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Composite scores and index construction

for cross cultural research

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Choice of weights for subjective variables

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Introduction

Introduction

Comparison between findings observed at
both macro (e.g. countries) and micro (cases or groups) level

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one of the more vexed issues in the field of social research and surely is among the much-discussed matters.

Difficulties:

- if and how the differences might be explained,
- if and how explanations could help in performing comparisons more accurately.

Introduction

Exemplificative field of this topic

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Comparison between different levels of <u>quality of life</u> in terms of subjective well-being.

Explanatory models → differences in well-being can be explaned by (Christoph and Noll, 2001)

→ objective characteristics, e.g. different living conditions (objective micro level) and different national structures (objective macro level)

→ different cultural traits and value orientations observed at micro level and next considered to perform comparisons at macro level (region, country, etc.).

In this perspective, the question could be how to carry on comparisons between individuals (or groups) by taking into account inter-individual (or inter-group) differences yielded by different contextual conditions, i.e. cultural traits and value orientations.

Introduction

Possible answers → definition of "subjective weights"

Example

Satisfaction with life defined as a combination of satisfaction with family, work, income, and so on (*formative model*).

Combination has to take into account the *importance* (in terms of "life value" or in terms of expectations) that each individual assigns to each ambit.

income	**Life as a whole**
career	
family	
neighbors	
friends	
physical aspect	
financial independence	
ideals	
health	
partner	
...	

Comparison of satisfaction scores

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by taking into account the importance that individuals can differently assign to each ambit according to their life experience

Introduction

Defining weights → traditional methodologies

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<u>Construction of composite indicators</u>

approaches for determining **differential *importance* weights**

→ derived from **<u>objective principle</u>** (Nardo et al., 2005; Ray, 2008; Sharpe, 2004)

→ trying to take into account and to maximize the agreement among individuals (**<u>subjective weights</u>**) concerning the importance to be assigned to indicator to be aggregated (Hagerty and Land (2007)

Introduction

Purpose

to identify procedures that provide a framework allowing differential weights to be determined and managed (*subjective/individualized weighting* procedure)

The framework must clarify how:

- to obtain importance weights at individual-subjective level through subjective judgments
- to assign the weights to the corresponding subjective scores

Introduction

This work intends to

1. introduce the **general underlying principles in obtaining weights**
2. analyze the **classical approaches for obtaining "objective" weights**, generally applied in the ambit of composite indicators construction:
   - (i) statistical approaches,
   - (ii) Multi-Attribute approaches,
   - (iii) expertise approaches.

   Pros and cons of these approaches in the perspective of subjective weighting will be discussed
3. analyze the classical approaches allowing subjective data to be managed and to be transformed in subjective [importance] weights (**scaling models**). Among these, the models able to
   - (i) handle subjective evaluations and judgments, explicitly or implicitly expressed
   - (ii) obtain subjective weights at group level and at individual level,

   will be identified and described in the perspective of obtaining subjective weights

1. Underlying principles in obtaining weights

Preliminary statement

the elementary indicators (sub-score) to be aggregated do not contribute with the same importance to the measurement and evaluation of the total variable (synthetic score)

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a weighting system needs to be defined before proceeding to the indicators aggregation (Ghiselli, 1964).

Weights will be used in the successive computation of each (i) individual aggregate score (IAS)

$$IAS_i = \sum_{j=1}^{K} x_{ij} w_{ij}$$

Preliminary statement

When an implicit weighting system can not be identified,

a criterion has to be adopted in order to define a weighting system,

which can reproduce as accurately as possible the contribution of each sub-score to the construction of the synthetic score.

In this perspective, defining a weighting system constitutes an improvement and refinement of the adopted model of measurement.

Preliminary statement

In order to proceed to the difficult choice among the different weighting approaches, researchers need to take into account (Nardo et al., 2005; Ray, 2008):

- rationale and theoretical framework on which the measurement of the complex characteristics is founded and that will consequently regard the synthetic score
- meaning and contribution of each sub-score to the synthesis
- quality of data and statistical adequacy of indicators

In this perspective, the developed and defined can be interpreted in terms of

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judgment values

Preliminary statement

The identification of a system of weights must make a **decision** in advance **on**:

1. proportional size of the weights → **equal** or **differential** weighting
2. aggregation technique adopted → **compensatory** or **non-compensatory**
3. level at which subjective weights are determined and applied → **individual** or **group** weights
4. scale on which the weights should be determined → **rescaling issue**

1.1 Equal vs. differential weighting

1.1 Equal vs. differential weighting

The first decision that needs to be made and that will be strongly influence the final results is between

Equal Weighting (EW) ⇔ **Different Weighting (DW)**

1.1 Equal vs. differential weighting

Equal weighting

Preferred procedure → mainly when

- **theoretical structure**
  - each indicator has the same adequacy in defining the variable
  - no hypothesis can be consistently derived on differential weightings
- **statistical and empirical knowledge** → inadequate
- correct **adoption and application** of alternative procedures → no agreement

1.1 Equal vs. differential weighting

Equal weighting

- ⇨ does not necessarily imply unitary weighting
- ⇨ is an explicit weighting scheme
- ⇨ makes the choice of weights apparently less subjective
- ⇨ allows elementary indicators to be treated more consistently (that is, as statistical objects that are not subject to further subjective numerical interpretation).

