Adaptive α-β tracking filters for ATC applications

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Abstract. In the ATC environment the characteristics of aircraft motion require adaptive filters to implement the tracking function of a TWS system, while the system processing constraints require low-complexity computational algorithms. Adaptive α-β tracking filters are therefore a convenient solution to the problem. In this paper four alternatives to the implementation of adaptive α-β tracking filters are presented, based on the selection of an appropriate set of values of the filter coefficients and on a suitable switching strategy among those values depending on the comparison between radar measurements and filter outputs. Their computational load is much lower than for adaptive Kalman filters, whose relevant structures are also discussed for comparison. The performance of the four proposed adaptive strategies is presented on the basis of computer simulations applied to two typical civil aircraft flight paths and compared with the performance of α-β non-adaptive tracking algorithms.

1. INTRODUCTION

In an air traffic control (ATC) system the continuous monitoring of aircraft motion is of course of fundamental importance for a regular navigation and for the collision avoidance problems. An automatic track-while-scan (TWS) system can achieve the performance required by the ATC operations for the tracking of aicrafts. In particular the efficiency of the collision avoidance techniques is strictly related to the precision of a TWS system and therefore determines the requirements of the tracking operation.

A TWS system consists of a radar sensor that measures the target position at discrete time instants, regularly spaced by the antenna rotation period, and of an estimation algorithm (the tracking filter) that, by processing the radar measurements, determines the target state in terms of position and velocity. In general the tracking algorithm could also estimate the target acceleration; however, in ATC applications a tracking algorithm estimating only position and velocity is preferred and, therefore, this is the version considered in this paper.

To be more precise, the operations of a tracking filter can be stated as follows: given the target range and azimuth radar measurements $r_m(i)$ and $\theta_m(i)$, respectively, at the time instants iT, $i = 0,1,\ldots,k$, T being the antenna scan period, the filter has to determine the following quantities, according to a chosen estimation criterion:

a) the estimates of range $\hat{r}(k|k)$, azimuth $\hat{\theta}(k|k)$, range velocity $\hat{\dot{r}}(k|k)$ and angular velocity $\hat{\dot{\theta}}(k|k)$ at the present time instant kT based on all radar measurements up to the same time instant kT (filtered estimates);

b) the estimates of range $\hat{r}(k+1|k)$, azimuth $\hat{\theta}(k+1|k)$, range velocity $\hat{\dot{r}}(k+1|k)$ and angular velocity $\hat{\dot{\theta}}(k+1|k)$ at the subsequent antenna scan time instant $(k+1)T$ based on all radar measurements up to the present time instant kT (one-step ahead predictions).

Again, in general, a TWS filter could also yield multi-step-ahead predictions; however only one-step estimates are considered here, because they are adequate for ATC applications.

Clearly, the tracking filter is supposed to operate in a bidimensional space, because the aircraft altitude is generally known, for example through the flight plan or through a secondary surveillance radar. Generally, the bidimensional space where the tracking algorithm operates is a cartesian coordinates system instead of the radar polar coordinates system, because the former is an inertial system where rectilinear uniform aircraft motions correspond to uniform motions along the coordinate axes. Indeed the simulations of the tracking filters applied to typical civil aircraft trajectories reported in [1] have verified that the algorithms give much better results when operating in an orthogonal coordinates system than in a polar one. Therefore only the former is

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considered in this paper. This means that a polar to Cartesian coordinates transformation at the input of the tracking filter and an inverse transformation at its output must be included in the whole TWS system.

2. AIRCRAFT FLIGHT PATHS AND TWS ALGORITHMS

Generally, in tactical and space applications stochastic, discrete, time-invariant models are assumed for both the motion of a maneuvering target and the radar measurements [1-3]. The state model for the target motion is defined by the matrix equation

(1) $X(k+1) = \Phi X(k) + GU(k)$

where $X(k) = [x_1(k)\ \dot{x}_1(k)\ x_2(k)\ \dot{x}_2(k)]'$ ('being the matrix transposition operator) is the system state vector, with $x_1(k)$ and $x_2(k)$ denoting the two position coordinates (Cartesian coordinates in our case) at the time instant kT and $\cdot$ denoting the differential operation with respect to time.

