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SOCIO-ECONOMIC EVALUATION WITH ORDINAL VARIABLES: INTEGRATING COUNTING AND POSET APPROACHES

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SUMMARY

The evaluation of material deprivation, quality of life and well-being very often requires to deal with multidimensional systems of ordinal variables, rather than with classical numerical datasets. This poses new statistical and methodological challenges, since classical evaluation tools are not designed to deal with this kind of data. The mainstream evaluation methodologies generally follow a counting approach, as in a recent proposal by Alkire and Foster pertaining to the evaluation of multidimensional poverty. Counting procedures are inspired by the composite indicator approach and share similar drawbacks with it, computing aggregated indicators that may be poorly reliable. A recent and alternative proposal is to address the ordinal evaluation problem through partial order theory which provides tools that prove more consistent with the discrete nature of the data. The goal of the present paper is thus to introduce the two proposals, showing how the evaluation methodology based on partial order theory can be integrated in the counting approach of Alkire and Foster.

Keywords: *Partial Order theory, Counting Approach, Evaluation, Material Deprivation, Quality of Life.*

1. INTRODUCTION

Evaluation studies pertaining to material deprivation, quality of life and well-being more and more frequently involve the analysis of multidimensional systems of ordinal variables. This poses new methodological challenges, since the statistical tools usually employed in evaluation procedures are designed to deal primarily with quantitative data, as in classical multivariate analysis. In many cases, ordinal data are turned into cardinal numbers, through more or less sophisticated scaling algorithms, so that classical tools can be formally applied. However, one is realizing that this approach is not satisfactory, both from an epistemological and a statistical point of

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view. Phenomena pertaining to quality of life are often ordinal in nature and one may legitimately ask what kind of knowledge is gained about them when they are conceptualized and addressed in cardinal terms.

When dealing with ordinal multidimensional phenomena, the relevant issue is not giving numerically ''precise'' figures, but providing faithful representations of them, reproducing, as far as possible, their complexity. However, socio-economic indicators serve primarily for policy making purposes and must necessarily be easy and clear to understand and to communicate. The statistical challenge is thus to produce simple, albeit not trivial, measures conveying as much information as possible to decision-makers, preserving at the same time the key features of socio-economic complexity. Many studies have appeared in the last twenty years pertaining to evaluation in an ordinal setting. Most of them are based on a counting approach as in the seminal paper of Cerioli and Zani (1990) and in a recent proposal of Alkire and Foster (2007). The counting approach is basically inspired by the composite indicator philosophy and shares with it some of its limitations. In particular, it implicitly defines compensation criteria among evaluation dimensions and produces aggregated indicators that may prove poorly reliable and not so easy to interpret (Freudenberg, 2003). Recently, a completely different approach has been proposed, based on partial order theory. It overcomes many of the limitations of the counting procedures and may provide a general framework for ordinal evaluation. Partial order theory is the natural setting for dealing with multidimensional ordinal data; in fact, as a branch of discrete mathematics, it provides the conceptual and formal tools needed for addressing the analysis of ordinal datasets in a consistent and effective way.

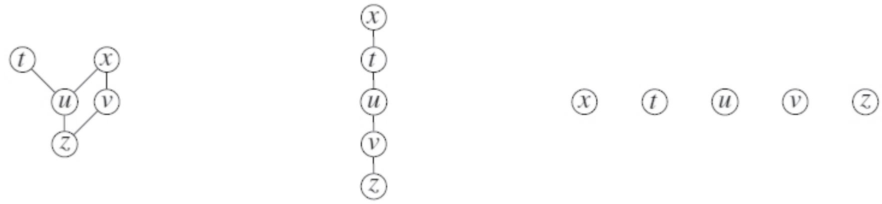
In this paper, we review Alkire and Foster's proposal and show how it can be combined with partial order theory, to obtain a more effective evaluation methodology. In particular, we focus on extending the binary evaluation function of Alkire and Foster, that classifies individuals just in ''deprived'' or ''non-deprived'', so as to assign to individuals deprivation degrees between 0 and 1. This is possible since partial order theory allows for taking into account and reproducing complexity, nuances and ambiguities of ordinal datasets in a natural way. The paper is organized as follows. Section 2 provides a very brief introduction to partial order theory, giving the essential definitions needed for the subsequent discussion. Section 3 shows how partial order theory helps in representing ordinal data, in view of the definition of evaluation procedures. Section 4 describes the counting approach of Alkire and Foster and casts it in poset theoretical terms. Section 5 presents the poset methodology and shows how it can be integrated with Alkire and Foster's procedure. Section 6 concludes. For sake of clarity, the discussion will be held referring to a simple example and not to real data. The principal aim of the paper is, in fact, methodological and this choice allows for graphical representations of the data, simplifying the presentation a lot. The methodologies discussed in the paper can indeed be applied to much more complex datasets.

