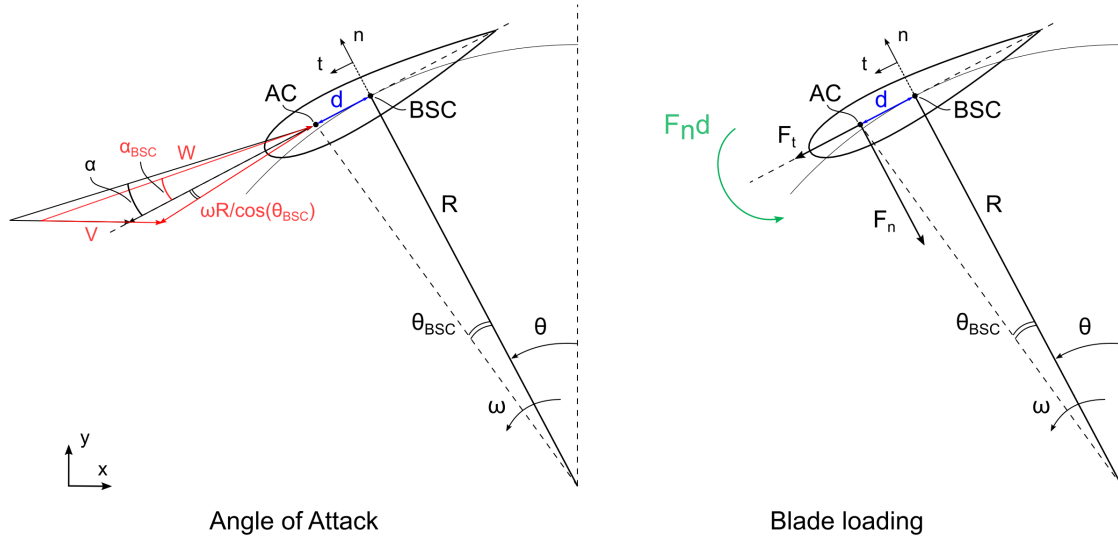


Figure 4 - Effects of shifting the AC with respect to the BSC on blade aerodynamics.

On one hand in fact, the AC peripheral velocity presents an angular shift θ_{bsc} with respect to the nominal one ωR . This provokes a deformation of the computed blade velocity triangles (see Figure 4), modifying the corresponding angle of attack, as in Eqs. 8, 9 and 10:

$$\theta_{bsc} = \tan^{-1} \left(\frac{c \cdot d}{R} \right) \quad (8)$$

$$W_n = -V_x \cdot \sin(\vartheta) + V_y \cdot \cos(\vartheta) + \omega \cdot R \cdot \tan(\vartheta_{bsc}) \quad (9)$$

$$W_t = -V_x \cdot \cos(\vartheta) - V_y \cdot \sin(\vartheta) - \omega \cdot R \quad (10)$$

where θ is the BSC angular position, d is the AC-BSC distance and R is the turbine radius. It must be noted how, if $AC \equiv BSC$, i.e. $d=0$, the last term in the computation of Eq. 10 vanishes, yielding the nominal relative velocity components. From a global point of view, this effect results in an apparent pitch applied to the turbine blade, of the same order magnitude of θ_{bsc} .

On the other hand, due the shifting of AC with respect to BSC, a lever arm is introduced between the blade normal force and the turbine axis of rotation, as evident from Figure 4, so that an additional contribution arises in the computation of the torque at the shaft, as in Eq. 11:

$$torque = F_t \cdot R + M + F_n \cdot d \quad (11)$$

3.2 Angle of attack sampling

In the ALM method, one of the core issues is the quality of the angle of attack signal coming from the resolved flow field, which is then used to compute the blade loading as in section 3.1. As a matter of fact, it is pivotal not only to sample the correct value of AoA from the CFD flow field, but also to minimize the corresponding signal-to-noise ratio. The latter aspect is particularly relevant when this signal needs to be numerically derived over time to be given as an input to a dynamic stall model, which in absence of ad hoc filtering strategies may lead to severe oscillations of the solution and, in extreme cases, to divergence [20].

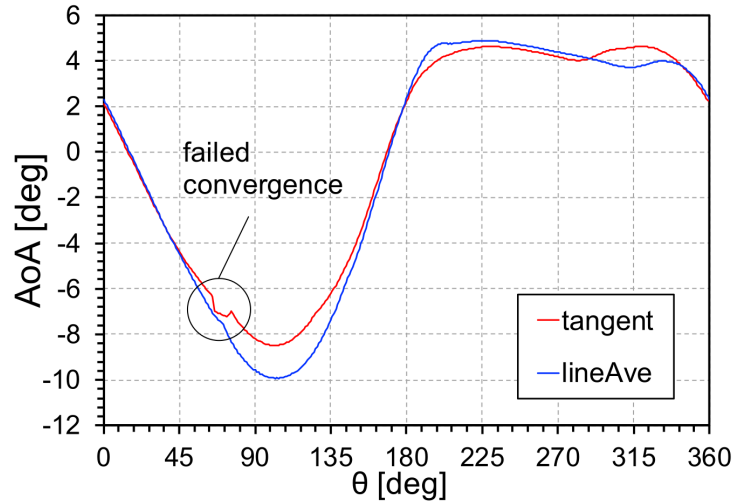
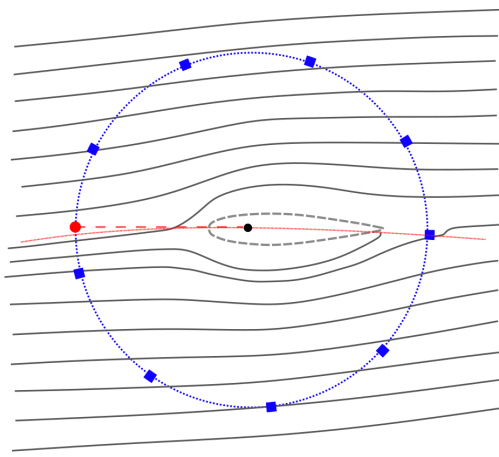


Figure 5 - Schematic comparison between the Tangent and the LineAverage methods.

In order to address these issues in the present work, the framework developed by the authors in a recent publication [28] has been adopted, which combines a novel velocity sampling method (*LineAverage*) with a low-pass filtering of the sampled data to remove the corresponding background noise. This is achieved with a dedicated *Cubic Spline Smoothing (CSS)* algorithm [44], which at the beginning of each rotor revolution is applied to the AoA signal coming from the previous one, so that no *real-time* processing of the signal is required.

The *LineAverage* method (LineAve), originally proposed by Jost et al. [29] for HAWTs, computes the undisturbed velocity V as the integral average of the flow velocity field along a closed line around the airfoil, as shown in Figure 5a. According to the creators of the method, this sampling strategy should be able to completely remove the effect of bound circulation on the local inflow velocity, since in the averaging process the induced velocity components on any pair of opposite points on the closed path are levelled out, still accounting for the net distortion associated to trailing vorticity.

As verified by the authors in a previous study [32], where the performance of different methods has been compared on fully-resolved CFD simulations of the same *R500* test rotor adopted for this work, this approach provides the best accuracy and robustness when applied to turbine stable operation, i.e. when the angle of attack does not exceed the static stall limit for the major part of one rotor revolution. In unstable conditions nonetheless, the interaction of the sampling line with the dynamic stall vortices shed in the airfoil wake results in an unstable AoA trend.

When applied to the ALM on the other hand, the *LineAverage* does not present the aforementioned issue, due to the absence of the vortices causing the instability in the resolved flow field. Conversely, additional benefits arise when compared to the methodologies available until now for the analysis of VAWTs ([15,17,18,20]):

1. *no dependency on input polar data*: the effect of bound vorticity is inherently accounted in the velocity averaging, so that no theoretical corrections based on lift data are needed, improving accuracy and robustness;
2. *spatial smoothing of the AoA signal*: by averaging a large amount of velocity data from the flow field, the corresponding background numeric noise is automatically filtered out, improving the AoA signal quality.

As an example, in Figure 5 the proposed methodology is compared with the one previously embedded in the code, i.e. the *Tangent method* [23], in which the undisturbed flow velocity is sampled in a single monitor point along the tangent to the blade trajectory. In the latter case, when the iterative correction required by the method to account for the airfoil bound vorticity effect [39] does not converge, the

sampled AoA azimuthal profile presents relevant discontinuities, undermining the simulation stability. This aspect is particularly critical when adopting a dynamic stall model. Regarding the method set-up, it has been chosen for the present study to use a circular sampling line, centered in the aerodynamic center of the airfoil (assumed as the quarter-chord) and with a radius $r=lc$ and a resolution of $N=80$ equally-spaced points along the sampling line.

3.3 Regularization Kernel

As anticipated in section 3.1, the aerodynamic forces have to be applied to the flow domain and properly spread on the mesh to avoid non-physical discontinuities [41]. As highlighted in numerous studies [16,45], the proper selection and calibration of such function is pivotal for the robustness and accuracy of the ALM approach.

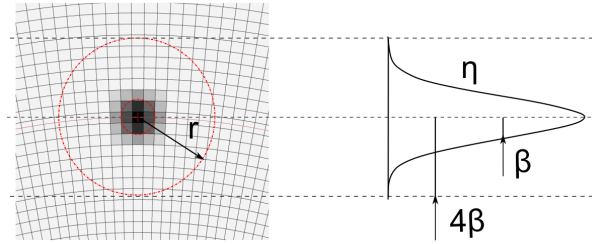


Figure 6 - Insertion of forces in the CFD domain via gaussian regularization kernel.

In the present work, the common *isotropic two-dimensional Gaussian kernel* η , defined in Eq. 12, has been used:

$$\eta(r) = \frac{1}{\pi\beta^2} e^{\left[-\left(\frac{|r|}{\beta}\right)^2\right]} \quad (12)$$

where r is the distance between the generic cell centroid and the actuator line, while β is a specific width parameter related to the kernel distribution. An example of Gaussian smearing application is shown in Figure 6.

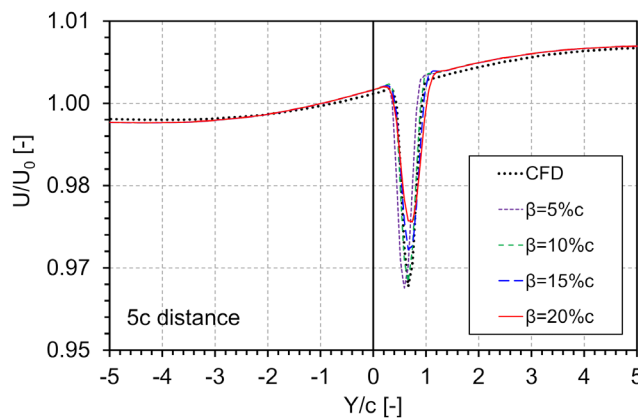


Figure 7 - Wake velocity comparison between CFD and ALM from [15].