The state transition matrix Φ is defined as

(2) $$\Phi = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$U(k) = [u_1(k)\ u_2(k)]'$, the maneuvering noise, is a forcing term which is modeled as a discrete-time stochastic process with a known probability density of the two components $u_1(k)$ and $u_2(k)$ and a known correlation matrix $Q(k)$, and G is the noise transition matrix defined as

(3) $$G = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}$$

In some cases an 'augmented' (and more realistic) version of this state model is considered, including the two accelerations $\ddot{x}_1(k)$ and $\ddot{x}_2(k)$ in the state vector $X(k)$, thus leading to a 6x6 state transition matrix Φ and a 6x2 noise transition matrix G [1]. Even though for illustrative purposes only the simplified version given by (2) and (3) is explicitly reported here, most of the subsequent considerations are valid for any version.

The measurement model for the radar system is also given by a matrix equation of the form

(4) $Z(k) = H\ X(k) + V(k)$

where $Z(k) = [z_1(k)\ z_2(k)]'$ is the vector of the two measured (Cartesian) coordinates $z_1(k)$ and $z_2(k)$, H is the measurement projection matrix defined as

(5) $$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and $V(k) = [v_1(k)\ v_2(k)]'$ is the measurement noise which is supposed a white, Gaussian, zero-mean stochastic process with a known correlation matrix $R(k)$. Moreover, the two stochastic processes $U(k)$ and $V(k)$ are supposed to be uncorrelated. Further details on these two models can be found in [2,4].

Based on these stochastic models many TWS algorithms for the filtered and one-step prediction estimates of the target position and velocity have been proposed in the literature [1-10]. Generally they can be classified as Kalman filters or as α-β filters depending on their respective structure.

The Kalman filter solution to the tracking estimation algorithm is based on the following set of recursive matrix equations

(6) $\hat{X}(k+1|k) = \Phi\ \hat{X}(k|k)$

(7) $\hat{X}(k|k) = \hat{X}(k|k-1) + K(k)[Z(k) - H\ \hat{X}(k|k-1)]$

(8) $P(k|k-1) = \Phi\ P(k-1|k-1)\ \Phi' + G\ Q(k-1)\ G'$

(9) $K(k) = P(k|k-1)\ H'\ [H\ P(k|k-1)\ H' + R(k)]^{-1}$

(10) $P(k|k) = P(k|k-1) - K(k)\ H\ P(k|k-1)$

where $\hat{X}(k|k) = [\hat{x}_1(k|k)\ \hat{\dot{x}}_1(k|k)\ \hat{x}_2(k|k)\ \hat{\dot{x}}_2(k|k)]'$ is the filtered estimate vector, $\hat{X}(k+1|k) = [\hat{x}_1(k+1|k)\ \hat{\dot{x}}_1(k+1|k)\ \hat{x}_2(k+1)|k)\ \hat{\dot{x}}_2(k+1|k)]'$ is the one-step prediction estimate vector, $K(k)$, defined by (9), is the 4x2 (simplified version) or 6x2 (augmented version) Kalman gain matrix and $P(k|j)$ is the covariance matrix of the estimation error $\bar{X}(k|j) = X(k) - \hat{X}(k|j)$, that is

(11) $P(k|j) \triangleq E\ [\bar{X}(k|j)\ \bar{X}'(k|j)]$

being $E\ [\cdot]$ the expectation operator.

The estimates given by the Kalman filter are optimum in the minimum mean-square error sense. The algorithm defined by equations (6) to (10) requires an initialization, for example the evaluation of $\hat{X}(1|0)$ and $P(1|0)$, and for a real-time processing the more restrictive computational constraint comes from the matrix inversion operation in (9).

The α-β filter solution, which is applicable only to the simplified model specified by the matrix definitions (2) and (3), is given by the following expressions

(12) $\hat{X}(k+1|k) = \Phi\ \hat{X}(k|k)$

(13) $\hat{X}(k|k) = \hat{X}(k|k-1) + \Gamma\ [Z(k) - H\ \hat{X}(k|k-1)]$

where

(14) $$\Gamma = \begin{bmatrix} \alpha & 0 \\ \beta/T & 0 \\ 0 & \alpha \\ 0 & \beta/T \end{bmatrix}$$

is the gain matrix defined by the values of the filter parameters α and β. The α-β filter still has a recursive structure and requires an initialization, but its computational load is greatly reduced with respect to the Kalman filter. Moreover, the form of the matrices Φ, H and Γ allows an independent processing along the two coordinates, that greatly simplifies the computational and implementation aspects of the algorithm.

