



# Comparison between Bayesian updating and approximate Bayesian computation for model identification of masonry towers through dynamic data

Silvia Monchetti<sup>1</sup> · Cecilia Viscardi<sup>2</sup> · Michele Betti<sup>1</sup> · Francesco Clementi<sup>3</sup>

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## Abstract

Model updating procedures based on experimental data are commonly used in case of historic buildings to identify numerical models that are subsequently employed to assess their structural behaviour. The reliability of these models is closely related to their ability to account for all the uncertainties that are involved in the knowledge process. In this regard, to handle these uncertainties and quantify their propagation, Bayesian inference is frequently employed being able to deal with the effects of parameter uncertainty, observation errors and model inadequacy. The computation of the posterior distribution through Bayesian inference needs—however—the evaluation of the likelihood function, which requires solving complex multi-dimensional integration problems. To bridge this shortcoming, the paper compares two Bayesian inference approaches to show how different approximations affect the results of simulated inference: a discrete approach for the likelihood computation in the Bayesian Model Updating (BMU) and a Monte Carlo likelihood-free method known as Approximate Bayesian Computation (ABC) are reported. As reference, the typology of historic masonry towers was considered by using their natural frequencies as experimental data for model updating. The two procedures provide very similar results supporting the validity of both methods despite ABC turns out to be a more flexible approach.

**Keywords** Bayesian inference · Approximate Bayesian computation · Model updating · Masonry tower · Uncertainty quantification · Experimental data

## 1 Introduction

In recent decades, the scientific community has paid increasing interest in identifying appropriate computational models for historic masonry buildings, with the aim of evaluating their structural behaviour and performance under exceptional loads. Several modelling strategies are proposed in literature, from block-based models to continuum models, and a recent detailed review of these approaches and a possible classification for masonry structures is shown in D’Altri et al. (2020).

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✉ Francesco Clementi  
francesco.clementi@univpm.it

Extended author information available on the last page of the article

Because of the uncertainties that may affect the geometry of the building, the mechanical response of the masonry and the restraint conditions, the identification of reliable numerical models (i.e. model updating) still is a critical issue which necessarily involves the use of experimental data. It is therefore essential the development of procedures for formally quantifying the uncertainty and for updating the knowledge on unobserved quantities as experimental data are collected. Bayesian inference represents a possible solution; it is a well-known approach in civil engineering and its application to existing buildings for model updating is motivated by the requirement to solve an inverse-problem, i.e. inferring the unobserved parameters governing a particular observed physics, when different sources of uncertainties has led to collect noisy and incomplete data. Furthermore, since different parameter values can lead to the same observed physics the problem becomes ill-conditioned and ill-posed. In this scenario, Bayes' theorem can be employed for addressing inverse problems by using evidence to update the probability of each unknown quantity. As a result, the obtained probabilistic solution is able to describe a whole interval of plausible model parameters.

However, while the approach is adequately formalized from the theoretical point of view, in practice some difficulties still need to be handled.

Firstly, the choice of experimental data and their acquisition procedure. Dynamic monitoring of the structural modal parameter under environmental vibration loading is widely recognized as a powerful procedure for investigating the evolution of structural health over time (Beck et al. 2001; Ubertini et al. 2018; Saisi et al. 2018; Standoli et al. 2021a, 2021b; Zini et al. 2022). The dynamic data provide information about the building global response, taking into account material properties as well as structural configuration, and variations in the dynamic response, compared to a reference configuration, can provide indication about damage detection (Pallarés et al. 2021). However, the need to adopt a limited number of sensors, or the necessity to employ no-contact techniques, can make these data unable of describing the structural complexity. Therefore, no experimental data is expected to provide an exact representation of how the system output behaves, as well as no model is expected to exactly represent the system input/output behaviour. Nevertheless, dynamic experimental data can be used to calibrate numerical models to describe not so much the single optimal parameters vector, as the family of all plausible values of model parameters based on the available data.

Secondly, the evaluation of the likelihood function in Bayesian inference. Indeed, despite Bayesian inference is often used in civil engineering for parameters estimation and model updating (Rocchetta et al. 2018; Huang et al. 2019; Pepi et al. 2020), in some cases the likelihood is analytically intractable, and an approximation is required. On the one hand, the likelihood can be computed by following a heuristic approach, using the deviousness of the probability distributions of measurement and modelling errors, when dealing with low-dimensional spaces and easy-to-evaluate distributions (Simoen et al. 2015). On the other hand, the difficulties in computing the likelihood function may be overtaken by using likelihood-free methods such as Approximate Bayesian Computation (ABC) (Tavarè et al. 1997; Pritchard et al. 1999). This latter solution is typically computationally intensive and often involves the employment of metamodels (Rutherford et al. 2005; Ren and Chen 2010; García-Macías et al. 2021; Teixeira et al. 2021) or the definition of empirical or synthetic likelihood functions (Mengersen et al. 2013; Wilkinson 2014; Price et al. 2018).

This paper aims to compare two Bayesian inference approaches for model updating by discussing a framework of realistic inference in case of inaccurate data. In particular, a discrete approach for the likelihood computation in the Bayesian Model Updating (BMU) and a Monte Carlo likelihood-free method are considered. The first, BMU, was proposed

in Monchetti et al. (2022) based on Monte Carlo simulations producing an approximation of the continuous problem to a discrete one. The second, known as Approximate Bayesian Computation (ABC), was investigated in Kitahara et al. (2021), Feng et al. (2020). The case of a historic confined masonry tower is analysed as a reference study, and the natural periods are selected as observations employed to update the parameters of the numerical model.

The rest of the paper is organized as follows: Sect. 2 reports the related works. Section 3 outlines the main concepts of Bayesian inference and the two considered approaches are defined. The application of these approaches is shown in Sect. 4 by highlighting their advantages and limitations through discussion of the results. Section 5 closes this paper by reporting conclusions and possible future developments of this study.