According to [46], this strategy, which results in a cylindrical force distribution when considering the whole blade span, allows to better model the geometrical discontinuity characterizing the blade tip, improving the resolution of the corresponding shed vortexes. Furthermore, it does not suffer from the element-to-element overlap issues that are typical of the more common three-dimensional spherical kernel. The Gaussian distribution width β on the other hand was selected based on the calibration campaign performed by the authors in a previous study [17]. In the latter, a test symmetric

NACA0018 airfoil was analyzed in static conditions, at given angle of attack and Reynolds number, with a 2D blade-resolved CFD approach. The CFD aerodynamic coefficients C_L and C_D were then used as inputs for the corresponding ALM simulation, which was carried out for a wide range of mesh resolutions β/h and kernel width β/c . It must be noted how, following the guidelines from [25,27], the kernel width has been related to a characteristic airfoil dimension, in this case the chord c . Upon examination of the velocity field sampled at a distance of $2c$ and $5c$ from the airfoil aerodynamic center, here reported for clarity in Figure 7, the authors recommend the values of $\beta/h \geq 3$ and $\beta/c=0.1$ as an optimal trade-off between robustness and accuracy.

For the current work however, the guideline kernel width was relaxed to $\beta/c=0.2$, together with a reduction of the ALM grid resolution to $\beta/h=2$. As demonstrated by the mesh sensitivity analysis reported in section 4 in fact, these settings allowed to considerably reduce the method computational cost, with a negligible variation in the overall accuracy.

3.4 Polar data

As for every aerodynamic code based on the blade element theory, the accuracy of the ALM methodology is largely based on the quality of the input polar data. For small VAWTs, such as the one under consideration, this issue is made critical by the operation of the turbine blades under the low-Re regime ($Re_c \lesssim 1000k$). As shown in Figure 8, where the aerodynamic data of the *R500* test rotor are reported as an example for the extremes of the Re_c range of interest, this corresponds to a large variability of the profile lift curve, both in terms of shape and maximum value. Furthermore, it is inherently challenging to obtain reliable data from experiments or CFD in those flow conditions [47]. In the present work, the pre-stall aerodynamic coefficients were calculated with the viscid panel method software XFOil [48] and then extended over the deep stall region, i.e. up to $AoA=\pm 180^\circ$, via the Viterna-Corrigan model [49], as recommended by Bianchini et al. [50] for VAWT applications.

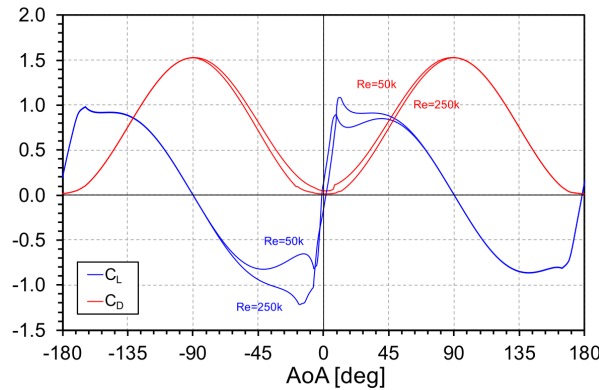


Figure 8 – Polar data for the R500 test rotor.

It must be noted how, in order to take the *flow curvature* phenomenon into account in the simulation campaign, the tabulated data for both *R515* and *R500* rotors were derived for a virtually cambered airfoil, obtained by proper *conformal mapping* from the geometric one according to the rotor chord-to-radius ratio c/R , blade-spoke connection point (*BSC*) and TSR [51]. The latter was selected as the highest available in the simulation campaign. This procedure is supposed to be more accurate than the ones used in the existing software, which are mainly based on an algebraic correction of the sampled AoA [18,19].

Flow curvature

In several studies [34,52,53], it has been observed how the cycloidal motion of VAWT blades with respect to the turbine axis of rotation is responsible for the deformation of streamlines around them, increasing their effective degree of curvature with respect to the freestream flow. This reflects on the aerodynamic performance of the blade, which behaves aerodynamically (i.e. in terms of lift,

drag and moment coefficients) like a virtually transformed equivalent airfoil, with a camber line defined by its arc of rotation and oriented outwards with respect to the turbine axis, as in Figure 9.

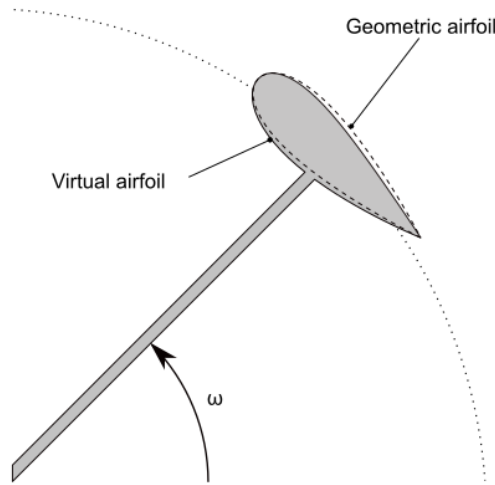


Figure 9 - Schematic representation of the flow curvature effect.

3.5 Dynamic stall model

The presence of a proper dynamic stall model, capable of accounting for the oscillating nature of VAWT loads during unstable operation, is pivotal for an accurate simulation of these machines, especially at medium-low TSRs [23,35]. For this work, the dynamic stall model from Berg [54], already implemented in the ALM code by the authors in a previous study [28], has been selected, as it represents a good trade-off between accuracy, calculation cost, and implementation effort [35]. In its most general form, the Berg model represents an adaptation to VAWTs of the semi-empirical Gormont one [55], originally developed for helicopter blades. Dynamic effects are embedded in the solution by entering the available polar data with a modified angle of attack α_{ref} (Eq. 13):

$$\alpha_{ref} = \alpha - K_I \Delta\alpha \quad (13)$$

where K_I is an empirical constant, accounting for the direction of the angle of attack variation, i.e. whether the flow is detaching or reattaching to the airfoil, via a discrete function of the derivative over time of the angle of attack $\dot{\alpha}$:

$$K_I(\dot{\alpha}) = \begin{cases} 1, & \dot{\alpha} \geq 0 \\ -0.5, & \dot{\alpha} < 0 \end{cases} \quad (14)$$

On the other hand, $\Delta\alpha$ is a dynamic correction to the static angle of attack α and varies linearly with an index of unsteady forcing S , derived from reduced frequency, via a set of sensitivity parameters γ_i that depend on the airfoil maximum thickness t/c and on the Mach number M :

$$\Delta\alpha = \begin{cases} \gamma_1(M) \cdot S \\ \gamma_1(M) \cdot S_C(t/c) + \gamma_2(M) \cdot (S - S_C) \end{cases} \quad (15)$$

where S_C is a threshold parameter depending on t/c . It must be noted how γ_i assumes different values whether lift or drag are considered, leading to different correction for the the two curves. The reduced frequency index S takes the form:

$$S = \sqrt{\left| \frac{c \dot{\alpha}}{2W} \right|} \quad (16)$$

It must be noted that, as anticipated in section 3.2, the $\dot{\alpha}$ value used here is the one sampled in the previous rotor revolution, after having been smoothed via a CSS-based low-pass filter.

Eventually, dynamic lift and drag coefficients are evaluated by plugging α_{ref} in the available static tabulated polars. In order to distinguish between the attached and separated flow regions, an ad-hoc extrapolation procedure is used for the lift coefficient:

$$\begin{aligned} C_L^{dyn} &= C_L(\alpha_0) + m(\alpha - \alpha_0) \\ C_D^{dyn} &= C_D(\alpha_{ref}) \end{aligned} \quad (17)$$

where the slope m is determined from the following equation:

$$m = \left[\frac{C_L(\alpha_{ref}) - C_L(\alpha_0)}{\alpha_{ref} - \alpha_0}, \frac{C_L(\alpha_{SS}) - C_L(\alpha_0)}{\alpha_{SS} - \alpha_0} \right] \quad (18)$$

where α_0 is an arbitrary reference angle of attack, in this case the zero-lift one. According to Massé [54], the Gormont model, being originally developed for helicopter blades, for which the maximum angle of attack is much lower than for VAWT ones, tends to overestimate dynamic stall effects in the deep stall region; as a consequence, the author proposed to compute the dynamic lift and drag coefficients as linear interpolation between the static and the Gormont ones:

$$C_L^{mod} = \begin{cases} C_L + m_\alpha(C_L^{dyn} - C_L), & \alpha \leq A_M \alpha_{SS} \\ C_L, & \alpha \geq A_M \alpha_{SS} \end{cases} \quad (19)$$

$$C_D^{mod} = \begin{cases} C_D + m_\alpha(C_D^{dyn} - C_D), & \alpha \leq A_M \alpha_{SS} \\ C_D, & \alpha \geq A_M \alpha_{SS} \end{cases} \quad (20)$$

$$m_\alpha = \left[\frac{A_M \alpha_{SS} - \alpha}{A_M \alpha_{SS} - \alpha_{SS}} \right] \quad (21)$$

where A_M is a tuning semi-empirical constant. For this study, the latter has been set to $A_M=6$, according to a sensitivity analysis performed by the authors on the same code [28].

For a complete description of the model formulation, here reported in its main aspects, please refer to [56].

4. CFD model

All three-dimensional simulations were carried out with an unsteady Reynolds-Averaged Navier-Stokes (URANS) approach via the commercial solver ANSYS® FLUENT®, in the 18.2 release. For turbulence closure, the $k-\omega$ Shear Stress Transport (SST) model was selected: compared to the other models available, it provides better accuracy and stability in unsteady flow conditions such as the ones experienced by VAWT blades, with massively separated flows and strong pressure gradients [57]. For detailed information about the settings for adopted numerical schemes, convergence definition, maximum inner iterations and residuals, please refer to [57].

4.1 Computational Domain

An open-field domain was chosen for the analysis. In order to reduce the number of elements, together with the overall computational cost, only the upper half of the rotor was modelled, exploiting the spanwise symmetry of the VAWT flow [7].