However differently from tactical targets characterized by highly varied maneuvers, in an ATC environment civil aircrafts generally follow only two types of trajectories, apart the take-off and landing flight phases, i.e. rectilinear paths and circular paths both with constant velocity. Therefore any civil trajectory can be thought of as formed by constant-parameters parts. This means that the supposed model for the maneuvering noise vector U(k) is not suitable for the application in a civil aircraft environment. Schematically the civil aircraft motion can be characterized as being in one of the two possible states:

a) absence of maneuver (rectilinear motion);

b) presence of maneuver (circular motion).

Therefore a unique (i.e. non-adaptive) algorithm for estimating the civil aircraft position and velocity based on the previous models cannot be chosen. For this reason adaptive tracking algorithms are more suitable for ATC applications. Ideally they can produce optimum or nearly optimum estimates in any state of the aircraft motion. They generally use some kind of decision or estimation procedure to adapt their structure and/or parameters to the characteristics of the target motion.

Both adaptive Kalman and α-β tracking filters have been considered in the literature [2-9]. They obviously require an increased computational load with respect to their non-adaptive versions, as will be shown in the next two sections.

3. ADAPTIVE KALMAN TRACKING FILTERS

The Kalman filters introduced in the previous section yield the "best" possible estimate of a linear dynamical system provided that

i) the system has been modeled correctly,

ii) the system parameters have the correct values.

If any of the two above conditions is not fulfilled, the Kalman filter looses its optimality properties and, in some cases, it may diverge. From this point of view a Kalman filter is not more insensitive to modeling errors or coefficient drifts than any other time-invariant filter. However the Kalman filter, providing also a statistical description of the prediction residuals, can be easily monitored and can also provide a more sound statistical basis for the development of adaptive filtering strategies.

For the above reasons, also in the ATC context, several adaptive tracking strategies based on the Kalman filter are reported in the literature. Two of them will be briefly recalled here in order to compare them with the adaptive tracker proposed in this paper. Due to the limits of the present work, only the basic assumptions and some practical implementations will be discussed. All mathematical details, than can be found in the literature, are skipped.

1) Adaptive trackers based on modeling and estimation of the aircraft maneuver.

In this class of adaptive trackers the target is modeled by

$$(15) \qquad X(k+1) = \Phi\, X(k) + G\, U(k) + W(k)$$

where the forcing term W(k) can only assume a finite set of values $\{W_i,\ i = 1,\dots,r\}$ and its time evolution is described by a transition probability matrix P with elements

$$(16) \qquad p_{ij} = \mathrm{Prob}[W(k) = W_i \,|\, W(k-1) = W_j]$$

As discussed in the previous section, typical civil aircraft maneuvers are better modeled by a discrete Markov chain (provided that P and W_i are appropriately chosen) than by a white or exponentially correlated zero-mean noise.

The recursive estimation of the state of the model (15) under the assumption (16) is a difficult task. Indeed, if we denote by Z^k the set of measurements up to time k, then

$$(17) \qquad E\,[X(k)|Z^k] = \sum_{j=1}^{r^k} \alpha_j\, E\,[X(k)|Z^k, \bar{W}_j^k]$$

where

$$E\,[X(k)|Z^k, \bar{W}_j^k]$$

is the estimate of X(k) conditioned to a given sequence $\bar{W}_j^k$ of maneuvers and is given by a Kalman filter, α_j is the "a posteriori" probability of the corresponding sequence. Hence (17) requires an increasing bank of Kalman filters. The simplest approach to reduce the complexity of (17) is to assume that

i) the conditional density $p(X(k)|Z^k)$ is normal for any k,

ii) the conditional probability $p_i(k-1) = \mathrm{Prob}[W(k-1) = W_i|Z^k]$ is available at time k.

Then the state estimate updating equation (7) is replaced by

$$(18) \quad \hat{X}(k+1)|k+1) = \hat{X}(k+1|k) + \hat{W}(k) + K(k+1)\,[Z(k+1) - H\,\hat{X}(k+1)|k) - H\,\hat{W}(k)]$$

where $\hat{W}(k) = \sum_{i=1}^{r} p_i(k) W_i$ is the average maneuver at time k, and the new conditional probability $p_i(k)$ can be computed through the matrix P and the normal density of the residual associated to each maneuver W_i.

Hence, this sub-optimal estimation procedure requires at each step:

1) the evaluation of r normal densities,

2) the computation of a discrete probability function,

3) the implementation of a standard Kalman filter.

This approach is promising and it has been applied to submarine [5] and aircraft [6] tracking. However, it must be stressed that the assumptions i) and ii) hold only if the average time duration of each maneuver W_j is longer than the dynamic response time of the resultant Kalman filter. Moreover, a better description of aircraft maneuvers can be obtained by expressing the forcing term in (15) by

$$W(k) = G_1\, X(k)\, w(k)$$

where w(k) is now a scalar quantity.