2. BASIC ELEMENTS OF PARTIAL ORDER THEORY

A partially ordered set (or a poset) $P = (X, \leq)$ is a set X equipped with a partial order relation $\leq$, that is a binary relation satisfying the properties of reflexivity, antisymmetry and transitivity (Davey and Priestley, 2002; Neggers and Kim, 1988; Schroeder, 2003):

1. $x \leq x$ for all $x \in X$ (reflexivity);
2. if $x \leq y$ and $y \leq x$ then $x = y$, $x, y \in X$ (antisymmetry);
3. if $x \leq y$ and $y \leq z$, then $x \leq z$, $x, y, z \in X$ (transitivity).

If $x \leq y$ or $y \leq x$, then x and y are called comparable, otherwise they are said to be incomparable (written $x \,||\, y$). A partial order P where any two elements are comparable is called a chain or a linear order. On the contrary, if any two elements of P are incomparable, then P is called an antichain. A finite poset P (i.e. a poset over a finite set) can be easily depicted by means of a Hasse diagram (Davey and Priestley, 2002; Patil and Taillie, 2004), which is a particular kind of directed graph, drawn according to the following two rules: (i) if $s \leq t$, then node t is placed above node s; (ii) if $s \leq t$ and there is no other element w such that $s \leq w \leq t$ (i.e. if t covers s), then an edge is inserted linking node s to node t. By transitivity, $s \leq t$ (or $t \leq s$) in P, if and only if there is a path in the Hasse diagram linking the corresponding nodes; otherwise, s and t are incomparable. Examples of Hasse diagrams are reported in Figure 1. An upset U of a poset P is a subset of P such that if $x \in U$ and $x \leq z$, then $z \in U$. In a finite poset P, it can be shown that given an upset U there is always a finite antichain $\underline{u} \subseteq P$ such that $z \in U$ if and only if $u \leq z$ for at least one element $u \in \underline{u}$. The upset is said to be generated by $\underline{u}$, written $U = \underline{u}\uparrow$. The subset $\{x, t, u, v\}$ of poset (1) in Figure 1 is an upset, generated by the antichain $\{u, v\}$. Similarly, a downset of P is a subset I such that if $x \in I$ and $y \leq x$, then $y \in I$. An extension of a poset P is a partial order defined on the same set X as P, whose set of comparabilities comprises that of P. A linear extension of a poset P is an extension of P that is also a linear order. Poset (2) of Figure 1 is a linear extension of poset (1) and, trivially, of the antichain (3). A fundamental theorem of partial order theory states that the set of linear extensions of a finite poset P uniquely identifies P (Neggers and Kim, 1988).



(1) (2) (3)

FIGURE 1. - *Hasse diagrams of a poset (1), a chain (2) and an antichain (3)*

3. REPRESENTING DEPRIVATION DATA THROUGH POSETS

Partial order theory allows for a very natural and effective way to represent multivariate systems of ordinal data and provides a general framework to treat and analyse them. In fact, since statistical units are differently ordered according to the different variables in the study, a partial order relation is naturally associated to the population of interest. Tools from partial order theory can then be employed to explore the relational structure of the data and extract information out of it (Fattore, Brüggemann and Owsínski, 2011).

In formal terms, let $v_1, \ldots, v_k$ be k ordinal variables. Each possible sequence of ordinal scores on $v_1, \ldots, v_k$ defines a different profile. Profiles can be (partially) ordered in a natural way, by the following dominance criterion:

Definition 1
Let s and t be two profiles over $v_1, \ldots, v_k$; we say that t dominates s if and only if $v_i(s) \leq v_i(t)\ \forall i = 1, \ldots, k$, where $v_i(s)$ and $v_i(t)$ are the ordinal scores of s and t on v_i.

Not all the profiles can be linearly ordered based on the previous definition, so that the set of profiles gives rise to a poset (in the following called the *profile* poset). It is easily checked that the profile poset is a lattice whose order relation is the product order of the linear orders defined by the each of the variables $v_1, \ldots, v_k$ (Davey and Priestley, 2002).

Example 1
Let us consider three ordinal variables v_1, v_2 and v_3. Suppose that v_1 and v_2 are recorded on a four-grade scale while v_3 is a two-grade variable. For example, in a material deprivation study, v_1 and v_2 could refer to receiving food or money donations (see variables DIFCIB and DIFDEN in the Italian version of the EU-SILC survey) and the four grades could correspond to ‘‘1 - Never’’, ‘‘2 - Seldom’’, ‘‘3 - Sometimes’’,

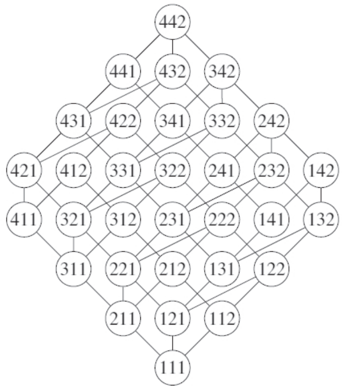


Figure 2. - *Hasse diagram for the deprivation poset built on variables v_1, v_2 and v_3 of Example 1*

''4 - Often''. Variables v_3 could instead pertain to the ownership of a good (e.g. ''1 - owning a car'', ''2 - not owning a car''). The 32 profiles resulting from considering all the sequences of scores over v_1, v_2 and v_3 can be partially ordered according to Definition 1. The Hasse diagram of the resulting poset is shown in Figure 2.

The poset has eight different levels[1], a top element (442) and a bottom element (111). The longest chain comprised in P has 8 elements. The length of P is defined as the number of edges connecting the elements of this chain, which is 7. The largest antichain comprised in P has 7 elements; this is, by definition, the width of P.

Example 2

For future reference, we introduce another simple poset. Let us consider three binary variables recorded on a 0-1 scale. The poset $\hat{P}$ obtained by Definition 1 comprises 8 profiles and is depicted in Figure 3 (this poset is usually referred to as the 'diamond').

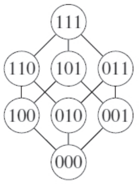


Figure 3. - *Hasse diagram of* $\hat{P}$

4. THE ALKIRE AND FOSTER APPROACH TO ORDINAL EVALUATION

The primary problem addressed by Alkire and Foster is that of defining a criterion to identify deprived individuals, when several material deprivation dimensions are jointly considered on a population. The task is conceptually non-trivial, since evaluation dimensions are often ordinal and quite independent of each other (Alkire and Foster, 2007). This last feature is essential to grasp the complexity of the problem. Evaluation studies often deal with phenomena that are unidimensional, although measured through multivariate systems of correlated variables. This leads to using dimension reduction tools, which exploit the covariances among evaluation dimensions, to compute overall evaluation scores. The problem faced by Alkire and Foster is, instead, truly multidimensional and cannot, neither conceptually nor operationally, be tackled just relying on associations among dimensions. As it will be clear later, in fact, it is basically a problem of comparison and (partial) ordering.

To illustrate Alkire and Foster's methodology, casting it in poset terms, we refer

[1] The deprivation poset P is in fact a 'lattice' (Davey and Priestley, 2002) satisfying the Jordan-Dedekind chain condition (Schröder, 2003). This ensures that the distance between two comparable nodes is well-defined as the number of edges in any chain connecting the two nodes. The level of a node is then defined as the distance between that node and element 111. In practice, nodes sharing the same level share the same number of 1s in the correspondent profiles.

to Example 1 of the previous section. Suppose to consider a population and to assess variables v_1, v_2 (both recorded on a $1-4$ scale) and v_3 (recorded on a $1-2$ scale) on each individual. According to the scores on the deprivation dimensions, individuals are assigned to one of the 32 profiles of the deprivation poset P. Alkire and Foster's procedure identifies deprived people directly assessing the corresponding profiles, through a simple two-step procedure, that, in the language of social choice theory, can be explained as follows:

1. A set of judges is selected; each judge determines on his own whether a profile (and thus each individual sharing it) is to be classified as deprived or not.
2. The number of judges classifying a profile as deprived is computed; if it is equal or higher than a predetermined threshold, the profile is definitely classified as representing a deprived situation.

As usual in the counting approach (Cerioli and Zani, 1990), also in Alkire and Foster's procedure judges coincide with the evaluation dimensions that is, with reference to Example 1, with variables v_1, v_2 and v_3. As an illustration, let us suppose that:

- Profile p is considered deprived on v_1 if $v_1(p) = 3$, deprived on v_2 if, similarly, $v_2(p) = 3$ and deprived on v_3 if (obviously) $v_3(p) = 2$.
- Profile p is considered as definitely deprived if it is deprived on two dimensions out of three.

Consequently, any profile ordered above $\tau_1 = 311$ will be regarded as deprived on v_1, any profile ordered above $\tau_2 = 131$ will be considered deprived on v_2 and any profile ordered above $\tau_3 = 112$ will be retained deprived on v_3. Indicating by $\tau_i \uparrow$ the upset of profiles classified as deprived on v_i $(i = 1, 2, 3)$, the set D of definitely deprived profiles is then given by

$$D = (\tau_1 \uparrow \cap \tau_2 \uparrow) \cup (\tau_1 \uparrow \cap \tau_3 \uparrow) \cup (\tau_2 \uparrow \cap \tau_3 \uparrow)$$

or, explicitly, by

$$\begin{aligned} D = \{&331, 332, 341, 342, 431, 432, 441, 442, \\ &312, 322, 412, 422, 132, 232, 142, 242\}. \end{aligned}$$

Elements of the set D are depicted in Figure 4 as grey nodes. The set D is easily seen to be an upset of the deprivation poset. This has a very nice consequence, leading to a different and more interesting definition of the set of deprived profiles. As any upset, D is generated by an antichain and, in fact, it is easily verified that D is generated by $\underline{d} = \{331, 312, 132\}$. That is, a profile p belongs to D if and only if it is above (or coincides with) an element of $\underline{d}$. Otherwise stated, an individual is definitely deprived if and only if it is as deprived as or more deprived than an individual having one of the profiles listed in D. The antichain $\underline{d}$ can then be seen as a deprivation threshold and its elements are drawn in Figure 4 as larger grey circles with a dot inside.

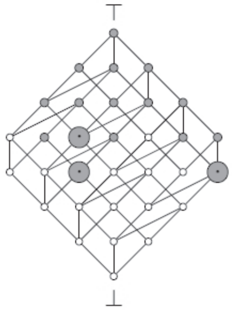


FIGURE 4. - *Deprivation threshold and deprived profiles according to Alkire and Foster's procedure*

It is interesting to notice that the existence of a deprivation threshold is a general result, whenever the set D of definitely deprived profiles is an upset. Since any reasonable way of identifying deprived profiles must satisfy the trivial requirement that if p is a deprived profile then any profile above it in the deprivation poset is also deprived, the set D is always an upset and the existence of a deprivation threshold may be assumed as generally valid.

