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Studying heterogeneity among fundamentalists in financial markets: A note

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ABSTRACT

In this note, developing a model with agents that act as fundamentalists, we show that adding simple stochastic components, a greater set of parameters can lead to complex dynamics. Moreover, we identify periods of financial distress between bubbles and crashes detected by Kindleberger [C.P. Kindleberger, Manias, Panics, and Crashes: A History of Financial Crisis, fourth ed., John Wiley, New York, 2000] from a series of famous speculative bubbles and crashes in world history.

1. Introduction

The fundamental value is quite far from being a clear and well-defined concept. On one hand, it depends on assumptions done and models employed by researchers and, on the other, it depends on possible computational errors. Paraphrasing Kirman [1] the choice of *one representative fundamental value* "is not simply an analytical convenience [but it can] lead to conclusions which are usually misleading and often wrong".

Theoretical works on financial markets with heterogeneous agents, generally trace complex dynamics of price fluctuations back to interactions amongst agents that stabilize the market (fundamentalists) and agents, which instead introduce instability into the system (chartists) (see [2] for a complete survey). Strongly aware that technical analysts are key actors in the modern financial markets, we have shown, in a deterministic framework [3,4], that complex dynamics can arise even if fundamentalists agents generate different fundamental values, despite the canonical model. We, therefore, aim primarily to stress the role that heterogeneity[1] has in explaining price fluctuations.

As in [6], our model "involves agents who may use one of a number of predictor which they might obtain from [two] financial gurus" (experts), and, that acting as fundamentalists, are able to dig independently out two different fundamental values. Moreover, agents can switch from one expert to the other following an adaptive belief system [7,8]: therefore an evolutionary competition generates price' fluctuations which may be triggered by differences in beliefs and amplified by dynamics among different schemes. Particularly, the switching mechanism is strictly linked to the experts' prediction ability, approximated by the distance between fundamental value and price. The switching mechanism is based on squared error: the less the margin of squared error, the higher the quota of agents that emulate that expert. We have shown that an increasing degree of heterogeneity, proxied by the difference between the estimated fundamental values, leads firstly (i) to the rise

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[1] Recently, De Grauwe and Kaltwasser [5] have analysed the coexistence of different fundamental values in the foreign exchange market. However, their switching mechanism is based on profitability, secondly they have chartists in analysis, moreover they assume that supply and demand are always equal and the former is exogenous. Finally, while they use extensively simulations, we have also an analytical approach.

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of multiples equilibriums and, secondly (ii) it generates, together with a larger reaction to misalignment of both market-makers and agents, the appearance of a periodic, or even, chaotic, price fluctuation, thus justifying the observed bull and bear behaviours in the market.

In this paper, using this same framework, we want to show that adding simple stochastic components a greater set of parameters can lead to complex dynamics. Moreover, as attempted by Gallegati et al. [9], we try to identify the period of financial distress between bubbles and crashes acknowledged by Kindleberger [10] in the world history.

In section two, we briefly present the deterministic model, discussing the existence and stability of fixed points, as well as the insurgence of a pitchfork bifurcation and the transition to a homoclinic bifurcation. In section three, we add a stochastic component in the agent's reaction coefficient, and in the market maker's perception on excessive demand. We firstly show that a larger set of parameters leads to complex dynamics and, secondly we analyse the existence of a period of financial distress. Finally, last section provides brief concluding remarks and suggestions for further research in this topic.

2. The deterministic model

For the purpose of our model, we assume that there are two gurus who, thanks to the available information on the economic system, independently formulate their expectations on future prices, therefore creating two different fundamental values. The two gurus are followed by other operators, who can switch from the prediction of one expert to another. The ability of the experts, measured throughout the distance between the hypothetical fundamental value and the current price, is the main driver for the switching process of the agents. Market-makers act as mediators in transactions, setting prices in response to excessive demand (supply). We explore a model in which two assets are considered: one is risky and the other one is risk-free. The latter has a perfectly elastic supply at the gross return ($R > 1$). The risky asset has, instead, a price per share ex-dividend at time t equal to X_t and it presents a (stochastic) dividend process equal to y_t. Defining $i = 1, 2$ the two groups of agents, their wealth at $t + 1$ is given by:

$$W_{i,t+1} = RW_{i,t} + R_{t+1}q_{i,t} = RW_{i,t} + (X_{t+1} + y_{t+1} - RX_t)q_{i,t} \tag{1}$$

where $R_{t+1} = (X_{t+1} + y_{t+1} - RX_t)$ is the excess capital gain/loss, and $q_{i,t}$ is the number of shares of the risky asset purchase at time t. Given a specific set of information, the agents of type i have two key structural *a priori* about their wealth: a conditional expectation and a variance. From equation (1) it follows that

$$E_{i,t}(W_{t+1}) = RW_{i,t} + E_{i,t}(X_{t+1} + y_{t+1} - RX_t)q_{i,t} \text{ and } V_{i,t}(W_{t+1}) = q_{i,t}^2 V_{i,t}(R_{t+1}).$$