This assumption would lead to a non-linear filtering problem and, consequently, to heavy filtering algorithms.

2) Adaptive trackers based on maneuver detection.

This class of adaptive trackers is based on a simpler model for the target dynamics

$$(19) \quad X(k+1) = \Phi\, X(k) + G\, U(k) + G\, W\, \delta_{k,k_o}$$

where the vector $W\,\delta_{k,k_o}$ represents a sudden unknown maneuver taking place at the unknown time k_o in any of the two Cartesian coordinates. This perturbing term will then produce a bias in the predictions of a standard Kalman filter, from which the maneuver can be detected. Applying a generalized likelihood ratio test, then the following quantity must be evaluated at each instant k

$$(20) \qquad \Lambda(k,L) = \frac{\left(\sum_{j=1}^{L} h(j)\ \varepsilon(k+1-j)\right)^2}{\left(\sum_{j=1}^{L} h(j)\right)^2}$$

where $\varepsilon(j)$ is the prediction residual for each coordinate and $h(j), j = 1,2,\ldots$ is a suitable sequence of scalar coefficients. Then the hypothesis of a maneuver will be accepted if

$$(21) \qquad \max_{L} \ \Lambda(k,L) \geq \lambda\sigma_{\varepsilon}^2$$

where σ_{ε}^2 is the residual variance and λ a threshold chosen according to a given false alarm probability. For computational simplicity the search is limited to the last M measurements. If $\hat{L}$ is the value of L which maximizes the right-hand side of (21) then it is assumed that the maneuver took place at time $\hat{k}_o \triangleq k-\hat{L}$. After that a maneuver is detected, several adaptive strategies can be adopted:

1) Reinitialize the Kalman filter at time k,
2) Reinitialize the Kalman filter at time $\hat{k}_o$,
3) Partially reinitialize the Kalman filter, waiting for subsequent observations in order to make a final choice [7],
4) Increase the value of the variance matrix Q(k) of the maneuvering noise.

The computational effort and the memory requirements of this adaptive tracker depend heavily on the evaluation of the maneuver detection test, since the last M residuals for each coordinate must be filtered with a bank of M finite memory filters. Therefore great savings in computer memory and time, can be obtained by approximating the impulse response of the filter with largest memory, that is M, with that of a first order recursive filter. Indeed, in this case, the test statistics $\Lambda(k,M)$ can be approximated by the recursive expression [3]:

$$(22) \qquad \Lambda(k) = \exp\left(-\frac{3}{M}\right)\Lambda(k-1) + \varepsilon(k)$$

and its absolute value compared with the above threshold.

4. ADAPTIVE $\alpha-\beta$ TRACKING FILTERS

In an ATC environment the tracking algorithm of a TWS system must be implemented for every aircraft under the radar control. The number of these monitored aircrafts can be very large (several tens and even hundreds). Because the processing equipment (typically a minicomputer) has generally to perform other ATC operations in addition to the tracking function, an algorithm with relatively low computational load is required in an ATC environment. Another characteristic of an ATC tracking algorithm is to have a fast response to sudden aircraft maneuvers. Hence any adaptive tracking strategy must underlay to the following constraints:

1) Simplicity of the aircraft dynamics model in terms of the filtering structure and of the adaptation mechanism (in order to reduce the computation time),
2) Limited number of past measurement data necessary to update filter estimates (in order to limit memory requirements and to speed up the filter readyness).

Therefore, the adaptive tracking based on maneuver detection presented in the previous section seems to satisfy all the TWS requirements, since the test variables are recursively computed and the adaptation mechanism is the simplest one, viz. the reinitialization of the filter gains. However a further reduction of the computational load can be obtained by giving up the use of a Kalman filter in the adaptive tracker.

Indeed the main benefit of the Kalman filter is that the filter gains are function of the error covariance matrix, which, in turn, is updated at every sampling time, if

i) the filter steady state has not yet been reached,
ii) the coefficients of the system model are time varying (in our case the noise covariance matrix is time varying since it is dependent on the aircraft position).

Notice moreover that the noise covariance matrix, being non-diagonal, introduces a coupling between filter estimates along the two Cartesian coordinates, and therefore increases the filter complexity.

