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Visual sustained attention and numerosity sensitivity correlate with math achievement in children

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ABSTRACT

In this study, we investigated in school-age children the relationship among mathematical performance, the perception of numerosity (discrimination and mapping to number line), and sustained visual attention. The results (on 68 children between 8 and 11 years of age) show that attention and numerosity perception predict math scores but not reading performance. Even after controlling for several variables, including age, gender, nonverbal IQ, and reading accuracy, attention remained correlated with math skills and numerosity discrimination. These findings support previous reports showing the interrelationship between visual attention and both numerosity perception and math performance. It also suggests that attentional deficits may be implicated in disturbances such as developmental dyscalculia.

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Introduction

Educational problems are often related to the inability to sustain attention even when there are no apparent cognitive impairments (Zentall, 1993). Attention acts as a filter to select and maintain relevant information while suppressing irrelevant distracters, improving the efficiency with which information arriving from the environment is acquired and processed and then memorized and learned (Posner & Rothbart, 2005). Recent studies have demonstrated a causal link between visual attention and reading acquisition; serial search and spatial cueing facilitation predict future reading acquisition in children (Franceschini, Gori, Ruffino, Pedrolli, & Facoetti, 2012). Furthermore, they showed that

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playing action video games improved both children's attentional skills and their reading speed (Franceschini et al., 2013).

The current research raises a simple and relevant question: What is the relationship between visual attention and perceptual capacities in general and the acquisition of math skills? Using a correlational approach, we tested whether visual sustained attentional capacity of children, as well as perceptual tasks related to the perception of number, correlates with formal school-acquired and nonsymbolic numerical skills after controlling for potential confounding variables such as age, nonverbal IQ, gender, and reading accuracy.

Although humans are the only species with a linguistically mediated code for numbers, humans share a nonverbal representation of numerical quantities with many animal species (Dacke & Srinivasan, 2008; Pepperberg, 2006; see Nieder, Diester, & Tudusciuc, 2006, for a review). In addition, human infants (Feigenson, Dehaene, & Spelke, 2004; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005), including newborns (Izard, Sann, Spelke, & Streri, 2009) and cultural groups with no words for numbers or any mathematic formal system (Dehaene, Izard, Spelke, & Pica, 2008; Gordon, 2004), can reliably discriminate numerical quantities. Whereas the representation of integers is exact, estimation of numerical quantities is approximate, with a certain degree of error associated with number estimation. Numerosity perception obeys Weber's law (Whalen, Gallistel, & Gelman, 1999), meaning that discrimination thresholds increase with stimulus intensity. Weber fraction reflects the *precision* with which two numerical quantities can be discriminated, an index of "number acuity."

A growing amount of evidence links the ontogenetically inherited nonverbal system with the culturally invented and linguistically mediated number code (Feigenson, Libertus, & Justin, 2013). Number acuity, which improves during development (Halberda & Feigenson, 2008), correlates with formal mathematics achievement (Mazzocco, Feigenson, & Halberda, 2011) and predicts math skills years later (Halberda, Mazzocco, & Feigenson, 2008). Even if the causal direction of influence has not been demonstrated, is clear that numerosity representation plays a key role in the acquisition of formal mathematical ability (Mazzocco et al., 2011; Piazza, 2010).

Number and space are intrinsically interconnected (Hubbard, Piazza, Pinel, & Dehaene, 2005). Conceptions of how numbers map onto space develop during school years (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003); kindergarten children represent numbers in space in a compressed, seemingly logarithmic scale (e.g., placing the number 10 near the midpoint of a 1–100 scale). The scale becomes progressively more linear over the first 3 or 4 years of schooling. Interestingly, dyscalculic children (those who suffer from a specific mathematical learning disability) show poor number acuity (Piazza et al., 2010) and a more logarithmic representation of the number line than controls (Ashkenazi & Henik, 2010; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008).

Like space representation, attention correlates with many aspects of numerosity and number processing. We recently demonstrated that*subitizing* (the errorless and rapid apprehension of collection of items up to four) strongly depends on visual, auditory, and haptic attention (Anobile, Turi, Cicchini, & Burr, 2012; Burr, Turi, & Anobile, 2010). Attention training (through video game playing) increases the subitizing range (Green & Bavelier, 2003), and under attentional load small numbers (inside the errorless subitizing range) become susceptible to adaptation (Burr, Anobile, & Turi, 2011). In line with these results, the event-related potential (ERP) component P2p, a signature of numerosity processing, emerges in the subitizing range under dual-task conditions (Hyde & Wood, 2011). In addition, the capacity to map number onto space requires attention; we recently showed that visual attentional load leads to a logarithmic-like number line mapping (Anobile, Cicchini, & Burr, 2012). Merely looking at numbers causes a shift in covert attention to the left or right side, depending on number magnitude (Fischer, Castel, Dodd, & Pratt, 2003). The connection between attention and number processing also finds support from recent functional magnetic resonance imaging (fMRI) studies of neural correlates of visual enumeration under attentional load. Ansari, Lyons, van Eimeren, and Xu (2007) showed that the temporal–parietal junction (rTPJ), an area thought to be involved in stimulus-driven attention (Corbetta & Shulman, 2002), is activated during a comparison task of quantities. This evidence reveals a strong connection among the representations of numbers, space, and attention.

