Parametric decay of linearly polarized shear Alfvén waves in oblique propagation: One and two-dimensional hybrid simulations

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[1] The parametric instability of a monochromatic shear Alfvén wave in oblique propagation with respect the ambient magnetic field is investigated in a kinetic regime, performing one-dimensional (1-D) and two-dimensional (2-D) hybrid simulations. The parallel component of the mother wave is found to be subject to a parametric decay which excites an ion-acoustic wave along the magnetic field and a backward propagating daughter shear Alfvén wave, as in the instability for a purely parallel mother wave. At the same time, the acoustic wave generation supports the acceleration of a velocity beam in the ion distribution function, due to the non-linear trapping of protons. Moreover, the instability leads to the generation of broad band oblique spectra of coupled Alfvénic and compressive modes with variable perpendicular wavevectors, and, as a consequence, the magnetic field after saturation is characterized by a strong transverse modulation. **Citation:** Matteini, L., S. Landi, L. Del Zanna, M. Velli, and P. Hellinger (2010), Parametric decay of linearly polarized shear Alfvén waves in oblique propagation: One and two-dimensional hybrid simulations, *Geophys. Res. Lett.*, *37*, L20101, doi:10.1029/2010GL044806.

1. Introduction

[2] Parametric instabilities characterize the evolution of non-linear Alfvén waves [e.g., *Goldstein*, 1978; *Malara and Velli*, 1996; *Del Zanna et al.*, 2001] and predict the coupling of Alfvénic fluctuations withcompressive modes. Kinetic effects can play an important role in parametric instability; ion dynamics influences both the growth rate of the instability and the range of unstable modes [e.g., *Araneda et al.*, 2007; *Nariyuki et al.*, 2007]. Moreover, when the feedback of the instability on ions is retained, we observe the deformation of the distribution function. It has recently been found that ion-acoustic waves supported by a modulational [*Araneda et al.*, 2008] and a decay [*Matteini et al.*, 2010] parametric instability can accelerate a velocity beam.

[3] Despite the coupling of the mother wave with ion acoustic fluctuations acts along the magnetic field, transverse modulation across **B** can be important, as shown by two-dimensional (2-D) numerical studies [e.g., *Ghosh et al.*, 1993; *Del Zanna et al.*, 2001; *Passot and Sulem*, 2003; *Nariyuki et al.*, 2008]. Also, parametric instability can destabilize fluctuations at oblique propagation [*Viñas and Goldstein*, 1991].

[4] On the other hand, few studies have considered the instability of obliquely propagating mother waves. In the framework of MHD, *Del Zanna* [2001] reported an analysis of the evolution of linear and arc-polarized Alfvén waves [*Barnes and Hollweg*, 1974; *Vasquez and Hollweg*, 1996]. The investigation of the evolution of non-strictly parallel waves is relevant for the solar wind, where the larger part of the magnetic power of fluctuations is observed at oblique angles [e.g., *Horbury et al.*, 2008].

[5] The aim of this paper is then to study the parametric decay of a mother shear Alfvén wave at oblique propagation within a kinetic regime. We focus on the properties of the resulting ion distribution and on the full description of the coupling along and across the magnetic field.

2. Simulation Results

[6] We performed 1-D and 2-D simulations using a hybrid numerical code [*Matthews*, 1994], describing electrons as an isothermal fluid and ions as particles. Units of space and time are $c/\omega_p = v_A/\Omega_p$ and Ω_p^{-1}, where ω_p and Ω_p are respectively the plasma and the cyclotron proton frequencies. We initialize the system using a monochromatic linearly polarized shear Alfvén wave $B_z = B_{z0} \cos(\omega t - k_0 x)$, with wave number $m = 10$, amplitude $B_{z0}/B = 0.1$, and propagating along the x axis with an angle θ_{kB} with respect the ambient magnetic field **B** lying in the x-y plane. The amplitude of the velocity component is $u_{z0} = B_{z0}$ and the adopted mother wave propagates with frequency $\omega = k_0 v_A \cos(\theta_{kB})$.