Now, from extensive computer simulations along typical aircraft trajectories, it has been found [1] that the noise covariance matrix is only slowly time varying and that the coupling between the two coordinates is very low. Moreover the transient behaviour of the Kalman filter can be obtained by an appropriate scheduling of the filter gains. These considerations motivate the introduction of an adaptive tracker based on the simplified structure of $\alpha-\beta$ filters, that requires a reduced computational load. In particular four adaptive tracking algorithms are considered in this paper. All of them are characterized by

1) a prespecified sequence of $\alpha-\beta$ gains,
2) a decision procedure for the maneuver detection,
3) an updating filter gain strategy.

Let us examine in more details these three items.

1) $\alpha-\beta$ gain sequence.

The β parameter has been chosen in order to satisfy the following relation

$$(23) \qquad \beta = \frac{\alpha^2}{2-\alpha}$$

which, as proved in [4], has several optimality properties. The highest value of the α gains was chosen in order to get both good initial tracking properties and fast response to sudden maneuvers; the lowest one was chosen in order to get satisfactorily good estimates along straight-line constant velocity trajectories. The number and the values of the intermediate gains were chosen in order to follow the typical decreasing behaviour of the Kalman gain. In particular, the $\alpha-\beta$ gains, used in our simulations, are reported in Tab. 1. Notice however that many other choices are possible as well.

Tab. 1 - Sequence of α-β gains

Index of the gains	1	2	3	4	5	6	7	8	9	10
α	0.83	0.6	0.45	0.36	0.3	0.26	0.19	0.15	0.11	0.09
$\beta = \frac{\alpha^2}{2-\alpha}$	0.58	0.25	0.13	0.079	0.052	0.038	0.019	0.012	0.006	0.004

Tab. 2 - Adaptive strategies

Adaptive Strategy	Test statistics	M	Threshold values	Gain updating strategy: If	then
A	R(k)	2	$\lambda_u=2.5$ $\lambda_\ell=0.5$	$R(k) \geq \lambda_u \Sigma(k)$	$\alpha_i(k) \rightarrow \alpha_{i+1}(k+1)$
				$R(k) \geq \lambda_u \Sigma(k)$, $R(k-1) \geq \lambda_u \Sigma(k-1)$	$\alpha_i(k) \rightarrow \alpha_1(k+1)$
				$R(k) \leq \lambda_\ell \Sigma(k)$	$\alpha_i(k) \rightarrow \alpha_{i-1}(k+1)$
				$\lambda_u \Sigma(k) > R(k) > \lambda_\ell \Sigma(k)$	$\alpha_i(k) \rightarrow \alpha_i(k+1)$
B	R(k)	2	$\lambda_u=5,10,15,20$ $\lambda_\ell=0$	$R(k) \geq \lambda_u \Sigma(k)$	$\alpha_i(k) \rightarrow \alpha_{i+1}(k+1)$
				$R(k) \geq \lambda_u \Sigma(k)$, $R(k-1) \geq \lambda_u \Sigma(k-1)$	$\alpha_i(k) \rightarrow \alpha_1(k+1)$
				$R(k) < \lambda_u \Sigma(k)$	$\alpha_i(k) \rightarrow \alpha_i(k+1)$
C	R(k)	1	as in B	as in B	
D	$\bar{R}(k)$	1	as in B	as in B ($R(k)$ replaced by $\bar{R}(k)$)	

2) Maneuver detection.

The following test statistics have been considered

$$(24a)\quad R(k) = \frac{1}{M}\sum_{j=k-M+1}^{k}\left(\varepsilon_1^2(j) + \varepsilon_2^2(j)\right)$$

and

$$(24b)\quad \bar{R}(k) = R(k) - \frac{1}{M}\sum_{j=k-M+1}^{k}\frac{\left(\varepsilon_1(j)\,\hat{\dot{x}}_1(j|j-1) + \varepsilon_2(j)\,\hat{\dot{x}}_2(j|j-1)\right)^2}{\hat{\dot{x}}_1^2(j|j-1) + \hat{\dot{x}}_2^2(j|j-1)}$$

where

$$\varepsilon_i(k) = z_i(k) - \hat{x}_i(k|k-1) \qquad i = 1,2\ldots$$

Notice that $R(k)$ is the average value of the square of the norm of the prediction residual vector, while in $\bar{R}(k)$ only the component of the residual vector orthogonal to the estimated velocity direction is considered. The above test variables are compared with $\lambda_u\Sigma(k)$ and $\lambda_\ell\Sigma(k)$, where λ_u and λ_ℓ are suitable threshold values $\lambda_u>\lambda_\ell>0$, and

$$(25a)\quad \Sigma(k) = K_p^2(k)\,(\sigma_r^2 + \hat{r}^2\sigma_\theta^2)$$

$$(25b)\quad K_p^2(k) = \frac{2\alpha^2(k) + \beta(k)(2+\alpha(k))}{\alpha(k)(4-\beta(k)-2\alpha(k))}$$