1.1 Equal vs. differential weighting

Equal weighting

Doubtful procedure → mainly when

- definition of the variable requires different components specified by different numbers of indicators; in this case, adopting equal weighting corresponds to **assigning higher weights to the components showing higher numbers of elementary indicators**; in these cases, the synthetic variable will have an unbalanced structure;
- **indicators exist measuring the same component** (high correlations between elementary indicators): the result corresponds to that obtained when higher weights are assigned to indicators showing higher correlation (*double weighted* o *double counting*).

1.1 Equal vs. differential weighting

Differential weighting

This approach corresponds to the

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selection of the most appropriate approach in order to identify the weights among the identified ones (Nardo et al., 2005).

1.1 Equal vs. differential weighting

Differential weighting

Doubtful procedure → mainly when **is not supported by**

- **theoretical reflections** endow a meaning on each indicator or consider its impact on the synthesis,
- **methodological concerns** helps to identify the proper techniques, consistently with the theoretical structure.

1.1 Equal vs. differential weighting

Differential weighting

Preliminary statement

a **whole set of weights**

able to express in a **perfect** way the contribution of each indicator

does not exist.

Independently from the technical approach adopted in order to define them, the **weights can be**

kept constant → adoptable when the aim is to analyze the evolution of the examined ambit

or

changed → adoptable when the aim concerns the definition of particular priorities.

In both cases, the researcher needs to rationalize the choice.

1.1 Equal vs. differential weighting

According to Bobko (2007; §), the literature indicates that

- unit weights have substantial predictive validity when compared with regression weights, but
- there is a lack of data on how other differential weighting strategies (e.g., weights generated by subject matter experts) compare to unit weights.

Moreover, they provide a primary and a meta-analytic study by which they show how in their applications data and findings indicate that unit weights can be a highly appropriate approach for weighting under many circumstances.

However, in our perspective, the decision about the importance and the preference concerning particular aspects should be left to the individual. The particular aspects will be aggregated by taking into account the subjective weights into a synthetic score supposing measuring a certain subjective characteristic.

§ Bobko et al. (2007) made a interesting review of the relevant literature across multiple disciplines and multiple decades on differential and unit weights.

1.2 Weights and aggregating techniques: compensatory and non-compensatory feature

1.2 Weights and aggregating techniques: compensatory and non-compensatory feature

Consistent aggregating technique

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to avoid incoherencies between

theoretical meaning of weights ←→ the actual application of weights

(Nardo et al., 2005)

The choice of the weighting system must consider

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compensability among the elementary indicators

Aggregation through a compensability approach yields a synthetic score that does not allow us to return to the original unit profiles.

In other words, two units relating with different realities turn out to be identical and not distinguishable from each other.

1.2 Weights and aggregating techniques: compensatory and non-compensatory feature

Aggregating technique → *compensatory*

low values are compensated by high values

Example: <u>aggregating table</u>

Aggregation of two indicators (A and B): all possible synthetic scores obtainable **by additive approach (simple addition):**

		B		
		1	*2*	*3*
A	*4*	5	6	7
	3	4	5	6
	2	3	4	5
	1	1	3	4

1.2 Weights and aggregating techniques: compensatory and non-compensatory feature

Aggregating technique → *compensatory*

low values are compensated by high values

Example: <u>aggregating table</u>

Aggregation of two indicators (A and B): all possible synthetic scores obtainable **by geometrical approach (multiplicative technique):**

		B		
		1	*2*	*3*
A	*4*	4	8	12
	3	3	6	9
	2	2	4	6
	1	1	2	3

Also multiplicative technique is compensatory, especially with reference to indicators showing low values.

1.2 Weights and aggregating techniques: compensatory and non-compensatory feature

caution

in order to make multiplicative functions more manageable, the values of involved indicators are logarithmically transformed (summing up logarithm values corresponds to multiplying the original values)

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problems of interpretation

1.3 Subjective weights obtained at individual or group level

1.3 Subjective weights obtained at individual or group level

In our perspective

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the **decision** about

the importance and the preference concerning particular aspects

should be **left to individuals**.

