A Power Allocation Strategy using Game Theory in Cognitive Radio Networks

Enrico Del Re and Gherardo Gorni
Department of Electronics and
Telecommunications (DET)
University of Florence
Florence, Italy
{enrico.delre, gherardo.gorni}@unifi.it

Luca Ronga and Rosalba Suffritti
National Inter-University Consortium for
Telecommunications (CNIT)
University of Florence
Florence, Italy
{luca.ronga, rosalba.suffritti}@cnit.it

Abstract— **The Cognitive Radio approach can be considered as a promising and suitable solution to solve in an efficient and flexible way the increasing and continuous demand of services and radio resources. This paper investigates how the adoption of a cognitive radio strategy can help in the coexistence problem of two wireless networks operating on the same spectrum of frequencies. A DVB-SH based satellite network will be considered as primary system, while an infrastructured wireless terrestrial network will constitute the cognitive radio based secondary system. In this work it will be presented a power resource allocation technique based on Game Theory, considering mainly Potential Games. We will show the proposed approach is suitable for distributed implementation, furthermore it provides performances comparable to an heuristic allocation method representing the optimum allocation. The comparison between these two resource allocation methods will be provided as result of this work.**

I. INTRODUCTION

Because of the increasing and continuous demand of services and radio resources, the traditional communication systems which imply an a priori association of the frequency band, the service assigned to it and the used technology, need to become much more flexible, efficient and easy-to-use dynamic systems able to cope with the requirements and constraints of the environment and the users. A Cognitive Radio (CR) approach can be considered as a promising and suitable solution to solve this problem. The term *cognitive radio* was introduced in [1] with reference to a communication system able to observe and learn from the surrounding environment as well as to implement and adapt its own transmission modalities also to user requirements. The concept of CR is originated from the contrast between an increasing demand of broadband services and the scarcity of radio resources. Recent studies of the FCC Spectrum Policy Task Force [2] demonstrated that a large amount of licensed bands are under-utilized, i.e., a lot of spectral resources are reserved for specific services, but, actually, they remain unused for most of the time or in several locations. From these studies, the possibility of a CR is envisaged, i.e., a system able to sense the electromagnetic environment (spectrum sensing), detect the spectral resources actually occupied in a given temporal interval and in a given location, and use the free bands (spectrum holes) for its own communications [3]. The search for available resources is not limited to spectrum portions dedicated to unlicensed communications, but is also extended to licensed bands. This paper shows the potential benefits of the adoption of a cognitive radio strategy to the coexistence problem. The developed cognitive radio strategy it has been formulated according the mathematical discipline of Game Theory, with particular reference to Potential Games [4].

Game Theory has already been considered in radio resource management of wireless networks as well as in the development of access control and routing techniques. In [5], an extensive study about the adoption of Game Theory based methods in wireless networks has been done. Potential Games applications in CDMA power control problems have been investigated in [6] and [7], while in [8] this mathematical framework has been used to model an interference avoidance scheme. However it is in Cognitive Radio networks analysis that Potential Games are being used widely, as firstly noticed in [9]. The PhD thesis [10] investigates various applications in Cognitive Radio contexts; it proposes as well a distributed frequency selection algorithm developed through Potential Games. In [11], a spectrum sharing game is formulated in order to perform a distributed adaptive channel allocation in a CR network, while in [12] the potential game framework encompasses both power control and channel selection.

In this paper a Game Theory based strategy is employed to perform power allocation in an OFDM Cognitive Radio network. Such power allocation is aimed to the up-link communication toward the CR local base station, in a scenario similar to the one envisaged by IEEE 802.22 Draft Standard for WRANs [13]. The main contribution of this work is to achieve the solution of the Potential Game in closed form for the case of a two-carriers OFDM system, and then, to extend such solution to the considered OFDM system. It will be also showed in Section III-A, how the considered potential function is readily available for distributed implementation.

The paper is organised as follows: in Section II the coexistence scenario will be described while Section III will explain the Game Theory based framework. In the Section IV the achieved results and the comparisons with the heuristic allocation method will be presented. The concluding remarks will be given in Section V.