Each group of agents has a CARA (Constant Absolute Risk Aversion) utility function, i.e. $u(W) = -e^{-aW}$, where a is the constant risk aversion equal for both groups of agents and strictly positive. Maximizing the expected wealth utility function:

$$Max_{q_{i,t}}\left\{E_{i,t}(W_{i,t+1}) - \frac{a}{2}V_{i,t}(W_{i,t+1})\right\} \tag{2}$$

investors, belonging to the group i, demand the following amount, $q_{i,t}$:

$$q_{i,t} = \frac{E_{i,t}(X_{t+1} + y_{t+1} - RX_t)}{aV_{i,t}(X_{t+1} + y_{t+1} - RX_t)} = \frac{E_{i,t}(X_{t+1} + y_{t+1} - RX_t)}{a\sigma^2} \tag{3}$$

We assume that agents have *common* expectations on *dividends* $(E_{i,t}(y_{t+1}) = E_t(y_{t+1}) = \bar{y})$ but *different* expectations on *future prices* $(E_{i,t}(X_{t+1}) = E_i(X^*_{t+1}) = F_i)$, with $i = 1, 2$. F_i represents the benchmark of the fundamental value detected by the gurus. Although, the assumption of *common expectations on dividends* is restrictive, it does not modify the qualitative dynamic behaviour of the model. Therefore, equation (3) can be rewritten as follows:

$$q_{i,t} = \alpha(F_i - P_t) \tag{3'}$$

where $P_t = RX_t - \bar{y}$ and $\alpha = \frac{1}{a\sigma^2}$ is the positive coefficient of the reaction of investors that are negatively related to risk aversion.[2] The asset price follows a market maker mechanism where, out of equilibrium, changes are possible. Particularly, market maker applies the following rule:

$$P_{t+1} = P_t + \beta[w_{t+1}q_{1,t} + (1 - w_{t+1})q_{2,t}] \tag{4}$$

where β is the positive market maker's reaction coefficient to excess demand and w_{t+1} is the proportion of agents who follow the prediction of expert 1. According to [6], "agents make rational decision between (gurus') predictors and tend to choose the predictor which yields the smallest prediction error (...)". Assuming that agents have a determined length of memory $(l + 1)$, then their evaluation on a guru's performance depends on the weighted average of the square errors given by the distance between the fundamental values and the price recorded in the last l periods. Let $SE_{i,t}$ be the arithmetic weighted mean:

[2] Different beliefs alter mainly the median steady state, without having any impact on dynamics, because these beliefs are unstable and can detect only the basins of attractions of the coexistent attractors.

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$$SE_{i,t} = \sum_{k=0}^{l} \rho_{t-k}(F_i - P_{t-k})^2 \tag{5}$$

where ρ_{t-k} is the weight relative to the period $(t-k)$. We assume that (a) $0 \leqslant \rho_{t-k} \leqslant 1$; (b) $\sum_{k=0}^{l}\rho_{t-k} = 1$ and (c) agents give an higher weight to the most recent data, that is $\rho_{t-k-1} < \rho_{t-k}$. Using an adaptive rational mechanism, as in [7] and [8], w_{t+1} can be defined as a frequency:

$$w_{t+1} = \frac{\exp[-\gamma\sum_{k=0}^{l}\rho_{t-k}(F_1 - P_{t-k})^2]}{\exp[-\gamma\sum_{k=0}^{l}\rho_{t-k}(F_1 - P_{t-k})^2] + \exp[-\gamma\sum_{k=0}^{l}\rho_{t-k}(F_2 - P_{t-k})^2]} \tag{6}$$

that, straightforward algebra, is equal to:

$$w_{t+1} = \frac{1}{1 + \exp\left[\gamma\sum_{k=o}^{l}\rho_{t-k}(F_1 - P_{t-k})^2 - \gamma\sum_{k=o}^{l}\rho_{t-k}(F_2 - P_t)^2\right]} \tag{7}$$

where γ represents the intensity of choice which assesses how quickly agents switch from one of the gurus' predictions, to the other. In the present work, the share of agents following expert i depends on the relative distance between the corresponding fundamental value, F_i and the current price. However this mechanism is not so clear-cut: when the fundamental value F_i is equal to current price, in the next period a share of agents still follow the j expert. This implies that share of agents following experts, varies from zero to one, with extremes not included.