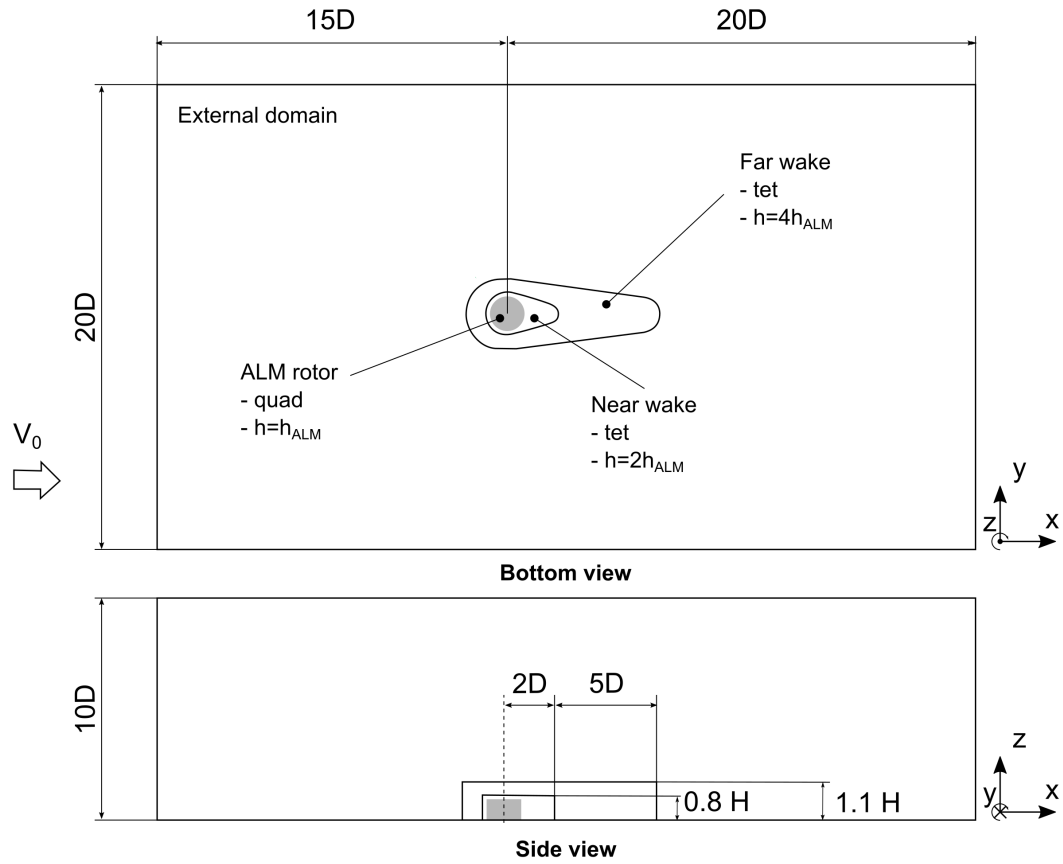


Figure 10 - Computational domain.

The corresponding dimensions, parameterized with respect to turbine diameter D and blade semispan height H (see Figure 10), were selected according to the sensitivity analysis from [20]. This allowed to minimize the disturbance induced by the turbine on the freestream flow, justifying the application of the standard *far field* boundary conditions commonly adopted for external flows. More in detail, a uniform velocity, turbulent kinetic energy k and frequency ω were applied at the inlet based on the testing conditions of the considered test case. The turbulent quantities, in particular, were estimated according to the test inlet turbulent intensity and length scale. At the outlet, a uniform atmospheric pressure was imposed, while for the other patches the *symmetry* condition was adopted.

4.2 Mesh

The computational domain is discretized with a hybrid meshing strategy. As shown in Figure 10, the CFD domain is divided in three separated fluid zones, according to the main features commonly found in a VAWT turbine operating in open-field environment:

- *Rotor ALM region:* The toroid encompassing the turbine rotor, where the actuator line forces are applied. In contrast with the rest of the domain, this region was discretized with a structured mesh, according to the requirements of the ALM code. The number of elements in the azimuthal direction are directly proportional to the timestep used for the CFD simulations, so that no discontinuity occurs when advancing the actuator line in the domain. The cell dimensions in the radial and spanwise direction, on the other hand, were chosen so that a cell Aspect Ratio as close as possible to 1 is achieved, at least on the turbine trajectory. This avoids the distortion of applied volume force distribution.
- *Wake region:* An unstructured quad-dominant mesh was used for the drop-shaped refinement region near the rotor, which is necessary to properly capture the turbine blockage and wake development. As shown, this region is divided in two separate zones: a *near-* and *far-wake* one.

The former, characterized by a cell dimension h_{NW} , comprehends the zone where the highest wake mixing is encountered. The latter, with $h_{FW} = 2h_{NW}$, serves to both resolve the far wake development and ensure a smooth transition from the rotor from the external domain.

- *External region*: The prismatic zone representing the external flow domain, discretized with tetrahedral elements.

In order to optimize the number of mesh elements required by the turbine CFD model, a dedicated grid sensitivity analysis was performed. To this end, the 2-blade *R500* turbine (see Table 1) at $TSR=4.5$ was selected as reference case, since it presents the highest blade loading among the tested configurations and as a consequence it is the most demanding in terms of downstream wake resolution. Following the strategy outlined in [7], such procedure was carried out in two steps, using as convergence index the *coefficient of determination* R^2 (Eq. 22) of each mesh with respect to the most refined one, indicated with the subscript “*ref*” [58]:

$$R^2 = 1 - \frac{\sum_{\theta=0^\circ}^{360^\circ} (F_t(\theta) - F_{t,ref}(\theta))^2}{\sum_{\theta=0^\circ}^{360^\circ} (F_t(\theta) - F_{t,avg})^2} \quad (22)$$

A first step (“*Step I*”) focused on the grid resolution required by the ALM on the two-dimensional plane corresponding to the turbine *midspan*. As reported in Table 3, five different meshes were generated, differing one from the other in terms of azimuthal resolution in the ALM rotor region. It must be noted how, considering the strategy adopted for the ALM rotor region discretization outlined in the previous paragraph, this corresponds to a progressive reduction of the kernel width β and the angular timestep $\Delta\theta$. Upon comparison of the different meshes blade tangential force profiles shown in Figure 11, it can be seen how convergence is already achieved with the *M03* mesh. Further relaxing the grid requirements in the wake, as for mesh *M03-1*, does not modify the predicted loading. Therefore, the latter set-up was used starting point for the following step.

Table 3 – Mesh sensitivity step I.

Value	M01	M02	M03	M03-1	M04
#cells azimuth	90	180	360	360	720
h_{NW}/h_{ALM} [-]	1	1	1	2	1
h_{FW}/h_{ALM} [-]	2	2	2	4	2
$\Delta\theta$ [deg]	4	2	1	1	0.5
# cells [k]	17	38	106	55	351

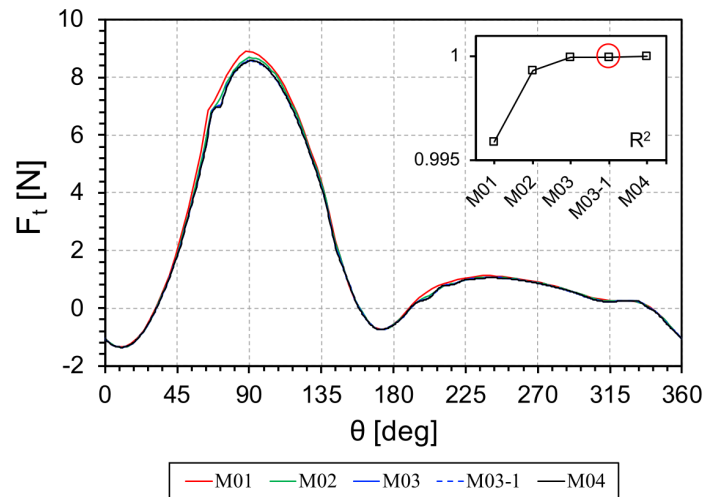


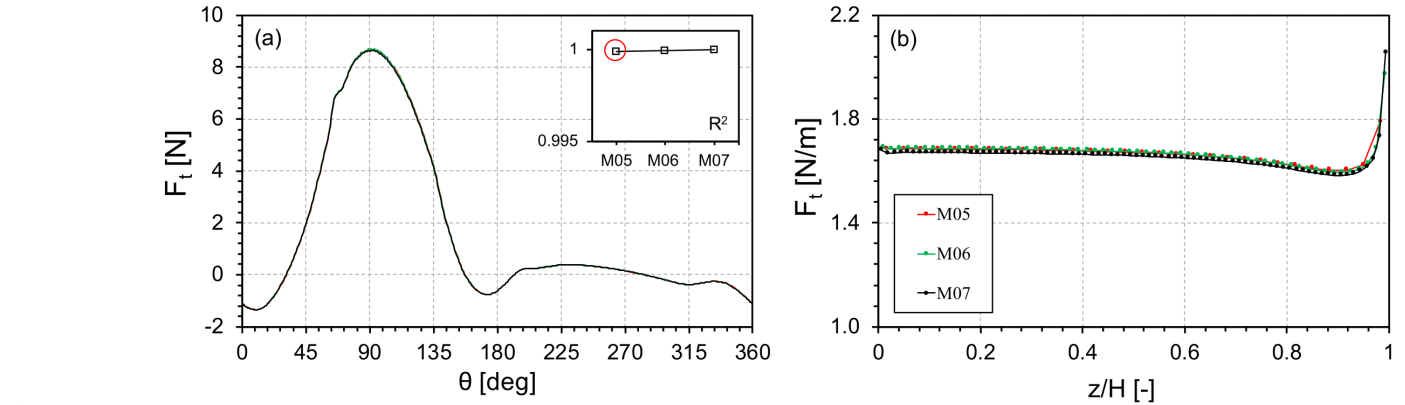
Figure 11 – Comparison between the different meshes in terms of tangential blade force (Step I).

486 A second step (“*Step II*”) on the other hand was used to assess the spanwise blade resolution
 487 required by the ALM simulation in the three-dimensional domain (see Table 4). As evident from
 488 Figure 12, no relevant modification in the predicted tangential force is achieved by increasing the
 489 number of span elements from 30 to 80, as the average blade loading distribution over the blade
 490 remains substantially unaltered. Only deviation between the different meshes can be observed in the
 491 last 10% of the blade, where the mesh *M07* offers a better resolution of the flow discontinuity at the
 492 blade tip. This effect is nonetheless negligible in terms of global performance. As a final result, the
 493 *M05* setup (see Figure 13) was selected as the reference one for the rest of the investigation.