2.1. One-Dimensional Geometry

[7] We first report 1-D results. In this case the simulation box, which the x direction, is long $300c/\omega_p$, with resolution $\Delta x = 1$, and we use $2 \cdot 10^4$ particles per cell (ppc). The proton and the electron beta are set $\beta_p = 0.01$ and $\beta_e = 0.1$, respectively, to avoid Landau damping and support the possible acceleration of velocity beams [see also *Matteini et al.*, 2010]. We have performed simulations at 3 different angles θ_{kB} between the magnetic field and the x axis, summarized in Table 1; all runs lead to analogous evolution. In Figure 1 (top left), we report the time evolution of the density rms (solid line) in the case of Run B. The increase of density fluctuations after $t \sim 500$ reveals the presence of a parametric instability, which then saturates at $t \sim 1400$; the Fourier analysis reveals the excitation of a compressive mode $m = 16$, shown in dashed line.

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Table 1. Growth Rate of Parametric Decay at Various Angles[a]

Run	θ_{kB}	k_0	k_s	k^-	γ	$\gamma_\parallel$
A	30	0.21	0.33	0.12	0.007	0.008
B	45	0.21	0.33	0.12	0.006	0.008
C	60	0.21	0.33	0.12	0.004	0.008

[a]For each angle we report the measured growth rate γ and the corresponding $\gamma_\parallel = \gamma/\cos(\theta)$.

[8] At the same time, we report in Figure 1 (bottom left) the corresponding evolution of the energies of the inward (dotted) and outward (solid) Alfvénic fluxes. Initially only the outward propagating mother wave is present, corresponding to the condition $E^+ = 1$ and $E^- = 0$. The parametric coupling with an ion-acoustic wave leads to the damping of the mother wave energy and to the generation of a reflected backward propagating daughter wave, with $E^- > 0$; this identify the process as a decay instability. As a result, the cross helicity $\sigma = (E^+ - E^-)/(E^+ + E^-)$ switches from positive (predominance of outward flux) to negative (inward dominates). This behavior is in good agreement with the result of *Del Zanna* [2001], where the same evolution for the parametric decay of an oblique Alfvén wave is investigated within the framework of the MHD. Our investigation also confirms that the growth rate for linearly polarized waves propagating at oblique angles is smaller than for analogous circularly polarized Alfvén waves at parallel propagation. Moreover, as we report in Table 1, we recover that the measured growth rate decreases with increasing propagation angle θ_{kB}; in particular, we observe that it scales with cos (θ_{kB}), as found by *Del Zanna* [2001], coherently with the fact that the phase velocity of the mother wave $v_{ph} = v_A \cos(\theta_{kB})$ follows the same scaling.

[9] The coupling between Alfvénic and ion-acoustic modes shows a three-wave interaction in the magnetic, electric, and density spectra (not reported here), that corresponds to the lower sideband resonance $k^- = k_0 - k_s$, between the mother wave k_0 ($m = 10$), the ion–acoustic wave k_s with $m = 16$, and a backward (or lower sideband) Alfvén wave k^- at $m = 6$. This is analogous to the dynamics of the parallel propagating case [e.g., *Matteini et al.*, 2010, Figure 2].

[10] It has been recently observed in hybrid simulations [*Araneda et al.*, 2008; *Matteini et al.*, 2010] that the ion-acoustic waves driven by a parametric instability can support, via proton trapping, the generation of a velocity beam aligned with the magnetic field in the proton distribution function. In the present investigation we found that such a process plays a role in the evolution of proton distribution also when the mother wave is obliquely propagating. Particle trapping and vortices in phase space characterize the saturation phase of the instability (on the contrary, in the MHD case the saturation is provided by the steepening and dissipation of the excited acoustic waves), and leads to the formation of ion velocity beams as in the case of parallel propagating mother waves.

[11] All these results suggest that the parametric decay of oblique waves remains essentially a parallel process, driven by the parallel component of the mother wave and that the excited fluctuations propagate along the magnetic field. To confirm this picture and to check the role of possible transverse couplings in the instability dynamics, we have extended our analysis to 2-D simulations.

