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(Article begins on next page)

# Efficiency of the Transformer Version of Class E Half-Wave Low $dv_D/dt$ Rectifier

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**Abstract** — An expression for the efficiency of a transformer version of a Class E, half-wave, low  $dv_D/dt$  rectifier has been derived in a closed form as a function of the output current. The equations giving the losses in capacitor ESRs, in the transformer windings, and conduction losses in the diode have been derived. Plots of the rectifier efficiency are given for several values of the output power, e.g., 50 W, 100 W, and 240 W, for output voltages of 5 V and 12 V, and using several values of capacitor ESRs. The Class E, half-wave rectifier is suitable for high-frequency applications that require outputs of 12 V. It is also suitable for applications with 5 V outputs only if the output power is lower than 50 W and the capacitor ESRs are low, e.g., 0.01  $\Omega$ . The presented method can be used for the design optimization of other rectifier topologies.

## I. INTRODUCTION

High-power density dc-dc converters used in power supplies can be obtained only if their operating frequency is increased above 500 kHz and if their efficiency maintains high values over the entire operating load range. Since the dc output voltages required by logic circuits are low, e.g., 3.3 V, 5 V, and 12 V, the output rectifier mostly affects the converter efficiency [1]. Many rectifier topologies suitable for high-frequency operation have been studied [2]-[9]. However, investigations are still needed to determine which applications allow the rectifiers to operate with a high efficiency. The Class E, half-wave, low  $dv_D/dt$  rectifier is particularly suitable for high-frequency operation because switching losses and noise are drastically reduced [2]-[8]. The purpose of the paper is to determine which levels of output voltage and power are most suitable for a high efficiency operation of the Class E, half-wave, low  $dv_D/dt$  rectifier and how the capacitor ESRs affect the rectifier efficiency. An equivalent circuit of the rectifier is derived to evaluate the losses in the capacitor ESRs, those in the transformer windings, and conduction losses of the diode. The values of the rectifier components and their parasitics are chosen according to those measured in a tested Class E

rectifier operating at a frequency of 1 MHz. The efficiency of the rectifier has been derived as a function of the load current and has been plotted for several values of the output power and capacitor ESRs, with outputs of 5 V and 12 V.

## II. EVALUATION OF THE RECTIFIER EFFICIENCY

A basic circuit of a Class E, half-wave, current-driven, low  $dv_D/dt$  rectifier is shown in Fig. 1 [3]. The rectifier is driven by a sinusoidal current source described by  $i = I_m \sin \omega t$ . The transformer magnetizing inductance  $L_m$  is assumed to be large enough to carry only the dc output current  $I_O$ . As shown in Fig. 2, the current flowing through the parallel combination  $C$ - $D$  is  $i_C + i_D = I_O + ni$  where  $n$  is the transformer turns ratio. Diode  $D$  turns-on at  $\omega t = \phi$  when the voltage  $v_D$  reaches the diode threshold voltage and turns-off at  $\omega t = \phi + 2\pi D$ , where  $D$  is the on-duty cycle. After this time, capacitor  $C$  shapes the voltage across diode  $D$ , according to  $v_D = v_C = C dv_C/dt$ . When  $\omega t = \phi + 2\pi$ , diode  $D$  turns-on again and a new switching period starts. In the Class E rectifier, the diode turns-on with a limited  $dv_D/dt$  and turns-off at zero voltage with  $di/dt = 0$ , resulting in theoretically zero switching losses. Actually, they can be neglected when evaluating the rectifier efficiency. Further assumptions used in evaluating the rectifier efficiency are as follows:

- 1) The rectifier is assumed to operate at constant frequency.
- 2) The transformer core losses are neglected, copper losses are considered by means of the equivalent resistance of the windings seen at the terminals of the primary  $r_{Cu} = r_{w1} + n^2 r_{w2}$ , where  $r_{w1}$  and  $r_{w2}$  are the resistances of the primary and secondary windings, respectively.
- 3) Diode  $D$  is modelled as a series combination of a dc battery with a voltage  $V_F$  and a resistance  $r_F$ , when it is in the on-state and as an open circuit when in the off-state.
- 4) The equivalent series resistances of capacitors  $C$  and  $C_f$  are  $r_{ESR}$  and  $r_{ESRf}$ , respectively.