494 **Table 4 – Mesh sensitivity step II.**

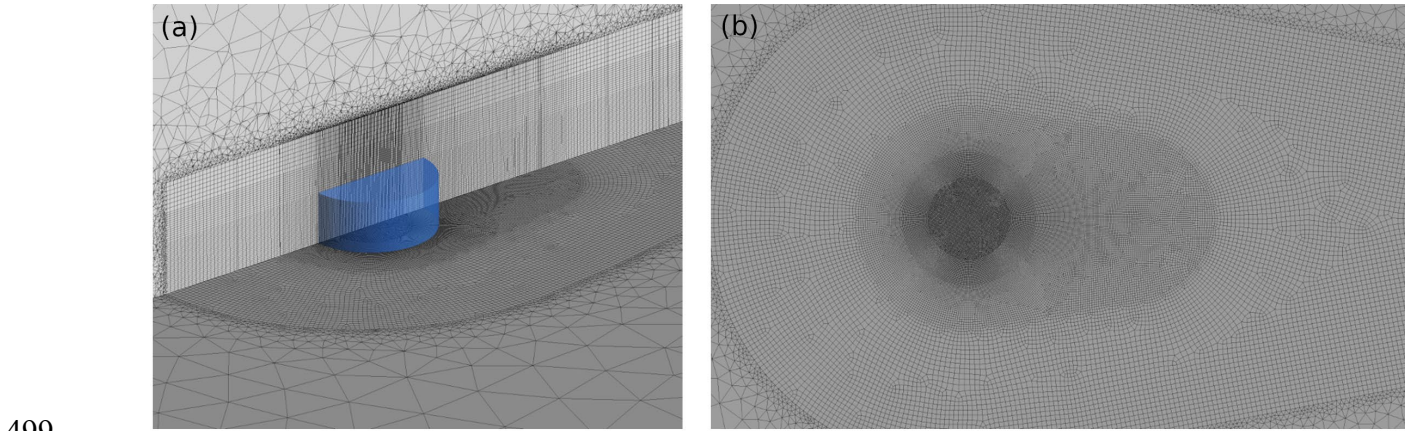
Value	M05	M06	M07
# cells azimuth	360	360	360
h_{NW}/h_{ALM} [-]	2	2	2
h_{FW}/h_{ALM} [-]	4	4	4
$\Delta\theta$ [deg]	1	1	1
# cells span	30	60	80
# cells [k]	4213	8577	10269

495



496
497
498

Figure 12 - Comparison between the different meshes in terms of tangential blade force: (a) azimuthal profile (b) average spanwise profile.



499

500 **Figure 13 – Overview of the M05 mesh selected for the analysis: (a) isometric view of the rotor zone (blue) (b) wake**
 501 **refinement on the midspan plane.**

5. Results

In this section, the main outcomes of the current study are presented. Subject of the first section is the one-blade *R515* rotor. Given the quality of the corresponding reference data, the analysis will focus on the capability of the proposed ALM approach to reproduce the effect of tip vortexes on the blade loading profile along the span and the rotor wake flow field. The sensitivity of the predicted performance to the combination of the different sub-models available in the tool is also performed.

The same investigation is then repeated for the two-blade *R500* rotor, this time shifting the attention to the rotor midspan plane, which the experimental and CFD blade load data refer to. In this context, the impact of the application of a dynamic stall model when simulating the turbine at an unstable operating point (TSR=2) is illustrated. Eventually, the three-dimensional rotor flow field computed with the ALM is compared to the available PIV measurements.

5.1 One-blade rotor – R515

Blade loading

Figure 14 reports the comparison between the ALM blade torque profiles both along one rotor revolution and in the spanwise direction, made non-dimensional according to Eq. 23, and the corresponding trends resulting from the blade-resolved CFD calculations of Balduzzi et al. [7]:

$$C_m = \frac{T_z}{0.5\rho c^2 V_0^2} \quad (23)$$

where T_z denotes the instantaneous torque per unit blade length.

At first glance, it can be seen how the direct application of the ALM approach without any specific sub-model, here referred to as *baseline*, yields already an overall fairly accurate prediction of the CFD benchmark data. In the first 80% of the blade span in particular, the lumped-parameter approach is capable to reconstruct the shape of the azimuthal torque trend, especially in the *upwind* sector ($0^\circ \leq \theta \leq 180^\circ$), where a good estimation of both magnitude and angular position of the torque peak is achieved. Some specific phenomena nonetheless, which characterize the rotor aerodynamic behavior, still fail to be captured. In the *downwind* region ($180^\circ \leq \theta \leq 360^\circ$) for instance, due to the well-known tendency of the ALM to underestimate the *upwind* rotor blockage effect, i.e. the slowing down of the flow due to energy extraction, torque production is overestimated over the whole angular range under consideration. Another issue, as it will be explained in a more detailed way in the next section, is the inability of the code to properly resolve the vortex-like structures that are shed from the blade extremities, i.e. the tip vortexes. As apparent from Figure 14 in fact, where the torque coefficient is averaged over one rotor revolution and reported as a function of the spanwise coordinate z , the ALM approach is not able to predict the corresponding blade loading drop characterizing the last 20% of the blade span with the same intensity encountered in the CFD benchmark data, where the computed C_m experiences a strong reduction down to negative values. Considering as an example two sections at 80% and 95% of the blade span, respectively, no substantial alteration of the ALM instantaneous C_m azimuthal profile can be observed with respect to the *midspan* one ($z/H=0$).

Taking into account the *flow curvature* phenomenon first and then the actual position of the *Blade-Spoke Connection (BSC)*, which for the *R515* rotor is exceptionally at *mid-chord* ($x/c=0.5$), led to a slight improvement in the computed torque azimuthal profile, especially in the *downwind* sector of the rotor. Behind this seemingly marginal variation in the predicted performance though, lies a substantial repartition of the blade aerodynamic loads with respect to the standard case of BSC at *quarter-chord* ($x/c=0.25$), for which the only contributor to the blade torque is the one related to the tangential force $F_t \cdot R$.

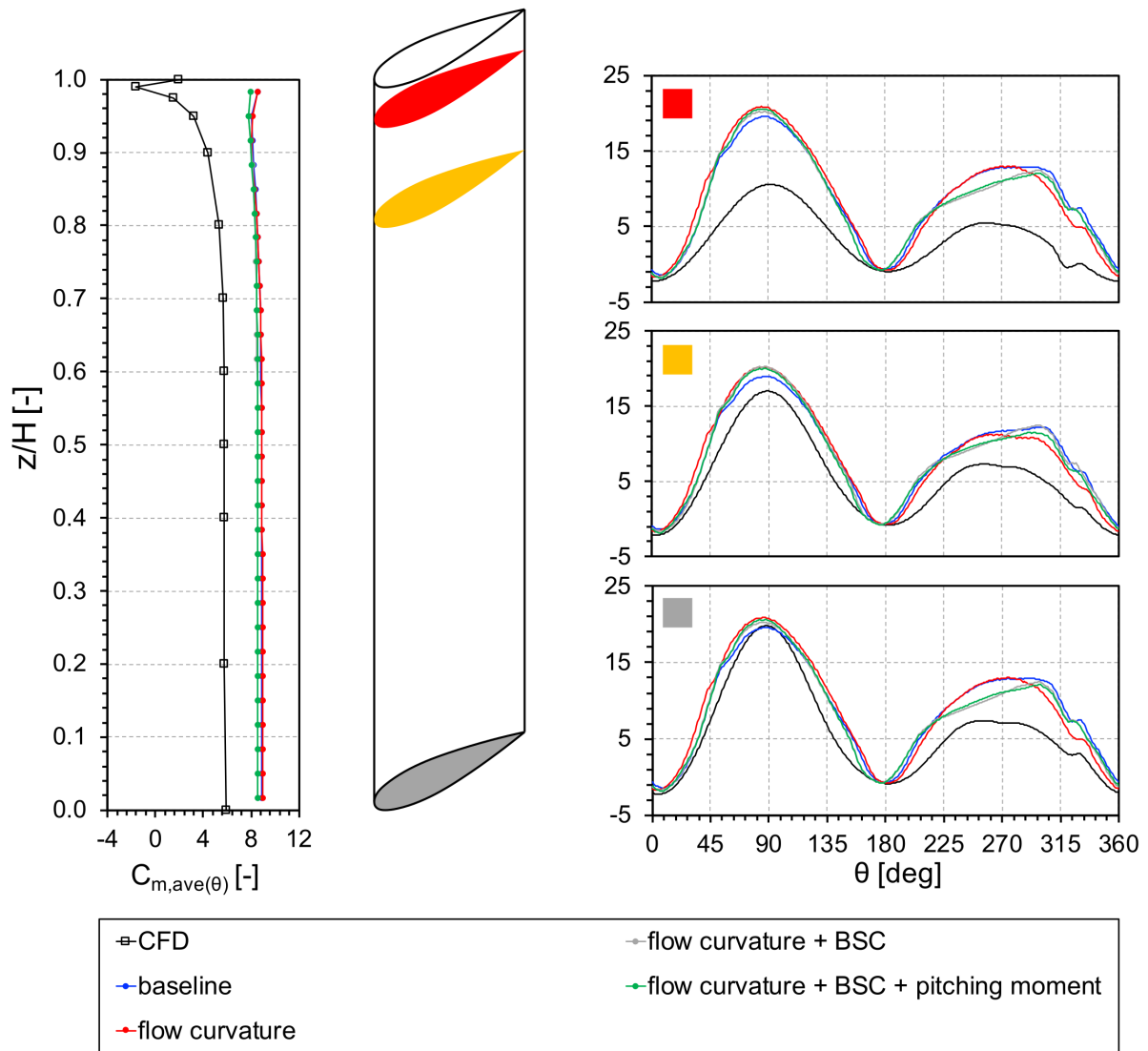


Figure 14 - Comparisons between CFD and ALM in terms of torque coefficient C_m .

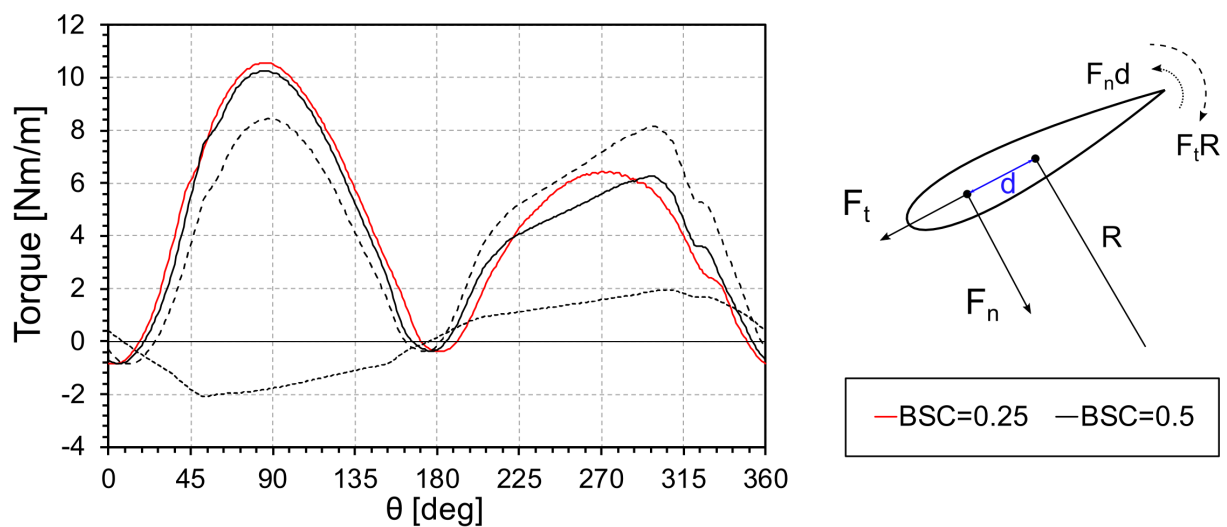


Figure 15 – Effect of BSC on blade loading repartition at rotor midspan.