Similarly to Kaizoji [11] the switching mechanism is based on forecasts' accurateness. However, in this case, the mechanism relies on differences between chartists and fundamentalist. Mainly in Kaizoji's model, *"according to the difference between the squared prediction errors of each strategy"*, agents prefer chartist strategy. Even the mechanism employed by [12] is based on agents's prediction ability; particularly, they assume that the larger the deviation of current price is from the fundamental values, the greater is the share of agents who choose the chartist's strategy. Hence, substituting (3′) and (7) in (4), the following dynamic system of dimension $(l+1)$ is achieved:

$$P_{t+1} = P_t + \alpha\beta\left\{(F_2 - P_t) - \frac{\Delta F}{1 + \exp\left[\gamma\Delta F\sum_{k=0}^{l}\rho_{t-k}(2P_{t-k} - F_1 - F_2)\right]}\right\} \tag{8}$$

where $\Delta F = F_2 - F_1 \geqslant 0$ is the degree of heterogeneity. Our main purpose is to show how heterogeneous beliefs can lead to chaotic price fluctuations, hence in a future work, we will analyse the issue of length of memory, limiting the present study just to the case in which agents do not have memory. Here, we consider mainly agents who try to infer which gurus' estimation is closer to the "real" price by considering an instantaneous square error, i.e. the best estimation for the fundamental price is the last observed price. Therefore, assuming $l = 0$, our map is the following one dimensional map:

$$P_{t+1} = P_t + \alpha\beta\left\{(F_2 - P_t) - \frac{\Delta F}{1 + \exp[\gamma\Delta F(2P_t - F_1 - F_2)]}\right\}, \tag{9}$$

where $\Delta F = F_2 - F_1 \geqslant 0$ represents the degree of heterogeneity. For the sake of simplicity we assume $F_1 \leqslant F_2$. With a positive degree of heterogeneity, at least a fixed point exists, $P_M = \frac{F_1+F_2}{2}$.[3] Moreover, a pitchfork bifurcation arises if $\Delta F = \sqrt{\frac{2}{\gamma}}$: given the intensity of switching, γ, a positive degree of heterogeneity exists, $\overline{\Delta F}$, such as the initial unique steady state becomes unstable and two new steady states, P_L and P_H, arise, with $P_L < P_M < P_H$.

It is worth noting that the existence of multiple steady states is not affected by reaction's coefficients: only the degree of heterogeneity and the transfer speed will determine the pitchfork bifurcation. Particularly: (a) the uniqueness can be achieved even in case of heterogeneity; (b) a higher degree of heterogeneity or an increase of γ leads to the insurgence of new steady states, closer to the fundamental values; (c) the higher is the intensity of switching, the lower is the degree of heterogeneity for which the pitchfork bifurcation arises (Fig. 1).

Finally, using numerical simulations the particular route to homoclinic bifurcation can be explored. Given symmetry of the map in relation to P_M, all qualitative dynamic changes (bifurcations, stability/instability, etc.) around the fixed points, P_L and P_H, occur due to the same set of parameters. We set up parameters as follows $\gamma = 0.8$; $F_2 = 8$; $F_1 = 6$; $\alpha = 1.1$, increasing the reaction coefficient of the market makers, β. Particularly, for $\beta \cong 2.7$ a period-doubling bifurcation arises and there are two symmetric stable cycles in period two. However, further increase of β leads initially to a new attractive period-four cycles, which is followed by a two symmetric chaotic attractors (Fig. 2). Mainly, for $\beta \approx 4.03$ a homoclinic bifurcation emerges. The new structure of the basins implies the synthesis between the basins of the two fundamental values: bull and bear price fluctuations appear.

From an economic point of view, in presence of an excess in demand (for example $P_o < F_1$), the overreaction of the market maker leads to a large price increase so that the price becomes higher than P_L. An excess in demand is transformed into an

[3] Proofs are available in [4].

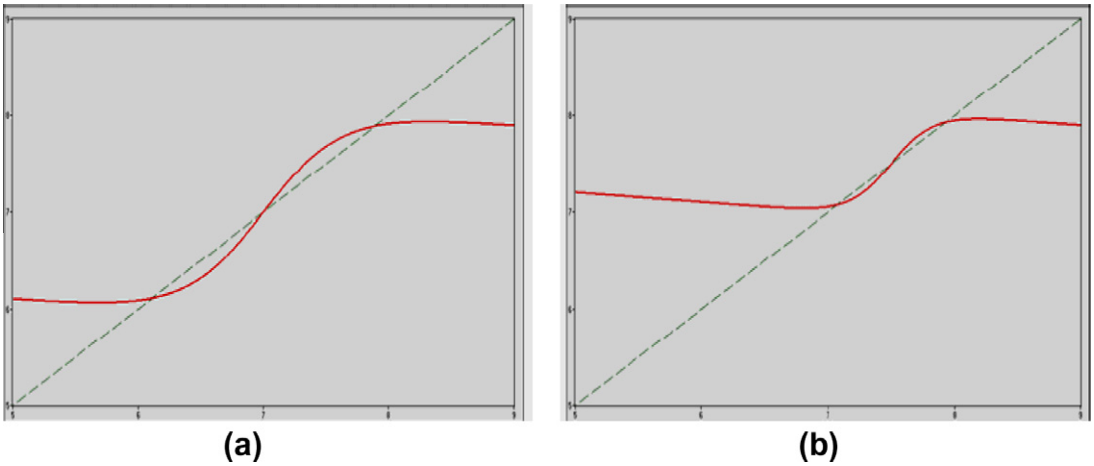

Fig. 1. Pitchfork bifurcation through an increase of the degree of heterogeneity (a) $\gamma = 0.8$; $F_2 = 8$; $F_1 = 6$; $\alpha = 1.1$; $\beta = 1$ or through an increase in the transfer speed (b) $\gamma = 3$; $F_2 = 8$; $F_1 = 7$; $\alpha = 1.1$; $\beta = 1$.

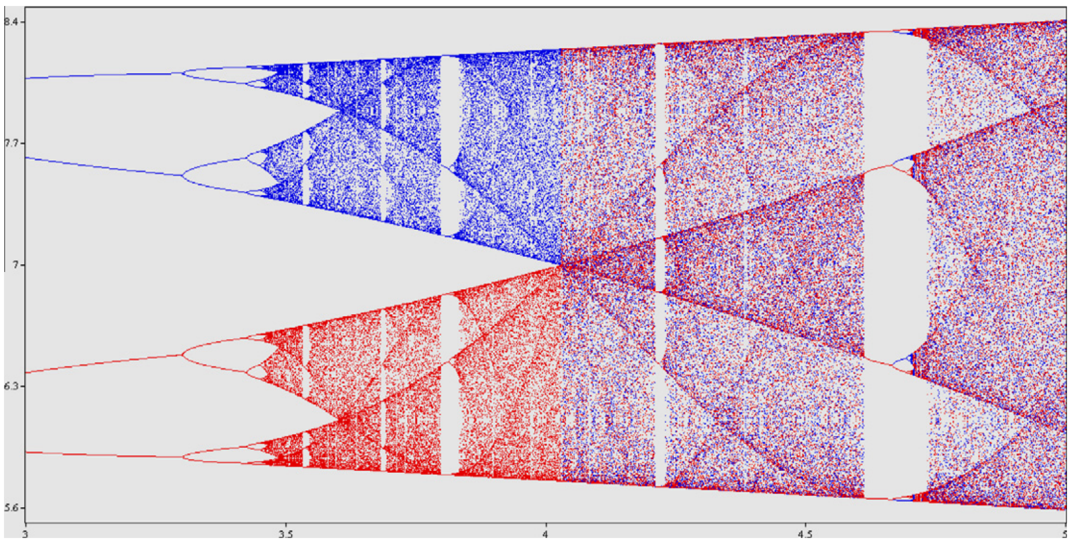

Fig. 2. Bifurcation diagram for $\gamma = 0.8$; $F_2 = 8$; $F_1 = 6$; $\alpha = 1.1$; $\beta \in [3,5]$.

surplus in supply. Even in this case, given a high β, agents which follow expert 1, provide a bulky quantity that leads the price down, particularly less than P_L. With a stable cycle the system fluctuates between excess of demand and excess of supply around the steady state P_L; on the other hand, when a homoclinic bifurcation appears, bull and bear price fluctuations (around the two fundamental values) become visible.

Finally, we reported in Fig. 3 the effects of an increasing degree of heterogeneity. It is interesting to notice that in this case the insurgence of heterogeneity does not entail the instantly insurgence of multiple equilibriums: on the contrary a low degree of heterogeneity, given this parameters ($\gamma = 0.1$; $\alpha = 1$; $\beta = 2.3$; $F_1 = 6$), stabilizes the system. With a low intensity of choice a pitchfork bifurcation arises for $\Delta F \approx 4.47$. Larger discrepancy between experts' evaluation leads to a flip bifurcation and then to a homoclinic bifurcation.

3. Adding a stochastic component

In this section we introduce in the model two stochastic elements. Through simulations we want to show that complex, and even chaotic, dynamics can appear for larger sets of parameters. We add a stochastic component firstly, to the agent's reaction coefficient and, secondly, to the market maker's perception of the excess of demand. Finally, the introduction of a stochastic component allows us to detect of a period of financial distress.

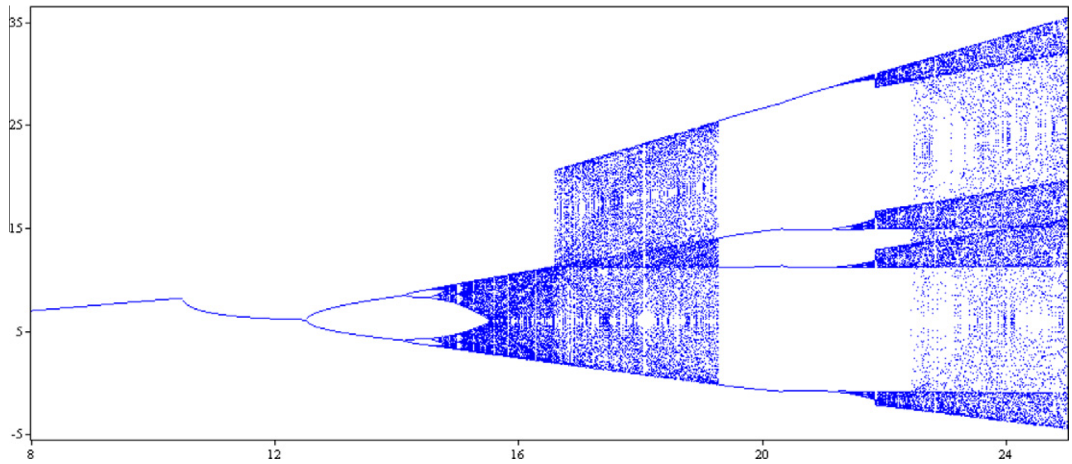

Fig. 3. Increasing degree of Heterogeneity $\Delta F \in [2,17]$; $\gamma = 0.1$; $\alpha = 1.1$; $\beta = 2.1$.

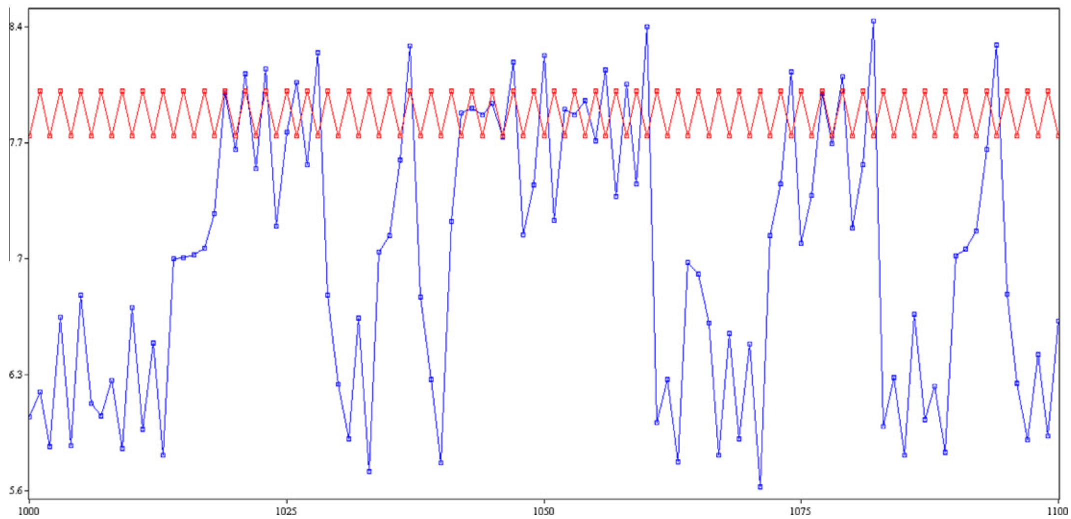

Fig. 4. Random component in reaction coefficient $F_1 = 6, F_2 = 8$; $\gamma = 0.8$; $\alpha = 1.1$; $\beta = 2.8$, 100 iterations after a transient of 1000 iterations.

3.1. A random reaction coefficient

We assume that agent's reaction coefficient is not constant: it has a fix component α and a (positive) random component that varies over time due to the use of available information. It is worth noting that these two parameters do not affect the existence of multiple steady states. The excess of demand (1) can be rewritten as follows:

$$q_{i,t} = (\alpha + \mu_t)(F_i - P_t) \tag{10}$$

μ_t is a random number that lies within a specified range (0, 1), with any number in the range with the same probability.[4] Hence the (stochastic) map becomes:

$$P_{t+1} = P_t + \beta(\alpha + \mu_t)\left\{(F_2 - P_t) - \frac{\Delta F}{1 + \exp[\gamma \Delta F(2P_t - F_1 - F_2)]}\right\} \tag{11}$$

In Fig. 4 we report time series of both deterministic and stochastic map fixing the parameters $\gamma = 0.8$; $F_2 = 8$; $F_1 = 6$; $\alpha = 1.1$, $\beta = 2.8$. Given these values, multiple equilibriums are depicted (for $\lambda = 0.8$ the pitchfork bifurcation arises for $\Delta F = 1.58$). While in the deterministic model there is a stable cycle of period two (the upper line in Fig. 4), simply adding a stochastic component allows the merge of the two attractors (the bottom line).

[4] We simply use the LUA random number generator available with E&F Chaos. More information are available in [13].

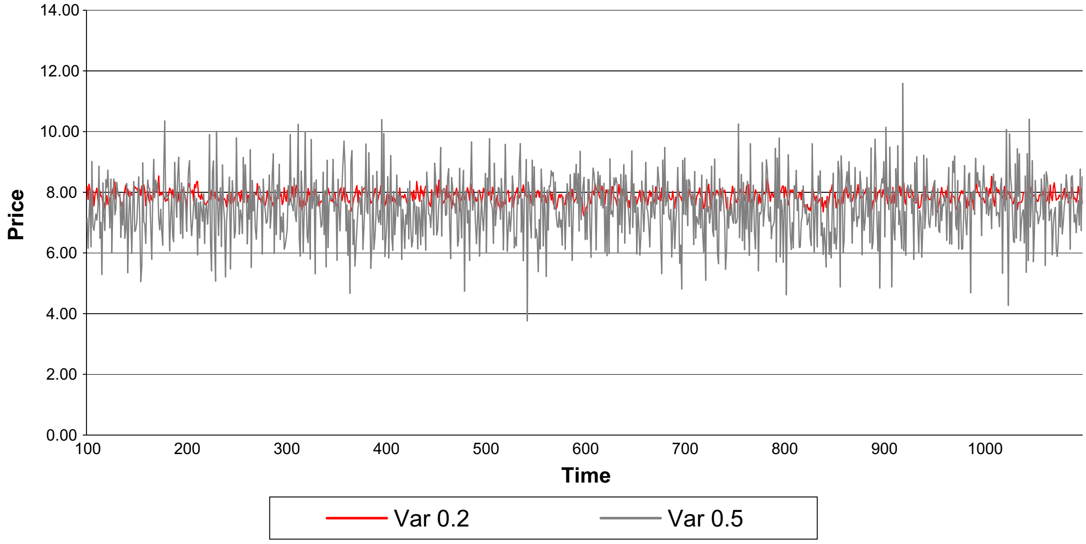

Fig. 5. Increasing standard deviation $F_1 = 6, F_2 = 8$; $\gamma = 0.8$; $\alpha = 1.1$; $\beta = 1$, 1000 iterations after a transient of 100 iterations.

Particularly, in the deterministic model the homoclinic bifurcation arises for $\beta \approx 4.03$; while (all else equal) with a stochastic element bull and bear price fluctuations can appear with a lower market maker's reaction ($\beta \cong 2.1$).

3.2. A noise in perceiving excess of demand

A different stochastic element can regard the market maker perception of the excess of demand. Assuming that there is an error, distributed as a Gaussian with zero mean, the new stochastic map is:

$$P_{t+1} = P_t + \alpha\beta\left\{(F_2 - P_t) - \frac{\Delta F}{1 + \exp[\gamma \Delta F(2P_t - F_1 - F_2)]}\right\} + \varepsilon_t, \tag{12}$$

where ε_t is the normally distributed error. What happen for increasing values of standard deviation (σ)? Fig. 5 shows the case for $\sigma = 0.2$ and $\sigma = 0.5$. Parameters are settled in such a way that the deterministic map converges to the upper steady state. A low stochastic noise ($\sigma = 0.2$) does not change qualitatively the price behaviour: values remain around the upper fundamental value $F_2 = 8$. On the other hand, different soars between the two attractors are possible when a slight increase in standard deviation ($\sigma = 0.5$) is allowed.

Another interesting case regards the effect of higher degree of heterogeneity. In Fig. 6 we employ the same set of parameters used in Fig. 3 but adding an error normally distributed with zero mean and standard deviation equal 0.2.

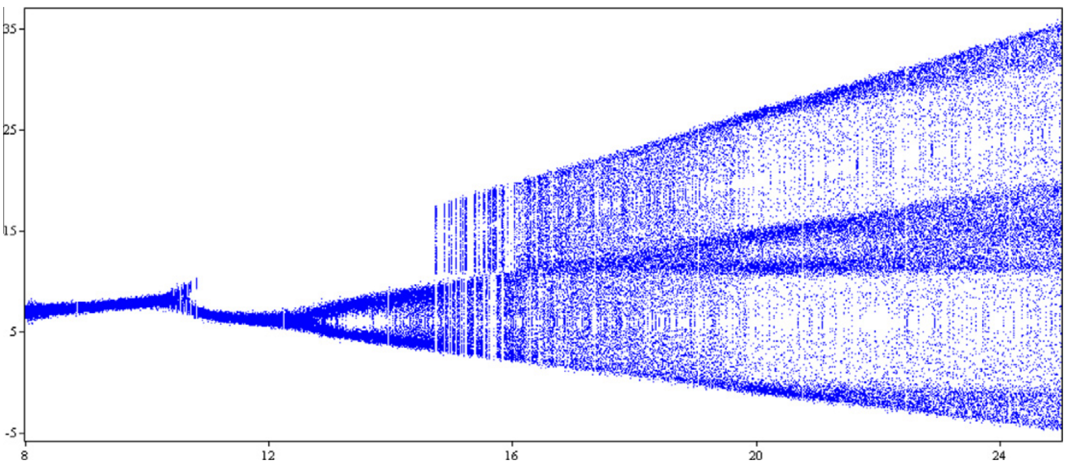

Fig. 6. Increasing degree of heterogeneity $\Delta F \in [2,17]$, $\gamma = 0.1$; $\alpha = 1.1$; $\beta = 2.1$, F_2 on X-axis; $\sigma = 0.2$.

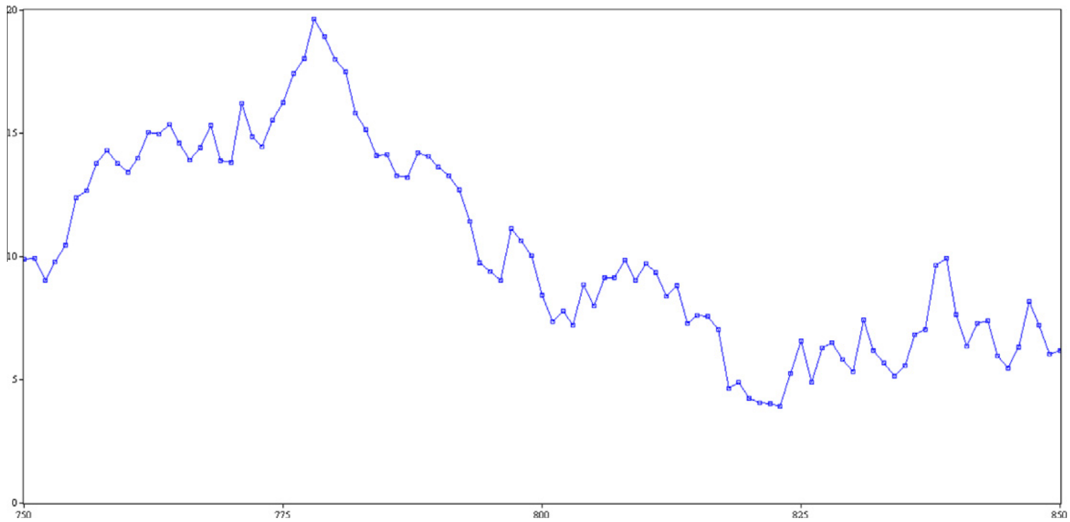

Fig. 7. PFD with increasing degree of heterogeneity $F_1 = 6, F_2 = 9$; $\gamma = 0.6$; $\alpha = 0.1$; $\beta = 0.4$, time on X-axis; $\sigma = 1$

It is interesting to mention that with stochastic component bull and bear price fluctuations appear for lower value of degree of heterogeneity. Specifically, while in the deterministic model ΔF has to be equal roughly 10.7, an even low standard deviation reduce this value by two points.

3.3. Period of financial distress

Charles Kindleberger [10], enumerating a series of famous speculative bubbles and crashes in world, reveals that roughly 80% of existing bubbles follow a specific pattern: after the bubble's peak, a period of gradual decline comes before the real financial crash. He names this interval as a period of financial distress. During this period, agents may acknowledge that a crash is not a unsuspected possibility. They, therefore, speed up to leave (the market) as soon as possible so triggering the crash.

Recently, Gallegati et al. [9] replicate and analyse the PFD using a stochastic model with herding and financial constraints. Our simple model is also able to explain the PFD.

Mainly, adding a stochastic component, a homoclinic bifurcation is not necessary to explain all the dynamics during financial crises. Fig. 7 shows a clear pattern with financial distress:

(1) A boom pushes agents to buy even if they are far from the fundamental values. An overreaction can be caused by pure speculation. In our case, the price reaches a level (almost 20) which is really far from the highest fundamental value;
(2) *"A few insiders decide to take their profits and sell out. At the top of the market there is hesitation, as new recruits to speculation are balanced by insiders who withdraw. Prices begin to level off*": particularly in few steps we move from 20 to less than 14.
(3) In a period of financial distress: *"the awareness on the part of a considerable segment of the speculating community that a rush for liquidity — to get out of other assets and into money — may develop, with disastrous consequences for the prices of goods and securities, and leaving some speculative borrowers unable to pay off their loans". As distress persists, speculators realize, gradually or suddenly, that the market cannot go higher. It is time to withdraw. The race out of real or long-term financial assets and into money may turn into a stampede."*
(4) Fear (or "revulsion") feeds itself until prices become really low: before prices reache P = 4and then they close the gap, moving to values closer to the lowest fundamental estimation.

4. Some conclusions

Heterogeneity in financial markets has been developed in various models which have been useful in explaining the intraday financial market dynamics. Unlike canonical models we focus on agents having the same trading rules (i.e. fundamentalists) where heterogeneity depends on different expectations on fundamental values, and agents can move from one expert to the other, following a switching mechanism. We show that an increasing degree of heterogeneity leads firstly (i) to the insurgence of a pitchfork bifurcation and, secondly (ii) together with a larger reaction to misalignment of both market makers and agents, to the generation of a period doubling.

We add simple stochastic components to the deterministic framework, showing that a greater set of parameters can lead to complex dynamics and identifying the period of financial distress between bubbles and crashes.

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Since our simple model has just two fundamentalists and the switch mechanism relies on the distance between current prices and fundamental values, a further interesting development would be to analyze this framework in the case of profitability-based imitative process.

References

[1] A.P. Kirman, Who and What Does the Representative Individual Represent?, Journal of Economic Perspectives 6 (2) (1992)
[2] C.H. Hommes, Heterogeneous agent models in economics and finance, in: L. Tesfatsion, K.L. Judd (Eds.), Handbook of Computational Economics, Agent-Based Computational Economics, Vol. 2, Elsevier Science B.V., 2006, pp. 1109–1186.
[3] A.K. Naimzada, G. Ricchiuti, Heterogeneous fundamentalists and imitative processes, Applied Mathematics and Computations 199 (1) (2008) 171–180.
[4] A.K. Naimzada, G. Ricchiuti, Dynamic effects of increasing heterogeneity in financial markets, Chaos, Solitons and Fractals 41 (4) (2009) 1764–1772.
[5] P. De Grauwe, P.R. Kaltwasser, Modeling optimism and pessimism in the foreign exchange market, CESifo Working Paper Series No. 1962 (2007).
[6] H. Follmer, U. Horst, A. Kirman, Equilibria in financial markets with heterogeneous agents: a probabilistic perspective, Journal of Mathematical Economics 41 (1–2) (2005) 123–155. February.
[7] W. Brock, C. Hommes, A rational route to randomness., Econometrica 65 (1997) 1059–1095.
[8] W. Brock, C. Hommes, Heterogeneous beliefs and routes to chaos in a simple asset pricing model, Journal of Economic Dynamics Control 22 (1998) 1235–1274.
[9] M. Gallegati, A. Palestrini, J. Barkley Rosser, The period of financial distress in speculative markets: interacting heterogeneous agents and financial constraints, Macroeconomic Dynamics 15 (February 2011) 60–79.
[10] C.P. Kindleberger, Manias, Panics, and Crashes: A History of Financial Crisis, fourth ed., John Wiley, New York, 2000.
[11] T. Kaizoji, Speculative bubbles and fat tail phenomena in a heterogeneous agent model, in: W. Barnett et al. (Eds.), Economic Complexity ISETE, vol. 14, Elsevier, Amsterdam, 2003, pp. 259–275. Chapter 10.
[12] X.Z. He, F. Westerhoff, Commodity markets, price limiters and speculative price dynamics, Journal of Economic Dynamics and Control 29 (2005) 1577–1596.
[13] C. Diks, C. Hommes, V. Panchenko, E&F Chaos : a User Friendly software package for nonlinear economic dynamics, Department of Quantitative Economics, University of Amsterdam, (2007).

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