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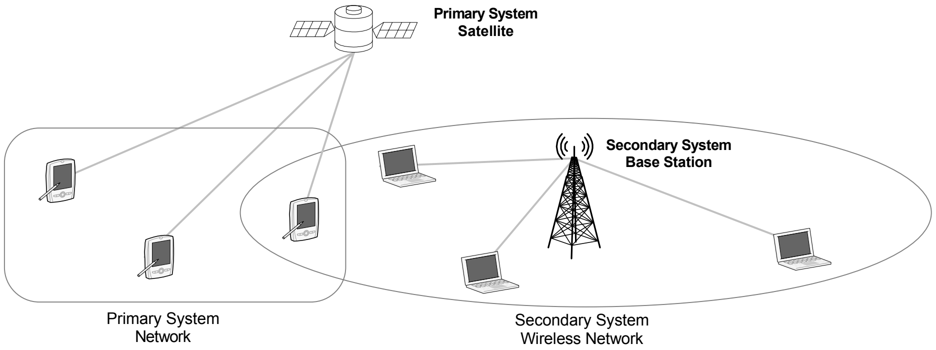

Fig. 1. The considered scenario

II. THE COEXISTENCE SCENARIO

The considered scenario is constituted by a primary licensed system and a secondary system whose terminals implement cognitive resource allocations, i.e. the Cognitive Radio based network. In particular it has been considered a mobile satellite system compatible with DVB-SH standard [14] as primary system in the envisaged scenario. As secondary system, it has been considered an infrastructured wireless terrestrial network, i.e. all the secondary terminals communicate with a local base station. The two systems work in the L band frequency spectrum (0,39-1,55 GHz), which in this context is considered as the main radio resource. In Fig.1 it is showed the proposed scenario. For both systems it has been supposed, as representation in the frequency domain of the transmitted OFDM symbols, a complex vector of length K, which is the actual number of subcarriers used to transmit the signal. Considering the DVB-SH standard specifics, K has been set equal to 853.

The two communication systems require the definition of two distinct channel models. For the satellite based primary system at L band it has been considered the Lutz et al. propagation model [15]. This model is based on a two state, GOOD-BAD, Markov chain for the fading process. Both slow fading and fast fading are kept into account. In particular, slow fading events due to large obstacles are modeled as a finite state machine; fast fading events instead, caused by irregular obstacles (e.g. vegetative shadowing) and multipath phenomena, they have been superimposed as a random variation with a specific probability density function for each state of Markov chain. The propagation model described shows a flat frequency response. A path loss term has been also considered in the primary system propagation model. The path loss is modeled by:

$$L = 10\log_{10}\left(\frac{4\pi d}{\lambda_0}\right)^{\alpha} dB \qquad (1)$$

where α is the attenuation depending on the distance, λ_0 is the wave-length at central frequency of the considered frequency band, and d is the distance between the transmitter and the receiver.

The propagation channel model considered for the terrestrial secondary system keeps into account mainly two phenomena: the path loss and the multipath fading. The path loss term is given by the expression used into the channel model for the primary system, i.e. the equation (1). The multipath fading due to the operating environment has been considered through a tapped-delay model with Rayleigh distributed coefficients. The power delay profile is exponential as follows:

$$\sigma_n^2 = e^{-\beta n} \qquad (2)$$

where σ_n^2 is the variance of the n-th coefficient and β is computed for a normalized mean power response. In the time domain the propagation model adopted exibits a finite impulse response of l samples resulting in a frequency selective channel response.

III. THE POWER ALLOCATION GAME

The Game Theory based allocation approach in the secondary network implies the definition of a proper framework. Indeed Game Theory, if formulated in the correct way, can lead to a stable equilibrium point known as Nash Equilibrium of the game. Such framework needs the definition of the players involved in the game that in this case are M cognitive radio terminals. Furthermore a set of available strategies has to be defined. The strategies are the actions each player can choose from in order to adapt both to the operating environment and to the opponent players choices. It has been considered as possible strategies for the players, the amounts of power that secondary terminals allocate on the K OFDM subcarriers to communicate effectively with the secondary system base station. The strategies actually played can be represented through a power allocation matrix, $\mathbf{P}^{II}$, containing all the amounts of power the cognitive terminals allocate. The power allocation matrix has some mandatory constraints to respect. In fact each cognitive radio terminal has to keep into account two constraints per subcarrier plus a general one due to the maximum power available on board for transmission, P^{II}_{max}. The constraint on the maximum power for the i-th user can be stated in the following way:

$$\sum_{k=1}^{K} p_i^{II}(k) \leqslant P_{max}^{II} \tag{3}$$

where the term $p_i^{II}(k)$ stands for the amount of power the i-th cognitive user is allocating on the k-th subcarrier. As far as it is concerned about the constraints on each subcarrier, two constraints are needed: one establishing the minimum amount of power to allocate in order to respect the secondary system target BER (i.e. the lower bound), the second one is needed in order to guarantee and protect the primary system functioning and it establishes the maximum amount of power on a certain subcarrier (i.e. the upper bound). The lower bound relative to the k-th subcarrier for the i-th terminal is given by equation (4) at the top of the next page, where $SINR_{min}^{II}$ is the signal-to-noise-interference ratio corresponding to the maximum BER tolerable by a cognitive terminal in order to use a QPSK modulation, $h_i^{II}(k)$ is the channel coefficient between the i-th terminal and the base station of the secondary network relative to the k-th subcarrier, $\sum_{j=1,j\neq i}^{M} h_j^{II}(k)p_j^{II}(k)$ is the interference on the k-th subcarrier due to the other terminals of the secondary network, $c^{I\rightarrow II}(k)P^{I}(k)$ is the disturb due to the transmission of the satellite system on the k-th subcarrier, and $N(k)$ is the noise power on the k-th subcarrier. The former constraints define a multidimensional real subspace within the available strategies stay. Given such particular constraints the real subspace considered is compact and convex.

The upper bound relative to the k-th subcarrier for the i-th terminal is given by equation (5), where $SINR_{min}^{I}$ is the signal-to-noise-interference ratio corresponding to the maximum BER tolerable by receivers of the primary system, $h^{I}(k)$ is the coefficient describing the channel between the satellite and the primary terminal nearest to the i-th secondary user, $P^{I}(k)$ is the power the primary system satellite is transmitting on the k-th subcarrier, $c_i^{II\rightarrow I}(k)$ is the channel coefficient considering the impairment effects of the power allocation $p_i^{II}(k)$ on the primary receiver nearest to the i-th terminal. In the case inequalities (4) and (5) don't have a common interval of real values, it is clear no power will be allocated on that subcarrier. This possibility identifies a situation where a primary system receiver is particularly near to a cognitive terminal or it is particularly sensible on some subcarriers to impairments effects due to cognitive terminals.

Finally opportune utility functions have to be defined. They represent the future benefit a player will achieve adopting a certain strategy, i.e. power allocation. In this case, the utility functions map the power allocation strategies for the i-th player into a real number considering also the power allocated by other terminals, i.e. $u_i : \mathbf{P}^{II} \rightarrow \mathbf{R}$. Making the assumption of rationality for all implies each player will choose the strategy producing the highest utility. Thus, it is a coherent hypothesis since the players are electronic devices suitably programmed. The choice of utility functions with specific properties has a crucial importance in the development of the game and especially in the existence of Nash Equilibrium Point. The utility function for the i-th cognitive player has been defined in the following way:

$$\begin{aligned} u_i(\mathbf{P}^{II}) &= \sum_{k=1}^{K} B \log_2 \Big[1 + SINR_i\left(\mathbf{P}^{II}(k)\right)\Big] \\ &\quad - \sum_{m=1}^{M} c_m^{II\rightarrow I}(k) p_m^{II}(k) \end{aligned} \tag{6}$$

where $SINR_i\left(\mathbf{P}^{II}(k)\right)$ represents the signal-to-noise-interference ratio the i-th terminal is achieving on the k-th subcarrier at the base station of the secondary network. $SINR_i\left(\mathbf{P}^{II}(k)\right)$ is so defined:

$$\begin{aligned} &SINR_i\left(\mathbf{P}^{II}(k)\right) = \\ &= \frac{h_i^{II}(k)p_i^{II}(k)}{\sum_{\substack{j=1,\\ j\neq i}}^{M} h_j^{II}(k)p_j^{II}(k) + c^{I\rightarrow II}(k)P^{I}(k) + N(k)} \end{aligned} \tag{7}$$