As shown in Figure 15, shifting the BSC to *mid-chord* (BSC=0.5) leads on the other hand to a significant re-balancing of the F_t between the upwind and downwind sectors of the rotor, due to the apparent pitch (in this case ca. 2°) related to the blade new kinematics (see section 3.1). In this case, the normal force directly affects the overall torque balance in the form of an oscillating pitching moment $F_d \cdot d$, increasing the blade energy extraction in the *upwind* region and decreasing it in the *downwind* one, as already reported in [33].

The eventual addition of the *pitching moment* sub-model resulted in no significant variations in the predicted blade loads, as evident in Figure 14. This confirms the negligible weight of such phenomenon during turbine stable operation, i.e. when the blade angle of attack remains confined in the pre-stall region (see section 3.1).

Wake flow field

In order to gain a deeper insight in the flow structure predicted by the ALM code and understand its effect on the computed blade loads, comparisons with the available benchmark CFD flow fields have been made in terms of velocity magnitude V/V_0 , normalized over the freestream wind speed V_0 , and vorticity. For the purpose, only the most complete ALM setup from the previous section, i.e. the “*flow curvature + BSC + pitching moment*”, has been considered.

Figure 16 shows the velocity fields on planes normal to turbine axis (X-Y in the adopted frame of reference) for $\theta=90^\circ$ and $\theta=270^\circ$. At midspan ($z/H=0$), ALM flow fields are fairly comparable to the CFD ones. The velocity deficit in the wake, as well as the position of the shear layers defining its borders with respect to the freestream, is properly captured. The same can be observed for the deceleration region in front of the rotor and the two acceleration spots on its sides, which are related to the blockage exerted by the turbine on the incoming flow [5]. In the downwind region nonetheless, due to the already mentioned difficulty of the ALM tool to properly quantify the upwind-downwind interaction, the flow encountered by the blades for $\theta=270^\circ$ is faster than the one predicted by CFD, with the corresponding overestimation of shaft torque (see Figure 14). Moving towards the tip region ($z/H=1$), the deviation with respect to the benchmark data increases sharply over the whole rotor revolution. More in detail, a large acceleration zone, following the trace of the rotating airfoil wake, is observed in the CFD flow fields. Such flow pattern is probably related to the distortion induced by the three-dimensional vortex-like structures shedding from the blade tip [7]. The effect of the tip vortexes is not confined to the local flow field near the blade extremities, but it influences the whole wake development downstream of the machine. According to various studies [36,37] in fact, the swirling velocity component induced by these structures in a plane normal to the wind speed direction is responsible for the entrainment of air from the freestream flow to the upper portion of the turbine wake. Such phenomenon is clearly visible in Figure 17, where the velocity fields on a plane normal to the wind direction (Y-Z) are reported at different streamwise positions. The blade, highlighted in grey for clarity, is positioned at $\theta=90^\circ$. The dotted line, on the other hand, refers to the rotor swept area. At the rotor location ($0 \leq x/D \leq 0.5$), the velocity defect in the region extending right above the blade tip is quickly reabsorbed by the freestream, so that it is confined again within the rotor height. As the wake develops downstream, the enhanced mixing induced by tip vortexes provokes a progressive reduction in height of the maximum velocity defect region behind the rotor, which for $x/D=1.5$ covers approximately 80% of the blade span. If, on the one hand, this behavior is properly captured by the high fidelity CFD data, on the other hand it is notably underestimated in the ALM ones, where the flow pattern generated in the wake right behind the rotor ($x/D=0.5$) is preserved up to 1D downstream.

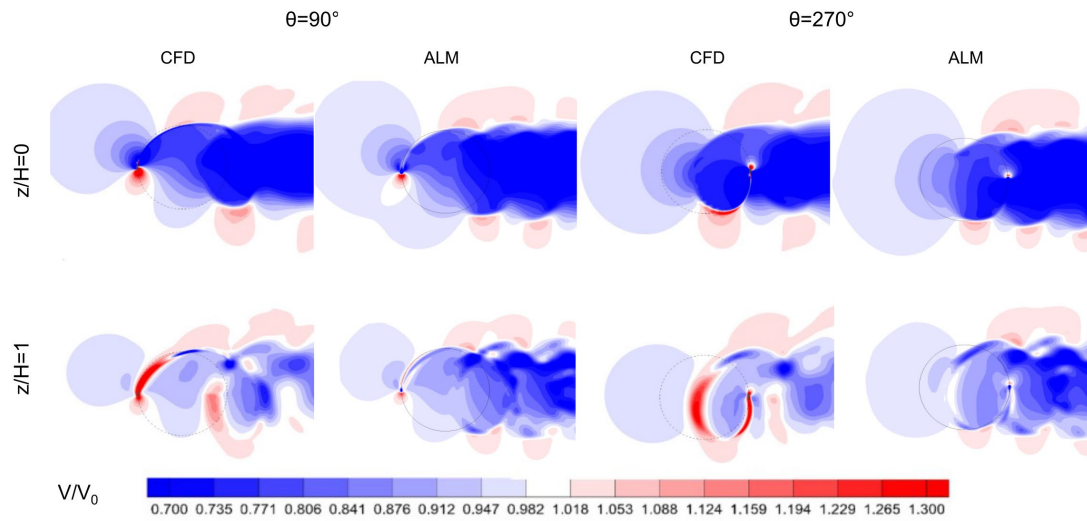


Figure 16 - Comparisons between CFD and ALM in terms of velocity V/V_0 at different X-Y planes.

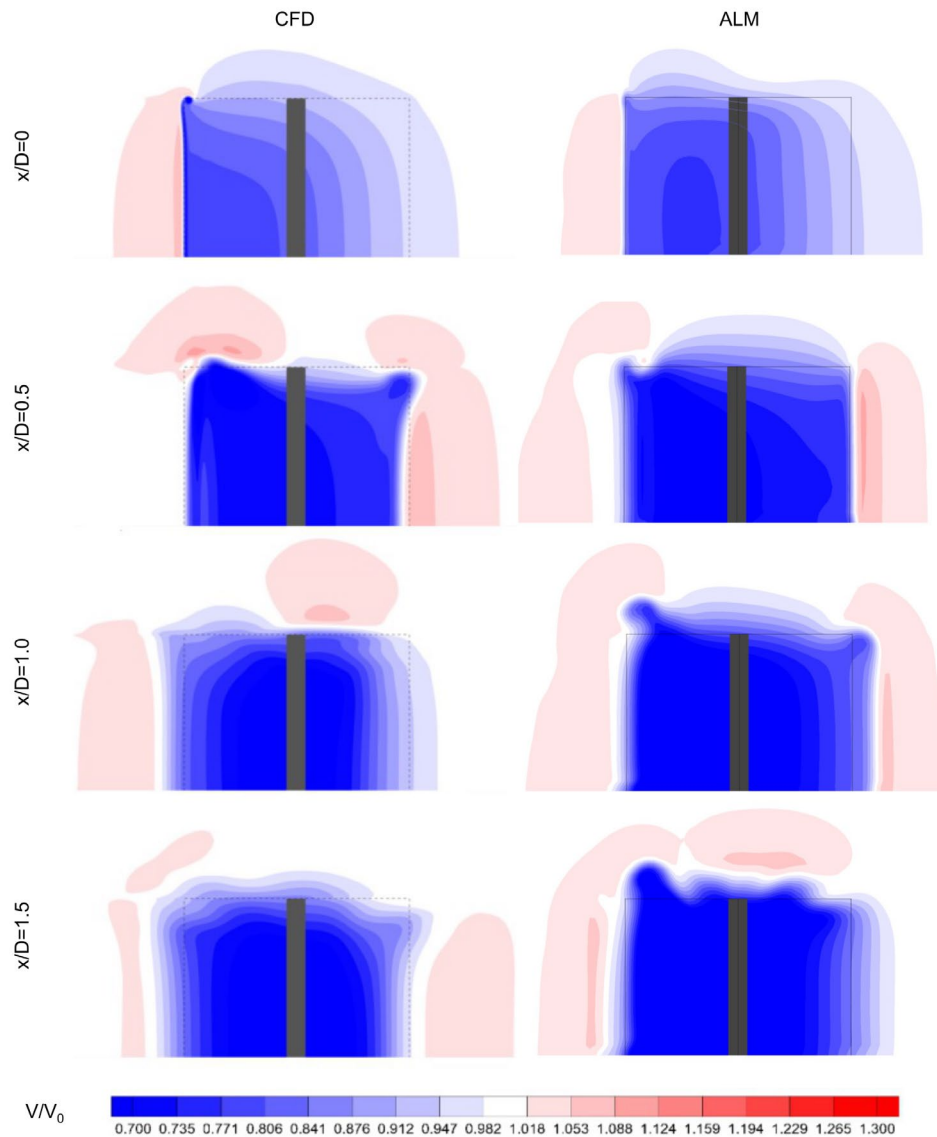


Figure 17 - Comparisons between CFD and ALM in terms of velocity V/V_0 for $\theta=90^\circ$, at different Y-Z planes.