- 5) The rectifier is assumed to operate with a maximum duty cycle  $D_{max}=0.5$  because this value maximizes the rectifier output-power capability [3].
- 6) The filter capacitor  $C_f$  is assumed to be large enough so that the output voltage is approximately constant.
- As given in [6], current  $i$  magnitude is

$$I_m = \frac{-I_o}{n \sin(\phi + 2\pi D)} \quad (1)$$

where  $\phi$  is the turn-on delay angle of diode  $D$ . The expression of the current through diode  $D$  is

$$i = I_o \left[ 1 - \frac{\sin \omega t}{\sin(\phi + 2\pi D)} \right] \quad (2)$$

for  $\phi \leq \omega t < \phi + 2\pi D$ . Expression (2) represents also the waveform of the current through capacitor  $C$  when  $\phi + 2\pi D \leq \omega t < \phi + 2\pi$ . The current through diode  $D$  and capacitor  $C$  is zero when  $\phi + 2\pi D \leq \omega t < \phi + 2\pi$  and when  $\phi \leq \omega t < \phi + 2\pi D$ , respectively. Diode  $D$  turn-on delay angle is expressed as a function of its on-duty cycle as follows

$$\tan \phi = \frac{\cos 2\pi D - 2\pi(1-D)\sin 2\pi D - 1}{2\pi(1-D)\cos 2\pi D + \sin 2\pi D}. \quad (3)$$

The relationship between the normalized load resistance and the duty cycle is given by

$$\omega CR_L = \frac{1}{\pi} - (1-D) \left[ \pi(1-D) + \cot(2\pi D + \phi) \right] - \frac{\sin \phi}{2\pi \sin(2\pi D + \phi)}. \quad (4)$$

Rectifier losses are expressed as functions of the output current  $I_o$  combining (1)-(4) with the equivalent circuits of the rectifier given in Fig. 3. Since  $P_o = R_L I_o^2$ ,

$$P_{Cu} = \frac{r_{Cu} I_m^2}{2} = \frac{r_{Cu} P_o}{2 R_L n^2 \sin^2(\phi + 2\pi D)} \quad (5)$$

gives the transformer loss. The conduction loss in diode  $D$  is

$$P_D = V_F I_o + r_F I_{Drms}^2 \quad (6)$$

where  $I_{Drms}$  is the rms value of the current through diode  $D$  when it is ON. Using (2), the rms current through diode  $D$  is

$$I_{Drms} = \sqrt{\frac{1}{2\pi} \int_{\phi}^{\phi+2\pi D} \left[ I_o \left( 1 - \frac{\sin \omega t}{\sin(\phi + 2\pi D)} \right) \right]^2 d\omega t}. \quad (7)$$

Substitution of the expression resulting from (7) and  $P_o = R_L I_o^2$  into (6) gives

$$P_D = P_o \left\{ \frac{V_F}{V_o} + \frac{r_F}{R_L} \left[ D + \frac{2\pi D + \sin \phi \cos \phi}{4\pi \sin^2(\phi + 2\pi D)} \right] + \frac{3}{4\pi \tan(\phi + 2\pi D)} - \frac{\cos \phi}{\pi \sin(\phi + 2\pi D)} \right\}. \quad (8)$$

Using (2), the rms value of the current through the parallel capacitance  $C$  is given by

$$I_{Crms} = \sqrt{\frac{1}{2\pi} \int_{\phi+2\pi D}^{\phi+2\pi} \left[ I_o \left( 1 - \frac{\sin \omega t}{\sin(\phi + 2\pi D)} \right) \right]^2 d\omega t}. \quad (9)$$