## 2 Related works

The use of dynamic response data in the problem of updating a computational model using a Bayesian statistical framework was originally addressed in the late 1990s by Beck and Katafygiotis (1998). Their research has fostered in the following years a growing interest in the use of Bayesian inference for model updating in practical engineering applications and, at the same time, has been accompanied by an increasing development of computational modelling capabilities. Starting from these early studies, to date, a variety of Bayesian methods for model updating have been proposed in literature to obtain a family of plausible model parameters tuned based on the available experimental information with the aim of overcoming the drawbacks of the deterministic optimization procedure.

These approaches were initially applied to numerical case studies and, more recently, have been extended in the field of civil engineering to existing structures (e.g. Box and Tiao 1992; Beck et al. 2001; Beck 2010; Goller and Schuëller 2011; Arendt et al. 2012; Brynjarsdóttir and O'Hagan 2014; Cheung and Bansal 2017; Conde et al. 2018; Rocchetta et al. 2018; Huang et al. 2019; Pepi et al. 2020; Pepi et al. 2021). However, only a few contributions are available for masonry structures (e.g. Atamturktur et al. 2012; Beconcini et al. 2016; Campostrini et al. 2017; Conde et al. 2018; De Falco et al. 2018; Ierimonti et al. 2021).

In particular, Atamturktur et al. (2012) investigating the choir vaults of the Washington Cathedral employed a Bayesian approach in order to assess the sources of modelling error and to quantify their contribution to the overall structural response prediction. Beconcini et al. (2016) proposed a probabilistic reliability assessment of heritage buildings under seismic and wind loads based on Bayes' theorem by discussing a practical application on a historical aqueduct in Italy. Campostrini et al. (2017), with the focus of the seismic vulnerability assessment at urban scale, proposed a probabilistic method based on a Bayesian approach, and applied it to a façade of a building structural unit in Italy. To investigate the causes that have originated damage in a masonry arch bridge, Conde et al. (2018) proposed an inverse analysis procedure based on the Bayesian approach formulating the damaged configuration as a parameter estimation problem. A comparison between two model updating procedures, one developed within a deterministic framework and the other based on a Bayesian probabilistic approach, was discussed by De Falco et al. (2018)

for the model calibration of a masonry bridge in Italy. More recently, a transfer Bayesian learning methodology for structural health monitoring of an historic masonry palace has been developed in Ierimonti et al. (2021).

These studies show the effectiveness of the application of the Bayesian framework in the field of heritage structures with the purpose of model identification as well as damage detection. They also show how for historic constructions the updating process needs to be addressed by considering the (usual) scarce amount of information, therefore it becomes crucial to understand how limited data can affect model calibration. Recently Rocchetta et al. (2018), discussing a procedure for crack detection in mechanical components, outlined the importance of explicitly accounting for the relevant sources of uncertainties. They presented a comparison between different likelihood functions to improve the accuracy and robustness of the whole procedure.

A comprehensive classification of the various sources of uncertainty was originally proposed in Kennedy and O'Hagan (2001) where the model discrepancy was formally introduced as an uncertainty in the simulator prediction. The central role of the model discrepancy was then deepened in Brynjarsdóttir (2014). They showed that an analyst that does not account for model discrepancy may lead to biased and over-confident parameter estimates and predictions. Indeed, as reported in Beck (2010), since no model of the system is expected to give perfect predictions, it is important to explicitly quantify the uncertain prediction errors in the likelihood evaluation. However, despite in literature are proposed Bayesian methods based on the likelihood evaluation, most of them rely on strong assumptions of normality and independence to overcome the difficulties in writing down analytically the likelihood function. It is therefore becoming increasingly important to define a procedure allowing to easily take into account many different sources of uncertainty to provide a realistic description of the behaviour of the system. Observed data are often linked to mechanical properties through complex systems of differential equations, thus the introduction of the uncertainty produces very complex likelihood functions. Furthermore, each point-wise evaluation of such functions would require running finite element (FE) models to use its solution in complex integral evaluations.

To overcome these difficulties one may rely on likelihood-free methods such as Approximate Bayesian Computation (ABC) to bypass any likelihood definition. Recently, this issue has been attracting the attention of other research groups (among others, Chaudhuri et al. 2018; Feng et al. 2020; Kitahara et al. 2021). In particular, Chaudhuri et al. (2018) reviewed the empirical likelihood ABC methods investigated in literature and proposed an easy-to-use method applied to some illustrative examples. Feng et al. (2020) resorted to ABC methods, but they did not adopt a Bayesian perspective instead, they aimed at finding optimal values of the model parameters. More recently, Kitahara et al. (2021) developed a two-step ABC updating framework using dynamic response data, by using the adaptive Kriging model applied to a numerical example in order to reduce the computational effort. To the best of the authors' knowledge, ABC methods have not yet been investigated in the model updating of existing buildings. A first attempt is provided in the present paper by overcoming the required computational effort through the definition of a metamodel, as shown in the following sections.