Notice that $\Sigma(k)$ is the expected value of $R(k)$, under the assumptions that the α-β filter with gains $\alpha(k)$, $\beta(k)$ is in its steady state and no maneuver had occurred. The statistics $R(k)$ and $\bar{R}(k)$, involving only the norm of the residual vector $\varepsilon(k)$ have been preferred to those involving separately each component $\varepsilon_1(k)$ and $\varepsilon_2(k)$, since its mean value does not depend on the aircraft bearing. This simplifies the detection procedure.

3) α-β gains updating strategies.

Four α-β gains updating strategies have been considered. They are summarized in Tab. 2, where the notation $\alpha_i(k)\rightarrow\alpha_j(k+1)$ means that at time k+1 the α gain has been changed from the i-th to the j-th value.

In all the adaptive strategies, the filter gains are increased as soon as $R(k)$ or $\bar{R}(k)$ is greater than the upper threshold $\lambda_u \Sigma(k)$. If this happens twice consecutively, then the target is assumed to be maneuvering and the filter is reinitialized. In the strategies B, C, D, the gain is decreased at each step if the upper threshold is not exceeded. Conversely, in the A strategy, the gain is decreased only if $R(k)$ is less than the lower threshold $\lambda_\ell \Sigma(k)$. The threshold values λ_u and λ_ℓ have been chosen empirically, though, in principle, they could be computed according to some false alarm probability. But the computation is based on the assumption, often violated, that the α-β filter is in the steady state. Finally notice the short memory length of the test statistics; this was necessary in order to get a fast filter maneuver detection and adaptation.

Any of the previous four adaptive α-β tracking filters can be implemented through a computational load much lower than that of any kind of adaptive Kalman filter. Their performance is discussed in the next section on the basis of extensive computer simulations. Adaptive Kalman tracking filters were not simulated, because, due to the computational constraints, their use was 'a priori' precluded in the ATC system for which this study was performed.

5. PERFORMANCE OF ADAPTIVE α-β FILTERS

The performance of the previously described adaptive α-β filters has been evaluated through computer simulations applied to two typical civil aircraft flight paths:

a) the first one (Fig. 1), formed by two rectilinear parallel paths joined by two semicircular paths with a radius of 1800 m, at a mean distance of about 30 Km from the radar site (this is a typical civil aircraft 'waiting' path before landing);

b) the second one (Fig. 2), formed by two rectilinear paths (air lanes) tilted by an angle of 10° joined by a short circular arc, at a mean distance of about 250 Km from the radar site (this a typical civil aircraft en-route path).

The following typical parameters for the two paths have been assumed:

1) aircraft velocity: 97.22 m/s (350 Km/h) for the waiting path and 250 m/s (900 Km/h) for the en-route path;

2) centripetal maneuver acceleration: 5 m/s^2 for the waiting path and 2 m/s^2 for the en-route path.

The two flight paths have been generated in polar coordinates and the radar measurements have been simulated by adding independent Gaussian, zero-mean, white stochastic processes to each coordinate, with the following standard deviations:

σ_r = 50 m (range)

σ_θ = 3° (azimuth)

Then the radar measurements have been transformed into a Cartesian coordinate system before supplying them to the input of the tracking filter.

The antenna scan time T has been set equal to 4s.

The different adaptive strategies have been compared by considering eight parameters. They have been averaged over N = 50 independent realizations and filter processings of each flight path, according to the following expressions:

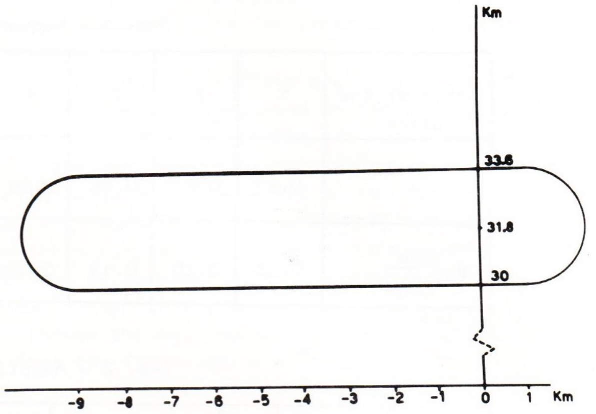

Fig. 1 - 'Waiting' path.