Substitution of expression resulting from this and  $P_o = R_L I_o^2$  into  $P_C = r_{ESR} I_{Crms}^2$  gives the power dissipated in the ESR of capacitor  $C$

$$P_C = P_o \frac{r_{ESR}}{R_L} \left[ 1 - D + \frac{2\pi(1-D) - \sin \phi \cos \phi}{4\pi \sin^2(\phi + 2\pi D)} - \frac{3}{4\pi \tan(\phi + 2\pi D)} + \frac{\cos \phi}{\pi \sin(\phi + 2\pi D)} \right]. \quad (10)$$

The current through the filter capacitor is that of the sinusoidal current source as it is seen at the terminals of the secondary winding of the transformer. Hence, losses in filter capacitor  $C_f$  are

$$P_{Cf} = \frac{r_{ESRf} (n I_m)^2}{2} = P_o \frac{r_{ESRf}}{2 R_L \sin^2(\phi + 2\pi D)}. \quad (11)$$

The efficiency of the rectifier circuit is

$$\eta = \frac{P_o}{P_o + P_{Cu} + P_D + P_C + P_{Cf}}. \quad (12)$$

Substitution of (5), (8), (10), and (11) into (12) gives the final expression of the rectifier efficiency. Using (3), (4), and  $V_o = R_L I_o$  efficiency  $\eta$  is expressed as a function of dc load current  $I_o$ .

According to the values measured at 1 Mhz in a tested Class E rectifier, the transformer windings equivalent resistance was assumed to be  $r_{Cu} = 0.38 \Omega$  and the transformer turns ratio was  $n = 6$ . The values of capacitors  $C$  and  $C_f$  ESRs were assumed to be variable from 0.01 to 0.04  $\Omega$ . The

resistance and the battery used in diode  $D$  equivalent circuit are  $r_F = 0.033 \Omega$  and  $V_F = 0.3 \text{ V}$ , according to the data sheet of a Motorola MBR3525 Schottky diode.

Fig. 4 shows that the efficiency of a 100 W Class E rectifier operating with an output of 5 V highly depends on the output current  $I_O$  and on  $r_{ESRf}$ . Plots of the efficiency of a 50 W rectifier with an output of 5 V are given in Fig. 5. These figures show that the Class E half-wave rectifier is suitable for application with 5 V outputs only when the output power is lower than 50 W and if capacitor ESRs are small. As shown in Fig. 6, the efficiency of a 240 W rectifier with an output of 12 V is highly affected by resistance  $r_{ESRf}$ . The efficiency of a 100 W rectifier operating with an output of 12 V is plotted in Fig. 7. Its maximum value is  $\eta = 94\%$  and only slightly decreases to  $\eta = 92\%$  when the load current increases from no load to the full load. Because of this, the Class E, half-wave, low  $dv_D/dt$  rectifier is particularly suitable for applications where a 12 V and 100 W output is required.

### III. CONCLUSIONS

A detailed analysis of power losses occurring in a Class E, half-wave, low  $dv_D/dt$  rectifier has been carried out. Several values of capacitor ESRs were considered in plotting the circuit efficiency. With applications requiring 5 V outputs, the rectifier operates with a sufficiently high efficiency only if the output power is reduced below 50 W and the capacitor ESRs are small. It operates with a high efficiency over a load range varying from 10% to 100% of the full load when the output voltage is 12 V, especially if the output power is lower than 100 W. A low ESR of capacitors extend the load range where the rectifier can operate with a high efficiency. The filter capacitor ESR affects the circuit efficiency more than that of parallel capacitor  $C$ .

The presented analysis can be applied to any other rectifier topologies to determine the applications where they operate with higher efficiency. Investigations on the efficiency of other rectifiers are recommended for future work.