### 3 Bayesian inference

Bayesian inference is aimed to obtain the posterior probability density function  $p(\bar{\theta}|D^E)$ , *i.e.* the distribution of the unknown quantities ( $\bar{\theta}$ ) given knowledge of the data ( $D^E$ ), by using Bayes' theorem:

$$p(\bar{\theta}|D^E) = \frac{p(D^E|\bar{\theta})p(\bar{\theta})}{\int p(D^E|\bar{\theta})p(\bar{\theta})d\bar{\theta}} \tag{1}$$

where  $p(\bar{\theta})$  denotes the prior probability of the unknown quantities, and  $p(D^E|\bar{\theta})$  is the probability of the data given the unknown quantities which is proportional to the likelihood. The denominator is a normalizing factor necessary to ensure that the integral of the posterior distribution equals one (Box and Tiao 1992). The prior distribution allows including the expert judgment in the analysis, and can be interpreted as a measure of relative plausibility of the initial hypothesis. The likelihood function results from the probabilistic model assumed for the data, and quantifies how likely are certain values of  $\bar{\theta}$  in the light of the observed data. The computation of the normalizing constant in Eq. (1) may be challenging, except for the special case of conjugacy, *i.e.* when the posterior distribution follows the same parametric form as the prior distribution (Gelman et al. (2013)). To solve this problem, a common strategy is to resort to Bayesian simulated inference, in particular to Monte Carlo (MC) and/or Markov Chain Monte Carlo (MCMC) methods (Robert and Casella (2013)). The computation of posterior quantities becomes even more complex when the likelihood function is intractable, *i.e.* its analytical form is elusive or it is computationally demanding to evaluate. The definition of a tractable likelihood function often requires simplification of the reality based on strong assumptions of independence and normality of the involved random variables. In general terms, it should be considered a numerical mechanical model  $M$  which reproduces the observed physical phenomenon. It requires, as an input, the vector of unknowns  $\bar{\theta} = (\theta, X) \in \mathbb{R}^{m+n}$ . It is characterized by  $m$  independent unknown random parameters  $\theta = (\theta_1, \dots, \theta_m)$ , and  $n$  latent variables  $X = (X_1, \dots, X_n)$ . Note that random parameters are unknown and unobservable characteristics, while latent variables appear as unknown but fixed quantities that are only observable in principle, not in practice.

In this framework, the experimental data represent observable random variables whose generative process can be reproduced through a numerical model, the so-called “*simulator*”. They, as reported in Kennedy and O’Hagan (2001); Arendt et al. (2012); Brynjarsdóttir (2014), can be expressed according to the following mathematical formulation:

$$D^E = d^M(\bar{\theta}) + b + \epsilon \tag{2}$$

where  $D^E = (D_1^E, \dots, D_k^E)$  is the vector of  $k$  random variables representing experimental data,  $d^M(\bar{\theta})$  denotes the outputs of the computer model, *i.e.* the simulator,  $b$  is a bias term, and  $\epsilon$  represents the random measurement error. Note that the input of the simulator is the vector of unknowns  $\bar{\theta} = (\theta, X)$ . Therefore, given a certain value of  $\bar{\theta}$ , Eq. (2), combines two sources of uncertainties (Kennedy and O’Hagan (2001): the model inadequacy, *i.e.* the difference between the simulator and the real behaviour of the system, and the observation

error. These terms represent the discrepancy between the reality and its approximations obtained from the simulator and from the acquisition of experimental data, respectively. From Eq. (2), it is apparent that  $D^E$  is a random vector whose randomness is induced by the randomness of the observation error and the bias term (given certain values of  $\bar{\theta}$ ). This random vector has a probability density function,  $p(D^E|\bar{\theta})$ , that can be obtained as the convolution of two probability distributions according to the total probability theorem:

$$p(D^E|\bar{\theta}) = \int p(D^E|D, \bar{\theta})p(D|\bar{\theta})dD = \int p(D^E|D)p(D|\bar{\theta})dD \tag{3}$$

where,  $D = d^M(\bar{\theta}) + b$  is the actual behaviour of the system and, given  $D$ , the values of the experimental data depend only on the measurement errors, being conditionally independent from  $\bar{\theta}$ .

The two probability density functions (PDFs)  $p(D^E|D)$  and  $p(D|\bar{\theta})$  derive from the distributional assumptions on  $\epsilon$  and  $b$ . It follows that the way in which the probability distributions for these two sources of uncertainty is specified affects the easiness of computation of the integral in Eq. (3). Alternatively, two additional hypotheses can be considered: (i) the object of the inference procedure is the vector of parameters ( $\theta$ ) and, (ii) the bias term is equal to zero because the whole discrepancy between the output of the model and the real behaviour of the system may be led back to latent random variables  $X$ . In this simplified scenario, the likelihood function is thus modelled by considering the PDF of the observation error.

Given the posterior distribution for  $\bar{\theta}$ , the posterior distribution for  $\theta$  is provided by marginalizing over  $X$ , as shown in the following equation:

$$\begin{aligned} p(\theta|D^E) &= \int p(\theta, x|D^E)dx = \frac{\int p(\theta)p(x)p(D^E|\theta, x)dx}{\iint p(\theta)p(x)p(D^E|\theta, x)dx} \\ &= c^{-1} \int p(\theta)p(x)p(D^E|\theta, x)dx \\ &= c^{-1}p(\theta) \int p(x)p(D^E|\theta, x)dx \\ &= c^{-1}p(\theta)L(\theta, D^E) \end{aligned} \tag{4}$$

where  $c = \iint p(\theta)p(x)p(D^E|\theta, x)dx d\theta$  is the normalizing constant and  $L(\theta, D^E) = \int p(x)p(D^E|\theta, x)dx$  is the likelihood. Note that both the computation of  $c$  and  $L(\theta, D^E)$  require numerical approximations and may be challenging. In particular, multiple pointwise evaluation of  $L(\theta, D^E)$  would be demanding because also the evaluation of  $p(D^E|\theta, x)$  itself may require numerical approximation of the integral in Eq. (3) based on multiple runs of FE model. In this framework, the computational cost of these evaluations often makes the implementation of standard MCMC methods prohibitive and requires novel likelihood-free approaches (Huang et al. (2019)).

Hereinafter, the solution of Eq. (4) is obtained following two approaches which differ in the computation of  $L(\theta, D^E)$ : the first, a BMU approach, is based on a discrete approximation of the likelihood and the second, an ABC method, overcomes the problem of computing the likelihood through a likelihood-free approach. Their applicability is discussed next (in Sect. 4).