2.2. Two-Dimensional Geometry

[12] We have performed simulations of previous section using a 2-D grid of size 300 × 300, with spatial resolution $\Delta x = \Delta y = 1$ and 1000 ppc. We report in Figure 1 (right) the evolution of the density rms (Figure 1, top), E^+, E^-, and σ (Figure 1, bottom), in the case of Run A; the evolution found is very similar to that in 1D. Also in 2D the growth rate of the instability is found to decrease with increasing θ_{kB}. The instability is observed to saturate through particle trapping and we observe the formation of a proton beam in the ion distribution, as a result of the interaction of particles resonant with the parallel electric field carried by the ion-acoustic fluctuations driven by the parametric decay. In Figure 2 we report the velocity distribution function for protons $f(v_\parallel, v_\perp)$ after the saturation of the instability, showing the presence of a beam with a secondary peak at $v \sim 0.5 v_A$.

[13] Figure 3 reports the density and B_z 2-D spectra at two different simulation times. We also report the $k_\parallel$ and $k_\perp$ directions encoding the framework parallel and perpendicular to the magnetic field, to better compare the modes in

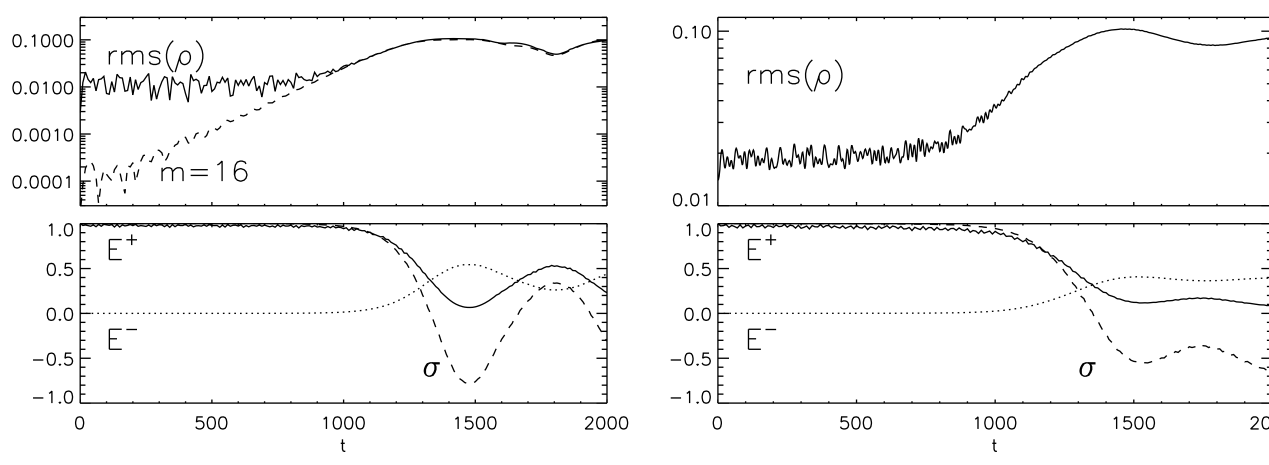

Figure 1. Time evolution of density fluctuations and wave energy for (left) a 1-D and (right) a 2-D simulation. (top) The evolution of the density rms; (bottom) the outward (inward) Alfvénic energy E^+ (E^-) and the cross helicity σ.

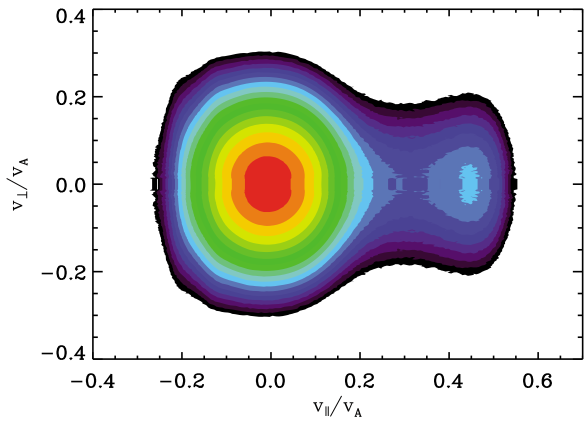

Figure 2. Proton velocity distribution function $F(v_\parallel, v_\perp)$ after the saturation of the parametric instability.

terms of their propagation along or across **B**. In Figure 3 (left), which reports the magnetic spectrum after few simulation steps, only the mother wave signature (M) at $k = 0.2$ is present. At the same time, two signals are present in the density spectrum (M_1 and M_2 in Figure 3, middle left); these are due to the almost immediate coupling of density with respectively the mother wave at $k = k_0$, since the initial wave is not an exact solution of the Vlasov system, and with B^2, at $k = 2k_0$, due to ponderomotive effects. However these modes are found to play no role in the following evolution.