The weights can be defined at

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- **individual level**: individual data will be used in order to construct weights that could be different for each subject,
- **group level**: individual data will be used in order to construct different weights for different group of individuals.

1.3 Subjective weights obtained at individual or group level

Subjective weighting at individual level

Let us define

X	a matrix with N rows (i=1 … N, individuals) and K columns (j=1 … K object variables) in which x_{ij} score that individual i assigned to j object (e.g. satisfaction for family)
W	a matrix with N rows (i=1 … N, individuals) and K columns (j=1 … K object variables) in which w_{ij} importance that individual i assigned to j object (e.g. importance of family)
Z	a new matrix with N rows (i=1 … N, individuals) and K columns (j=1 … K weighted object variables) in which z_{ij} weighted score for individual i concerning j object

1.3 Subjective weights obtained at individual or group level

Subjective weighting at group level

Let us define

X	a matrix with N rows (for i=1 … N, individuals) and K columns (for j=1 … K object variables) in which x_{cij} score that individual i belonging to the c group assigned to j object (e.g. satisfaction for family) The group can be predefined or can be determined through clustering methods
W	a matrix with G rows (for c=1 … G, groups) and K columns (for j=1 … K object variables) in which w_{cj} importance that group c assigned to j object (e.g. importance of family)
Z	a new matrix with N rows (for i=1 … N, individuals) and K columns (for j=1 … K weighted object variables) z_{ij} weighted score for individual i concerning j object

1.3 Subjective weights obtained at individual or group level

The object is

⇒ to determine the values of the W matrix (in the two versions, weights for individual and weights for groups)

⇒ to determine the interpretable values in Z matrix

⇒ to sum up the K weighted scores in a unique individual synthetic score.

We need to identify **methods supporting the two perspectives**, individual and group weighting.

1.4 Conditions for obtaining weights

Rescaling weights

- normalises weights to have an identical range (0; 1)
- could distort the transformed indicator in presence of extreme values/or outliers
- could widen the range of indicators lying within a small interval → increasing effect on weights.

2. Approaches for obtaining weights

2.1 Statistical approaches for obtaining "objective" weights

Statistical methods are the preferred approaches when the choice of weights relies preferably on "objective" principle:

1. **Correlation Analysis (CA)**
2. **Principal Component Analysis (PCA)**
3. **Data Envelopment Analysis (DEA)**

(Nardo et al., 2005; Ray, 2008; Sharpe, 2004)

2.1.1 Correlation Analysis

Premise → assigning equal weights to elementary indicators that are highly correlated can introduce.

Goal → less importance to be assigned to indicators that are highly correlated to the others (*double counting* effect)

Methods → since high correlation is considered as a sign of *double counting*, the procedure requires

i. averaging the correlation values registered between all the selected elementary indicators
ii. defining the weight (inversely proportional to the correlation level)

Problem → which is the limit value?

The limit can not be defined at a statistical level because there is no statistical rule on this matter; in any case, such decision can not be made on a statistical base but in the ambit of the adopted conceptual framework.

2.1.2 Principal Component Analysis

Premise → the group of elementary indicators define a multidimensional variables

Goal → for each elementary indicator, definition of one set of weights - components scores (*) – one for each dimension/component defining the latent variable. Weights allow one synthetic indicator for each component to be calculated. The resulting synthetic indicators will be consistent and independent from each other.

Methods →

i. identification of the components explaining the greatest portion of total variance.
ii. for each elementary indicator, calculation of component weight by removing the part of the elementary indicator's contribution explained by its correlation with the other elementary indicators.

Problem → which is the interpretation of the weights?

The adoption of this approach has to consider that the meaning of the weights (component scores) is exclusive statistical.

(*) Component scores measure the independent and not-correlated contribution of each elementary indicator in defining each component.

2.1.3 Data Envelopment Analysis

Premise → The elementary indicators define particular dimensions, like capacity

Goal → Identifying weights to be assigned to elementary indicators, with particular reference to concepts like "capacity".

DEA estimates the *efficiency frontier* that can be used as a benchmark in order to measure and evaluate the relative performance of observed units.

Methods → The set of weights for each unit depends on its position defined in terms of its distance from

- the efficiency frontier (corresponding to the best registered performance) or
- the benchmark (corresponding to an ideal point) or different priority defined according to those aspects that turned out to be good performances (in some sense this requires the individual identification of a strategic- or priority-objective).