Despite the growing number of studies demonstrating the relationship between attention and acquisition of reading skills (Franceschini et al., 2012, 2013), surprisingly few studies have examined the connection between attention and formal (school-acquired) and nonsymbolic math abilities, especially during development.

In this study, we measured the relationship between visual sustained attention and performance on both formal and nonsymbolic numerical tasks. We focused on 8- to 11-year-old children because this is a particularly sensitive period for learning. It is the age range where dyscalculia is commonly diagnosed, and excellent standardized and validated diagnostic neuropsychological batteries are available. Moreover, previous studies performed in similar cohorts of children have shown large interparticipant variability in the ability to perceive and manipulate numerical quantities, encouraging a correlational study.

In brief, we tested whether interindividual variability in both formal and nonsymbolic mathematical skills is related to participants' performance in allocating and maintaining visual spatial attention. We tested children with psychophysical and neuropsychological tests, measuring their abilities in math, numerosity discrimination (comparison task) and mapping (nonsymbolic number line), sustained visual attention (multiple objects tracking), text reading, and general visuospatial reasoning (Raven's matrices). We performed correlations among these measures after controlling for the influence of many generic and neuropsychological variables such as age, gender, nonverbal IQ, and reading skills.

Methods

Participants

A total of 68 typically developing children, aged 8 to 11 years (mean age = 9.7 years), participated in the study. Participants were recruited from local schools, and only those who returned a signed consent from parents were included. None had a diagnosis of learning or attention disorder, and all had nonverbal intelligence in the normal range (as measured by Raven's matrices; Belacchi, Scalisi, Cannoni, & Cornoldi, 2008) and normal visual acuity (as measured by the Snellen chart).

Materials and procedure

All visual stimuli were presented in a dimly lit room on a 17-inch LG touch screen monitor with 1280 × 1024 resolution at refresh rate of 60 Hz, viewed binocularly from 57 cm. Stimuli were generated and presented under Matlab 7.6 using PsychToolbox routines (Brainard, 1997).

Numerosity acuity

Two patches of dots were briefly (500 ms) and simultaneously presented on either side of central fixation. Children performed a comparison task touching the side of the screen with more dots (Fig. 1A; see also online Demo 1 in supplementary material). Standard numerosity was fixed at 24 dots while the probe (of varying number) adaptively changed, according to participant responses, with numerosity determined by the QUEST algorithm (Watson & Pelli, 1983). The sides of the standard and probe were counterbalanced. Patches consisted in nonoverlapping dots (0.27° area), half white and half black, that were constrained to fall within a virtual circle of 6° visual angle. The procedure consisted of two blocks of 45 trials (90 trials in total). Each trial was initiated by the experimenter to ensure that children maintained fixation throughout. To ensure that neither density nor area was a consistent and reliable cue to numerosity, three different conditions were simultaneously interleaved: equal area (density and numerosity correlated), equal density (area and numerosity correlated), and a minimal increase in both. In the equal area condition, area was kept constant at 6° with density varying accordingly with numerical changes, whereas in the equal density condition, density was fixed at 7.3% and area increased with numerosity accordingly. In the minimal increase condition, an increase in number was associated with half the equal density condition increase in area and half the equal area increase in density. The proportion of "more" trials was plotted against probe numerosity and fit with a cumulative Gaussian function where the 50% point provided an estimate of

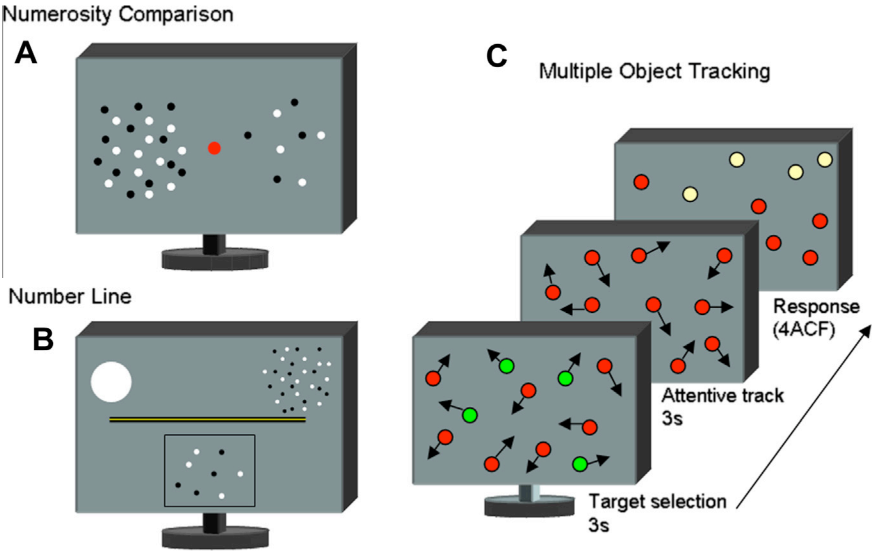

Fig. 1. Illustration of tasks and stimuli. (A) Numerosity comparison. Two patches of dots were briefly (500 ms) presented on either side of the central fixation point. Participants were asked to touch the side of the screen with more dots. (B) Number line. At the onset of each trial, observers viewed a number line marked at each end with a single dot to the left and 30 dots to the right. On key press, the dot stimulus appeared; after 500 ms, a binary pixel random noise mask was displayed until participants responded. Participants touched the screen at the position on the number line they thought corresponded to the dot cloud. (C) Multiple object tracking. Eleven disks—four green targets and seven red distracters—moved randomly on the screen (4°/s) for a duration of 3 s. The green targets then turned red (like the distracters), which participants tracked with their attention for 3 s. Participants then identified (by touching the screen) which of four possible items (highlighted in yellow) was the target (4ACF). (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

the point of subjective equality (PSE) and the normalized difference between this and the 75% point gave an estimate of the Weber fraction.

Mapping numerosity onto space

We measured the ability to map numbers using a “nonsymbolic number line” task (Fig. 1B; see also online Demo 2 in supplementary material). Participants viewed a cloud of dots and positioned their quantity on a line demarcated by two sample numerosities. Each trial started with participants viewing a 22-cm “number line” with sample dot clouds representing the extremes, one dot on the left of the number line and 30 dots on the right, which remained throughout the trial. Dot stimuli were presented for 500 ms, followed by a random noise mask that remained until participants responded. The dots were half-white/half-black and positioned in pseudo-random positions on a gray background without overlap, within a virtual circle of 8° diameter. Participants touched the touch screen at the position on the number line they thought corresponded to the dot cloud. Each block measured one of the nine different numerosities that were each presented once in random order. Participants performed three blocks with numerosities of 2, 3, 4, 6, 10, 14, 18, 20, and 27. To discourage observers using strategies other than numerosity (e.g., texture density), on each block we kept constant either the total covered area (varying individual dot size) or individual dot size (varying total area covered), alternatively trial by trial. Thus, on average, neither dot size nor total covered area correlated with numerosity. In the sample dot clouds of the number line, we kept constant total covered area. It follows that the left extreme (numerosity of 1) was represented by a dot with a diameter greater than the dots that represented the right extreme. We quantified performance by computing the root mean square error (RMS), an index that takes into account both *variance* (average standard deviation of trials at a particular numerosity) and *bias* (average distance of the mean response from the physical numerosity):

$$\mathrm{RMS} = \frac{1}{N}\sqrt{\sum_{i}^{N}\left(\frac{X_i - R_i}{X_i}\right)^2}, \tag{1}$$

where X_i is the tested numerosity on the ith trial, R_i is the response location to that numerosity, and N is the number of trials.

Visual sustained attention

Visual sustained attention was measured by a multiple object tracking task (Pylyshyn & Storm, 1988). Eleven disks of 0.9° diameter—four green targets and seven red distracters—moved randomly on the full screen at 4°/s for a period of 3 s (Fig. 1C; see also online Demo 3 in supplementary material). The green targets then turned red (like the distracters), which participants tracked with their attention for 3 s. They then identified (by touching the screen) which of four possible items (highlighted in yellow) was the target (4ACF). Each experimental session comprised 10 trials, and participants performed three sessions, for a total of 30 trials. No feedback was provided. Performance was measured as proportion correct and was converted to d' as a measure of sensitivity.

Math achievement

Mathematical achievement was measured by an age-standardized Italian battery (Biancardi & Nicoletti, 2004) that explores several aspects of math with several different tasks in which accuracy and/or total time are recorded. There were 10 separate tasks. First, in *Arabic numeral reading*, the child reads aloud 36 or 48 Arabic numbers—depending on chronological age—arranged in four different lists, each composed of 12 integer numbers of three, four, five, or six digits (e.g., 193, 1832, 31,020, 142,634). Both accuracy (total of numeral stated correctly) and speed are measured. