

Practical Limits for Large-Momentum-Transfer Clock Atom Interferometers

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Atom interferometry on optical clock transitions is being pursued for numerous long-baseline experiments both terrestrially and for future space missions. Crucial to meeting the required sensitivities of these experiments is the implementation of large momentum transfer ($> 10^3 \hbar k$). Here, we show that to sequentially apply such a large momentum via π pulses places stringent requirements on the frequency noise of the interferometry laser and we find that the linewidth is required to be considerably lower than the previous estimate of approximately 10 Hz. This is due to imperfect pulse fidelity in the presence of noise and is apparent even for an atom at rest interacting with resonant light, making this a fundamental constraint on operational fidelity for a given laser and pulse sequence. Within this framework, we further present and analyze two high-power frequency-stabilized laser sources designed to perform interferometry on the 1S_0 – 3P_0 clock transitions of cadmium and strontium, operating at 332 nm and 698 nm, respectively.

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I. INTRODUCTION

Atom interferometry utilizing single-photon optical transitions represents an emerging technology with the ability to probe physics in a variety of previously untested regimes [1–3]. This includes experiments looking for tests of the quantum twin paradox [4], gravitational red shift [5], and tests of the weak equivalence for atoms in quantum superpositions [6]. In particular, however, multiple experiments based upon the 1S_0 – 3P_0 clock transition of Sr at 698 nm have been proposed to search for a very wide set of fundamental physics goals, but especially for ultra-light dark-matter searches and gravitational wave detection [7–12]. These experiments are predicated on the crucial fact that for interferometers based upon a single-photon transition, the laser phase is set at the point of emission and is therefore identical for all regions of the interferometer, regardless of their spatial separation, and cancels out in the readout stage for a gradiometric configuration [7,8].

This allows for much larger interferometer baselines to be employed than would be possible with counterpropagating multiphoton interferometry beams, as used in Raman or Bragg interferometers, where the finite speed of light introduces a noise term [13].

The use of large baselines and the enabled very large interferometry times are not, however, sufficient in themselves to achieve the sensitivity required for the physics goals of these experiments. Therefore, it is necessary to enhance the device sensitivity by employing the technique of large momentum transfer (LMT) [8], with very large enhancements of 10^3 – 10^4 ultimately proposed for terrestrial experiments currently in the development stage [11,12]. In practice, this typically means increasing the momentum separation between the two paths of the wave packet by applying a series of π pulses, the number of which is of the same order as the required enhancement [3,8]. For these schemes, it is therefore crucial that each π pulse is as efficient as possible if interferometry contrast and fringe visibility are to be maintained, as a loss of fringe visibility leads to a decrease in device sensitivity and therefore acts in opposition to the metrological gain that would otherwise be achieved by LMT. From a practical standpoint, this requirement translates as needing an ultracold velocity-selected atom source illuminated by an intense and homogeneous beam. Furthermore, although laser frequency noise cancels out in the phase readout in the clock-transition gradiometer configuration [1,2], laser noise can nevertheless degrade device performance by reducing π -pulse efficiency and interferometer contrast

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and visibility [14], as has been experimentally observed for clock atom interferometers [1,2].

In this paper, we concentrate on the effect of laser noise, both intensity but especially frequency noise, on pulse fidelity, and hence on interferometer fringe visibility. We determine the fidelity for the Mach-Zehnder interferometry schemes proposed for large-baseline clock-transition interferometers [8], showing that the induced imperfections of fidelity lead to practical difficulties in scaling LMT to the desired levels of large-baseline clock atom interferometers, especially at the proposed laser linewidth of approximately 10 Hz [12]. It is further shown that, in general, the fidelity of the pulse sequence is improved for a given noise level with increasing Rabi frequency, highlighting the need for a high-intensity interferometry beam, which is an important technological difference between using optical-clock transitions for atom interferometry and frequency metrology. As we consider the case of a single two-level atom interacting with a resonant laser, these results hold regardless of the atom-cloud temperature or spatial spread and represent the maximum achievable fidelity for a given laser system and pulse sequence.

We further apply this fidelity formalism to two newly developed high-power hertz-level clock lasers and show the practical LMT limits in principle achievable with these lasers when performing a standard clock-interferometry sequence. These two systems have been specifically developed for performing atom interferometry on the 1S_0 – 3P_0 clock transitions of Cd and Sr at 332 nm and 698 nm, respectively [15]. For the case of the UV transition of Cd, we build upon on recently demonstrated technology for a laser source centered at 326 nm [16], extending that work by demonstrating the capability of obtaining high coherence via successive frequency-stabilization stages on optical cavities with increasing finesse.

This paper is structured as follows. In Sec. II, we discuss the basics of LMT clock interferometry and visibility loss. In Sec. III, we present the basic theory of pulse fidelity, with further details in Appendix A, and show the results of the pulse fidelity calculations applied to a LMT sequence. In Sec. IV, we discuss the developed laser systems and analyze them within this framework. Our conclusions are reported in Sec. V.

II. LMT CLOCK ATOM INTERFEROMETER SENSITIVITY

The leading-order phase shift of a clock atom interferometer is $\phi \sim nkgT^2$, where n is the LMT order, k is the wave number of the transition, g is the local gravitational acceleration, and T is the interferometry time [1,8]. Single-shot-interferometry sensitivity can be characterized by the ratio $\Delta\phi/\phi$, where $\Delta\phi$ is the phase difference between the two interferometer arms that can be experimentally

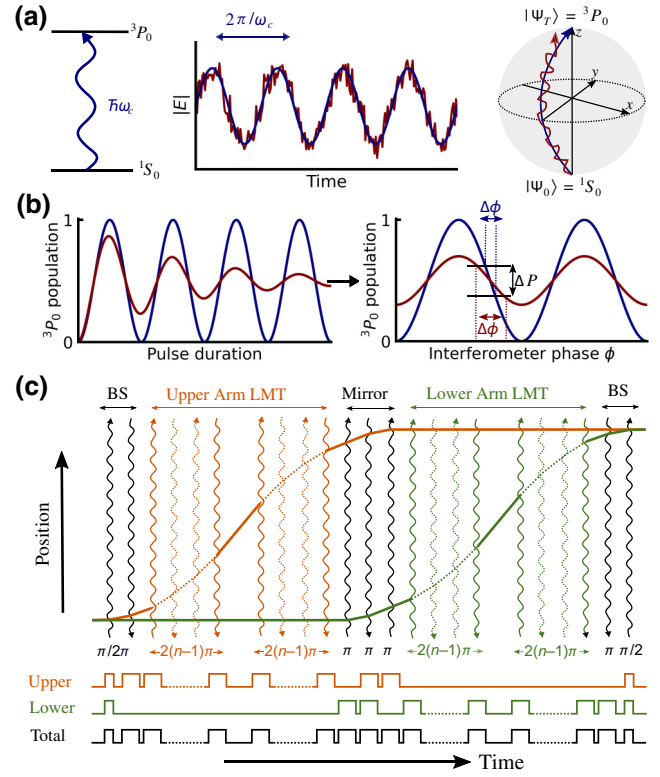


FIG. 1. (a) Clock atom interferometers operate on the metastable 1S_0 – 3P_0 transition, which is ideally driven by a noiseless laser (blue line). A noisy laser system (red line) will instead introduce errors when applying operations, such as for a π pulse as shown here in the Bloch-sphere representation of the two-level system. (b) These imperfections lead to damping in the temporal domain and also to a loss of interferometry visibility and hence interferometric sensitivity. (c) The proposed LMT scheme for a large-area clock atom interferometer [8]. The black lines represent the basic interferometry scheme, while the orange and green lines represent the LMT pulses applied to the upper and lower arms, respectively. The upper and lower arms of the interferometer receive different pulses but with a pattern that is symmetric in time.

resolved. It is therefore expected that, in principle, device-sensitivity enhancement should scale linearly with LMT order n and it has consequently been suggested that clock-atom interferometers should be improved by performing a succession of π pulses propagating in alternating directions [8]. In brief, a LMT enhancement of n is achieved by applying two additional stages of $2(n-1)\pi$ pulses independently resonant to the upper and lower interferometry arms and temporally mirrored about the standard interferometry-mirror pulse sequence (Fig. 1).

However, in experimental conditions, $\Delta\phi$ itself is typically found to be dependent upon the parameters n and T . This is because $\Delta\phi$ is extracted from the relative population difference and is therefore proportional to the slope and hence the visibility of the interferometry fringe (see

Fig. 1). In a typical atom interferometer, sources of visibility loss relating to the interferometry pulses can be roughly divided into two categories: those that lead to the different constituent atoms of the ensemble becoming out of phase with each other; and those that degrade the operational fidelity on the single-atom scale [17]. The former is well studied and often represents the leading practical constraint on atom interferometers, with finite atom-cloud temperature and size, as well as interferometry beam imperfections (e.g., wavefront and intensity inhomogeneities), causing different atoms within the ensemble to interact with effectively different laser beams during the sequence [14,18,19]. The resulting atomic dephasing causes a loss of contrast and visibility, as the readout of the interferometer is effectively the average over all the individual atoms. Not all such contrast losses, however, can necessarily be considered as due to a fundamental loss of coherence, as demonstrated by related spin-echo techniques [20,21]. Nevertheless, such effects are known to be both crucial and a key limiting factor for many current atom interferometers [14].

In addition to beam and cloud temperature imperfections reducing visibility, however, imperfect application of the desired quantum operation (e.g., $\pi/2$ and π pulses) will lead to contrast loss and other systematic errors. One source of such imperfections is laser noise and, crucially, LMT exacerbates the problem as it increases the number of necessarily imperfect pulses that are applied. In investigating this problem, previous analyses have tended to account for laser frequency noise by considering the excited population fraction from a π pulse to be $P_e \approx 1 - \delta^2/\Omega_R^2$ [8,12], where δ is the effective detuning and Ω_R is the Rabi frequency. In this regime, it is the small changes in δ caused by the nonzero laser linewidth that are considered. Such calculations have concluded that, for $\Omega_R/2\pi \sim 10^3$ Hz, the laser linewidth needs to be kept to the order of 10 Hz for contrast loss on the percent level for an interferometer with LMT order $n \sim 10^3$ [12]. As 10 Hz is readily achievable with current technologies, other effects such as finite atom temperature and intensity variations across the atom cloud are considered as the main source of dephasing and thus contrast and visibility loss.

However, it is also well known that the laser noise in terms of both intensity [22] and, especially, frequency [23,24] is an important parameter in determining pulse fidelity in quantum systems, as has been extensively studied in the field of quantum computing, which targets 10^4 operations on a single-qubit system [25,26]. Furthermore, it has recently been shown that to correctly determine π -pulse fidelity, it is not sufficient to consider only the line width of the interrogating laser; rather, the whole power spectrum of fluctuations should be considered [27]. Imperfect pulse fidelity arising from laser noise reduces the visibility as above but in an arguably more fundamental

way, as it is a damping of a single atom rather than a washing-out across the ensemble.