2.1 Statistical approaches for obtaining "objective" weights

2.1.3 Data Envelopment Analysis

Benefits → it allows us to

- avoid an explicitly specification of a mathematical form describing the production function and the performance model
- uncover relationships that remain hidden for other methodologies
- handle many elementary indicators at the same time
- use any kind of input-output measurement
- identify, analyse, and quantified the sources of inefficiency, for every evaluated unit (in other words, it allows us to identify the elementary indicator showing the worse performance)

Problem →

The adoption of this approach is not always possible.

2.1 Statistical approaches for obtaining "objective" weights

Applicability of statistical models to obtain subjective weights

Statistical approaches are **useful**

to identify differential relative weights

to be assigned to each sub-score that will be considered in the synthetic score

and

expressing subjective importance with regard to different objects

These weights should be assigned to different evaluations (expressed – for example – in terms of satisfactions) corresponding to the considered objects (ambits of life).

The weighted evaluations could be subsequently combined into a total subjective evaluation concerning certain object.

2.1 Statistical approaches for obtaining "objective" weights

Applicability of statistical models to obtain subjective weights

Adoption of statistical methods in weighting components of indices has to be considered

carefully

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removing any control over the weighting procedure from the analysts gives a

false appearance of mathematical objectivity

that is actually difficult to achieve in social measurement

(Sharpe, 2004)

2.1 Statistical approaches for obtaining "objective" weights

Applicability of statistical models to obtain subjective weights

In particular, as we know, our perspective needs

⇨ to identify a criterion of importance or preference,

⇨ to define a model allowing

- subjective evaluations and judgments to be collected at individual level consistently with the criterion of importance/preference expressed by subjects (explicitly or implicitly)
- subjective importance/preference continuum to be constructed in order to transform evaluations and judgments into data analyzable and interpretable in terms of importance/preference weights

In this perspective

statistical models do not completely meet our requirements.

2.2 Multi-attributes approaches for obtaining "subjective" weights

2.2 Multi-attributes approaches for obtaining " subjective" weights

In order to

- define importance of a group of elements (elementary indicators) to be identified at subjective level and consequently
- identify subjective weights

methods are required able to manage a certain number of combined comparisons.

These comparisons can be managed by applying methods aimed at making decision among different available alternatives.

These methods are encompassed among Multi-Attribute Models.

Weights obtained through these methods are considered more stable than those produced by direct evaluations.

Among these models we can distinguish:

1. **Multi-Attribute Decision Making**: branch of the wider field of *Multiple Criteria Decision Making (MCDM)*, referring to making preference decisions over available alternatives that are characterized by multiple conflicting attributes.

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Analytic Hierarchy Process (AHP) (pairwise comparison of attributes) represents one of the techniques used in this ambit. It derives the “importance” of an alternative by summing up the scores of the elementary indicators,

Among these models we can distinguish:

2. **Multi-Attribute Compositional Models**: these models are based upon a statistical de-compositional approach through which it is possible to manage subjective comparisons of attributes on different levels. Its goal is to determine which combination is preferred by the subject

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Conjoint Analysis (CA) is able to manage subjective comparisons of attributes on different levels by disaggregating the preferences which are expressed by the subject in combination.

2.2.1 Analytic Hierarchy Processes (AHP)

Structured technique for dealing with complex decision

by **decomposing the problem related to the decision in hierarchical terms**

AHP provides a comprehensive and rational framework for

- ⇨ structuring a problem,
- ⇨ representing and quantifying its elements,
- ⇨ relating those elements to overall goals, and
- ⇨ evaluating alternative solutions

Many solutions, provided by pros and cons, can be analyzed and compared.

2.2.1 Analytic Hierarchy Processes (AHP)

AHP → compensative methodology

↓

since the identified alternatives can turn out to be efficient with regard to one or more objectives which counterbalance their performances.

AHP is based upon three basic principles:

- Interacting and interrelated attributes (objects) are not allowed (independency of criteria)
- Attributes can be hierarchically organized and the score for each level of the hierarchy can be calculated by summing up the weighted scores of the lower levels
- Scores can be calculated for each level from paired comparisons data; this can be performed only if the number of items is quite low

2.2.1 Analytic Hierarchy Processes (AHP)

Applicability of AHP in order to obtain subjective weights

Pro → possibility to obtaining subjective weights at individual level by a quite straightforward approach.

Con → need to construct a hierarchy with many nodes might make this approach non-applicable in the context we are dealing with (large surveys).

It is used to understand how respondents develop preferences for certain objects (products, services, ideas, ambits and so on)

2.2.2 Conjoint analysis approach

Basic premise → Individuals evaluate the value of an object (real or hypothetical) by combining separate amounts of value provided by each objects’ attribute.