### 3.1 Bayesian model updating (BMU): a discrete approximation of likelihood

Model updating allows the identification of the unknown system properties whenever new information is acquired. Different techniques are proposed in the literature to compute the high-dimensional integrals which usually are involved in the definition of the posterior distributions. For small dimensional problems, when the number of involved latent variables and variables to be inferred is limited, their continuum joint-space can be discretized and the integrals involved in Eq. (4) can be solved via numerical solution. This approach is proposed in Monchetti et al. (2022) where the large number of required simulations necessary to reproduce the continuum joint-space is decreased by using Latin Hypercube sampling technique. Note that this procedure is a simplification of the problem which is applied when the dimension of the vector of unknowns is limited. Moreover, this approach is feasible only when one assumes known and tractable PDFs for  $b$  and  $\epsilon$ , otherwise the evaluation of the integral in Eq. (3) would become too challenging. However, in this simplified scenario, as discussed in Sect. 4, the aim is to obtain a prompt solution for the Bayesian inference, by taking the following actions:

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Algorithm 1: *BMU*

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- Draw  $\theta_1, \dots, \theta_j, \dots, \theta_N$  from  $p(\theta)$ .
  - Simulate  $X_1, \dots, X_i, \dots, X_N$  from  $p(X) = \text{log}N(\mu_X, \sigma_X^2)$  through Latin Hypercube Sampling.
  - Run the numerical model to compute  $d^M(\theta_j, x_i) = f(\theta_j, x_i)$  for each  $j \in [1, N]$  and  $i \in [1, N]$ .
  - If  $b \neq 0$ , compute  $D_{ij}^E$  as:  $D_{ij} = d^M(\theta_j, x_i) + b_{ij} + \epsilon_{ij}$  by simulating  $b_{ij}$  and  $\epsilon_{ij}$  from their distributions.
  - Evaluate the likelihood function approximating it as  $\hat{L}(\theta_j, D_{ij}^E) = \sum_{i=1}^N p(x_i)p(D_{ij}^E|\theta_j, x_i)$  for each  $j \in [1, N]$ .
  - Compute the posterior in Eq. (4) using the approximate likelihood  $\hat{L}(\theta_j, D^E)$ .
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To sum up, the discrete approximation of the likelihood function is workable when (i) a parametric setting of the errors and the latent variables is available and, (ii) the continuum joint-space of the parameters is simple enough to solve the integral  $L(\theta, D^E)$  (i.e. Equation (4)) via numerical solutions. It is worth noticing that assuming the numerical model as perfect ( $b = 0$ , with probability 1) and that there are not observation errors ( $\epsilon = 0$ , with probability 1), the probability in Eq. (3) is equal to 1 only when  $D^E$  is exactly equal to the model output with  $\theta$  given as an input. The posterior distribution thus becomes a point of mass over the exact solution of the inverse problem. This particular condition underlines that, even an exact solution to the deterministic optimization problem is reliable only when any discrepancies from the numerical model is completely impossible. In summary, the proposed BMU shows the benefit of not needing the use of metamodels but, as a deficiency, the number of involved variables to be inferred must necessarily be limited.

### 3.2 Approximate Bayesian computation (ABC): a likelihood-free approach

ABC (Sisson et al. (2018)) is a class of likelihood-free methods for drawing posterior inference when the likelihood function is unavailable, intractable or computationally demanding to be computed. The key idea of this method traces back to Rubin (1984) and is to convert samples from the prior distribution into samples from the posterior. The only requirement is the ability to produce samples from a simulator, beyond that from the prior. The method is based on the comparison between observations and data produced by the simulator, the so-called pseudo-data,  $D^S$ . In particular, the primal ABC algorithm (Tavaré et al. 1997; Pritchard et al. 1999) proceeds by: (i) drawing  $N$  parameter proposals  $\theta_1, \dots, \theta_N$  from the prior distribution, (ii) giving each parameter proposal as input to the simulator to get  $N$  pseudo-dataset  $D_1^S, \dots, D_N^S$  and, (iii) accepting only parameters  $\theta_j$  such that  $D_j^S = D^E$ .

The described procedure returns a sample from the exact posterior distribution. However, the sampling scheme described above may be very inefficient with high-dimensional data, or infeasible when dealing with continuous data. Thus, the exact equality constraint at the final step is usually relaxed by introducing a distance function,  $d(D^S, D^E)$ , and a positive threshold,  $\delta$ .

The resulting algorithm can be summarized as follows:

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Algorithm 2: ABC

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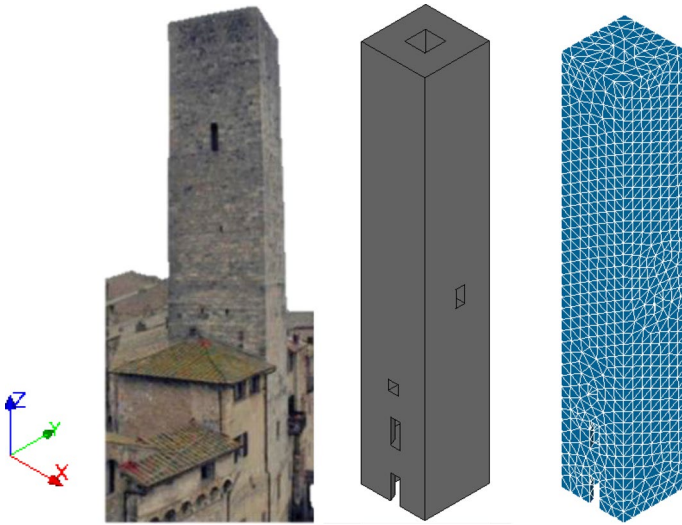
- Draw  $N$  parameter proposals  $\theta_1, \dots, \theta_j, \dots, \theta_N$  from the prior distribution  $p(\bar{\theta})$ .
  - Give each parameter proposal as input to the simulator to get  $D_1^S, \dots, D_j^S, \dots, D_N^S$ .
  - Accept  $\theta_j$  if  $d(D_j^S, D^E) < \delta \forall j \in [1, N]$ .
- 