We note that although this laser noise also affects the fidelity of, e.g., Raman transitions [27], this represents a crucial practical difference between clock atom interferometers and more standard multiphoton-transition configurations. In the latter case, the relevant noise term is the *relative* phase noise between the counterpropagating interferometry beams, while for clock interferometers it is the *absolute* laser phase noise itself that is the relevant parameter [2]. The phase noise of Raman and Bragg beams can be easily reduced to orders of magnitude less than the phase noise that is currently achievable with even the narrowest clock lasers [28,29].

III. PULSE FIDELITY AND COMPUTATIONAL RESULTS

In determining the effect of the laser noise, we consider the fidelity of the whole interferometry sequence, where the fidelity of a quantum operation is defined as the overlap of the target operation with the actually applied operation. This is quantified by the square modulus of the Hilbert-Schmidt inner product of these operations [17]. Laser noise causes the state evolution on the Bloch sphere, defined here by the two-level system formed by the 1S_0 – 3P_0 clock transition, to deviate from the ideal case and thus for the fidelity to deviate from unity (Fig. 1). Specifically, in the standard case of applying a rotation about the x or y axes, frequency and intensity noise, respectively, produce undesired rotations about the z axis and the x and y axes [27]. In the presence of weak noise, the fidelity can be written as [17,23,27]

$$\mathcal{F} \simeq \frac{1}{2} [1 + e^{-\chi}], \quad (1)$$

where χ is the fidelity decay constant, which is related, to first order, to the noise power spectral density (PSD) by the filter function ($F(\omega)$) of the operation being performed:

$$\chi = \sum_j \frac{1}{\pi} \int_0^\infty \frac{d\omega}{\omega^2} S_j(\omega) F_j(\omega), \quad (2)$$

where $S_j(\omega)$ is linearly related to the PSD in a manner dependent upon the type of noise source and the summation is over the different sources of noise. Specifically, for frequency and intensity noise $S_j(\omega) = \frac{1}{4} S_{\omega_L}(\omega)$ and $S_j(\omega) = \frac{1}{16} \Omega_R^2 S_I(\omega)$, respectively, where $S_{\omega_L}(\omega)$ is the laser frequency noise, $S_I(\omega)$ the relative intensity noise (RIN), and Ω_R is the Rabi frequency [27].

In addition to the fidelity, the parameter $W = \exp[-\chi]$, which varies from 0 to 1, can also be considered. For a standard Mach-Zehnder sequence, this W parameter scales linearly with the interferometer visibility. Therefore, assuming that the interferometer sensitivity is linearly

dependent upon both this value and LMT order (see Sec. II), the use of this parameter allows for the sensitivity relative to an interferometer with $W = 1$ and $n = 1$ to be determined. For both \mathcal{F} and W , the problem of determining the fidelity of a quantum operation is reduced to the knowledge of the relevant PSDs and calculation of the appropriate filter functions. We note that this more general approach differs from the sensitivity-function method, which instead quantifies only the expected readout phase fluctuations as a function of the pulse sequence and the interferometric laser phase noise [30–32].

We determine the filter function for laser frequency noise for a sequence of square pulses mimicking the proposed LMT sequence in a clock-transition interferometer for detecting gravitational waves [8] [Fig. 1(c)]. These calculations follow a known method [17], with the key results for this application summarized in Appendix A. Figure 2 shows the filter function calculated for a variety of LMT values and shows a structure with a low-frequency cut-on, two principal resonances around Ω_R , and then a flatter distribution at higher frequencies with some harmonics on top. As can be seen, the filter-function structure is largely unmodified with increasing LMT, except that the function resonances about the Rabi frequency are amplified approximately quadratically with LMT order n . An increase in the interferometry time increases the sensitivity to low-frequency noise and determines the position of the nodes of the function at low frequency. The angular frequency of the

first node is found to have a $1/\log_{10}(T)$ dependence, with subsequent nodes occurring at harmonics of this frequency.

The above framework can be used to compute the effect of the laser frequency noise and the RIN on the operational fidelity. We calculate the effects independently but in the real case they must be combined through the summation in Eq. (2). When analyzing these effects, we in all cases assume a spectrally flat distribution, i.e., the PSD is a constant value, and integrate Eq. (2) in the range 10^{-2} – 10^7 rad/s. Although ideally Eq. (2) should be integrated over all angular frequencies, we find that, in our case, when increasing or decreasing both the upper and lower integration limits by two decades, the resultant fidelity is only altered by approximately 1%.

In the case of the RIN, it is observed that the pulse fidelity from a single π pulse is relatively unaffected by the laser noise [Fig. 2(d)], with excellent fidelities above 0.999 achievable even with high RIN values of -70 dBc/Hz for a wide range of Rabi frequencies. However, it is important to note that the Ω_R^2 dependence in Eq. (2) may make the RIN contribution experimentally important should alternative schemes to increase the Rabi frequency be used [3,33]. Nevertheless, we therefore ignore the contribution of the RIN in the following analysis.

