NEAR-PR DESIGN OF NON-UNIFORM FILTER BANKS

F. Argenti, B. Brogelli and E. Del Re

Dipartimento di Ingegneria Elettronica, Universitá di Firenze
Via di Santa Marta, 3 - 50139 Firenze - Italy
Tel.: +39 55 4796 424 - Fax: +39 55 4796 485
e-mail: argenti@cosimo.ing.unifi.it

ABSTRACT

In this work a new method to design filter banks with rational decimation factors is proposed. It aims at the cancellation of the main component of aliasing in the output signal; this imposes a set of conditions on the filters of the analysis/synthesis banks. If cosine-modulation of different linear phase prototypes is used, the aliasing cancellation condition constrains the prototypes relative to adjacent branches to become dependent on each other. A procedure to design the prototypes based on these constraints is proposed and examples of cosine-modulated non-uniform filter banks are presented.

1. INTRODUCTION

Splitting the spectrum of a digital signal can be useful in several applications, for example data compression. Most of the literature in the field of subband coding filter banks design is concerned with uniform width subbands. However, in some cases a non uniform splitting is more suitable, for example in audio coding [1], where non-uniform width subbands could match better the *critical bands* of the human auditory system.

The problem of designing non-uniform filter banks has been addressed, for example, in [2]-[5]. In this work filter banks with rational decimation factors are considered, so extending the work done in [6] related only to integer decimation factors.

The method can be considered a Near Perfect Reconstruction (Near-PR) one since it is based on the cancellation of the main component of aliasing, like in Pseudo-QMF banks [7]. In the case of rational decimation factors banks, however, more than one coupling of the aliasing components of adjacent branches that lead to their cancellation is possible. If cosine modulation is used, the aliasing cancellation constraints involve the prototype filters of each branch. A design procedure is proposed and numerical examples are presented to show the effectiveness of the method.

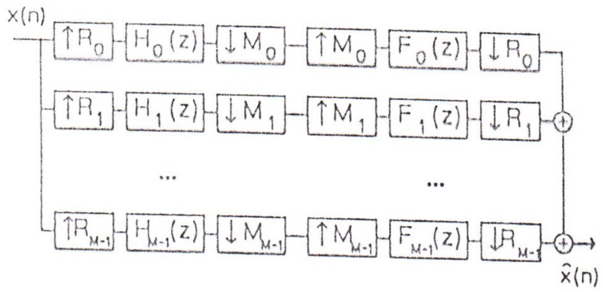

Figure 1: Non-uniform bank with rational decimation factors

2. ALIASING CANCELLATION IN FILTER BANKS WITH RATIONAL SAMPLING FACTORS

Consider the system in Fig. 1, where a non-uniform bank having rational sampling factors R_m/M_m, $m = 0, \ldots,$ M-1, is depicted. The input-output relationship in the z-domain is given by:

$$\begin{aligned}\hat{X}(z) = &\sum_{m=0}^{M-1} \frac{1}{R_m}\frac{1}{M_m} \sum_{p=0}^{R_m-1} F_m(z^{\frac{1}{R_m}} W^p_{R_m}) \cdot \\ &\cdot H_m(z^{\frac{1}{R_m}} W^p_{R_m}) X(z) + \\ &+ \sum_{m=0}^{M-1} \frac{1}{R_m}\frac{1}{M_m} \sum_{l=1}^{M_m-1} \sum_{p=0}^{R_m-1} F_m(z^{\frac{1}{R_m}} W^p_{R_m}) \cdot \\ &\cdot H_m(z^{\frac{1}{R_m}} W^p_{R_m} W^l_{M_m}) X(z W^{lR_m}_{M_m}) \end{aligned} \qquad (1)$$

where $W_M = e^{-j2\pi/M}$. Eq. (1) highlights the reconstruction transfer function and the aliasing components.

Consider the analysis stage of each branch shown in

Fig. 1. Real coefficients filters are taken into account and, therefore, the frequency response of each filter has passbands located at positive and negative frequencies, symmetrically with respect to the origin. The passband at positive (negative) frequencies has width π/M_m and is centered in $(k_m + 0.5)\pi/M_m$ $(-(k_m + 0.5)\pi/M_m)$. The value k_m is an integer and selects which part of the spectrum of the R_m-fold upsampled input signal must be extracted. For example, to extract the spectrum in the frequency interval $[\pi/5, 3\pi/5]$ ($[-3\pi/5, -\pi/5]$) we use R_m=3, M_m=5 and k_m=2. If we consider the frequency response of each filter of the analysis/synthesis banks approximately equal to zero in their stopbands, then the filters transfer functions can be expressed as:

$$H_m(z) = U_m(z) + V_m(z) \tag{2}$$

$$F_m(z) = \hat{U}_m(z) + \hat{V}_m(z) \tag{3}$$

where $U_m(\omega)$ and $\hat{U}_m(\omega)$ have a passband for $\omega > 0$, while $V_m(\omega)$ and $\hat{V}_m(\omega)$ have a passband for $\omega < 0$.

Due to the M_m-fold upsampler in the synthesis stage, *images* of the *m-th* subband spectrum are filtered by $F_m(z)$. The main aliasing terms are created at the high-frequency and at the low-frequency edges of the passband of $F_m(z)$. These components have been described for a cosine-modulated uniform bank in [7]. If we consider that in a rational decimation factors bank each branch operates on an R_m-fold upsampled version of $x(n)$ and we retain only the more relevant terms, then the aliasing terms can be written as:

$$\begin{aligned} A_m^{(low)}(z) = \\ &\frac{1}{M_m}\Big[\hat{U}_m(z)V_m(zW_{M_m}^{k_m})X(z^{R_m}W_{M_m}^{k_m R_m})+ \\ &+\hat{V}_m(z)U_m(zW_{M_m}^{-k_m})X(z^{R_m}W_{M_m}^{-k_m R_m})\Big] \end{aligned} \tag{4}$$

$$\begin{aligned} A_m^{(high)}(z) = \\ &\frac{1}{M_m}\Big[\hat{U}_m(z)V_m(zW_{M_m}^{(k_m+1)})X(z^{R_m}W_{M_m}^{(k_m+1)R_m})+ \\ &+\hat{V}_m(z)U_m(zW_{M_m}^{-(k_m+1)})X(z^{R_m}W_{M_m}^{-(k_m+1)R_m})\Big] \end{aligned} \tag{5}$$

In [7] it is shown that for uniform cosine-modulated filter banks the component $A_m^{(high)}(z)$ of the *m-th* branch is canceled by the component $A_{m+1}^{(low)}(z)$ of the *(m+1)-th* branch. In the non-uniform case, we have to consider that the cancellation may occur also by coupling the *(high)-(high)*, *(low)-(low)* or *(low)-(high)* aliasing terms coming from the *m-th* and the *(m+1)-th* branch, i.e., the following cases must be taken into account:

a) $A_m^{(high)}(z) \downarrow R_m + A_{m+1}^{(high)}(z) \downarrow R_{m+1} = 0$

b) $A_m^{(low)}(z) \downarrow R_m + A_{m+1}^{(low)}(z) \downarrow R_{m+1} = 0$

c) $A_m^{(low)}(z) \downarrow R_m + A_{m+1}^{(high)}(z) \downarrow R_{m+1} = 0$

d) $A_m^{(high)}(z) \downarrow R_m + A_{m+1}^{(low)}(z) \downarrow R_{m+1} = 0$

where $Q(z) \downarrow M$ stands for the z-transform of the M-fold subsampled version of $q(n)$. For example, consider the bank $\{1/5, 3/5, 1/5\}$ that can be implemented using filters having a passband equal to $\pi/5$ and centered, on the positive frequency axis, in $\pi/10$, $\pi/2$ and $9\pi/10$. The aliasing term $A_0^{(high)}(z)$ produced in the m=*0* branch at the synthesis stage must be canceled by $A_1^{(high)}(z) \downarrow 3$.

Consider the *(high)-(high)* case: substituting (5) into case *a)* equation yields an expression that can be split into two systems: if $W_{M_m}^{(k_m+1)R_m} = W_{M_{m+1}}^{(k_{m+1}+1)R_{m+1}}$ then the following must be verified

$$\begin{cases} \frac{1}{M_m}[\hat{U}_m(z)V_m(zW_{M_m}^{(k_m+1)})] \downarrow R_m + \\ +\frac{1}{M_{m+1}}[\hat{U}_{m+1}(z)V_{m+1}(zW_{M_{m+1}}^{(k_{m+1}+1)})] \downarrow R_{m+1} = 0 \\ \\ \frac{1}{M_m}[\hat{V}_m(z)U_m(zW_{M_m}^{-(k_m+1)})] \downarrow R_m + \\ +\frac{1}{M_{m+1}}[\hat{V}_{m+1}(z)U_{m+1}(zW_{M_{m+1}}^{-(k_{m+1}+1)})] \downarrow R_{m+1} = 0 \end{cases} \tag{6}$$

otherwise, if $W_{M_m}^{(k_m+1)R_m} = W_{M_{m+1}}^{-(k_{m+1}+1)R_{m+1}}$ then the following must be verified

$$\begin{cases} \frac{1}{M_m}[\hat{U}_m(z)V_m(zW_{M_m}^{(k_m+1)})] \downarrow R_m + \\ +\frac{1}{M_{m+1}}[\hat{V}_{m+1}(z)U_{m+1}(zW_{M_{m+1}}^{-(k_{m+1}+1)})] \downarrow R_{m+1} = 0 \\ \\ \frac{1}{M_m}[\hat{V}_m(z)U_m(zW_{M_m}^{-(k_m+1)})] \downarrow R_m + \\ +\frac{1}{M_{m+1}}[\hat{U}_{m+1}(z)V_{m+1}(zW_{M_{m+1}}^{(k_{m+1}+1)})] \downarrow R_{m+1} = 0 \end{cases} \tag{7}$$

Similar equations can be written also for the other possible couplings of aliasing components.

3. COSINE-MODULATED NON-UNIFORM BANKS

The use of cosine modulation simplifies the fulfillment of the aliasing cancellation condition. Suppose that each filter of the analysis/synthesis banks is obtained as follows:

$$\begin{aligned} h_m(n) &= 2g_m(n)\cos((2k_m+1)\tfrac{\pi}{2M_m}(n - \tfrac{N_m-1}{2}) + \theta_m) \\ f_m(n) &= 2g_m(n)\cos((2k_m+1)\tfrac{\pi}{2M_m}(n - \tfrac{N_m-1}{2}) - \theta_m) \\ &= h_m(N_m - 1 - n) \end{aligned} \tag{8}$$

for $m = 0, 1, \ldots, M-1$. N_m is the length of $g_m(n)$. The prototypes $g_m(n)$ have a linear phase and satisfy $g_m(n) = g_m(N_m - 1 - n)$. The phase terms θ_m are chosen to satisfy the aliasing cancellation constraints.

In the case of cosine-modulated banks, the following relationships hold:

$$
\begin{aligned}
U_m(z) = & \; G_m(zW_{2M_m}^{(k_m+(1/2))})W_{2M_m}^{(k_m+(1/2))(N_m-1)/2} \cdot \\
& \cdot e^{j\theta_m} \\
V_m(z) = & \; G_m(zW_{2M_m}^{-(k_m+(1/2))})W_{2M_m}^{-(k_m+(1/2))(N_m-1)/2} \cdot \\
& \cdot e^{-j\theta_m} \\
\hat{U}_m(z) = & \; G_m(zW_{2M_m}^{(k_m+(1/2))})W_{2M_m}^{(k_m+(1/2))(N_m-1)/2} \cdot \\
& \cdot e^{-j\theta_m} \\
\hat{V}_m(z) = & \; G_m(zW_{2M_m}^{-(k_m+(1/2))})W_{2M_m}^{-(k_m+(1/2))(N_m-1)/2} \cdot \\
& \cdot e^{j\theta_m}
\end{aligned}
\tag{9}
$$

Consider, for example, the *(high)-(high)* case. Substituting the above expressions into the aliasing cancellation constraints (6) and (7) yields a relationship between the prototypes of adjacent branches. In [8] it is shown that for $W_{M_m}^{(k_m+1)R_m} = W_{M_{m+1}}^{\pm(k_{m+1}+1)R_{m+1}}$ the choice $e^{-j2\theta_m} + e^{\mp j2\theta_{m+1}} = 0$ allows aliasing cancellation if the following relationship holds (the same result is obtained considering the cases *b)-d)*, but with a different relationship between the phase terms θ_m):

$$
\begin{aligned}
& \frac{1}{M_m}\frac{1}{R_m}\sum_{p=0}^{R_m-1} G_m(z^{1/R_m}W_{R_m}^p) \cdot \\
& \cdot G_m(z^{1/R_m}W_{R_m}^p W_{2M_m}) = \frac{1}{M_{m+1}}\frac{1}{R_{m+1}}\sum_{p=0}^{R_{m+1}-1} \\
& G_{m+1}(z^{1/R_{m+1}}W_{R_{m+1}}^p W_{4M_m}^{R_m/R_{m+1}} W_{4M_{m+1}}^{-1}) \cdot \\
& \cdot G_{m+1}(z^{1/R_{m+1}}W_{R_{m+1}}^p W_{4M_m}^{R_m/R_{m+1}} W_{4M_{m+1}})
\end{aligned}
\tag{10}
$$

Moreover, it is possible to demonstrate the following facts [8], which outline also the steps of the procedure to design rational sampling factors filter banks.

By using the zero-phase representation of the prototypes, i.e.,

$$
G_m^{(zp)}(\omega) = G_m(\omega)e^{j\frac{N_m-1}{2}\omega} \tag{11}
$$

and by imposing the condition

$$
\frac{N_m-1}{R_m} = \frac{N_{m+1}-1}{R_{m+1}} \tag{12}
$$

on the lengths of the prototypes, the constraint of aliasing cancellation reduces to the following relationship between the zero-phase frequency responses of the prototype filters

$$
\begin{aligned}
& \frac{1}{M_m}\frac{1}{R_m}G_m^{(zp)}(\frac{\omega}{R_m})G_m^{(zp)}(\frac{\omega}{R_m}-\frac{\pi}{M_m}) = \\
& \frac{1}{M_{m+1}}\frac{1}{R_{m+1}}G_{m+1}^{(zp)}(\frac{\omega}{R_{m+1}}-\frac{\pi R_m}{2M_mR_{m+1}}+\frac{\pi}{2M_{m+1}}) \cdot \\
& \cdot G_{m+1}^{(zp)}(\frac{\omega}{R_{m+1}}-\frac{\pi R_m}{2M_mR_{m+1}}-\frac{\pi}{2M_{m+1}})
\end{aligned}
\tag{13}
$$

Let $\omega_{c,m} = \pi/(2M_m)$ be the cut-off frequency of $G_m(\omega)$. Suppose the transition band, having width $\Delta\omega_m$, is centered in $\omega_{c,m}$ and let $\omega_{p,m} = \omega_{c,m}-(\Delta\omega_m/2)$ and $\omega_{s,m} = \omega_{c,m}+(\Delta\omega_m/2)$ be the upper bound of the passband and the lower bound of the stopband, respectively, of $G_m(\omega)$. Therefore, the constraint (13) is satisfied if $R_m\Delta\omega_m = R_{m+1}\Delta\omega_{m+1}$ and if $G_{m+1}^{(zp)}(\omega)$ is chosen as follows

$$
G_{m+1}^{(zp)}(\omega) =
$$

$$
= \begin{cases}
0 & -\pi < \omega \le -\omega_{s,m+1} \\
\sqrt{\frac{R_{m+1}M_{m+1}}{R_mM_m}}\, G_m^{(zp)}(-\omega_{p,m}+(\omega+\omega_{p,m+1}) \cdot \\ \quad \cdot \frac{\omega_{s,m}-\omega_{p,m}}{\omega_{s,m+1}-\omega_{p,m+1}}) & -\omega_{s,m+1} < \omega \le -\omega_{p,m+1} \\
\sqrt{R_{m+1}M_{m+1}} & -\omega_{p,m+1} < \omega \le \omega_{p,m+1} \\
\sqrt{\frac{R_{m+1}M_{m+1}}{R_mM_m}}\, G_m^{(zp)}(\omega_{p,m}+(\omega-\omega_{p,m+1}) \cdot \\ \quad \cdot \frac{\omega_{s,m}-\omega_{p,m}}{\omega_{s,m+1}-\omega_{p,m+1}}) & \omega_{p,m+1} < \omega \le \omega_{s,m+1} \\
0 & \omega_{s,m+1} < \omega \le \pi
\end{cases}
\tag{14}
$$

Assuming the aliasing components have been completely eliminated, the input-output relationship shown in (1) can be expressed by

$$
\begin{aligned}
\hat{X}(z) = & \Big[\textstyle\sum_{m=0}^{M-1}\frac{1}{R_m}\frac{1}{M_m}\sum_{p=0}^{R_m-1}F_m(z^{\frac{1}{R_m}}W_{R_m}^p) \cdot \\
& \cdot H_m(z^{\frac{1}{R_m}}W_{R_m}^p)\Big] X(z) = T(z)X(z)
\end{aligned}
\tag{15}
$$

Phase error is absent if the synthesis filters are a time reversed version of the analysis filters, while the magnitude error is maintained at low levels if $T(z)$ is approximately allpass. The reconstruction error is reduced choosing prototype filters with high stopband attenuation and also with a proper behavior in the transition band.

A first prototype is designed (by using known techniques, for example those shown in [9][10][5]). This prototype is relative to the *m-th* branch, where m must be chosen so that $R_m/M_m=\min\{R_k/M_k,\ k=0,\ldots,M-1\}$. Its cut-off frequency is $\omega_{c,m} = \frac{\pi}{2M_m}$ and $\sqrt{R_mM_m}$ is the gain in the passband. $G_m(\omega)$ must have a *power complementary* transition band, i.e., satisfies

$$
\begin{aligned}
& |\, G_m(\omega)\,|^2 + |\, G_m(\tfrac{\pi}{M_m}-\omega)\,|^2 = R_mM_m \\
& for\ \omega_{p,m} < \omega < \omega_{s,m}
\end{aligned}
\tag{16}
$$

The prototypes in the other branches are obtained by using (14) and by adding the correct linear phase term to determine $G_{m+1}(\omega)$; $g_{m+1}(n)$ is obtained by the inversion of $G_{m+1}(\omega)$.

It can be shown that prototypes designed by using this procedure make $T(z)$ approximately allpass.

4. EXPERIMENTAL RESULTS

To show the effectiveness of the design procedure described in the previous section we consider three examples of non-uniform banks. Example 1 and 2 are relative to banks with rational sampling factors, while Example 3 refers to an integer sampling factors bank, suitable for audio coding applications, that has been proposed in [11]. We indicate with K and Θ the sets { k_m, m=0,...,M-1 } and { θ_m, m=0,...,M-1 }, respectively.

Example 1: Bank { 1/5, 3/5, 1/5 }. Two prototypes need to be designed ($g_0(n) = g_2(n)$); the couplings of aliasing components that must be considered are *(high)-(high)* and *(low)-(low)* between the branches *0-1* and *1-2*, respectively; K={ 0, 2, 4}; Θ = { $\pi/4$, $\pi/4$, $\pi/4$ }.

Example 2: Bank { 2/7, 2/7, 2/7, 1/7}. Two prototypes have to be designed ($g_0(n) = g_1(n) = g_2(n)$). In this example more than one choice is possible for K. We will use K={ 0, 5, 4, 6 } to show the largest variety of couplings of aliasing components (*(high)-(high)*, *(low)-(high)*, *(low)-(low)*, in the order). In this case Θ = { $\pi/4$, $\pi/4$, $-\pi/4$, $-\pi/4$ }.

Example 3: Non-uniform bank having 16, 32 and 64 as possible decimation factors and allowing the splitting of an audio signal sampled at 48 kHz as shown in Fig. 2.

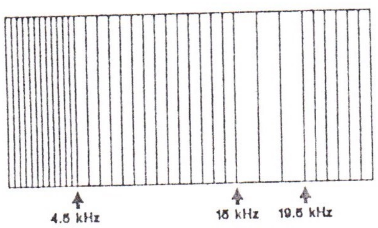

Figure 2: Subband splitting relative to Example 3

The performance of the presented design method is evaluated in terms of both the overall distortion function $T(\omega)$ and the residual aliasing error. As to the latter error, a global measure relative to the whole structure is used in this work. According to the input-output relationship in (1), the aliasing contribution relative to $X(zW_{M_m}^{lR_m})$ can be written as

$$A_{l,m}(z) = \frac{1}{R_m}\frac{1}{M_m}\sum_{p=0}^{R_m-1} H_m(z^{\frac{1}{R_m}} W_{R_m}^{p} W_{M_m}^{l})\cdot F_m(z^{\frac{1}{R_m}} W_{R_m}^{p}) \tag{17}$$

with m=0,...,M-1, l=0,...,M_m − 1. The functions $A_{l,m}(\omega)$ are 2π-periodic functions. All the aliasing terms $A_{l,m}(z)$ that refer to the same shifted version of $X(z)$, i.e., having the same value of $W_{M_m}^{lR_m}$, must be summed up, so that the following aliasing error can be defined:

$$E_a(\omega) = \sqrt{\sum_{r=1}^{M_{max}-1} \left| \sum_{m=0}^{M-1} \sum_{l=1,(lR_m) \bmod M_m = r}^{M_m-1} A_{l,m}(\omega) \right|^2} \tag{18}$$

where $M_{max} = \max\{M_m, m=0,\ldots,M-1\}$ and where the inner summation in (18) is evaluated only for the values of l and m satisfying the condition $(lR_m) \bmod M_m = r$.

Therefore

$$E_{p-p} = \max_{0\leq\omega\leq\pi} |T(\omega)| - \min_{0\leq\omega\leq\pi} |T(\omega)| \tag{19}$$

$$E_{a,max} = \max_{0\leq\omega\leq\pi} E_a(\omega) \tag{20}$$

can be used as measures of the quality of the designed banks.

Tables 1 and 2 report the results obtained for Example 1 and 2, respectively, for different lengths of the prototypes. As can be seen, both the magnitude distortion and the aliasing error are kept small

In Fig. 3 the frequency responses of the final cosine modulated analysis filters relative to Example 2 and obtained with prototypes having 82 and 163 coefficients are shown: from the inspection of this figure it can be seen that the design based on (14) does not degrade the passband and the stopband characteristics of the new prototypes.

In Fig. 4 the final bank relative to Example 3 and obtained with filter lengths equal to 512 is shown: the reconstruction and the aliasing error are $E_{p-p} = 3.88E-03$ and $E_{a,max} = 8.99E-03$, respectively.

5. CONCLUSIONS

In this work a method to design non-uniform filter banks with rational sampling factors has been presented. Aliasing cancellation constraints have been applied to cosine-modulated banks. A simple procedure, that requires numerical optimization of only one prototype, being the others derived in a straightforward way from this one, has been proposed.

6. REFERENCES

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Table 1: Results relative to Example 1

N_0, N_2	N_1	E_{p-p}	$E_{a,max}$
36	106	3.42 E−03	1.82 E−02
46	136	4.33 E−03	3.79 E−03
56	166	1.52 E−03	2.93 E−04

Table 2: Results relative to Example 2

N_0, N_1, N_2	N_3	E_{p-p}	$E_{a,max}$
67	34	6.67 E−02	3.00 E−02
83	42	3.06 E−02	1.20 E−02
123	62	6.45 E−03	4.36 E−03
163	82	4.04 E−03	9.15 E−04

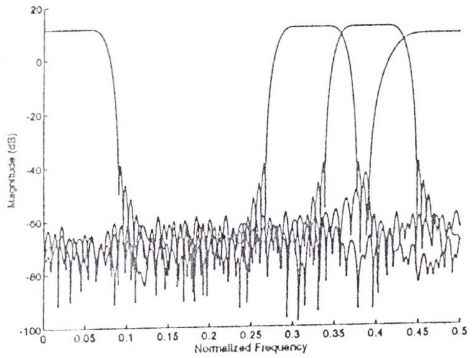

Figure 3: Cosine-modulated bank relative to Example 2 ($N_0 = N_1 = N_2 = 163$, $N_3 = 82$)

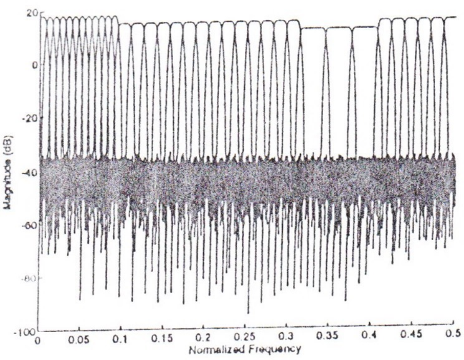

Figure 4: Filter bank relative to Example 3 (N=512)