[14] At a later time ($t = 1100$), when according to Figure 1 the linear phase of the instability takes place, we observe a more complicated structure. Both density (Figure 3, middle right) and magnetic (Figure 3, right) field show the presence of a developed activity, which corresponds to the excitation of two families of oblique modes at different propagation angles. The temporal Fourier analysis (k-ω) of those signals reveals Alfvénic and acoustic-like nature, respectively for magnetic and density fluctuations. Moreover, magnetic fluctuations follow the shear Alfvén wave dispersion $\omega = k_\parallel v_A$.

[15] The remarkable property of such wide spectra is that all excited modes lie on a straight line aligned with the $k_\perp$ axis, namely, each family of modes is characterized by a single parallel wave-number $k_\parallel$; as a consequence, along **B** this scenario identify a three-wave coupling, as in the 1-D case. At the same time, we observe that in the 2-D case a large range of perpendicular components $k_\perp$ for the daughter waves, is also excited during the instability.

[16] The generation of oblique daughter waves is a direct consequence of the fact that the mother wave is not strictly parallel. The 3-wave interaction $\mathbf{k}^- = \mathbf{k}_0 - \mathbf{k}_s$ implies:

$$k_\parallel^- = k_{0\parallel} - k_{s\parallel}, \tag{1}$$

and

$$k_\perp^- = k_{0\perp} - k_{s\perp}. \tag{2}$$

Since $k_{0\perp} \neq 0$, at least one of the excited modes $\mathbf{k}^-$ and $\mathbf{k}_s$ must have a nonzero perpendicular component. (Note that if $k_{s\perp} = 0$, then $k_\perp^- \neq 0$, and vice versa, so that it is not possible to excite pure parallel daughter waves only). Along **B** the decay is driven by the parallel interaction (1) and the coupling of the excited modes is selected by the maximum growth rate of the instability; we then observe the interaction between only three parallel wavevectors. Across **B** there is no similar constraint so that any perpendicular wavevector satisfying condition (2) is equally excited. In this framework, spectra shown in Figure 3 correspond to the oblique modes that satisfy equations (1) and (2); these are characterized by a constant component along the magnetic field, $k_\parallel^-$ and $k_{s\parallel}$, and a variable perpendicular component, $k_\perp^-$ and $k_{s\perp}$. Moreover, we have verified that the resonant interaction is satisfied also by wave frequency ω (Note that for the fluctuations involved, ω depends only on $k_\parallel$).

[17] To conclude our analysis, Figure 4 reports the 2-D spatial profile of the density and magnetic field component B_z. In Figure 4 (left) we report the density field at $t = 1300$, corresponding to the linear growth of the instability: an excited ion-acoustic wave propagating along **B** is clearly distinguishable. At the same time, B_z (Figure 4, middle)

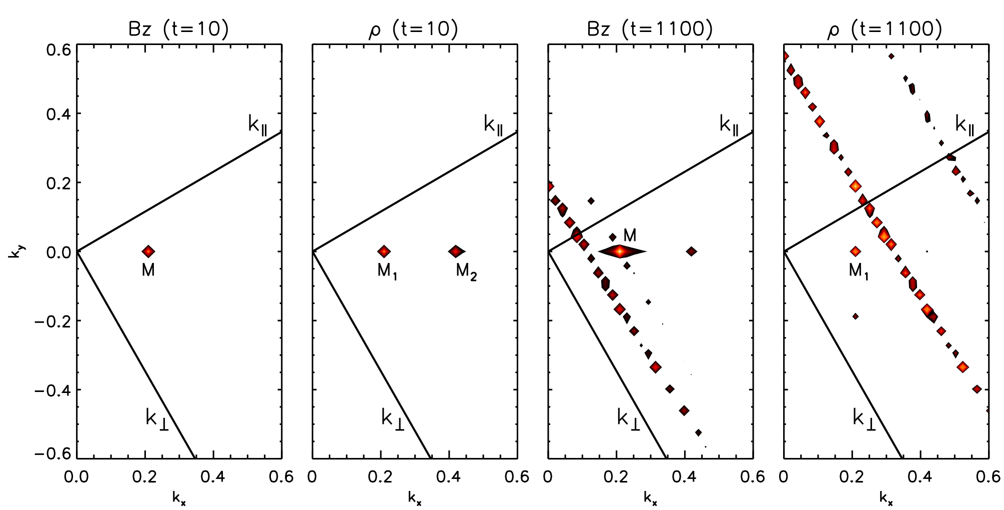

Figure 3. Fourier 2-D spectra of B_z and density fluctuations at (left and middle left) $t = 10$ and (middle right and right) $t = 1100$.