Elements of the deprivation poset are thus partitioned into two disjoint groups; the group D of definitely deprived profiles and its complement N, corresponding to the white nodes of Figure 4. Profiles in D are given a deprivation score equal to 1, while profiles in N receive a deprivation score equal to 0. Alkire and Foster's procedure thus leads primarily to a binary evaluation of deprivation[2]. Alternatively, the evaluation procedure can also be seen as defining an order-preserving map[3] $\varphi(\cdot)$ between the deprivation poset P and the diamond $\hat{P}$. Each element $p = p_1p_2p_3$ of P is in fact turned first into a binary sequence $b_1b_2b_3$, where $b_i = 0$ or $b_i = 1$ according to whether p_i is below τ_i $(i = 1, 2, 3)$ or not. Binary profiles with at least two 1s are then classified as deprived. Let us partition the elements of P in the following subsets (here, A_1^c stands for the complement of the set A_1)

$$
\begin{aligned}
A_1 &= (\{3**\} \cup \{4**\}) \cap \{*3*, *4*\} \cap \{**2, **2\} \\
A_2 &= (\{3**\} \cup \{4**\}) \cap \{*3*, *4*\} \cap A_1^c \\
A_3 &= (\{3**\} \cup \{4**\}) \cap \{**2, **2\} \cap A_1^c \\
A_4 &= (\{**2\} \cup \{*3*, *4*\}) \cap A_1^c \\
A_5 &= \{222\} \cup \{212\} \cup \{122\} \cup \{122\} \\
A_6 &= \{321\} \cup \{421\} \cup \{311\} \cup \{411\} \\
A_7 &= \{241\} \cup \{231\} \cup \{131\} \cup \{141\} \\
A_8 &= \{221\} \cup \{121\} \cup \{211\} \cup \{111\}
\end{aligned}
$$

[2] In the original paper of Alkire and Foster (2007), a further distinction is made among deprived profiles, based on the number of deprivations they share.

[3] An *order-preserving map* $\varphi(\cdot)$ between two posets $P_1 = (X, \leq_1)$ and $P_2 = (Y, \leq_2)$ is a map between X and Y such that if $a \leq_1 b$ in P_1, then $\varphi(a) \leq_2 \varphi(b)$ in P_2.

where $*$ stands for any value of the corresponding variable[4]. The map $\varphi(\cdot)$ between P and $\hat{P}$ is then given by:

$$\begin{array}{ccc} A_1 & \longrightarrow & 111 \\ A_2 & \longrightarrow & 110 \\ A_3 & \longrightarrow & 101 \\ A_4 & \longrightarrow & 001 \\ A_5 & \longrightarrow & 001 \\ A_6 & \longrightarrow & 010 \\ A_7 & \longrightarrow & 100 \\ A_8 & \longrightarrow & 000 \end{array}$$

The set of deprived binary profiles in $\hat{P}$ is depicted in grey in Figure 5. The deprivation threshold is simply $\{110, 101, 011\}$, that is the set of profiles with at least two (binary) deprivations.

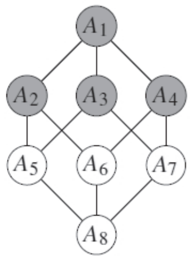


FIGURE 5. - *Image of P under the map φ. In grey, deprived binary profiles*

An inspection into the structure of the deprivation poset reveals that the binary partition of P into deprived and non-deprived profiles is not as satisfactory as it may seem at first. In fact, while it is clear that a profile which is more deprived than an element of the deprivation threshold must be classified as deprived, it is not that clear why elements of N should be classified as non-deprived. Apart from profile 111, which is less deprived than any element of $\underline{d}$, all of the other elements of N are incomparable with at least one element of the deprivation threshold. Profile 241 is even incomparable with all of the elements of $\underline{d}$. This means that it cannot be asserted that elements of N are less deprived that any element of the deprivation threshold. This is quite paradoxical: definitely non-deprived elements should be unambiguously less deprived than any element of $\underline{d}$. In a sense, profiles in N should not get deprivation scores equal to 0, neither should they get scores equal to 1. They should in fact be given a score in $]0, 1[$. To find out a formal and consistent way to extend in this direction Alkire and Foster's binary evaluation function, partial order theory is needed, as discussed in the next Section.

[4] For instance, $\{3**\}$ stands for $\{311, 321, 331, 341, 312, 322, 332, 342\}$.

