

# **Difference Equations, Special Functions and Orthogonal Polynomials**

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Proceedings of the International Conference

# Difference Equations, Special Functions and Orthogonal Polynomials

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Professor B. Aulbach

## PREFACE

The theory and application of difference equations, special functions and applications encompass a diverse variety of mathematical fields. This includes, but is not limited to, dynamical systems, special functions, orthogonal polynomials, numerical analysis, combinatorics, integrable systems, and applications to biology, physics and economics. This proceedings volume contains papers on these (and other) topics that were presented at the International Conference on Difference Equations and Applications held in Munich on July 25–30, 2005. Among others this conference brought together researchers from three societies and organizations: the International Society of Difference Equations (ISDE), Orthogonal Polynomials, Special Functions and Applications (OPPSFA), and Symmetries and Integrability of Difference Equations (SIDE). The goal was to exchange ideas and explore new directions in these fields. This kind of collaborative effort is of paramount importance in promoting communication among the various groups using difference equations and special functions in their research. We hope that this volume will well serve junior and senior researchers in all fields related to difference equations, special functions and integrability.

*The Editors*

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