In the previous equations $\mathbf{P}^{II}$ is the power allocation matrix concerning all the secondary terminals. $\mathbf{P}^{II}$ has dimensionality KxM, where K is the number of subcarriers and M is the number of secondary terminals. Moreover in equation (6) the term B is the channelization bandwidth value considered for every subcarrier. The sum of the terms $c^{I\rightarrow II}(k)P^{I}(k)$ and $N(k)$ in (7) has been assumed the same for all the secondary users; the reason is due to the mathematical conditions required to develop analitically the game. The chosen utility functions have as domain the real subspace given by the power constraints introduced before. It has to be pointed out the utility functions chosen have an attractive property: they are twice differentiable. Concerning the meaning of (6), the first part is the sum of capacity values achieved by i-th player with the power allocation vector $\mathbf{p}_i^{II}$: higher this sum will be, higher the bit rate achieved by a terminal in the uplink will be. The sum $\sum_{m=1}^{M} c_m^{II\rightarrow I}(k)p_m^{II}(k)$, in the second part of (6), considers instead the disturbs caused on the primary users and it counterbalances the secondary terminal tendency to use as much power as possible.

The defined game will be since now referred synthetically as $\Gamma = (M, \mathbf{P}, \{u_i\})$. In order to determine the solution point of the game Γ, i.e. the Nash Equilibrium, the utility functions have been modified to highlight the considered game is a Potential Game. According [4], a game, that respects the definition of Potential Game, has a potential function encompassing the gains all the players perceive changing their own strategy. The only technical requirement to assure the convergence of the game is that at each stage at most one player can change its strategy. The last condition is mandatory in order to achieve a best response strategy increasing at each stage the value of the potential function. In the considered scenario the requirement can be fullfilled, imposing to terminals to allocate power in a round robin way for example. For potential games it has been also demonstrated that the Nash Equilibrium point is corresponding to the strategies maximizing such potential function. Potential games have been analyzed for applications in wireless networks by MacKenzie and DaSilva in

$$p_i^{II}(k) \geqslant \frac{SINR_{min}^{II}\left(\sum_{\substack{j=1,\\ j\neq i}}^{M} h_j^{II}(k)p_j^{II}(k) + c^{I\to II}(k)P^I(k) + N(k)\right)}{h_i^{II}(k)} \tag{4}$$

$$p_i^{II}(k) \leqslant \frac{h^I(k)P^I(k)}{SINR_{min}^{I} c_i^{II\to I}(k)} - \frac{\sum_{\substack{j=1,\\ j\neq i}}^{M} c_j^{II\to I}(k)p_j^{II}(k)}{c_i^{II\to I}(k)} - \frac{N(k)}{c_i^{II\to I}(k)} \tag{5}$$

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[16]. Afterwards some interesting results in [16] will be used to prove that Γ is a potential game and to identify the associated potential function.

Through opportune manipulations over the utility functions, it is possible to show the following condition is satisfied for all i and j:

$$\frac{\partial^2 u_i}{\partial p_i^{II} \partial p_j^{II}} = \frac{\partial^2 u_j}{\partial p_i^{II} \partial p_j^{II}} \tag{8}$$

where p_i and p_j are the power allocation strategies on the k subcarriers made by respectively the i-th and j-th users. A property required to satisfy the former condition is the twice differentiability of utility functions, but the chosen set owns it as already stated. In [4] authors have presented equation (8) as mandatory condition for a game to be an Exact Potential Game. The considered game for cognitive secondary terminals will be then managed as a potential game. It has to be underlined equation (8) is verified because of the former assumption about the sum of $c^{I\to II}(k)P^I(k)$ and $N(k)$.

The potential function $V : \mathbf{P}^{II} \to \mathbf{R}$ for the game Γ has to be, for all players and strategies, such that:

$$V(\mathbf{p}_i^{II}, \mathbf{p}_{-i}^{II}) - V(\mathbf{p'}_i^{II}, \mathbf{p}_{-i}^{II}) = u_i(\mathbf{p}_i^{II}, \mathbf{p}_{-i}^{II}) - u_i(\mathbf{p'}_i^{II}, \mathbf{p}_{-i}^{II}) \tag{9}$$

where $\mathbf{p}_i^{II}$ and $\mathbf{p'}_i^{II}$ are two different power allocation vectors for the i-th user, while $\mathbf{p}_{-i}^{II}$ is the power allocation matrix for all the opponents of the i-th user in the game. Exploiting a result reported in [16] about the definition of potential function for exact potential games, it has been determined the potential function $V(\mathbf{P}^{II})$ for the proposed game. This function is given by:

$$\begin{aligned} V(\mathbf{P}^{II}) = \sum_{k=1}^{K} B \log_2 \Bigg[& N(k) + c^{I\to II}(k)P^I(k) + \\ & + \sum_{m=1}^{M} h_m^{II}(k)p_m^{II}(k)\Bigg] - \sum_{m=1}^{M} c_m^{II\to I}(k)p_m^{II}(k) \end{aligned} \tag{10}$$