5. EXTENDING THE COUNTING APPROACH THROUGH POSET THEORY

The methodology proposed by Alkire and Foster (2007) can be seen as a way to accomplish two different tasks: i) identifying definitely deprived profiles and ii) assigning a deprivation degree to any profile, defining an evaluation function on the deprivation poset P. In practice, the two tasks coincide, since the evaluation function chosen by Alkire and Foster is binary. The identification of the set D of definitely deprived profiles is attained through the two-step procedure described in Section 4, which results in determining a deprivation threshold $\underline{d}$ and considering the upset $D = \underline{d}\uparrow$. This is typical of socio-economic evaluation studies and the identification procedure of Alkire and Foster is just one possibility out of many. The definition of the evaluation function is instead more problematic. As discussed in the previous Section, while it is reasonable to assign a deprivation score equal to 1 to profiles belonging to D, it is not consistent to assign deprivation degree equal to 0 to any profile in N. To overcome this issue, it is useful to link the binary evaluation function of Alkire and Foster to the structure of the Hasse diagram of the deprivation poset.

The binary evaluation function assigns deprivation degree 1 to profiles in $\underline{d}\uparrow$. Consistently, it assigns degree 0 to the intersection of all the downsets of the elements of $\underline{d}$ that, in our example, reduces to profile 111. Thus, we may say that the evaluation function assigns deprivation degrees to profiles based on their position with respect to the threshold: 1 if the profile is above $\underline{d}$, 0 if the profile is (unambiguously) below $\underline{d}$. The problem of defining the evaluation function on the remaining set $M = N - \{111\}$ can then be turned into the problem of quantifying to what extent an element of M can be considered as above (or, complimentary, below) the deprivation threshold. This question can be answered formally through partial order theory and this is the key to extend the evaluation function proposed by Alkire and Foster.

Let us consider the set of all the linear extensions of the deprivation poset, $E(P)$, that, as previously mentioned, uniquely identifies P. By virtue of the definition of linear extension, it is clear that the profiles in $D - \underline{d}$ are the only elements of P that are ranked above an element of the threshold in any linear extension of P. Similarly, element 111 is the only profile being ranked below any element of the threshold in any linear extension of P. Instead, elements of M are ranked above elements of the threshold in some linear extensions and below them in others. Adopting a social choice terminology, linear extensions can be viewed as judges ranking elements of P in terms of deprivation. Since elements of $\underline{d}$ are deprived, any profile ranked by a judge above an element of the threshold will be considered, by that judge, as deprived. It is thus natural to define the evaluation function $\eta(\cdot)$ on a profile p as the fraction of linear extensions ranking p above at least one element of the deprivation threshold, in formulas

$$\eta(p) = \frac{|\{\ell \in E(P) : \exists d \in \underline{d} : d \leq p \in \ell\}|}{|E(P)|} \quad (1)$$

By construction, $\eta(\cdot)$ coincides with the evaluation function of Alkire and Foster on D and 111, but differs on M, assuming values in $]0, 1[$.

The exact computation of the evaluation function is very hard, although some efficient algorithms are available to compute mutual ranking frequencies without listing all of the linear extensions of P (de Loof, de Beats and de Meyer, 2006). So the evaluation function must be estimated, based on a sample of linear extensions. The most effective algorithm for sampling (quasi) uniformly from $E(P)$ is the Bubley-Dyer algorithm (Bubley and Dyer, 1999), which we have indeed employed in the paper. Following the original paper of Bubley and Dyer, we have extracted a preliminary set of linear extensions, until the algorithm reached the uniform sampling regime, within a ''distance from uniformity'' that we have chosen equal to 10^{-7}. Then we have sampled 10^9 linear extensions, computing the evaluation function for any element of P. The results are listed in Table 1 and depicted in Figure 6 (we have reported up to 3 decimal digits to show how the procedure is capable, in principle, to extract information from the partial order structure; anyway, it is clear that in practice such a precision is not needed).