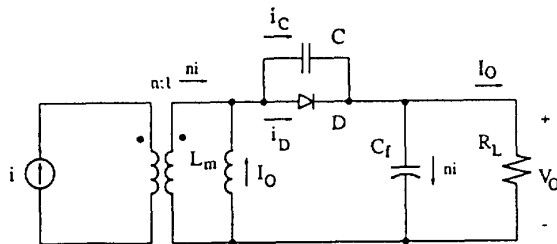


Fig. 1 Class E, half-wave, low  $dv_D/dt$ , rectifier circuit

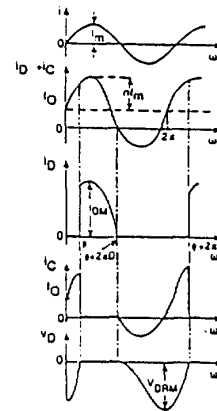


Fig. 2 Idealized waveforms in the Class E, half-wave, low  $dv_D/dt$ , rectifier

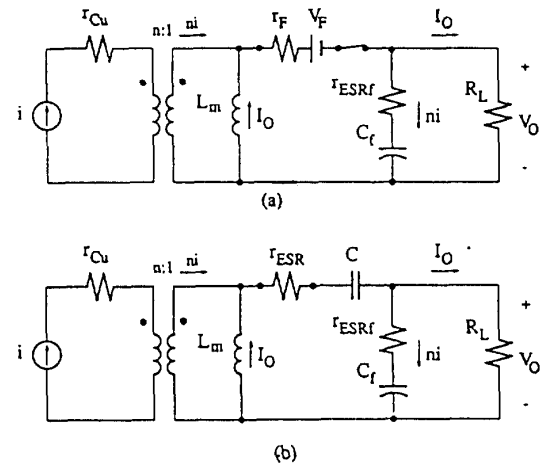


Fig. 3 Class E, half-wave, low  $dv_D/dt$ , rectifier equivalent circuit including parasitic components  
a) Equivalent circuit when the diode is ON  
b) Equivalent circuit when the diode is OFF

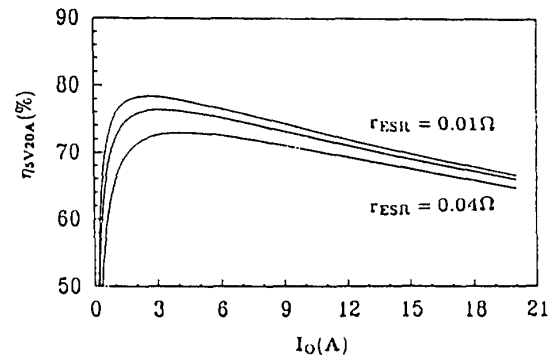
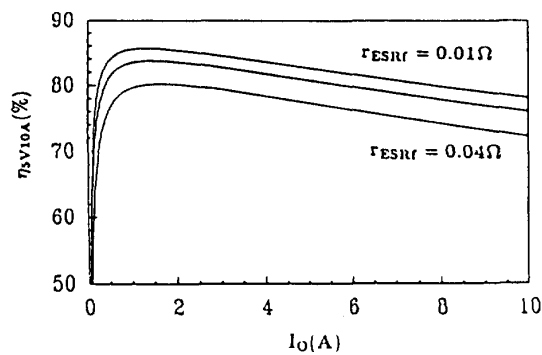
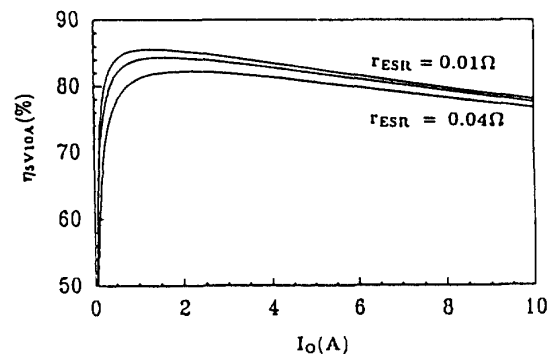


