Editorial

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What will the mathematics of tomorrow look like?

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Abstract: In this preface of the Special Issue on Future Directions of Further Developments in Mathematics, we discuss about mathematics, how it should be, what it was and how we hope it will develop.

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When we were asked to be editors of a special issue celebrating 20 years of the journal *Open Mathematics*, https://www.degruyter.com/math, we wondered what subject could be capable of including the very numerous lines of research in current mathematics. After a long reflection, we agreed that the topic that could unify the many souls of mathematics was to discuss the perspectives of our discipline.

Designing a volume on the future of mathematics ultimately means asking what it is for and what it is about. Many great mathematicians have tried to give an answer, among others Courant and Robbins [1], Hardy [2], and Penrose [3]; see also Bartocci and Oddifreddi [4]. Even one of the authors of this article recently attempted to tackle this fascinating but extremely difficult subject. We cite his contribution [5] because it extensively deals with all the historical ideas mentioned in this very brief note and, for this reason, it can be taken as a reference by the readers.

To tackle this difficult task, we decided to brainstorm in a fabulous fish restaurant in Acireale (Sicily). We let a little wine, while keeping us lucid, help give us the courage to undertake increasingly visionary reasoning. This sweet philosophizing in front of the sea and in the moonlight reminded us of the poems of the great mathematician Khayyam [6], who drew inspiration from similar environments for profound reflections on human nature. Furthermore, as a friend and teacher of ours, Professor E. DiBenedetto, used to say (cf. [7]), mathematics is, together with philosophy, the convivial discipline par excellence. The deepest ideas are often born by discussing informally, a bit like what happened in ancient Greece between Socrates and his disciples [8].

We began by agreeing that mathematics reaches its apex when it is poised between applications and pure abstraction. According to Hardy, the beauty of mathematics has the same nature as that of painting and poetry. Hardy went so far as to list some characteristics that make a mathematical theory "beautiful" and these are "unpredictability, inevitability and economy." Simplicity and elegance are characteristics closely linked to the least possible number of assumptions. This makes mathematics very similar to physics. Newton, in the third volume of the Principia [9], as the first of the rules of philosophizing establishes that "Of natural things, more numerous causes must not be admitted than those which are true and sufficient to explain the phenomena." But mathematics is not just aesthetics. According to Dirac, theories when they are too "beautiful" cannot be discarded and do not need experimental confirmation. In fact, if a theory is sufficiently universal and elegant, it will end up describing phenomena for which it was not originally conceived and directed.

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But where does this ability of mathematics to anticipate the description of nature come from? From the fact that, as Galilei said [10], "Philosophy is written in this huge book which is continually open before our eyes (I mean the universe), but it cannot be understood unless one first learns to understand the language and know the characters in which it is written. It is written in mathematical language, and the characters are triangles, circles, and other geometric figures, without which means it is impossible to humanly understand a word of it; without these it is a vain wandering through a dark labyrinth." The idea that the universe is written in a simple and clear language is the postulate, almost a truth of faith, on which modern science is based: the same vision of the greatest modern physicist Hawking [11], who was perpetually in search of the theory of everything. The whole universe can be perfectly described by simple and elegant principles that appear complex only because of their interactions and the enormous number of physical particles/bodies involved. Why this is true is a mystery. As De Giorgi used to say, "At the beginning and at the end we have the mystery. We could say that we have God's plan. Mathematics brings us closer to this mystery, without penetrating it."

However, mathematics differs from physics in one essential respect: mathematics, like theology, is eternal. Mathematical language is the only language where one can distinguish the true from the false. As written as an epitaph on Vitali's tomb "Empires die, but Euclid's theorems retain eternal youth." This is not true for the experimental sciences where knowledge is negative: we know if a theory is false, but we can never say that a theory is true. At most, we can say that a theory is consistent with experimental observations. This "timelessness" of mathematics makes ridiculous the applied-to-measure metrics currently used to measure the "productivity" of mathematicians. A theorem should not be measured by its immediate impact, but by how it remains alive even decades after its publication. Galileo compared human beings to flasks full of wine [12]; mathematical theorems are also subjected to the same comparison: the really good ones age well and increase their flavor and strength over the years.

This publish or perish mentality has been harmful for mathematics (cf. also the Code of Practice of the European Mathematical Society Ethics Committee), which may be fascinating, aesthetically beautiful, and useful in its apparent uselessness (as everyone says), but it is experiencing a difficult moment. It was the protagonist of the third scientific revolution (Von Neumann, Turing and Shannon were all mathematicians) although it was unable to become the protagonist of the technological revolution currently taking place. In fact, it is not convenient to get involved to study new and significant problems that will probably be mentioned in the decades to come, but it is essential for one's career to throw oneself on fashion topics where even a marginal contribution will certainly be mentioned. The number of citations is more important than the depth of thought. So relatively few mathematicians have the courage to do what mathematicians have always done: starting from the phenomena that surround us, through abstraction define, through induction, a model, study its properties and, finally, deduce the properties of the starting phenomenon.