The output of this algorithm is a sample from the following approximate posterior distribution:

$$p_{\delta,d}(\theta|D^E) \propto \int p(\theta)p(D^S|\theta)\mathbb{1}\{d(D^S, D^E) < \delta\}dD^S = p(\theta) \int p(D^S|\theta)\mathbb{1}\{d(D^S, D^E) < \delta\}dD^S \tag{5}$$

where  $d(D^S, D^E)$  and  $\delta$  are the two sources of approximation. Note that as long as  $d(D^S, D^E)$  is a proper distance function, as  $\delta$  goes to zero the approximate posterior distribution converges to the true posterior distribution. The reader is referred to Sisson et al. (2018) for further details on the method.

This approach avoids the analytical (or numerical) evaluation of  $p(D^E|\theta)$  allowing to specify whatever probability distribution for each of the considered sources of uncertainty. Furthermore, many latent variables can be introduced in the model to take into account all the sources of uncertainty, thus making the simulator as close as possible to the reality. However, realistic simulators are often computationally demanding, and Algorithm 2 requires a huge number of simulations to get an adequate number of accepted parameter proposals. Hence, a possible strategy to reduce the computational burden is to approximate the simulator through an emulator, i.e. a metamodel. Note also that the procedure reported in Algorithm 2 represents the primal ABC algorithm, but more sophisticated sampling schemes can be implemented to improve the efficiency of the method (e.g. Sisson et al. (2018)).



**Fig. 1** Illustrative example (left) and its numerical model (right)

### 3.3 Experiment

In this section, as illustrative example, a confined medieval masonry tower is considered to make a comparison between the two approximate Bayesian inference BMU and ABC approaches previously introduced. The considered tower has a quite regular geometry (see Fig. 1): the height is about 38 m and the base cross-section sizes  $6.8 \times 6.9$  m which gradually narrows along the height. Geometric of the tower was acquired combining terrestrial laser scanning techniques and traditional surveys. The walls are constituted by a multi-leaf stone masonry whose thickness ranges from 2.3 to 1.5 m. The internal and external faces of the walls are made with a soft stone masonry, whereas the internal core (despite it is not visible) is likely composed of heterogeneous stone blocks tied by a good mortar. At the lower levels the tower is integrated to an urban context built in the same or later period. This characteristic architectural context interacts with the tower and affects its dynamic behaviour by introducing some complexities in the model identification. The effective height of the tower, i.e. the length of the portion of the tower that is free from the restraint offered by adjacent buildings, strongly affects the estimation of the natural periods as discussed in Bartoli et al. (2017a); Bartoli et al. (2019). However, its determination may not be straightforward since it involves the quality of the restraints provided by the adjacent buildings and this parameter is usually uncertain (additional information on this parameter could be provided by adding as information the mode shapes).

Available experimental data for this tower are the first two natural periods along the two main directions; they are  $T_1 = 0.73s$  and  $T_2 = 0.60s$  (Pieraccini 2017).

As shown in Fig. 1, a three dimensional FE model is built to represent the masonry tower and to reproduce its dynamic behaviour. Even though the numerical model is always a rough simplification of the reality, it is essential to understand the system behaviour and to evaluate the effect of the input parameters on the structure output. Indeed, there are quantities usually considered as known and fixed despite their

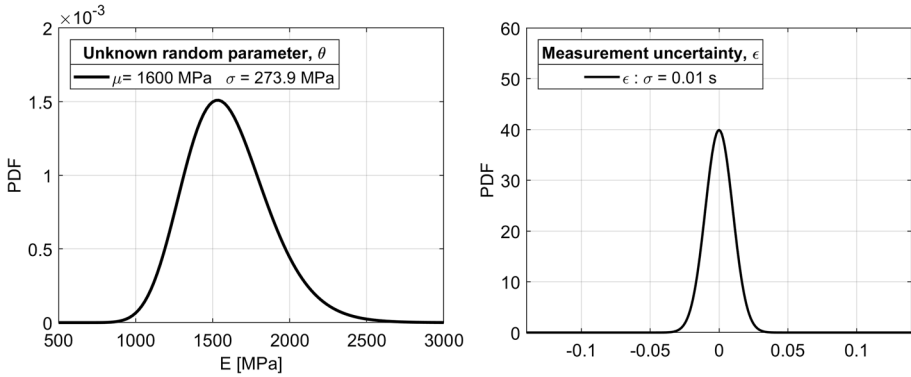


Fig. 2 Prior distribution of the Elastic Modulus of the masonry (left) and measurement uncertainty (right)

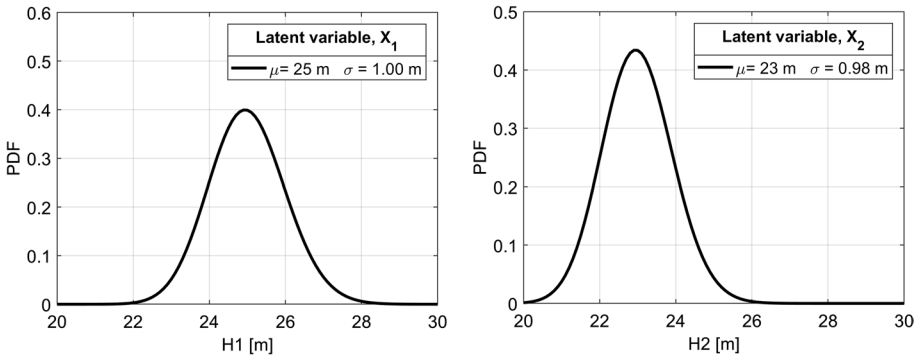


Fig. 3 Latent variable corresponding to North–South direction (left) and East–West direction (right)