Goal → to determine which combination of attributes is that preferred by the individual (Hair, 1998; Louviere, 1988; Malhotra, 1993).

Conceptual basis →Utility is a subjective judgment of preference unique to each individual. It is assumed to be based on the value placed on each of the values of the attributes and expressed in a relationship reflecting the manner in which the utility is formulated for any combination of attributes.

2.2 Multi-attributes approaches for obtaining " subjective" weights

2.2.2 Conjoint analysis approach

Procedure

1. Identification of ***factors*** describing the specific object of interest,
2. Identification of ***levels*** values defining each factor.
3. Identification of different configuration of the object by combining different values (levels) for each factor.

   Each combination is named ***scenario***.
4. ***Evaluation*** of the alternative scenarios according to a given criterion by respondents through one of the following approaches:
   - ***ranking***: respondent ranks scenarios in order of preference,
   - ***rating***: respondent assign to each scenario a level of preference expressed on a rating scale.

2.2 Multi-attributes approaches for obtaining " subjective" weights

2.2.2 Conjoint analysis approach

Procedure

5. Determination – through a de-compositional process – of
   - importance and weight of each factor in the total subjective decision,
   - how much each level of each factor has influenced the total preference (utility).

   The ***total worth***, expressed by a respondent with regard to an object, is formed of partial values (*part-worth*) relating to each level for each factor. The conjoint model can be formalized as following:

$$total \cdot worth = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(part - worth_{ij} \right)$$

where
m number of factors
n number of levels for each factor (value that changes for each factor).

2.2 Multi-attributes approaches for obtaining " subjective" weights

2.2.2 Conjoint analysis approach

Procedure

Estimates of part-worths allow the respondent's preference for any combination of factors to be assessed. The preference structure could reveal which is/are the factor/s determining the total utility and the final choice.

The analysis can be performed at both individual and group level. In particular, the choices expressed by a group of subjects can be combined in order to represent a "competitive" ambient.

This approach is considered *compensatory* and consequently requires a careful evaluation of its applicability.

2.2 Multi-attributes approaches for obtaining " subjective" weights

2.2.2 Conjoint analysis approach

Applicability of conjoint analysis to obtain subjective weights

Pro → the approach
- allows obtained proportions to be assigned to objects in terms of weights
- does not require the rescaling procedure to be applied
- does not allow a continuum of importance to be obtained
- meets the requirement of the sum of weights (sum of the obtained proportions is equal one)
- can be applied for obtaining subjective weights at both individual and group level.

Con → the approach should be applied with great caution since the obtained weights strongly depend upon the definition of the levels for each factor.

2.2 Multi-attributes approaches for obtaining " subjective" weights

2.2.2 Conjoint analysis approach

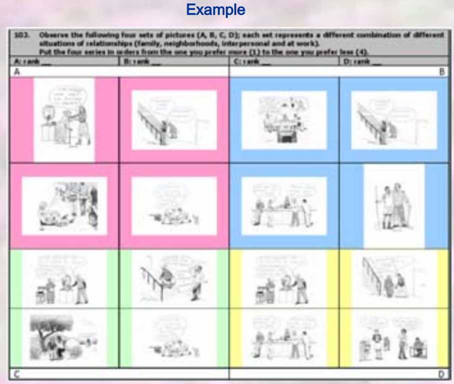
Example

Ambit: RELATIONS		Factors			
		Family	Neighbors	Friends	Colleagues
Stimuli	A	Supporting	Formal	Indifferent	Collaborative
	B	Intolerant	Formal	Superficial	Friendly
	C	Utilitarian	Intolerant	Open	Collaborative
	D	Utilitarian	Supporting	Superficial	Competitive

2.2 Multi-attributes approaches for obtaining " subjective" weights

2.2.2 Conjoint analysis approach

Example



2.2.2 Conjoint analysis approach

Example

In the table, proportions of factor are reported for each respondent.

The analysis has taken into account the polarity of the expressed judgments (which is negative: subjects assigned lower scores to the preferred scenario and vice versa).

Consequently, proportional values express the “importance”.

RESPONDENTS	NON-IMPORTANCE WEIGHTS expressed in terms of proportion (sum of weights = 1)			
	family	neighbors	friends	colleagues
Q1	0.064	0.350	0.427	0.159
Q2	0.087	0.467	0.216	0.231
Q3	0.301	0.246	0.253	0.200
Q4	0.297	0.190	0.401	0.113
Q5	0.303	0.203	0.258	0.237
Q6	0.176	0.293	0.172	0.359
Q7	0.301	0.246	0.253	0.200
Q8	0.267	0.287	0.187	0.260
Q9	0.176	0.293	0.172	0.359
Q10	0.301	0.246	0.253	0.200
Q11	0.301	0.246	0.253	0.200
Q12	0.301	0.246	0.253	0.200
Q13	0.157	0.308	0.336	0.199
Q14	0.087	0.467	0.216	0.231
Q15	0.266	0.287	0.187	0.260
Q16	0.266	0.287	0.187	0.260
Q17	0.266	0.287	0.187	0.260
Q18	0.266	0.287	0.187	0.260
Q19	0.265	0.125	0.154	0.456
Q20	0.297	0.190	0.401	0.113
...	...	...	...	...