A. The solution of the game

The definition of the potential function implies that the solution of Γ can be found maximizing $V(\mathbf{P}^{II})$. Such power allocation matrix, representing a profile of strategies use by players, will lead to the Nash Equilibrium. It has to be pointed out that we aren't dealing anymore with a game in the classic mathematical sense, in fact the definition of a potential function allows to treat Γ as an optimization problem with several constraints given by equations (3), (4) and (5). Furthermore this optimization problem is a convex problem since the potential function $V(\mathbf{P}^{II})$ is convex and the real subspace defined by constraints is convex as well. Recalling the assumption that only one player can change its power strategy at a certain time, we have that the value of potential function will increase step by step till the maximum value will be reached. This means the maximization of (10) is carried out by each cognitive terminal optimizing a potential function where the terms depending on the other players are considered constant. Then the potential function each cognitive terminal will maximize will be:

$$\begin{aligned} V(\mathbf{p}_i^{II}) &= \sum_{k=1}^{K} B \log_2 \left[\alpha_i(k) + h_i^{II}(k)p_i^{II}(k)\right] - \\ &\quad - c_i^{II\to I}(k)p_i^{II}(k) - \beta_i(k) \end{aligned} \tag{11}$$

where $\alpha_i(k)$ and $\beta_i(k)$ are costant terms, for the k-th subcarrier, of the function relative to the i-th user. The terms $\alpha_i(k)$ and $\beta_i(k)$ are actually the following quantities:

$$\begin{aligned} \alpha_i(k) &= N(k) + c^{I\to II}(k)P^I(k) + \\ &\quad + \sum_{\substack{m=1,\\ m\neq i}}^{M} h_m^{II}(k)p_m^{II}(k) \end{aligned} \tag{12}$$

$$\beta_i(k) = \sum_{\substack{m=1,\\ m\neq i}}^{M} c_m^{II\to I}(k)p_m^{II}(k) \tag{13}$$

In order to show a fully distributed resource allocation technique has been achieved, equations (12) and (13) need an explanation. In (12) there are three terms at second member of the equation, the first two terms, $N(k)$ and $c^{I\to II}(k)P^I(k)$, give a constant sum as formerly assumed for mathematical necessities. The third term $\sum_{m=1,m\neq i}^{M} h_m^{II}(k)p_m^{II}(k)$ is actually the interference the signal from the i-th user is perceiving at the base station. In order to achieve a fully distributed technique, the value of this sum can be communicated each time by the base station to the terminal performing the allocation during the downlink. Differently the term $\sum_{m=1,m\neq i}^{M} c_m^{II\to I}(k)p_m^{II}(k)$ in (13) encompasses the channel state coefficients, $c_m^{II\to I}$, between secondary terminals and their relative nearest primary receiver. It is reasonable to assume that each secondary terminal achieve the channel state information regarding its nearest primary devices through specific sensing and monitoring activities. Then it can be assumed the estimation done is communicated to secondary system base station as information in the packet overhead.

The solution vector for the i-th terminal, $\mathbf{p}_i^{II}$, can be found or with iterative methods, like gradient based methods, or even in closed form through Lagrange multipliers mehod, [17], when the value of K is low. In fact the complexity of the problem is asymptotic with $2 \cdot 3^K$. Since the number of carriers K constitutes an issue for the optimization, we have found the closed form solution when $K = 2$ and we have extended to higher number of subcarriers, $K \gg 2$, through a *Divide and Conquer* based algorithm.

IV. SIMULATION RESULTS

The whole system has been simulated in MatLab® environment. Three different working points, in terms of E_b/N_o, have been considered for the primary system receivers, i.e. 10 dB, 15 dB, 20 dB. The working points represent three alternatives for the primary system operations. The primary receivers achieved rate depends only on the E_b/N_o, since the secondary system allocation strategy has always to preserve the primary system users. As far as secondary system is concerned, different BER target values have been considered in the range $(5 \cdot 10^{-5} \div 10^{-2})$ for the secondary system terminals.