Fig. 4 Efficiency of a 100 W Class E rectifier with a 5 V output  
 $r_{ESRf} = 0.01 \Omega$ ;  $r_{ESR} = 0.01 \Omega$ ,  $r_{ESR} = 0.02 \Omega$ , and  $r_{ESR} = 0.04 \Omega$



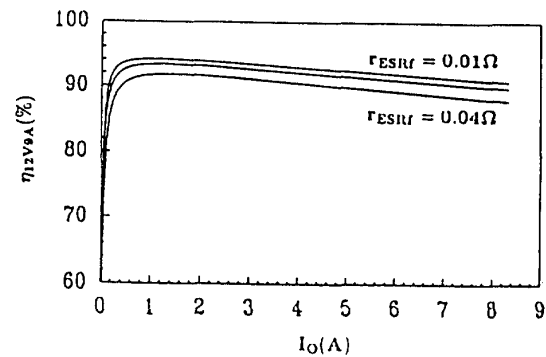
(a)



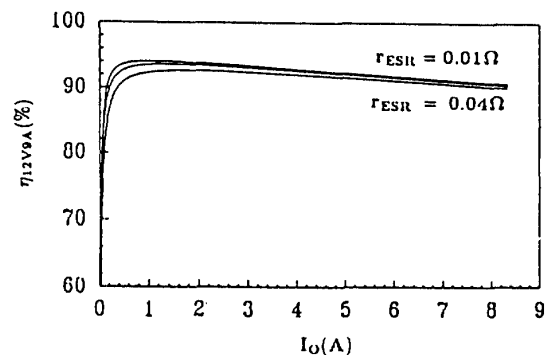
(b)

Fig. 5 Efficiency of a 50 W Class E rectifier with a 5 V output

a)  $r_{ESR} = 0.01 \Omega$ ;  $r_{ESRf} = 0.01 \Omega$ ,  $r_{ESRf} = 0.02 \Omega$ , and  $r_{ESRf} = 0.04 \Omega$   
b)  $r_{ESRf} = 0.01 \Omega$ ;  $r_{ESR} = 0.01 \Omega$ ,  $r_{ESR} = 0.02 \Omega$ , and  $r_{ESR} = 0.04 \Omega$



(a)



(b)

Fig. 7 Efficiency of a 100 W Class E rectifier with a 12 V output

a)  $r_{ESR} = 0.01 \Omega$ ;  $r_{ESRf} = 0.01 \Omega$ ,  $r_{ESRf} = 0.02 \Omega$ , and  $r_{ESRf} = 0.04 \Omega$   
b)  $r_{ESRf} = 0.01 \Omega$ ;  $r_{ESR} = 0.01 \Omega$ ,  $r_{ESR} = 0.02 \Omega$ , and  $r_{ESR} = 0.04 \Omega$

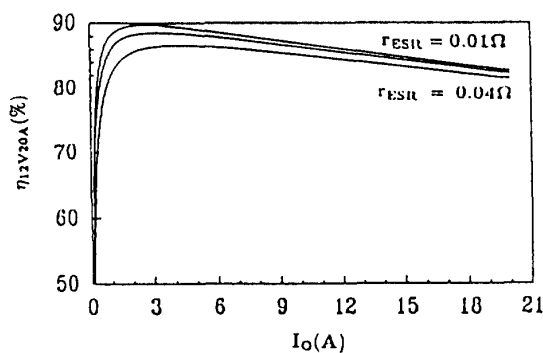


Fig. 6 Efficiency of a 240 W Class E rectifier with a 12 V output

$r_{ESRf} = 0.01 \Omega$ ;  $r_{ESR} = 0.01 \Omega$ ,  $r_{ESR} = 0.02 \Omega$ , and  $r_{ESR} = 0.04 \Omega$

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