2.3 Obtaining “subjective” weights through scaling approaches

2.3.1 Scaling models classification

As known, the traditional approaches that enable to deal with subjective evaluations and judgments are the “**scaling models**”. Let recall the features that can describe and characterize each scaling model (McIver, 1979):

- **Dimensionality**
- **Nature of data**
- **Scaling technique**
- **Criterion for testing the model**
- **Standard of measurement**
- **Contribution to the measurement of each multiple measures**

2.3.1 Scaling models classification

- **Dimensionality**, concerning the variable to which the combined individual score/s will be referred. Each dimension is related to different aspects of the defined variable. Two different dimensionalities can be distinguished:
  - uni-dimensionality: the definition of the considered variable assumes an unique and fundamental underlying dimension;
  - multidimensionality: the definition of the considered variable assumes several underlying aspects (dimensions).

2.3.1 Scaling models classification

- **Nature of data**, which depends on the researcher’s interpretation, expressed in terms of appropriateness and consistency. Different interpretations lead to different scaling procedures. Let us examine the scaling models applicable according to the classical classification of subjective data, theorized by Coombs (Coombs, 1950, 1953, 1964; Flament, 1976; Jacoby, 1991; McIver, 1979):
  - *Single stimulus*. Many scaling models were conceived for this kind of data; they are very often applied, such as the *additive model* and the *cumulative models* (deterministic and probabilistic) (Flament, 1976; McIver, 1979; Torgerson, 1958).
  - *Stimulus comparison*. The reference scaling models for this kind of data are the Thurstone model (Arcuri, 1974; McIver, 1979; Thurstone, 1927, 1959) and the Q methodology (McKeown, 1988).
  - *Similarities*. The reference scaling model for this kind of data is the *multidimensional scaling* (Cox, 1994; Kruskal, 1978; Torgerson, 1958).
  - *Preferential choice*. One of the reference scaling models is the *unfolding model* (McIver, 1979).

2.3.1 Scaling models classification

- **Scaling technique**, comparative or non-comparative.
- **Criterion for testing the model**. It is finalized to check the fitting of the model to data and it is different from model to model. The rationale of the testing procedure is common to all the models but the criteria are different according to the chosen model.

2.3 Obtaining "subjective" weights through scaling approaches

2.3.1 Scaling models classification

- **Standard of measurement**, concerning the treatment of the multiple measures and the assignment of the synthetic value (the final score can be assigned to individuals or to stimulus), according to the following pattern:

Standard of measurement		Multiple measures	With regard to the variable the objective of the measurement is to classify	Final score assigned to
The multiple measures allow to measure in more accurately	individual	**Stimulus (item)**	the individuals	**Individual**
	indicator	**Individual**	the elementary indicators	**Stimulus (item)**

2.3 Obtaining "subjective" weights through scaling approaches

2.3.1 Scaling models classification

- **Contribution to the measurement of each multiple measures**: it can be
  - *uniform* (all the multiple measures contribute through the same evidence)
  - *differential* (the multiple measures contribute through different evidence);

The characteristic of each measure can be defined by the **trace line**, that defines the relationship between the identified continuum and the frequency observed for each value of that continuum.

This frequency can be interpreted in terms of "probability to obtain each value" (McIver, 1979). In particular, two frequency distributions can be associated to each item, corresponding to two different probabilities respectively:

- *alpha*, probability relating to the expected value ("correct answer " or "agreement with the submitted sentence" o "answer that is in the direction of the measured variable")
- *beta*, probability relating to the not-expected value ("incorrect answer" or "disagreement with the submitted sentence" or "answer that is in the opposite direction to the measured variable")

Appendix A summarizes the characteristics of the well-known scaling models (Maggino, 2007).

2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

Our goal is the **identification of subjective weights** able to give back the idea of "subjective importance" attributed to each element (item) in comparison with the other elements composing the set.

Characteristics of the models allows us to identify those that better can help us in pursuing this goal.

In particular we have to select those models that utilize data

- whose nature is comparative or preferential (marked in yellow in Appendix A)
- produced by a comparative scaling technique (marked in pink in Appendix A).