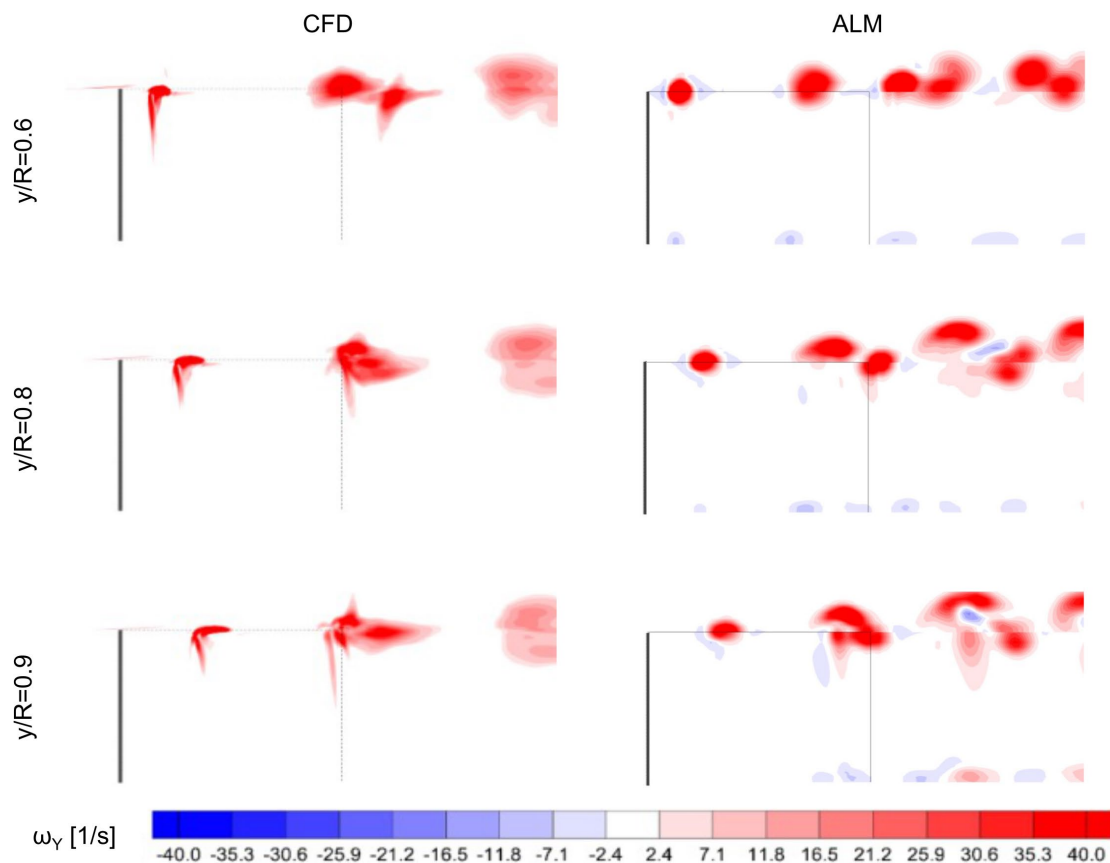


Figure 18 - Comparisons between CFD (left) and ALM (right) in terms of Y-vorticity for $\theta=90^\circ$, at different X-Z planes.

In order to understand better the reason why the presented ALM tool fails in properly capturing the intensity of the shed tip vortexes, the transversal component of the vorticity vector (Y-vorticity) has been plotted on different planes tangential to the wind direction (see Figure 18). It is evident how the vortex shedding in the CFD flow field begins to occur at approximately 60% of the blade span, for then concentrating in coherent structures at its extremity. The flow structures predicted by the ALM instead are abruptly generated at the tip and are spread over a wider area around the shedding point, maintaining their integrity for a longer downstream distance before being dissolved in the freestream flow. This fact is probably related to the intrinsically two-dimensional nature of the tool formulation, i.e. not taking into account the spanwise velocity component in the computation of aerodynamic forces, as well as to the kernel function. The latter, being homogeneous along the blade span (cylindrical distribution), is not capable to reproduce the spanwise vorticity gradient encountered in the real blade flow [26,46].

Computational cost

One of the main advantages of using the ALM approach is that, notwithstanding the issues highlighted in the previous sections, a reasonable estimation of the rotor performance and wake development can be achieved at a computational cost considerably lower than blade-resolved CFD.

In order to quantify this gain, Table 5 reports in a synthetic way the comparison in terms of computational cost between the two methodologies for the *R515* turbine computation. More in detail, resolving the full geometry of the rotor took 650 hours on a supercomputer having 16384 cores for only 12 revolutions and using a research code able to parallelize almost linearly the calculation. This resulted qualitatively in a reference specific calculation cost of approximately 887500 hours per revolution on a single core. Using the ALM on the other hand, the same specific cost was approximately 20 hours per revolution on a single core, so resulting in a reduction of more than five orders of magnitude.

Table 5 – Comparison in terms of computational cost between blade-resolved CFD and ALM for the *R515* rotor.

Value	CFD	ALM
# CPU cores	16384	24
# revolutions	12	25
Simulation time [h]	650	21
Hours/rev/core	887500	20

5.2 Two-blade rotor - R500

In order to assess the capability of the presented ALM tool to handle a geometry representative of a real Darrieus rotor, where solidity effects such as blade-wake interaction and rotor blockage are more relevant, the analysis moved to the second test case available, i.e. the *R500* two-blade Darrieus machine.

Blade loading

Figure 19 shows the comparison in terms of blade loading at the rotor midspan between the ALM, the experimental and the two-dimensional blade-resolved CFD data from [39] over one rotor revolution. It must be noted how the benchmark lift coefficient has been extracted from the numerical flow field with the same *LineAverage* technique adopted in this work. All forces are on the other hand here normalized over the freestream momentum $q_0=0.5\rho V_0^2$, scaled over the turbine radius R .

In general, the direct application of the ALM approach without any specific sub-model, here referred to as *baseline*, yields a quite poor prediction of the blade loading profile in all considered operating conditions. At TSR=2, the typical features of the dynamic stall phenomenon observed in the CFD lift data are completely absent. In the *upwind* region ($0^\circ \leq \theta \leq 180^\circ$), the lack of the stall delay and hysteresis effects in the lift curve, corresponding to the formation of the Leading Edge Vortex (LEV) on the airfoil suction side, reflects on the underestimation by the ALM of the load peak position and intensity. In the *downwind* ($180^\circ \leq \theta \leq 360^\circ$) one instead, where experiments and high-fidelity simulations predict the disruption of the blades performance due to their interaction with the vortices shed *upwind*, ALM simulations tends to overestimate the torque production. Going towards a stable operating point, i.e. TSR=4.5, the weight of dynamic effects on the blade loading profile is progressively reduced, although it does not fully disappear. In this condition nonetheless, the discrepancy between reference data and *baseline* simulations is still evident. As shown in Figure 19, the ALM lift loop presents on average a deviation in terms of slope and an offset with respect to the one sampled from CFD. This corresponds in turn to a positive offset of both F_n and F_t along the whole rotor revolution, which is particularly pronounced in the *downwind* sector.

The lift loop misalignment described so far can be compensated by applying the *flow curvature* model, i.e. by taking into account the *virtual camber* and *virtual incidence* characterizing the airfoil while rotating in VAWT-like motion. As anticipated in section 3.4, this procedure corresponds in practice to the replacement of the original airfoil polar data with that of the virtually transformed one. In terms of blade loads, the achieved accuracy improvement is particularly pronounced at TSR=4.5, where the observed offset with respect to CFD in the predicted tangential and normal forces is considerably reduced. The ALM is able to capture the corresponding *upwind* peak value, both in intensity and angular position, as well as the overall force reduction in the *downwind* section. The experimental data on the other hand, as stated by the same authors of the study [38], are still biased by a high degree of uncertainty due to issues in the force extrapolation method from PIV data. As a consequence, no rigorous comparison can be made. Decreasing the TSR, the weight of this correction on turbine performance becomes progressively lower, due to the increasing width of the considered AoA interval. Nonetheless, a better matching between ALM and reference forces is shown in the *downwind* region.

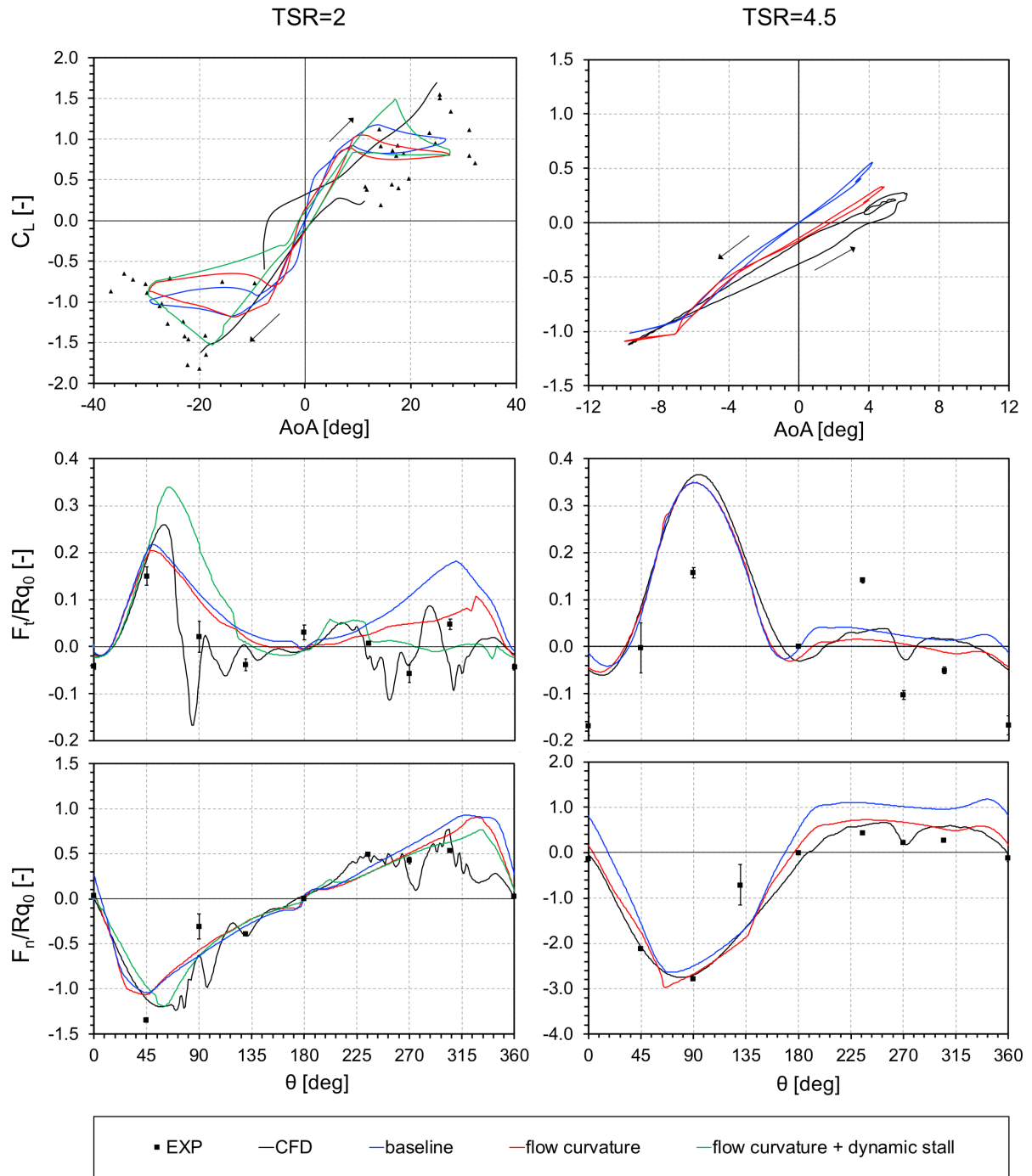


Figure 19 - Comparison between experimental, ALM and 2D CFD load data [35].