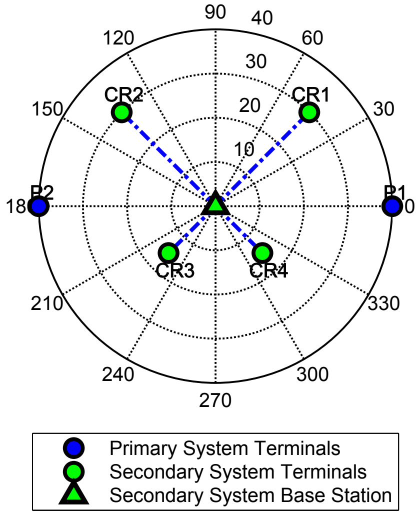

Fig. 2. The terminals placement.

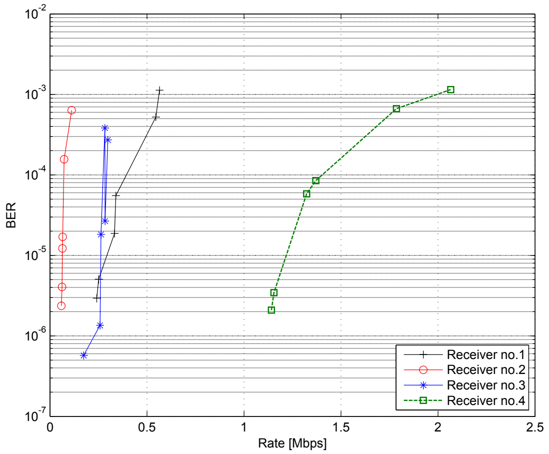

Fig. 3. The Bit Error Rate actually achieved at the base station by secondary terminals as function of the average rate.

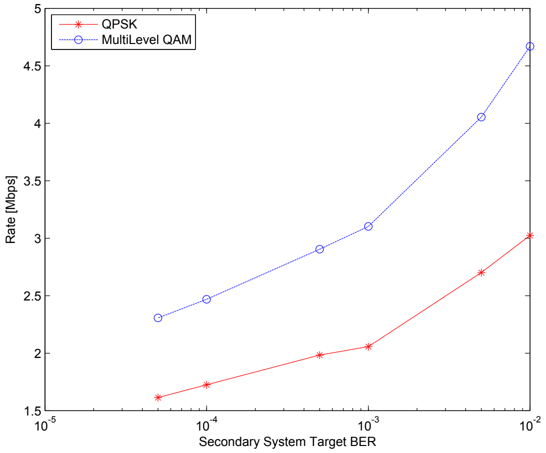

Fig. 4. The aggregate rate of the secondary system as function of the imposed target BER.

The scenario considered for the simulations is the one represented in Fig.2: two primary system terminals and four cognitive radio terminals located around the secondary system base station at the centre of scenario. The power allocation has been performed over the 853 subcarriers mentioned in Section II. Fig.3 shows the actual BER achieved by cognitive terminals. All the curves are widely below the imposed target BER. The target BER required in advance has been satisfied with a relevant margin. In Fig.4 the aggregate rate of the cognitive radio network has been plotted as function of the target BER a-priori imposed. In the graph two aggregate rates have been compared: a first one achieved using the QPSK as originally assumed, and a second one achieved employing the multilevel QAM modulation with 2, 4 or 6 bits transmitted per symbol according the signal-to-noise ratio obtained at the base station. Multilevel QAM demonstrates to be more efficient than the simple QPSK, and then it will be considered later on for the comparisons with an heuristic allocation algorithm. In Fig. 4 is also evident a second aspect, the graph in fact shows how the aggregate rate rises up relaxing the requirement for the secondary system target BER. This trend is valid also for each cognitive radio. The two graphs in Fig.5 show the signal-to-noise-and-interference ratio of receivers no.1 and no.4 over the considered frequency band. Cognitive terminals no.2 and

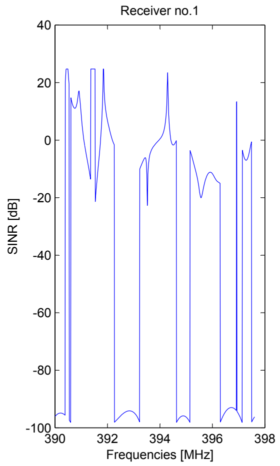

Fig. 5. Signal-to-noise-interference ratio versus frequency for the secondary terminals no.1 and no.4