Selected models are:

- *Thurstone model* (*differential scale*) and *Q* methodology, comprised among the cumulative approaches,
- *unfolding model* and *conjoint model*, comprised among the "perceptual mapping" approaches.

2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

Since we need also to identify a procedure that can be applied in a survey context without particular efforts, the *Q* methodology will be excluded by our consideration.

In our perspective, these models can be distinguished with reference to the possibility to define subjective weights at individual level or at group level (last column of the previous table), in particular:

- **individual weighting**: *conjoint model* (again)
- **group weighting**: *Thurstone model* (*differential scale*), *unfolding model*.

2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

A. Cumulative approach

Rationale → *law of comparative judgments*: theoretical construct representing the evaluation expressed by an individual in comparing two objects with reference to the attribute. The basic assumption underlying the law of comparative judgment is that the degree to which any two objects can be discriminated is a direct function of the difference in their status as regards the attribute in question.

Goal → to "create" a continuum on which the elements (items) concerning a certain characteristic are positioned.

In order to pursue this goal, the judgments expressed by a group of individuals are employed. The judgments can be expressed using the "paired comparison" scaling technique or the "rank ordering".

2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

A. Cumulative approach

Scales created by this method are called *Thurstone scales* or *differential scales*.

Many analytical versions exist according to the experimental model adopted (assumptions) and on the number of cases and the number of objects involved.

Values calculated through the application of particular and simple analytical procedure, allow defined elements to be placed on the continuum and can be considered in terms of group subjective weights.

Historically, Louis Thurstone (1927, 1959) was the first researcher that was engaged in the creation a continuum with a increasing intensity concerning a certain characteristic by using the judgments expressed by a group of "judges" (Arcuri, 1974; McIver; 1979; Torgerson, 1958).

In particular, Thurstone was mainly concerned with the fundamental problem of how psychological stimuli could be measured and compared with one another.

2.3 Obtaining “subjective” weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

A. Cumulative approach

In order to discover the “weight” of each of a set of objects (non-physical) the respondents need to order the objects by their relative importance.

Multiple subjective judgments could be collected through two different procedures:

(a) **Ranking →** each individual is asked to arrange objects according to a given criterion

(b) **Pairwise comparison →** objects presented in all possible pairs. In each pair, individuals point out the one that in the dyad better represents the criterion (e.g. importance)

2.3 Obtaining “subjective” weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

A. Cumulative approach

Applicability of cumulative model to obtain subjective weights

The cumulative approach

- allows a continuum of importance to be obtained
- requires the continuum to be interpreted in terms of polarity
- allows the objects to be positioned on this continuum according to a quantitative value interpretable in terms of weights
- produces weights that should be rescaled in order to meet the weights’ conditions presented above.

2.3 Obtaining “subjective” weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

A. Cumulative approach

Example

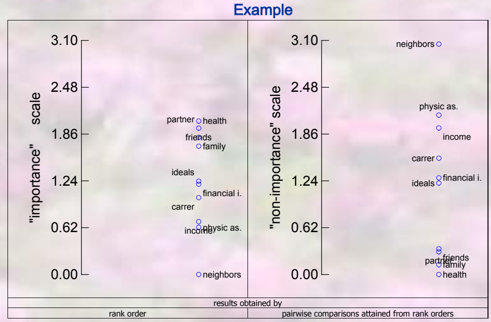
	ambits	rank
Put the listed individual life aspects in order, from the one you believe is the most important (1) to the one you consider the less important (10). (Don’t assign the same rank to two ambits)	income	
	career	
	family	
	neighbors	
	friends	
	physical aspect	
	financial independence	
	ideals	
	health	
	partner	

2.3 Obtaining “subjective” weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

A. Cumulative approach

Example

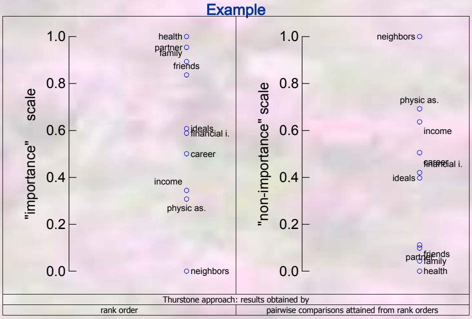


2.3 Obtaining “subjective” weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

A. Cumulative approach

Example



2.3 Obtaining “subjective” weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

B. Unfolding approach

It represents one of the models developed for the preferential choice data

Goal → to represent subjects and objects (said stimuli) in a common space – usually unidimensional – such that the relative distances between them reflect the psychological proximity between objects and individuals.