At the end of the dinner, we had the courage to invite mathematicians who had the same idea of mathematics as us and who were our friends. Indeed, as DiBenedetto said (cf. [7]), mathematics is a frustrating activity. Theorems, the real ones, are difficult to demonstrate, their proof often eludes us for years and years. Without the comfort of friends, it is difficult to manage this wait that seems to never end. The mathematician often experiences the sensations described by Beckett [13] or Buzzati [14].

Our friends have not betrayed us and have gifted us seven papers [15–21] of excellent and profound mathematics which may become ground for reflection and further discussion about the future directions of developments in the discipline.

References

[1] R. Courant and H. Robbins, *What is Mathematics? An Elementary Approach to Ideas and Methods*, Oxford University Press, New York, 1996.

[2] G. H. Hardy, *A Mathematician's Apology*, Cambridge University Press, Cambridge, 2012.

[3] R. Penrose, *Fashion, Faith and Fantasy in the New Physics of the Universe*, Princeton University Press, Princeton, 2016.

[4] C. Bartocci and P. Oddifreddi (Eds.), *La matematica. Pensare il mondo* (in Italian), Einaudi, Turin, 2010.

[5] V. Vespri, *Le anime della matematica* (in Italian), Diarkos, Sant'Arcangelo di Romagna, 2023.
[6] O. Khayyam, *The Rubaiyat of Omar Khayyam*, Dover Publ., New York, 2018.
[7] V. Vespri, *What I learnt from Emmanuele DiBenedetto*, in: V. Vespri, U. Gianazza, D. D. Monticelli, F. Punzo, D. Andreucci (Eds.), Harnack Inequalities and Nonlinear Operators, Proceedings of the INdAM conference to celebrate the 70th birthday of Emmanuele DiBenedetto, Springer INdAM Series, Vol. 46, Springer Cham, 2021, pp. 1–27.
[8] Plato, *Symposium*, StreetLib, Milan, 2018.
[9] I. Newton, *The Mathematical Principles of Natural Philosophy*, Published by Daniel Adee, 45 Liberty Street, New York, 1846.
[10] G. Galilei, *Il Saggiatore* (in Italian), Universale Economica I Classici, Milan, 2015.
[11] S. Hawking, *The Theory of Everything: The Origin and Fate of the Universe*, Laurier Books Ltd., Waterloo, Canada, 2008.
[12] G. Galilei, *Contro il portar la toga* (in Italian), ETS, Pisa, 2005.
[13] S. Beckett, *Waiting for Godot: A Tragicomedy in Two Acts*, Grove Press/Atlantic, Inc., New York, 2011.
[14] D. Buzzati, *Il deserto dei tartari. Nuova Edizione* (in Italian), Mondadori, Milan, 2021.
[15] G. M. Coclite and L. di Ruvo, *On H^2−solutions for a Camassa-Holm type equation*, Open Math. **21** (2023), 20220577, DOI: https://doi.org/10.1515/math-2022-0577.
[16] M. Winkler, *Classical solutions to Cauchy problems for parabolic-elliptic systems of Keller-Segel type*, Open Math. **21** (2023), 20220578, DOI: https://doi.org/10.1515/math-2022-0578.
[17] B. Piccoli, *Control of multi-agent systems: results, open problems, and applications*, Open Math. **21** (2023), 20220585, doi: https://doi.org/10.1515/math-2022-0585.
[18] J. Lopez-Gomez, E. Munoz-Hernandez, and F. Zanolin, *Subharmonic solutions for a class of predator-prey models with degenerate weights in periodic environments*, Open Math. **21** (2023), 20220593, DOI: https://doi.org/10.1515/math-2022-0593.
[19] R. Triggiani and X. Wan, *Luenberger compensator theory for heat-Kelvin-Voigt-damped-structure interaction models with interface/boundary feedback controls*, Open Math. **21** (2023), 20220589, DOI: https://doi.org/10.1515/math-2022-0589.
[20] S. Mosconi, *A non-smooth Brezis-Oswald uniqueness result*, Open Math. **21** (2023), 20220594, DOI: https://doi.org/10.1515/math-2022-0594.
[21] H. Hosni and J. Landes, *Logical perspectives on the foundations of probability*, Open Math. **21** (2023), 20220598, DOI: https://doi.org/10.1515/math-2022-0598.