uncertainty (e.g. some geometric data or the restraint conditions offered by the soil) and others whose uncertainty is directly modelled through probability function (e.g. the lateral buildings interaction and the mechanical properties of the masonry). The uncertainty around these parameters fosters the discrepancy between the model output and the reality and it is taken into account in the bias term  $b$ . However, some of these variables may be explicitly modelled as latent variables allowing the definition of their posterior distributions. In such a case, as described in Sect. 3, to get the posterior distribution of the unobservable variables it is needed to marginalize with respect to the all the latent variables considered in the model. More in detail, by considering the specific application herein introduced, the elastic modulus ( $E$ ) of the masonry and the effective height ( $H$ ) of the tower (i.e. the height of the unrestrained part of the tower, Bartoli et al. 2017b) were chosen to represent the vector collecting the unknowns. In particular, the effective height is considered as the factor determining the discrepancy from the simulator and the reality. It is explicitly modelled as a latent variable, but its posterior distribution is not evaluated.

Looking at Eq. (4),  $\theta$  represents the elastic modulus of the masonry ( $E$ ) and  $X$  the effective height ( $H$ ). For these unknown quantities the chosen probability distributions are shown in Fig. 2 and Fig. 3. Note that, the prior probability distribution of the elastic

modulus of the masonry is selected according to the reference values reported in NTC (2018) and CNR (2013) and a sensitivity analyses on the effect of different prior distributions on the posterior ones is reported in Monchetti et al. (2022).

Figure 2 shows the prior distribution of the elastic modulus of the masonry and the measurement uncertainty of the experimental observations. Note that the elastic modulus of the masonry is hypothesized as homogeneous along the height of the tower as a lognormal distribution with standard deviation equal to 273.98 MPa and mean value 1600 MPa. For both the natural periods the measurement error is considered as a zero mean Gaussian distribution with standard deviation equal to 0.01 s. Figure 3 shows the latent variables along the two main directions of the tower (North–South and East–West). In the North–South direction the effective height is simulated through a lognormal distribution with standard deviation corresponding to approximately 1 m and mean equal to 25 m. In the East–West direction the urban context is different and, keeping standard deviation 0.98 m, the mean value is equal to 23 m.

In the following, BMU and ABC methods are applied and compared by considering the same probability distributions.

### 3.4 Bayesian model updating (BMU): application

BMU method has required the evaluation of the likelihood function and the run of the FE model, following the Algorithm 1 previously introduced. In this case, the limited number of parameters to be inferred allows to discretize the continuum joint-space of  $\theta$  and  $X$ . In this manner, the integrals which define the likelihood function and the normalizing factor are solved via numerical solution. The high number of required simulations is reduced by using Latin Hypercube Sampling (LHS) technique to  $N = 40$  thousand runs of the FE model. The results, in terms of posterior distributions of the elastic modulus are discussed next.

### 3.5 Approximate Bayesian computation (ABC): application

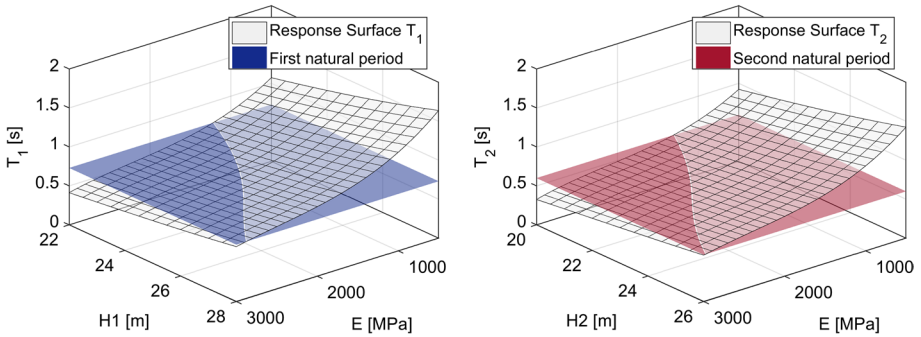
The ABC procedure is implemented as described in Algorithm 2. In principle, the simulator should be a function which (i) takes as input the elastic modulus of the masonry,  $\theta = E$ ; (ii) imputes the values of the latent variables ( $X = [H_1, H_2]$ , the effective heights); (iii) run the FE model to compute the first two natural periods of the tower; (iv) returns the simulated data with an additive error of measurement. All the random variables are sampled from the probability distributions illustrated in Fig. 2 and 3. However, the computational burden of the FE model does not allow to run the high number of simulations necessary in the ABC method. Thus, a third-order cubic Response Surface Method (RSM) is used to build a metamodel or emulator (Myers et al. 2016) in order to reproduce the  $k$ -th natural period of the tower; see Eq. (6):

$$T_k(\theta, X) = p_{00,k} + p_{10,k} \cdot \theta + p_{01,k} \cdot X + p_{20,k} \cdot \theta^2 + p_{11,k} \cdot \theta \cdot X + p_{02,k} \cdot X^2 + p_{30,k} \cdot \theta^3 + p_{21,k} \cdot \theta^2 \cdot X + p_{12,k} \cdot \theta \cdot X^2 + p_{03,k} \cdot X^3 + \omega \quad (6)$$

The polynomial metamodel is built by obtaining the coefficients  $p_{ij,k}$  (Table 1) through the least-square estimator (Han and Zhang 2012); in this way the trend of the FE model performance is reproduced (Fig. 4) and  $\omega$  represents the error. This latter term, herein negligible, could be treated as a statistical error normally distributed with zero mean.