Eventually, the addition of the Berg dynamic stall model with tuning constant $A_M=6$ (see section 3.5) is associated to a more physical representation of the lift coefficient behavior observed in the benchmark dataset at turbine unstable operation (TSR=2). As shown in Figure 19, both the stall delay and hysteresis phenomena are properly captured. Shifting the focus on the blade tangential force, it can be seen that this corresponds to a notable improvement in the code capability of predicting the *downwind* force intensity and *upwind* peak angular position. The magnitude of the latter however, as well as the overall force for $90^\circ \leq \theta \leq 125^\circ$, is largely overestimated. As demonstrated in [28], this issue is related to the intrinsic incapability of the Berg model to capture the drag rise associated to the LEV formation event.

Wake flow field

The availability of high-fidelity PIV wake velocity data from [37] represented a valuable opportunity to gain a deeper insight into the capability of the ALM tool in reproducing the flow pattern of a real VAWT machine. For the purpose, only the most accurate ALM version from the previous section, i.e. with the *flow curvature* correction, has been considered for TSR=4.5.

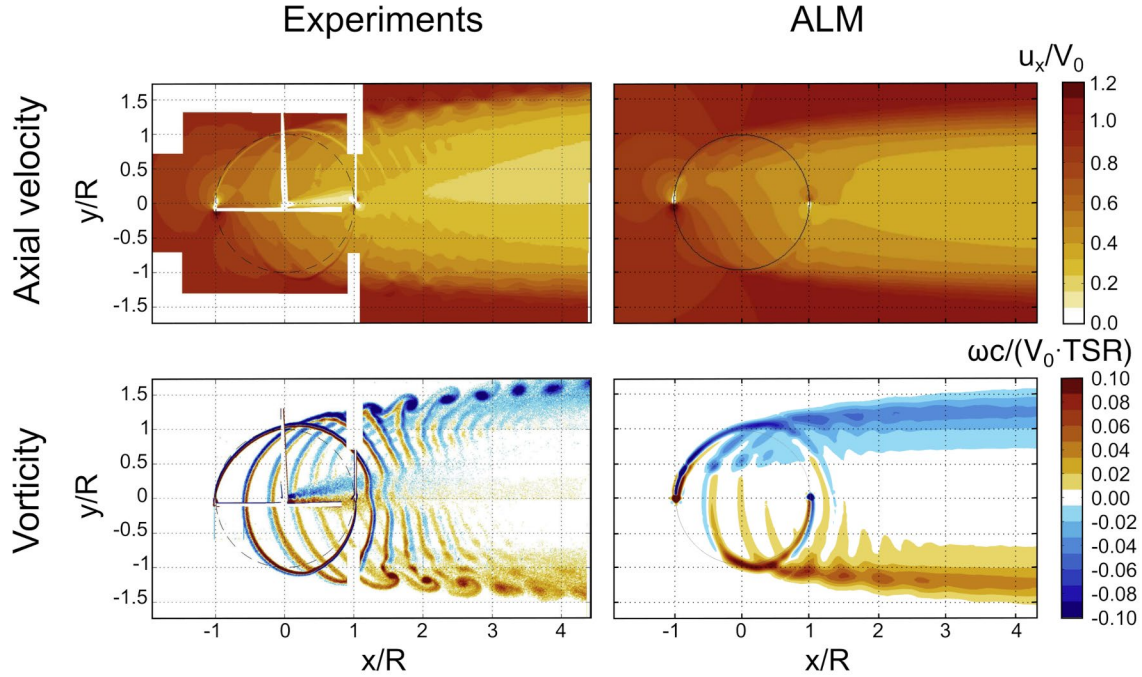


Figure 20 - Comparison between experimental and ALM flow fields.

Figure 20 reports the comparison in terms of streamwise velocity between experimental and ALM data, at the rotor midspan plane ($z/H=0$). The considered blade angular position is $\theta=90^\circ$. As for the single-blade machine (see section 5.1), the presented tool is able to reproduce all the main features of the wake in a fairly accurate way, even for a higher solidity rotor such as the one under consideration. More in detail, both the intensity of the turbine blockage on the incoming flow, i.e. the deceleration region in the front and the two acceleration ones on the sides are correctly captured, as well as the wake extension downstream of the rotor. The ALM nonetheless does not predict the overall wake skewness observed in the PIV flow fields, with the right border ($y/R > 0$) $0.5R$ larger than the left one. These features are even more evident when looking at the Z-vorticity on the same plane, here made non-dimensional to be compared with experimental data (see Figure 20). The shear layers defining the wake borders with respect to the freestream flow are properly captured by the ALM data, even though they tend to be more diffused with respect to their PIV counterparts. This is probably related to the use of a smearing function (see section 3.3) inside the code. For the same reason, second-order aerodynamic phenomena such as the *Kelvin-Helmoltz* instabilities in the wake shear layer, cannot be properly resolved.

Shifting to a three-dimensional perspective, on the other hand, the ALM shows a noteworthy accuracy reduction, as already observed for the one-blade machine (see section 5.1). More in detail, such approach, although able to reconstruct the overall wake shape, fails in taking into account the tip vortexes effect on the flow downstream of the rotor. This is particularly evident from Figure 21 and Figure 22, where both experimental and numerical wake structures are reported on different planes tangential to the wind direction (X-Z).

At the wake lateral borders ($y/R=\pm 1$), the agreement with experimental data is fairly good over the whole blade span, although numerical data tend to slightly overestimate rotor blockage with respect to experimental one, in terms of both flow acceleration (see Figure 21) and deflection (see Figure 22). In the near-tip region though ($z/H > 0.5$), the flow disruption and mixing due to tip

vortexes is completely absent in the ALM velocity fields. Moving towards the central sector of the wake ($-0.8 \leq y/R \leq 0.8$), this absence is even more evident. The mixing of the tip flow with the freestream one, already observed for the one-blade machine, is underpredicted. As a matter of fact, whilst the PIV wake starts lowering in height after $2R$ from the turbine shaft, the ALM one maintains the same spanwise extension for at least $3R$, before slightly flattening as it develops downstream.

In spite of the aforementioned issues, the presented hybrid model still manages to outline some of the typical structures characterizing the presence of vortex shedding from the blade extremities. In Figure 21 and Figure 22, for instance, alternated acceleration and deceleration spots can be observed in the tip region for both streamwise and transversal velocity components, corresponding to the influence of a three-dimensional swirling flow structure. Once again, the predicted intensity is much lower than in the available PIV data, confirming the failure of the ALM approach to capture these phenomena. If, on the one hand, this issue might be partially solved via *ad hoc* strategies, such as advanced regularization kernel functions or algorithm formulation capable of considering the velocity spanwise component in the force calculation (as anticipated in section 5.1), on the other hand it is probably intrinsically related to the ALM modelling strategy. As a matter of fact, one of the key elements in tip vortexes generation is the physical separation of the blade suction and pressure sides at its extremity [59,60], which naturally causes the transition of the fluid from the second to the first and so the vortex roll-up. In the hybrid approach instead, the profile is virtually replaced by a linear distribution of points along the span. As a consequence, these phenomena are inevitably neglected.

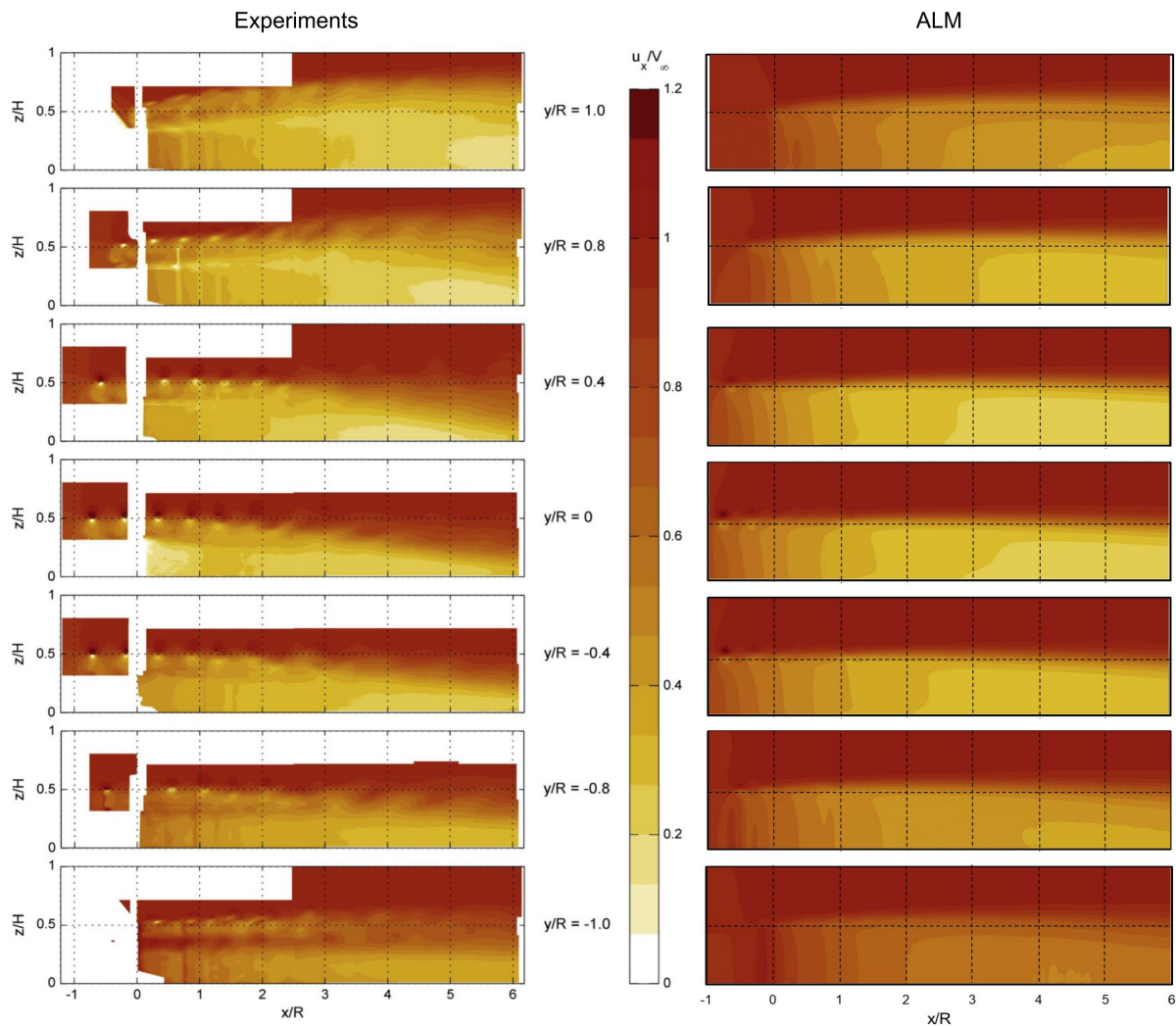


Figure 21 - Comparison of streamwise velocity between experiments and ALM for $\theta=90^\circ$, at different X-Z planes.