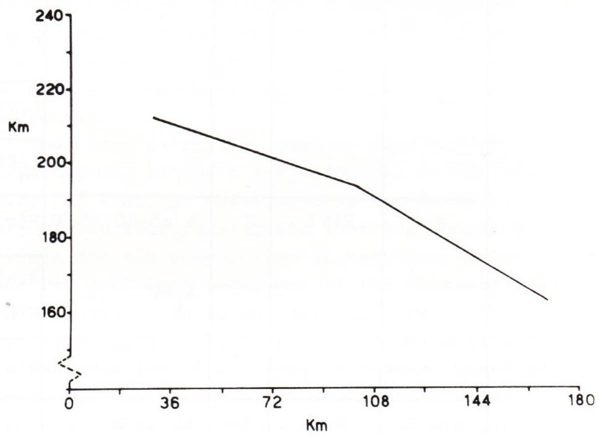

Fig. 2 - En-route path.

1) mean distance error of the filtered estimate

$$(26)\quad ERMS(k) = \frac{1}{N}\sum_{i=1}^{N}\left\{\left[\hat{x}_1(k|k)_i - x_1(k)\right]^2 + \left[\hat{x}_2(k|k)_i - x_2(k)\right]^2\right\}^{1/2}$$

2) standard deviation of 1)

$$(27)\quad ERDS(k) = \left\{\frac{1}{N}\sum_{i=1}^{N}\left(\left[\hat{x}_1(k|k)_i - x_1(k)\right]^2 + \left[\hat{x}_2(k|k)_i - x_2(k)\right]^2\right) - ERMS^2(k)\right\}^{1/2}$$

3) mean distance error of the one-step prediction estimate

$$(28)\quad ERMP(k) = \frac{1}{N}\sum_{i=1}^{N}\left\{\left[\hat{x}_1(k|k-1)_i - x_1(k)\right]^2 + \left[\hat{x}_2(k|k-1)_i - x_2(k)\right]^2\right\}^{1/2}$$

4) standard deviation of 3)

$$(29)\quad ERDP(k) = \left\{ \frac{1}{N}\sum_{1}^{N}{}_i \left(\left[\hat{x}_1(k|k-1)_i - x_1(k) \right]^2 + \left[\hat{x}_2(k|k-1)_i - x_2(k) \right]^2 \right) - ERMP^2(k) \right\}$$

5) mean error of the velocity magnitude v

$$(30)\quad ERMV(k) = v - \frac{1}{N}\sum_{1}^{N}{}_i \left[\hat{\dot{x}}_1^2(k|k)_i + \hat{\dot{x}}_2^2(k|k)_i \right]^{\frac{1}{2}}$$

6) standard deviation of 5)

$$(31)\quad ERDV(k) = \left\{ \frac{1}{N}\sum_{1}^{N}{}_i \left(v - \left[\hat{\dot{x}}_1^2(k|k)_i + \hat{\dot{x}}_2^2(k|k)_i \right]^{\frac{1}{2}} \right)^2 - ERMV^2(k) \right\}^{\frac{1}{2}}$$

7) mean error of the velocity phase angle ϕ

$$(32)\quad ERMF(k) = \phi - \frac{1}{N}\sum_{1}^{N}{}_i \, tg^{-1} \left[\hat{\dot{x}}_2(k|k)_i \,/\, \hat{\dot{x}}_1(k|k)_i \right]$$

8) standard deviation of 7

$$(33)\quad ERDF(k) = \left\{ \frac{1}{N}\sum_{1}^{N}{}_i \left(\phi - tg^{-1} \left[\hat{\dot{x}}_2(k|k)_i / \hat{\dot{x}}_1(k|k)_i \right] \right)^2 - ERMF^2(k) \right\}^{\frac{1}{2}}$$

In the preceding expressions the index i refers to the results of the ith simulation, while the quantities $x_1(k)$ and $x_2(k)$ are the known Cartesian coordinates of the flight path under consideration, of course without the measurement noise superimposed.