**Table 1** Least square estimates of the coefficients of the response surfaces in Eq. (6)

$k$	$P_{00,k}$	$P_{10,k}$	$P_{01,k}$	$P_{20,k}$	$P_{11,k}$	$P_{02,k}$	$P_{30,k}$	$P_{21,k}$	$P_{12,k}$	$P_{03,k}$
1	-1.115	$3.941e^{-4}$	$2.082e^{-4}$	$2.231e^{-7}$	$-1.446e^{-8}$	$-7.171e^{-9}$	$-6.359e^{-11}$	$8.944e^{-12}$	$6.992e^{-13}$	$1.416e^{-13}$
2	-0.1617	$-3.058e^{-4}$	$8.226e^{-5}$	$2.059e^{-7}$	$-1.973e^{-8}$	$-1.709e^{-9}$	$-5.475e^{-11}$	$7.772e^{-12}$	$-5.059e^{-13}$	$6.101e^{-14}$



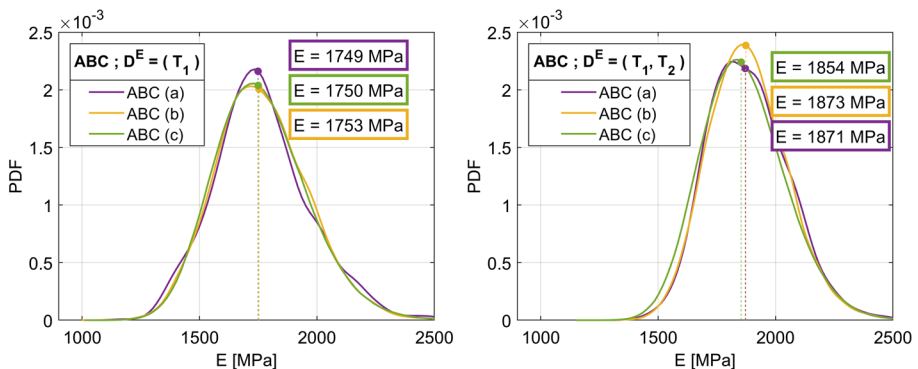
**Fig. 4** Response surfaces of the first two natural periods in the range of the unknown parameters

In order to assess the goodness of fit of the response surface in Eq. (6), the coefficient of determination is evaluated. For the present application it approaches 1 for both the two natural periods considered.

For the sake of comparison with BMU, the two posterior distributions are computed: the one based only on the first observed period, and the one based on the first and the second observed periods. The two distributions have been approximated running two independent ABC procedures. Recalling that the quality of the approximation of the posterior distributions provided by ABC algorithms depends on the tolerance threshold  $\delta$ , the sensitivity of the results to different choices were also investigated. Each value of  $\delta$  was computed using the  $\alpha$ -quantile method (Beaumont et al. (2002)). Table 2 displays the value of the threshold and the number of retained simulations corresponding to three different choices of  $\alpha$ . The absolute value of the difference between observed and simulated data was employed as distance metric when using only the first period. When considering both periods the

**Table 2** ABC tolerance thresholds and sample sizes corresponding to three different quantiles

ABC	$\alpha$	Sample sizes	$\delta_1$	$\delta_2$
(a)	0.0001	$10^3$	$1.4 \times 10^{-5}$	$1.4 \times 10^{-3}$
(b)	0.001	$10^4$	$1.4 \times 10^{-4}$	$1.4 \times 10^{-2}$
(c)	0.1	$10^6$	$1.4 \times 10^{-1}$	$4.63 \times 10^{-2}$

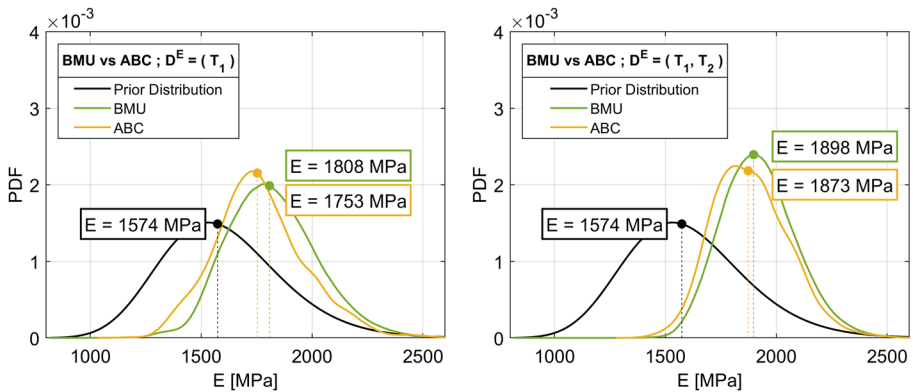


**Fig. 5** Posterior distributions of  $E$  derived by ABC with three different tolerance thresholds

Euclidean distance was employed. Figure 5 show the posterior distributions. All the results are obtained with  $N = 10$  millions of iterations.

### 4 Results and discussion

The results of the application of the two BMU and ABC Algorithms introduced in Sect. 3 are shown in Fig. 6 where the posterior distributions of the elastic masonry ( $E$ ) of the masonry are compared to the prior ones. The results of BMU are represented through a green line and the results of ABC with a yellow one. On the left, the figure shows the results of the two algorithms in the case of using the first observed period and, on the right, the figure represents the results in the case of using both the first and the second



**Fig. 6** Comparison between BMU and ABC **b** in terms of posterior distributions of  $E$

**Table 3** BMU versus ABC—Percentiles and IQR of the posterior PDFs of  $E$  [MPa] in the case of  $D^E = (T_1)$

E [MPa]	Percentiles			IQR
	25th	50th	75th	
<i>Prior</i>	1396	1576	1756	360
<i>BMU</i>	1664	1808	1934	270
<i>ABC(a)</i>	1636	1749	1883	243
<i>ABC(b)</i>	1631	1753	1892	261
<i>ABC(c)</i>	1624	1750	1888	264

**Table 4** BMU versus ABC—Percentiles and IQR of the posterior PDFs of  $E$  [MPa] in the case of  $D^E = (T_1; T_2)$

E [MPa]	Percentiles			IQR
	25th	50th	75th	
<i>Prior</i>	1396	1576	1756	360
<i>BMU</i>	1790	1898	2024	234
<i>ABC(a)</i>	1761	1871	1993	232
<i>ABC(b)</i>	1765	1873	1991	226
<i>ABC(c)</i>	1739	1854	1979	240

observed periods. Figure 6, as well as Tables 3 and 4, show that BMU is comparable to ABC in terms of median value and interquartile range (IQR). All the posterior distributions give evidence that the value of the elastic modulus is greater than what is suggested only by prior knowledge. Moreover, the introduction of the second period produces a further increase in the median value.