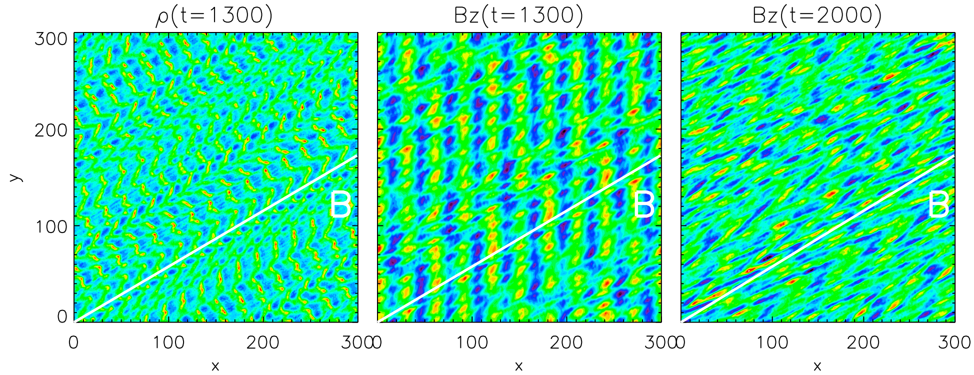

Figure 4. Two-dimensional spatial profile of (left) density and (middle) magnetic field component B_z at t = 1300 during the linear phase of the mother wave decay, and of (right) B_z at t = 2000 after saturation.

shows the damping of the mother wave m = 10 propagating along x which results modulated in the transverse direction due to the coupling with the daughter waves. Figure 4 (right) finally reports B_z at the end of the simulation when the mother wave has completely disappeared and B_z is dominated by the transverse modulation across the mean magnetic field.

3. Conclusion

[18] We have presented numerical simulations of the parametric instability of shear Alfvén waves at oblique propagation with respect the direction of the ambient magnetic field. Using a hybrid framework we have simulated the 1-D and 2-D properties of the decay instability.

[19] In 1-D our results are in agreement with the work of *Del Zanna* [2001] which concerned the MHD regime; oblique waves are found to decay with analogous properties as the parallel waves and the growth rate of the instability is found to scale with $\cos(\theta_{kB})$.

[20] Also we have found that the proton distribution function is influenced by the instability. The parallel electric field carried by the ion-acoustic wave excited by the three-wave parametric coupling, is able to accelerate resonant particles and produce a velocity beam, aligned with the ambient magnetic field, in agreement with the results obtained in the parallel propagating case [*Araneda et al.*, 2008; *Matteini et al.*, 2010].

[21] Extending our investigation to 2-D simulations we have investigated more in detail the nature of the coupling. Despite the parametric decay is driven by the resonant interaction between parallel wavevectors, we also observe in the transverse direction the generation of a wide range of oblique modes in both magnetic (Alfvénic) and density (ion-acoustic) fluctuations. As a result, while along **B** we still observe the preferential acceleration of protons and the formation of a velocity beam, the final magnetic field structure is modulated mainly in the transverse direction.

[22] The present investigation is relevant for astrophysical plasmas, such as the solar wind. Solar wind fluctuations appear to be characterized by Alfvén waves [*Belcher and Davis*, 1971] with most of the power contained in modes with large angle wavevectors respect to the mean magnetic field [*Horbury et al.*, 2008]. Our study indicates that the non-linear evolution of a moderately non-parallel Alfvén wave can lead to non-linear couplings that produce particle acceleration along the magnetic field and strong modulation in the perpendicular direction, with generation of extended spectra of oblique modes as observed in the solar wind. Investigations including more realistic solar wind parameters (non monochromatic waves, $\beta_p \sim \beta_e$) are planned.

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