TABLE 1. - *Deprivation scores of the profiles in the deprivation poset P*

prog.	1	2	3	4	5	6	7	8
p	442	441	432	342	431	422	341	332
$\eta(p)$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
prog.	9	10	11	12	13	14	15	16
p	242	312	412	331	322	142	232	132
$\eta(p)$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
prog.	17	18	19	20	21	22	23	24
p	241	421	222	141	231	321	411	122
$\eta(p)$	0.787	0.772	0.650	0.450	0.448	0.435	0.417	0.119
prog.	25	26	27	28	29	30	31	32
p	212	221	131	311	221	121	112	111
(p)	0.105	0.086	0.050	0.043	0.002	0.002	0.001	0.000

As can be noticed, the evaluation function smoothly decreases from 1 to 0, based on the ''relational'' position of profiles with respect to the deprivation threshold. Profiles near the bottom of the poset receive deprivation degrees smaller than profiles which occupy higher levels. Anyway, there is no correspondence between the levels of the Hasse diagram of P and the value of the evaluation function. In fact, elements sharing the same level need not share the same relational position in the poset, given $\underline{d}$. These differences are captured by the poset procedure and are turned into the different deprivation degrees assigned to the profiles. This shows how the poset procedure is capable to extract information out of the poset structure, reproducing the nuances and the complexity of the data in a much more effective way than simple counting approaches.

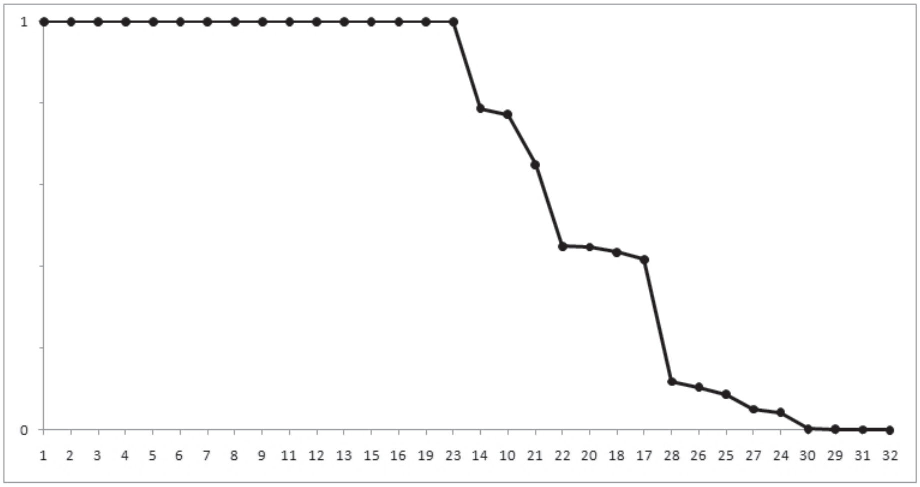


FIGURE 6. - *Evaluation function for the profiles in the deprivation poset P. Profiles are listed on the x axis according to decreasing deprivation degrees*

6. CONCLUSIONS

In this paper, we have addressed the problem of improving the counting approach to socio-economic evaluation in a multidimensional ordinal setting, The proposal of Alkire and Foster has been reviewed and integrated with partial order theory, improving the informative power of the classical evaluation procedures based on simple countings. In particular, tools from partial order theory prove capable to exploit the partial order structure of multidimensional ordinal datasets and to extract information directly, so that final evaluation scores reproduce more faithfully the nuances and the complexity of the original data. The use of poset techniques in socio-economic evaluation is at an inital stage (Fattore, 2008; Fattore *et al.*, 2011; Annoni and Brüggemann, 2009). Some proposals have been made, but a lot of research is still needed to build sound evaluation methodologies for ordinal data. However, results are quite promising. Poset theory is in fact the right formal framework in which multidimensional systems of ordinal variables can be treated and analysed. Through partial order theory, many limitations of the classical statistical procedures can be overcome and ordinal data can be addressed in full consistency with their discrete nature, without forcing them in inadequate conceptual frameworks. A great part of socio-economic indicators built on ordinal data and used by decision-makers in their daily practice are based on methodologies and assumptions that can be argued in many respects (Maggino, 2009). Still, they have a big impact at public level, directly entering policy-making. Using partial order theory in socio-economic evaluation studies is thus of primary interest, both for researchers and policy-makers; partial order theory can in fact help getting new and more meaningful insights into the complexity of socio-economic facts and can provide a sound basis for developing more consistent social indicators. For this reasons, we hope that the topic may attract more and more research both at theoretical and applied level.

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