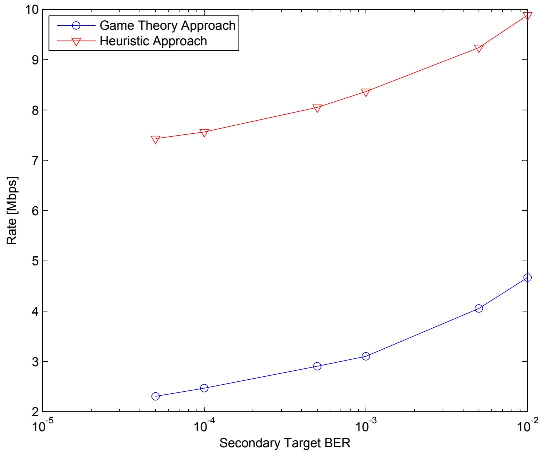

Fig. 6. The comparison between game theoretic approach and heuristic approach in terms of aggregate rate.

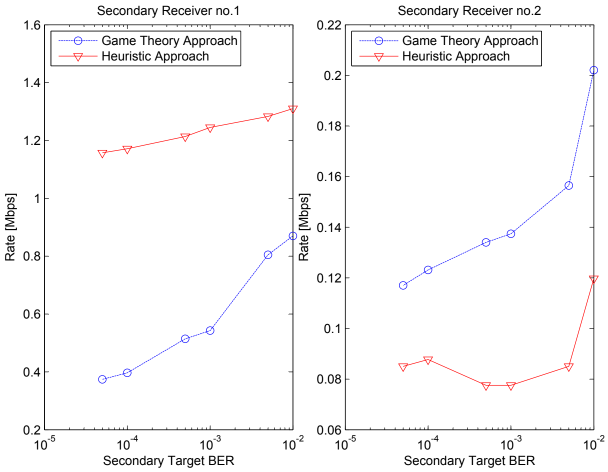

Fig. 7. The comparison between game theoretic approach and heuristic approach in terms of rate for terminals no.1 and no.2 .

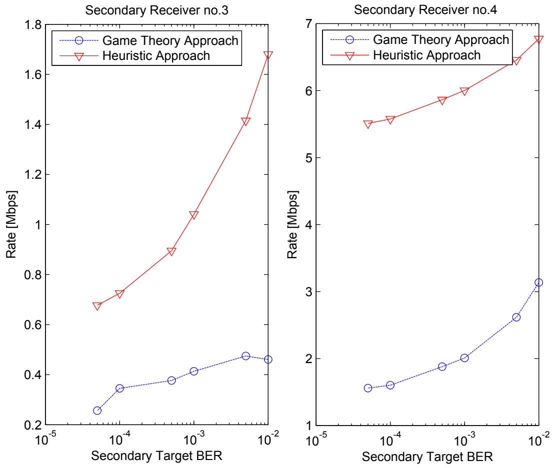

Fig. 8. The comparison between game theoretic approach and heuristic approach in terms of rate for terminals no.3 and no.4 .

no.3 have not been considered in this representation due to the scarce resources they were achieving. This fact has to be remarked since both receivers no.1 and no.4 have primary terminal no.1 as nearest primary receiver. It can be explained assuming that in this representation the primary no.1 leaves unoccupied wide portions of the spectrum due to a channel particularly impaired. Finally Figs.6-8 display the comparisons in terms of terminal rate, Figs.7-8, and aggregate rate, Fig.6, between the game theoretic approach presented in this paper and an heuristic power allocation method. The heuristic approach considered, it is an extension to a multi-user context of the power allocation strategy proposed in [18]. Differently from the analyzed game theory approach, the heuristic method has a centralized implementation and it needs a huge quantity of information to be forwarded on the network. Fig.6 clearly shows what is the difference of performance between the two methods: the heuristic allocation outperforms the Game Theory based allocation by a consistent amount, e.g. it doubles game theory approach aggregate rate. This fact has not to be interpreted as a failure for the Game Theory allocation because the heuristic method is actually representing the optimal allocation, i.e. the one achieving the highest rate. The detail of the comparison for each terminal is given in Figs.7-8, where it is possible to notice how the difference of rate between the methods increases severely when the rate achieved is above 1 Mbps, i.e. receivers no.3 and no.4; receiver no.1 manifests a similar behaviour, while receiver no.2 shows an unusual occurrence achieving higher rate through game theory approach than through heuristic method. This result is going against the general tendency, and it lets assume that game theory strategy is more fair than heuristic method toward those users experiencing lower rates.