Assumption → **one common latent attribute** (referred to as joint scale or *J scale*) exists underlying the different observed preference orderings of a group of individuals.

More than one latent variable can be assumed (*multidimensional hypothesis*) : theoretical approach remains the same even if the geometric structure is obviously more complex. The goal is to place the points regarding both the objects and the respondents in a *R*-dimensional space by using the distances, Euclidean and not.

Multi-dimensional approach should be carefully considered → possibility to obtain degenerate solutions (local minimum).

2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

B. Unfolding approach

Procedure →

1. Each individual is asked to order a series of stimuli according to a preference criterion. Each individual's preference ordering is called *I scale*.

2. The analytic approach aims at obtaining a single *J* scale from the rankings of the objects made by the subjects (*I scales*)

Unfolding: process of evaluating the consistency of the individual *I* scales to be represented on a common *J* scale.

2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

B. Unfolding approach

Possible results → through application of goodness-of-fit algorithms

a. a single *J* scale **can** be determined → subjects employ a common criterion in evaluating the various stimuli

b. a single J scale **can not** be determined → two different possibilities:

- subjects employ multiple criteria in the evaluation of the stimuli
- subjects respond to the stimuli in a personal way, in other words, a common underlying attribute does not exist

2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

B. Unfolding approach

Applicability of unfolding model to obtain subjective weights

The unfolding approach

- allows a continuum of importance to be obtained
- requires the continuum to be interpreted in terms of polarity
- allows the objects to be positioned on this continuum according to a quantitative value interpretable in terms of weights
- produces weights that should be rescaled in order to meet the weights' conditions presented above.

2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

B. Unfolding approach

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	friends	
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	ideals	
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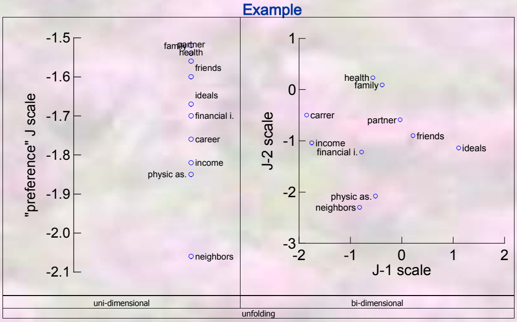
2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

B. Unfolding approach

Example

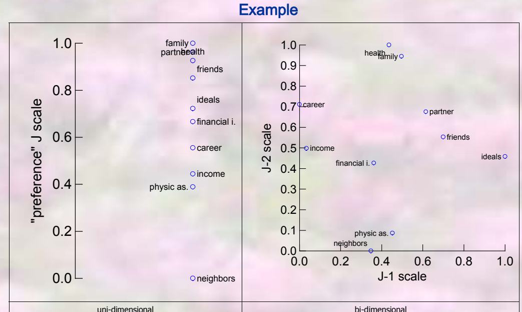


2.3 Obtaining "subjective" weights through scaling approaches

2.3.2 Scaling models allowing subjective weights to be obtained

B. Unfolding approach

Example



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References

Appendix A

Scaling model			Dimensionality	Nature of data	Scaling technique	Criterion for testing the model	Standard of measurement: final (synthetic) score assigned to
Additive	*Uni-dimensional*		Uni	Single-stimulus	Not-comparative	Internal consistency	Cases
	Multidimensional		Multi	Single-stimulus	Not-comparative	*Dimensionality of the items*	Cases
Cumulative	*Thurstone model* (*differential scale*)		Uni	Stimulus comparison	Comparative (pair comparison or rank-order)	Metrics between items	Items
	Q methodology		Uni	Stimulus comparison	Comparative (rank-order or comparative rating)		Items
	Deterministic	Guttman	Uni	Single-stimulus	Not-comparative	Scalogram analysis: reproducibility, scalability and ability to predict	Cases and items
		Multidimensional Scalogram Analysis (MSA)	Bi			Regionality and contiguity	Cases and items
		Partial Ordered Scalogram Analysis (POSA)	Bi			Correct representation	Cases and items
	Probabilistic	Monotone (one or more parameters)		Single-stimulus	Not-comparative	• parameters estimation (maximum likelihood) • goodness of fit (*misfit* and residuals analysis)	Cases and items (without condensation)
Perceptual Mapping	*Multidimensional scaling*		Multi	Similarities	Comparative (pair comparison)	Goodness of fit of distances to proximities (stress, alienation)	Items
	Unfolding		Uni & Multi	Preferential choice	Comparative	Goodness of fit of distances to ordinal preferences	Cases and items
Conjoint model			Multi	Preferential choice	Comparative (rank-order)	Goodness of fit of the model (part-worth) to the ranking	Items at individual level