The extensive simulation results are reported in detail in [10]. They have shown that:

a) the lower threshold is of little significance, because in strategy A the switching of the filtering parameters occurs at about every step;

b) the strategy C, which has a memory length shorter than the strategy B, under the same threshold values λ_u is a little more sensitive to the noise peaks, but leads to shorter delays in detecting maneuvers and therefore to smaller estimation errors, especially when the aircraft velocity is greater as in the en-route path;

c) the strategy D gives the lowest number of false maneuver detections and better estimates than the other strategies for the waiting path;

d) for the en-route path, which is at a great distance from the radar site and has a relatively short duration and small intensity of the maneuver, the estimation errors are mainly due to the measurement noise and not to the possible presence of a maneuver; therefore a strategy with small probabilities of false maneuver detection is more suitable in this case.

Tables 3 and 4 summarize and compare the simul-

Table 3 - Performance of α-β non-adaptive strategy

		Waiting path		En-route path	
ALPHA		0.7	0.4	0.7	0.4
ERMP(m)	r m	185 335	50 790	978 1100	575 700
ERMV (m/s)	r m	0.6 7	0.2 11.25	6.25 29	2 8
ERMF (°)	r m	3 21	2 47	10 10	0 8
Δt(s)		9	38	17	41
R_p		1.62	0.23	1.6	0.6

Table 4 - Performance of α-β adaptive strategies

		Waiting path				En-route path
Strategy		A	B	C	D	C
ERMP(m)	r m	135 365	112.5 470	95 430	75 223.7	500 1062
ERMV(m/s)	r m	1.41 9.69	1 11.82	0.6 10.2	0.4 14.9	8 10
ERMF (°)	r m	2 21	2 30	2 27	1 27	7 7
Δt(s)		25	27	26	32	51
R_p		1.13	0.63	0.75	0.25	0.34

ation results for the non-adaptive and adaptive α-β filtering algorithms respectively. Instead of all the evaluation parameters listed above, only the most critical ones for a TWS system are reported, i.e. the mean one-step prediction error (ERMP), the mean error on the velocity magnitude (ERMV) and phase (ERMF). For each parameter the average value in a rectilinear path (r) and the maximum value in presence of a maneuver (m) are indicated. Other parameters reported in Tables 3 and 4 are the transient time (i.e. the time to arrive at the steady state) after a maneuver has been completed (Δt) and the reduction factor (when less than one) or amplification factor (when greater than one) of the mean value of the square of the one-step prediction error with respect to the mean value of the square of the radar measurement error (R_p).

The α values are indicated in the Tables, whereas the β values are always given by the relation (23).

According to the preceding observations only the strategy C has been reported for the en-route path (with $\lambda_u = 20$). For the waiting path (with $\lambda_u = 10$), the reported results show that strategy A is the less efficient one, strategies B and C are essentially equivalent and strategy D achieves the best performance.

Tables 3 and 4 show that for the waiting path the adaptive strategies, in particular C and D, achieve a more satisfactory overall performance than non-adaptive strategies for maneuvering and non-maneuvering aircrafts. Moreover, in addition to a transient time of the same order with respect to the non-adaptive strategy (α = 0.4), the adaptive strategies present much lower errors during the transient period. For the en-route path, Tables 3 and 4 show essentially the same performance for both adaptive and non-adaptive strategies. This behaviour depends mainly on the characteristics of the en-route path where the contribution from the measurement noise is comparable, and even greater, than that from the maneuver.

The overall performance of the adaptive strategies, in particular C and D, essentially achieves the same results of a non-adaptive α-β filter with low-value gains for a rectilinear motion and of a non-adaptive α-β filter with high-value gains in the presence of a maneuver. Of course this is the desired behaviour of adaptive α-β strategies simply based on maneuver detection and filter gain re-initialization.

6. CONCLUSIONS

Four adaptive algorithms for the implementation of the tracking function of a TWS system have been presented and discussed, suitable for the application in air traffic control. The project for which this study was carried out excluded the use of adaptive Kalman tracking filters because of their too heavy computational load for an ATC system. Therefore only the performance of adaptive strategies based on the structures of the α-β tracking filters have been evaluated through computer simulations of typical civil aircraft flight paths. The adaptive α-β strategies presented herein achieve some improvements in comparison to non-adaptive strategies. The results presented in section 5 and the computer simulations reported in [10] suggest that a smaller number of filter gains could be used probably leading to simpler implementation and smoother tracker performance. Also different sets of filter gains would be more appropriate to the two different flight paths; however this would considerably complicate the adaptation strategy.

In conclusion, adaptive α-β tracking filters can represent an appropriate approach to the tracking function in ATC applications. Of course adaptive Kalman filter approaches could be more efficient in terms of estimate precision. However, their actual implementation would require technological choices (compatible with the present state of specialized hardware processing structure) different from those assumed in the present work.

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