V. CONCLUDING REMARKS

In this paper it has been presented a power allocation strategy for Cognitive Radio networks. The framework of the proposed technique is based on Game Theory. It has been showed how the game concerning power allocation is a Potential Game. The potential function has been found and it has been discussed how it can be implemented in a distributed way. Results achieved with the proposed technique have been presented for a specific scenario. Performances of the game theoretic approach have been given in terms of rate, actual BER and signal-to-noise ratio. Finally the method has been compared with the results achieved by an heuristic approach to the power allocation, representing the optimum. In the comparison the heuristic method outperformed game theory based one; however this result has not to be interpreted as a failure for the Game Theory allocation since, respect to heuristic method, it can be implemented in a distributed way. A final issue emerged, and it lets assume the proposed method is intrinsically more oriented to fairness than the heuristic one.

REFERENCES

[1] J. Mitola III and G. Q. Maguire Jr., "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[2] FCC Spectrum Policy Task Force, "Report of the spectrum efficiency working group," Tech. Rep., Nov. 2002.

[3] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[4] D. Monderer and L. S. Shapley, "Potential games," *Games and Economic Behavior*, vol. 14, no. 1, pp. 124–143, 1996.

[5] V. Srivastava, J. Neel, A. B. Mackenzie, R. Menon, L. A. DaSilva, J. E. Hicks, J. H. Reed, and R. P.Gilles, "Using game theory to analyze wireless ad hoc networks," *IEEE Commun. Surveys Tuts.*, vol. 7, no. 4, pp. 46–56, 2005.

[6] A. R. Fattahi and F. Paganini, "New economic perspectives for resource allocation in wireless networks," in *Proc. American Control Conference 2005*, vol. 6, Portland, OR, Jun. 2005, pp. 3960–3965.

[7] G. Scutari, S. Barbarossa, and D. P. Palomar, "Potential games: A framework for vector power control problems with coupled constraints," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP 2006*, vol. 4, Toulouse, France, May 2006, pp. 241–244.

[8] J. E. Hicks, A. B. MacKenzie, J. Neel, and J. H. Reed, "A game theory perspective on interference avoidance," in *Proc. IEEE Global Telecommunications Conference, GLOBECOM '04*, Dallas, TX, Dec. 2004, pp. 257–261.

[9] J. Neel, R. M. Buehrer, B. H. Reed, and R. P. Gilles, "Game theoretic analysis of a network of cognitive radios," in *Proc. 45th Midwest Symposium on Circuits and Systems, MWSCAS 2002*, vol. 3, Tulsa, OK, Aug. 2002, pp. 409–412.

[10] J. Neel, "Analysis and design of cognitive radio networks and distributed radio resource management algorithms," Ph.D. dissertation, Sep. 2006.

[11] N. Nie and C. Comaniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks," in *Proc. First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks, DySPAN 2005*, Baltimore, MD, Nov. 2005, pp. 269–278.

[12] M. Bloem, T. Alpcan, and T. Başar, "A stackelberg game for power control and channel allocation in cognitive radio networks," in *Proc. 2nd International Conference on Performance evaluation methodologies and tools*, Nantes, France, Oct. 2007, article no. 4.

[13] *IEEE P802.22/D1.0 Draft Standard for Wireless Regional Area Networks Part 22: Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Policies and Procedures for Operation in the TV Bands*, IEEE Draft Standard, Apr. 2008.

[14] *Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for Satellite Services to handheld devices (SH) below 3GHz*, ETSI EN 302 583, Rev. 1.1.1, Mar. 2008.

[15] E. Lutz, D. Cygan, M. Dippold, F. Dolainsky, and W. Papke, "The land mobile satellite communication channel-recording, statistics, and channel model," *IEEE Transaction on Vehicular Technology*, vol. 40, no. 2, pp. 375–386, May 1991.

[16] A. B. MacKenzie and L. A. DaSilva, *Game Theory for Wireless Engineers*. Morgan & Claypool Publishers, 2006.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.

[18] E. Del Re, F. Argenti, L. S. Ronga, T. Bianchi, and R. Suffritti, "Power allocation strategy for cognitive radio terminals," in *Proc. Cognitive Radio and Advanced Spectrum Management, CogART 2008, First International Workshop on*, Aalborg, Denmark, Feb. 2008, pp. 1–5.