Although not as insensitive as for the RIN case, we find that good single π -pulse fidelities can be achieved when considering frequency noise, with $\mathcal{F} = 0.9988$ for a linewidth of 10 Hz and a Rabi frequency of

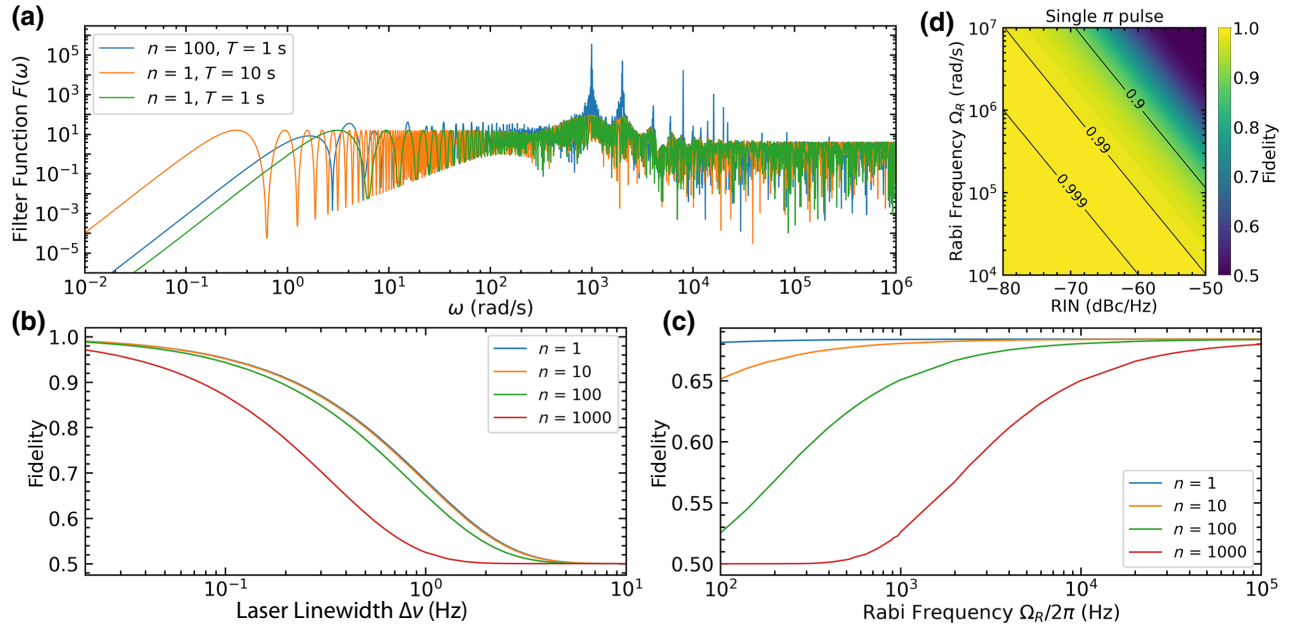


FIG. 2. (a) The filter function calculated for different LMT values [see Fig. 1(a)]. In these calculations, the interferometry time $T = 1$ s or 10 s, the Rabi frequency $\Omega_R = 10^3$ rad/s, and the pulse separation $\tau = 1$ μ s. (b),(c) The fidelity of a LMT sequence (b) as a function of the laser linewidth for a fixed Rabi frequency ($\Omega_R/2\pi = 10^3$ Hz) and (c) as a function of the Rabi frequency at a fixed laser line width ($\Delta\nu = 1$ Hz). In both cases, $\tau = 1$ μ s and $T = 1$ s. (d) The fidelity of a single π pulse as a function of the RIN level and the Rabi frequency.

$\Omega_R/2\pi = 1$ kHz, in line with previous estimates [8]. Conversely, however, the fidelity of a LMT sequence is highly degraded by the presence of laser frequency noise, unless the laser noise is reduced to experimentally difficult linewidths [Fig. 2(b)]. As mentioned, these calculations assume white noise that we relate to a Lorentzian linewidth via $\Delta\nu = \pi S_\nu = S_{\omega_L}/2$. We focus on Lorentzian linewidths for generality, whereas in reality laser linewidths typically take the form of a central Gaussian region and a Lorentzian component at higher frequencies [34], with an additional servo bump from the employed feedback circuit. The effect of these spectral features has been studied in a quantum computing context [27] and allows for the possibility of potentially reducing the degrading effect of the laser noise on the interferometry fidelity by suppressing the noise around peaks of the filter function or by judicious selection of the Rabi frequency [32]. Therefore, for any real application, the measured noise spectra of the laser sources should therefore be used for analysis, as we show later in Fig. 5.

As an example of the difficulties posed, however, for a Rabi frequency $\Omega_R/2\pi = 1$ kHz and $T = 1$ s, the fidelity already approaches its minimum value of 0.5 for $n = 1000$ even with a linewidth of $\Delta\nu = 1$ Hz and is at this value already for $n = 1$ when $\Delta\nu = 10$ Hz [Fig. 2(b)]. For a given LMT and Ω_R , the fidelity is initially relatively unaffected by increasing $\Delta\nu$ but decays exponentially following a cutoff linewidth. In this regime, the fidelity decays from approximately 0.9 to the minimum 0.5 with an approximate factor-of-10 increase in $\Delta\nu$, meaning that it is important to keep $\Delta\nu$ at, or lower than, this cutoff value to achieve the highest fidelities.

The dependence of fidelity on LMT order n at a fixed linewidth $\Delta\nu$ is similarly not seen to be initially critical for the realistic Rabi frequency $\Omega_R/2\pi = 1$ kHz in Fig. 2(b). As n is increased, the fidelity stays close to the $n = 1$ value until $n \sim 100$. This is due to the value of the χ parameter being dominated by the low-frequency region of the spectrum for small values of n and the insensitivity of the filter function to n in this region [Fig. 2(a)]. When the integration of the resonances around Ω_R begins to become significant in comparison to the low-frequency noise, then the fidelity begins to decrease significantly with n . When the fidelity is close to its maximum and minimum values of 1 and 0.5 for $n = 1$, the dependence on n is also decreased.