As already noted, the quality of the approximation of the ABC's results depends both on the value of the tolerance threshold,  $\delta$ , and on the goodness of fit of the metamodel. The sensitivity of the results to different tolerance thresholds was investigated (see Table 2). Generally speaking, the lower is the threshold, the higher is the quality of the approximation. Looking at Fig. 5 and Table 2 it is possible to observe that the posterior distributions corresponding to lower  $\delta$ -values tend to be slightly less diffuse. On the other hand, a small threshold corresponds to a lower size of the sample involved in the computation of posterior quantities. In the case at hand, the results are only slightly affected by  $\delta$  (see Table 2 and Fig. 5).

It is worth noting that the results of the two methods are very similar in terms of shape of the posterior distributions (see Fig. 6). This fact gives support to the validity of both the approaches and guarantees that the metamodel produces a good approximation of the FE model. Indeed, BMU does not involve any metamodel but rather uses simulations outputted by the FE model. Thus, ABC results that resemble BMU results give strong evidence that the error induced by the metamodel, *i.e.* discrepancies due to the term  $\omega$  in Eq. (6), is negligible.

Both the methods are sensitive to the choice of the prior distributions and the parametric form of the bias,  $b$ , and measurement error  $\epsilon$ . An analysis of this sensitivity was performed for the BMU approach in Monchetti et al. (2022), but results are extendable to the ABC approach.

All the comparisons between the two methods are effective only because the number of parameters to be inferred is restricted and the model is simple enough to allow the computation via numerical solution of the integrals involved in the BMU algorithm. In more complex cases, BMU becomes infeasible, whereas ABC continues to be a very flexible solution. Indeed, even when introducing a greater number of latent variables in the model, ABC only requires the ability of imputing their values when running the simulator. This means also that one can assume whatever PDFs for these random variables, since no analytical evaluations of that functions are needed.

## 5 Conclusions and future research

This paper was aimed at investigating strengths and weaknesses of two different Bayesian methods: BMU and ABC. The comparison was carried out by considering a representative case study dealing with the structural typology of confined historic masonry towers. Both BMU and ABC were implemented to infer the posterior distribution of the elastic modulus by using natural period data. The results show the effectiveness of the two procedures that have led to very similar results.

Both methods provide approximate posterior distributions, but they have different sources of approximations. BMU relies on a numerical approximation of the integrals involved in the computation of the likelihood function and of the normalizing constant of the posterior distribution. This approximation is achieved through a discretization of the space on which the unknown quantities are defined and the more finely is the discretization

the smaller is the approximation error. ABC avoids the computation of that integrals relying on a Monte Carlo approximation of the likelihood function based on a rejection scheme. Therefore, the quality of the posterior distribution depends on the value of the chosen threshold,  $\delta$ . Both the methods require multiple evaluations of the FE model and a very low value of the threshold, as well as a very fine discretization, can be achieved at the cost of a high number of simulations. However, ABC algorithms require a huge number of iterations to get a proper sample size. Accordingly, to reduce the computational cost, the simulator involving the FE model was approximated through a statistical model: the emulator. This solution introduced a further source of approximation in the ABC procedure but the results show that (in the case at hand), the ABC procedure is only slightly affected by these sources of uncertainty as it leads to results consistent with those of the BMU approach.

It is worth noting that the comparison between the two methods is feasible in this framework only because the number of parameters and latent variables is restricted. Furthermore, the PDF assumed for all the unknown quantities are characterized by the easiness of evaluation. Otherwise, numerical solutions of the integrals involved in the BMU algorithm would be infeasible. More sophisticated models may be developed in order to reduce the discrepancy between the real behaviour of the system and the observations. One solution would be involving statistical models in the formalization of the generative process of data. On the one hand, it would allow to introduce additional sources of uncertainty besides the additive errors; on the other hand, replacing mathematical models (such as the FEM considered so far) with complex stochastic relationships may make the likelihood function more complex. This latter aspect needs to involve specific Bayesian simulated inference approaches and an example is provided by Approximate Bayesian Computation.

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**Data availability statement** Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

## Declarations

**Conflict of interests** The authors declare that they have no conflict of interest.

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## Authors and Affiliations

Silvia Monchetti<sup>1</sup>  · Cecilia Viscardi<sup>2</sup>  · Michele Betti<sup>1</sup>  · Francesco Clementi<sup>3</sup> 

Silvia Monchetti  
silvia.monchetti@unifi.it

Cecilia Viscardi  
cecilia.viscardi@unifi.it

Michele Betti  
michele.betti@unifi.it

- <sup>1</sup> Department of Civil and Environmental Engineering, University of Florence, via di S. Marta 3, 50139 Florence, Italy
- <sup>2</sup> Department of Statistics, Computer Science, Applications “G. Parenti”, University of Florence, Viale Morgagni 65, 50139 Florence, Italy
- <sup>3</sup> Department of Civil and Building Engineering and Architecture, Polytechnic University of Marche, Via Breccia Bianche 12, 60131 Ancona, Italy