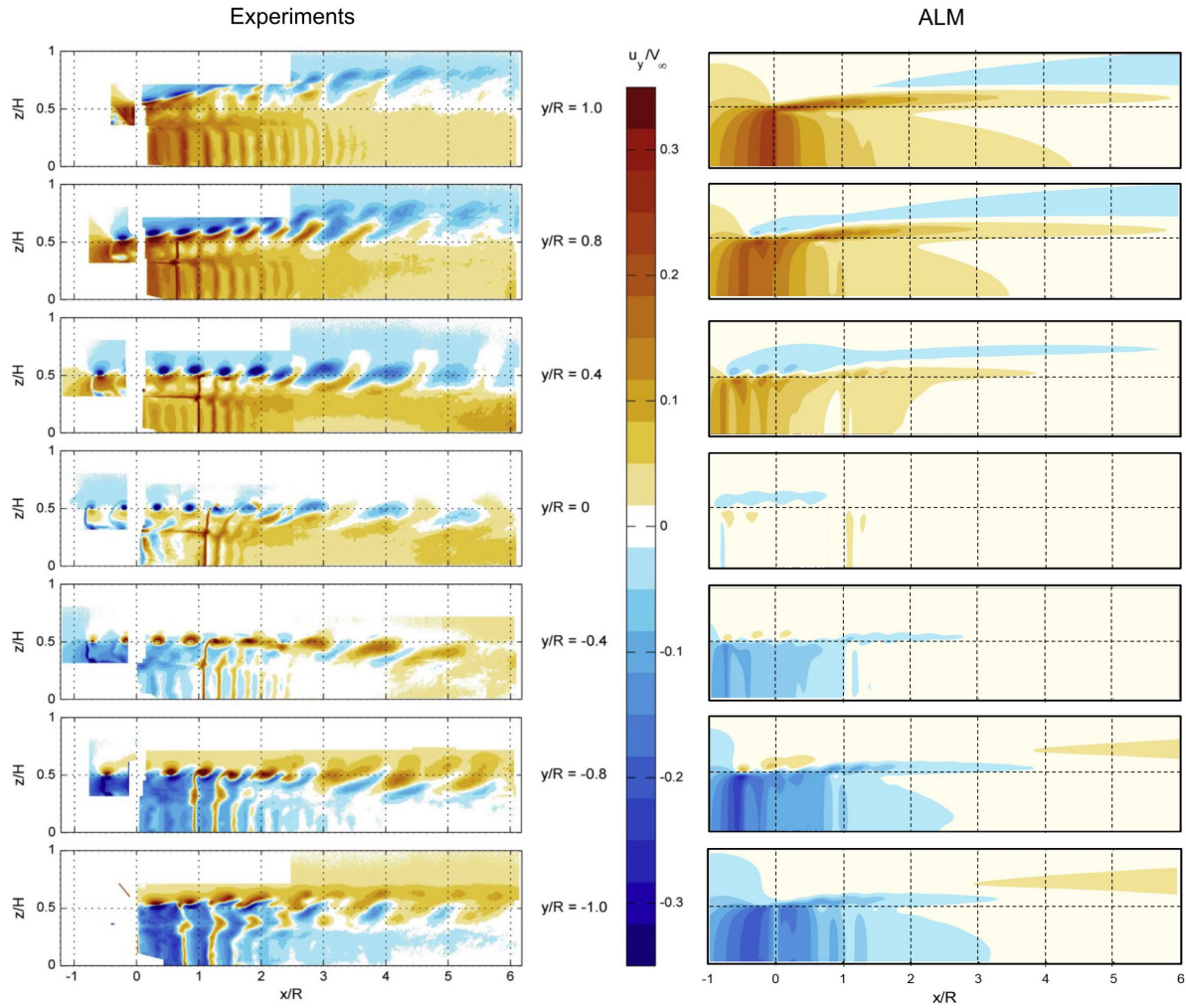


Figure 22 - Comparison of transversal velocity between experiments and ALM for $\theta=90^\circ$, at different X-Z planes.

6. Discussion

Overall, the ALM code presented in the study has been able to provide fairly accurate predictions of the turbine blade loads and of the main features of the flow field in comparison to blade-resolved CFD simulations.

The key point is that this was achieved at a computational cost notably lower than that of the higher order method. Simulating the theoretical half-blade of the *R515* turbine for instance took approximately 887500 hours per revolution on a single core, against the 20 hours of the ALM, resulting in a reduction of more than five orders of magnitude of the required computation resources.

More importantly, the proposed methods and models all proved to have a positive impact on the accuracy of the simulations, leading to an overall increase in the correlation (R^2) between the ALM torque profile and the benchmark one with respect to the base formulation. The latter amounted to ca. 16% for the one- and 3% for the two-bladed rotor, respectively. More specifically, for all considered test cases, the highest accuracy has been achieved at the turbine *midspan* plane, where the flow behavior can be considered inherently two-dimensional. For the one-blade rotor, a fair estimation of the *upwind* performance has been achieved, with an error on the peak torque value ($\theta=90^\circ$) of only 3.5%, whilst the torque profile is overestimated in the *downwind* sector due to the well-known tendency of the ALM approach to underestimate the upwind flow blockage effect. Given the

particular architecture of this rotor, the Blade-Spoke Connection (BSC) model is the one with the highest influence on predicted performance, especially if the different contributions to torque production are taken into consideration. For the two-blade machine on the other hand, a good agreement with reference data in terms of blade loading has been obtained over the whole rotor revolution, especially at the stable operating point ($TSR=4.5$). In this case, accounting for the *flow curvature* effect has revealed to be pivotal for the tool accuracy, raising the correlation R^2 between ALM and blade-resolved CFD torque trends up to ca. 0.977. Considering instead the turbine unstable operation ($TSR=2$), it has been observed how the introduction of the Berg dynamic stall model, coupled with the new methods to sample the angle of attack and to smooth it, respectively, has notably improved the description of the blade behavior in severely unsteady conditions with respect to the baseline approach. Nonetheless, the model still fails in capturing the airfoil drag increase associated to the Leading Edge Vortex event, leading to a large overestimation of blade torque in the upwind region. At the peak, such deviation can be ca. 25%. Thus, ad hoc modifications are still needed via the development in the future of a more advanced model.

Moving towards the blade tip region, in particular towards the last 20% of the blade span, a progressively increasing accuracy degradation has been observed, both in terms of predicted loads and flow fields. More in detail, the presented ALM formulation, although capable of outlining the main characteristics of the vortex-like structures in the tip region, tends to severely underestimate their intensity, as well as the corresponding effects on the wake flow pattern. For instance, the flattening of the wake in the spanwise direction, observed in the numerical and experimental flow fields as a consequence of the entrainment of air from the freestream to the upper portion of the wake by the tip vortices, is almost absent in the ALM ones. In the same fashion, the spanwise torque drop present in the 1-blade turbine CFD data taken as reference is not predicted by the ALM model of the rotor, in which the predicted forces tend to remain almost constant over the whole blade span. As a consequence, the maximum correlation obtained for the torque profile of this case remains limited to ca. $R^2=0.7$. All these issues are well-known by the scientific community working on this type of simulation tools; however, the proposed methods and models are thought to represent a milestone in the path leading hybrid tool for VAWTs to a level of maturity comparable to that of their counterparts for HAWTs.

7. Conclusions

In the present study, several methods and models to tailor the Actuator Line Method (ALM) to the simulation of VAWTs have been proposed and discussed.

In particular, a novel methodology for sampling of the angle of attack from the resolved flow field (*LineAverage*) has been introduced, together with a dedicated set of *aerodynamic sub-models* to tune the airfoil polars and a dedicated sensitivity analysis to the *spreading of forces in the fluid domain*. An extensive validation of the code has been carried out on selected test cases, for which both high-fidelity CFD and experimental data were available: a theoretical 1-blade machine and a real two-blade H-Darrieus rotor.

All the proposed methods and models showed a positive impact on the accuracy of the simulations, which was raised to values up to 97.7% (R^2 with reference data in terms of azimuthal torque profile). The robustness of the theory when rotors with different features are considered, e.g. when the blade-spoke connection point changes along the chord, was also increased. While the importance of dedicated sub-models to tune the airfoil polars (e.g. flow curvature effects) was known from studies on the Blade Element Momentum Theory, the introduction of a new method to sample the angle of attack showed to be key for the accuracy of the ALM, enabling not only the proper selection of aerodynamic coefficients, but also a more robust implementation of a key figure like the dynamic stall model.

802 In the future, further efforts will be devoted at developing ad hoc strategies to solve the issues
803 outlined in the present work. In particular, more advanced models for *three-dimensional effects* and
804 *dynamic stall* are required. This would make the ALM a fast and reliable tool for full three-
805 dimensional simulations of one- or multi-rotor configurations of Darrieus turbines.

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809 **9. Nomenclature**

810			
811	<u>Acronyms</u>		
812	AC	Aerodynamic Center	
813	AoA	Angle of Attack	
814	BEM	Blade Element Momentum	
815	BSC	Blade-Spoke Connection	
816	CFD	Computational Fluid Dynamics	
817	CSS	Cubic Spline Smoothing	
818	LEV	Leading Edge Vortex	
819	PIV	Particle Image Velocimetry	
820	SST	Shear Stress Transport	
821	TSR	Tip-Speed Ratio	
822	U-RANS	Unsteady Reynolds-Averaged Navier-Stokes	
823	HAWTs	Horizontal Axis Wind Turbines	
824	VAWTs	Vertical Axis Wind Turbines	
825			
826	<u>Greek symbols</u>		
827	$\dot{\alpha}$	Angle of Attack Derivative Over time	[rad/s]
828	α	Angle of Attack (symbol)	[deg]
829	β	Gaussian Kernel Width	[m]
830	ϑ	Blade Angular Position	[deg]
831	ρ	Freestream Flow Density	[Kg/m ³]
832	ω	Rotational speed	[rad/s]
833			
834	<u>Latin symbols</u>		
835	A_M	Berg Model Tuning Constant	[-]
836	c	Blade Chord	[m]
837	C_D	Drag Coefficient	[-]
838	C_L	Lift Coefficient	[-]
839	C_M	Moment Coefficient	[-]
840	D	Rotor Diameter	[m]
841	F_N	Blade Normal Force	[N]
842	F_T	Blade Tangential Force	[N]
843	H	Blade Semispan Height	[m]

844	h	Cell Characteristic Dimension	[m]
845	R	Rotor Radius	[m]
846	Re_c	Chord-based Reynolds Number	[-]
847	R^2	Coefficient of Determination	[-]
848	t/c	Dimensionless Maximum Airfoil Thickness	[-]
849	V	Undisturbed Flow Velocity Vector	[m/s]
850	V_0	Freestream Wind Speed	[m/s]
851	W	Blade Relative Flow Velocity Vector	[m/s]

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