Unlike in the case of the RIN, however, the situation is improved with an increasing Rabi frequency [Fig. 2(c)], as this increases the frequency of the resonances [Fig. 1(b)]. Combined with the $1/\omega^2$ dependence of Eq. (2), this leads to a relative suppression of the laser noise. This is in agreement with the simple picture of considering the ratio of the effective detuning and the Rabi frequency. Increasing Ω_R has additional benefits for real systems with finite temperature [33], although the RIN will become increasingly important, as discussed above.

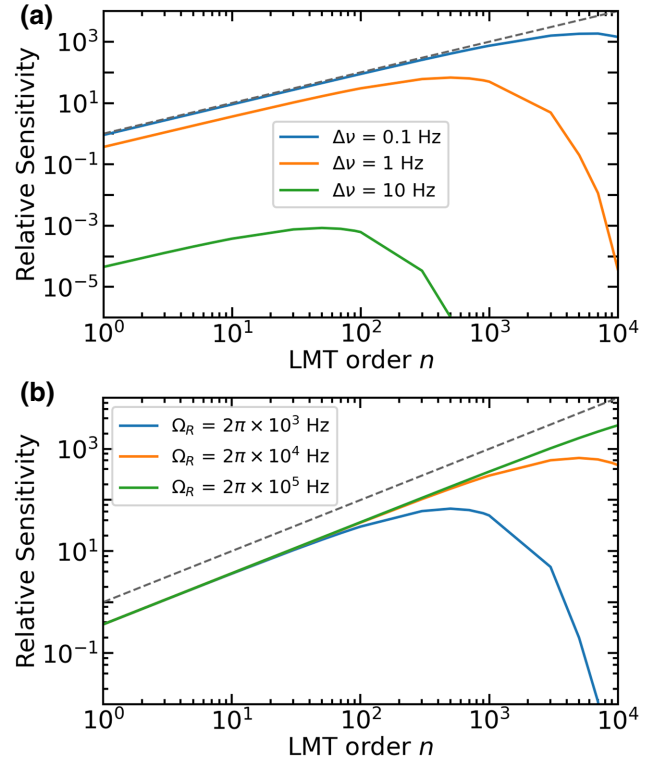


FIG. 3. The relative interferometer sensitivity, defined as nW , as a function of LMT for different values of (a) the laser linewidth $\Delta\nu$ (with $\Omega_R/2\pi = 10^3$ Hz) and (b) the Rabi frequency Ω_R (with $\Delta\nu = 1$ Hz). The dashed gray lines show linear scaling with LMT and $W = 1$ for comparison. In all cases, $\tau = 1$ μ s and $T_1 = 1$ s.

To quantify the reduction in interferometer performance, we consider the parameter W that we define earlier and that is closely related to the fidelity. In the ideal case of perfect pulse fidelity, the relative sensitivity enhancement of the interferometer would scale linearly with n but this is seen to not be the case in the presence of laser frequency noise. As shown in Fig. 3, there is, in fact, an optimal value of n for a given $\Delta\nu$ and Ω_R in terms of maximizing single-shot sensitivity. This framework represents a practical guide for finding the optimum LMT order n to maximize interferometer sensitivity in a real case. For example, for a laser with $\Delta\nu = 1$ Hz and $\Omega_R/2\pi = 10^3$ Hz, this occurs for $n \sim 500$ and at $n = 10^4$ the sensitivity is lower than at $n = 1$ [Fig. 3(b)]. For a 10-Hz-linewidth laser at the same Rabi frequency, however, this optimum is at $n < 100$ and, moreover, the relative sensitivity is significantly less than 1 in all cases [Fig. 3(a)], highlighting the necessity of low-noise lasers if the potential sensitivity gains from LMT are to be realized.

IV. CLOCK-INTERFEROMETRY LASER SOURCES

As can be deduced from the above results, the principal requirements for a laser system for

performing LMT clock-transition interferometry are high optical power and high coherence (Fig. 4). For the Sr system, the 1S_0 – 3P_0 transition is at 698 nm, meaning that commercial systems can provide the basis of this, in addition to providing good spatial mode quality and low RIN. We use a commercial Ti:sapphire system (Matisse TS, Sirah), pumped with up to 25 W at 532 nm, which is locked to a super-high-finesse ultralow expansion (ULE) optical Fabry-Pérot cavity with a finesse $\mathcal{F}_{698} = 4.1 \times 10^5$ [35]. A high feedback bandwidth (approximately 300 kHz) is achieved by utilizing an intracavity electro-optical modulator (EOM), at the cost of a slight reduction in output power (approximately 20%). Nevertheless, a total optical power of 4 W is achieved at 698 nm in this configuration, more than an order of magnitude more than has previously been used for interferometry on the 1S_0 – 3P_0 transition [1]. Stabilization is performed by prestabilizing the laser to a medium-finesse Fabry-Pérot cavity, using the EOM and the piezoelectric transducers (PZTs) on two of the laser mirrors. This cavity is in turn locked to the super-high-finesse cavity by controlling its length with a PZT on one of the cavity mirrors. For both cavities, approximately 1 mW of light is sent via polarization-maintaining fiber and modulated at approximately 10 MHz by external EOMs for Pound-Drever-Hall (PDH) locking [36]. The PSD of the in-loop measurement of the error signal is shown in Fig. 5 and yields a linewidth of 10 Hz.

The 1S_0 – 3P_0 transition of Cd, however, is at 332 nm and a UV light source capable of 1 W of optical power and coherence at the hertz level is not currently commercially available. It is therefore necessary to develop a system internally, optimizing its performance *ad hoc* for this application. We follow the same basic scheme as has recently been reported for a laser for the 1S_0 – 3P_1 Cd transition at 326 nm [16], where intense UV radiation is produced by two successive stages of second-harmonic generation (SHG), i.e., it is quadrupled from the infrared emission of a high-power semiconductor diode and fiber amplifier. In brief, a Littrow-configuration extended-cavity-diode master laser (ECDL) based on a gain chip is used to inject a commercial Raman fiber amplifier, which amplifies the 35-mW input to the 10-W level. SHG of this light is performed in a periodically poled lithium niobate (PPLN) crystal, producing around 3 W at 664 nm, before being doubled again in a resonant bow-tie cavity using a Brewster-cut beta-barium borate (β -BaB₂O₄, BBO) crystal as the nonlinear doubling medium [37]. This cavity is locked via the Hänsch-Couillaud technique [38] for total produced UV powers up to approximately 1 W.

The key difference with the previous scheme is the requirement of greater coherence than is necessary for the 1S_0 – 3P_1 Cd transition ($\Gamma = 2\pi \times 67$ kHz). This is achieved first by optimizing the prestabilization lock of the ECDL onto a medium-finesse optical Fabry-Pérot cavity ($\mathcal{F}_{\text{pre}} = 5 \times 10^3$). The error signal thus generated is

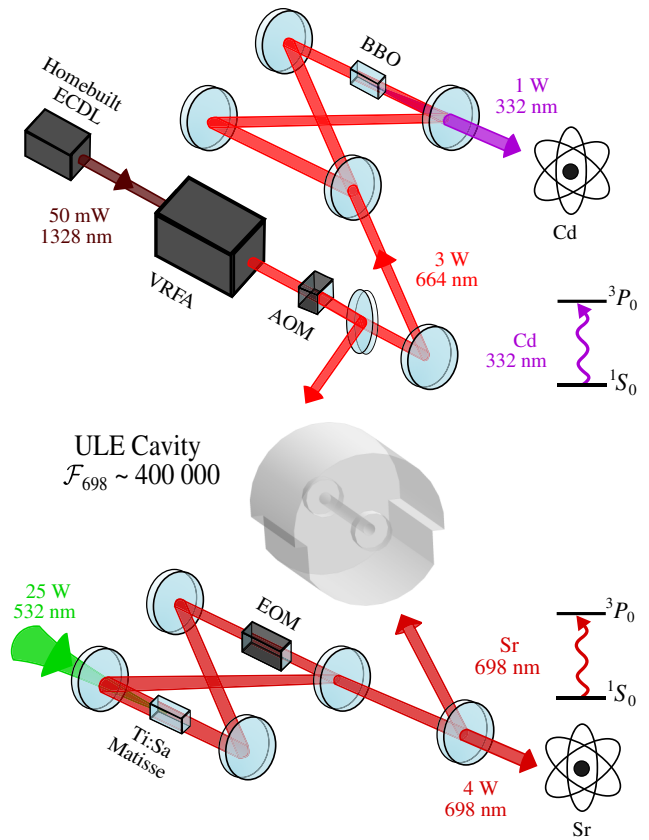


FIG. 4. The setup of the clock atom interferometry laser sources for Sr and Cd at 332 nm and 698 nm, respectively. The laser for Sr is derived from a Ti:sapphire laser with an intracavity EOM. For Cd, the laser is a frequency-quadrupled ECDL that is amplified by a Raman fiber amplifier. The lasers can both be locked to the same super-high-finesse ULE clock cavity. See the text for details of the locking procedures.

engineered by optimized proportional-integral-derivative controllers (PIDs) and filters before being used to close the stabilization loop. Two independent transduction channels are used: the PZT, which defines the grating orientation of the ECDL (low frequency—1 kHz); and the modulation input superimposed on the current supply (high frequency—700 kHz). In-loop noise analysis conducted on this stage shows that it is capable of reducing the source noise to equivalence to an estimated linewidth of approximately hertz level in the 100-kHz band [39].

To really reduce the linewidth further to the hertz level, the laser must be locked to the ULE cavity, for which a second feedback stage is applied in the visible region following SHG. The observation of a 20-Hz beat between the prestabilized and the amplified radiation allows us to conclude that the coherence length of the source is so far sufficiently large to neglect the frequency noise introduced by the Raman fiber amplification stage. We also observe the presence of low-frequency noise (10 dB at 100 Hz, 30 dB at 5 kHz with respect to the central emission peak),

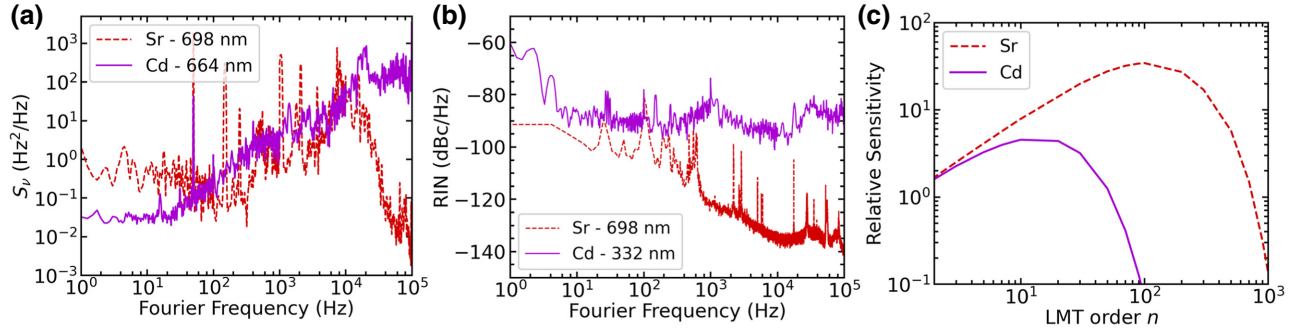


FIG. 5. (a) The measured in-loop $S_v(f)$ for the two clock-interferometry lasers at 698 nm and 664 nm for Sr and Cd, respectively: the equivalent fast linewidths are approximately 10 Hz and approximately 2 Hz, respectively. (b) The free-running RIN of the two clock-interferometry lasers. (c) The relative interferometer sensitivity as a function of LMT for the developed laser sources with $\tau = 1 \mu\text{s}$ and $T = 100 \text{ ms}$ and $\Omega_{\text{Sr}}/2\pi = 1.7 \text{ kHz}$ and $\Omega_{\text{Cd}}/2\pi = 740 \text{ Hz}$. In all of the plots, the dashed red lines are for Sr and the solid purple lines are for Cd.

to which this second stabilization stage is directed. We use the same cavity and setup as for the Sr laser, with the only difference being a reduction in cavity finesse to $\mathcal{F}_{664} = 1.3 \times 10^5$, as measured by the ring-down method. The PDH error signal is split and the low-frequency component is sent to the PZT of the prestabilization cavity to compensate drifts. An acousto-optical modulator (AOM), in single pass on the visible SHG, is used for higher frequencies up to around 50 kHz. The AOM is also used to additionally stabilize the intensity and reduce the amplitude noise added by the amplification stage [16]. With this method, the in-loop measured linewidth is 2 Hz at 664 nm, which corresponds to around 8 Hz after the doubling stage to the UV [Fig. 5(a)]. This value represents the lower limit for the linewidth, although the main noise contribution of the doubling stage should be frequency-to-amplitude conversion. Direct experimental validation of the linewidth or stabilization at 332 nm itself would require an evacuated high-finesse cavity in the UV regime, for which the production of the necessary substrates and coatings remains an active area of research [40].

Finally, the radiation at 664 nm is coupled (through a $T = 1.5\%$ input mirror) into a commercial bow-tie optical cavity, specifically designed for the application [37]. A production of UV power up to 1.2 W is achieved, which matches well with the expected values for the measured finesse $\mathcal{F}_{\text{BBO}} = 323 \pm 3$ and the nonlinear conversion coefficient $E_{\text{nl}} = 1.21 \times 10^{-4}$ calculated from the nominal system parameters [41]. The RIN of the UV [Fig. 5(b)] is kept low (approximately -90 dBc/Hz for $10\text{--}10^5 \text{ Hz}$) by the AOM stabilization stage and by passively shielding the BBO cavity to reduce acoustic resonances and no further active stabilization is required.

The measured intensity and frequency noise of these laser sources can be used to analyze their expected performance in a LMT clock atom interferometer. We input these measured values into Eq. (2) and integrate in the

measurement range of $1\text{--}10^5 \text{ Hz}$. Inputting the measured RIN values for our two laser sources [Fig. 5(b)] gives single-pulse fidelities of $\mathcal{F}_\pi > 0.9999$ for $\Omega_R < 10^6 \text{ rad/s}$ in both cases, suggesting that the RIN will pose negligible problems as expected. To instead estimate the effect of the frequency noise, however, we first fix the expected Rabi frequency to a reasonable value given our state-of-the-art laser powers. For a Gaussian beam radius of 5 mm, which is well below the maximum permissible size to keep the intensity ripple at the 1% level for diffraction from the standard DN100CF viewports that are to be employed [42], we estimate achievable $\Omega_R/2\pi$ values of 1.7 kHz and 740 Hz for the Sr and Cd sources, respectively. We use these Rabi frequencies to estimate optimal LMT parameters of $n_{\text{Sr}} \sim 100$ and $n_{\text{Cd}} \sim 10$ for Sr and Cd, respectively [Fig. 5(c)], based on the measured frequency noise [Fig. 5(a)] and $T = 100 \text{ ms}$, which is a reasonable interferometry time for our proposed experiments [15]. In the case of Sr, in principle, such a LMT-based enhancement in performance represents an improvement upon what has previously been demonstrated with clock-atom-interferometry schemes [3].

V. CONCLUSION AND OUTLOOK

We provide a novel general analysis for the noise requirements for a system to perform LMT clock atom interferometry with a standard pulse sequence by investigating the operational fidelity in the single-atom case. These analyses show the challenging nature of proposed long-baseline experiments for, e.g., gravitational wave detection and the difficulties in performing clock atom interferometry with LMT parameters up to $10^4 \hbar k$. In order to fully understand the effect on interferometer performance, especially in a gradiometer configuration where some common-mode noise cancellation may persist, it would be necessary to extend the above calculations to

a full quantum simulation incorporating parameters such as the atom-cloud temperature and spatial spread, loss mechanisms, and the external degrees of freedom of the system. Nevertheless, the outlined formalism will be useful for determining the interferometry laser requirements for the future experiments both on Earth [11,12] and in space [9,10] which seek to use clock atom interferometry to perform fundamental physics experiments. The maintenance of contrast and visibility is also crucial for achieving metrological gain in experiments that seek to use squeezing to go below the quantum projection noise limit [43,44].

The analysis presented above further focuses solely on square pulses and we note that it is possible to enhance π -pulse population transfer efficiency by techniques such as adiabatic rapid passage [45,46] or composite pulses [47,48], which can further be optimized by pulse shaping [49–51]. Alternative schemes, such as using the $^1S_0 \rightarrow ^3P_1$ transition for the LMT portion of the interferometer [3], using Floquet atom optics [52], or using a cavity to enhance the optical power and increase the Rabi frequency [33], have also previously been discussed and may help to circumnavigate some of these problems. Trapped-atom configurations that use Bloch oscillations to enhance the interferometry sensitivity may also offer a route toward a solution [53–55]. In all cases, however, our analysis suggests that the noise of the interferometry laser will play an important role and should not be discounted but, rather, integrated into these approaches.

Within this operational-fidelity framework, we present and analyze two laser systems suitable for performing clock atom interferometry—one for the 698-nm transition of Sr and one for the 332-nm transition of Cd—possessing the required characteristics of high power, low RIN, and an approximately hertz-level linewidth. In the case of the 332-nm laser, we establish the capability to produce high-coherence and high-power light in the UV by frequency quadrupling and amplifying an ECDL at 1328 nm. These lasers will be implemented in a dual-species atom interferometer, with potential applications to tests of the weak equivalence principle and quantum time dilation [15].

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APPENDIX: FILTER-FUNCTION CALCULATION

The filter transfer function $F(\omega)$ is calculated according to the procedure outlined in [17], briefly summarized here. The transfer function results from the norm of the *control*

matrix, generally introduced to describe the effect that laser noise sources, such as phase and intensity noise, have on the fidelity of a quantum operation. The loss of fidelity of a quantum operation is in fact proportional to the integral of the noise spectrum describing the radiation that induces the operation, averaged through the function $F(\omega)/\omega^2$ [Eq. (2)]. In the particular case of *dephasing* noise, the control matrix is reduced to a three-dimensional row vector ($\vec{R}(\omega)$), the squared norm of which gives $F(\omega)$:

$$F(\omega) = |\vec{R}(\omega)|^2. \quad (\text{A1})$$

The *control vector* $\vec{R}(\omega)$ for a sequence of pulses is calculated according to the formula

$$\vec{R}(\omega) = \sum_{l=1, \dots, N} e^{i\omega t_{l-1}} \vec{R}^{P_l}(\omega) \Lambda^{(l-1)}, \quad (\text{A2})$$

where the row vector $\vec{R}^{P_l}(\omega)$ is introduced and $\Lambda^{(l-1)}$ indicates a component, which is a matrix, of the $\vec{\Lambda}$ vector.

In this case of dephasing noise and for a piecewise defined control sequence, $R_i^{P_l}(\omega)$ is given by

$$R_i^{P_l}(\omega) = \frac{\omega}{\omega^2 - \Omega_R^2} \left\{ \delta_{zi} (i\Omega_R^l g_l(\omega) - \omega f_l(\omega)) + \frac{1}{2} (\Omega_R^l f_l(\omega) - i\omega g_l(\omega)) \text{Tr}(\sigma_{\phi_l} \sigma_z \sigma_i) \right\}, \quad (\text{A3})$$

where

$$f_l(\omega) = \cos(\Omega_R^l (t_l - t_{l-1})) e^{i\omega(t_l - t_{l-1})} - 1, \quad (\text{A4a})$$

$$g_l(\omega) = \sin(\Omega_R^l (t_l - t_{l-1})) e^{i\omega(t_l - t_{l-1})}, \quad (\text{A4b})$$

$$\sigma_{\phi_l} = \sigma_x \cos \phi_l + \sigma_y \sin \phi_l, \quad (\text{A4c})$$

in which δ_{ij} is the Kronecker delta function and σ_i denotes the usual Pauli matrices, with $i = x, y, z$.

The coefficients of the matrix Λ^l are then calculated according to

$$\Lambda_{ij}^l = \frac{1}{2} \text{Tr} \left(Q_{l-1}^\dagger \sigma_i Q_{l-1} \sigma_j \right), \quad (\text{A5a})$$

$$Q_l = P_l P_{l-1} \dots P_1, \quad (\text{A5b})$$

$$P_l = \exp \left(-\frac{i}{2} \Omega_R^l (t_l - t_{l-1}) \sigma_{\phi_l} \right), \quad (\text{A5c})$$

where the *cumulative* matrices Q_l are introduced and P_l is the propagator of the l th operation of a quantum sequence. Observe that for the case $l = 0$, $Q_0 = P_0 = \mathcal{I}$.

The calculation of the transfer function therefore requires the definition of a chain of N operations ($l = 1, \dots, N$) of *pseudorotation*, sequenced through a vector of operation times $\vec{t} = \{t_l | l = 1, \dots, N\}$, a vector of Rabi frequencies $\{\Omega_R^l\}$, describing the amplitudes of the rotations, and a vector of azimuthal angles $\{\phi_l\}$ describing the axes of the rotations, with $\phi = 0$ for a rotation about the x axis and $\phi = \pi$ for a counterpropagating beam. For the case of free propagation, $\Omega_R^l = 0$ and $P_l = \mathcal{I}$.

The required sum on l described in Eq. (A2) requires the calculation of N functions, each obtained by performing the calculation described by Eqs. (A3)–(A5) relative to step l , which must be iterated for values $l = 1, \dots, N$.

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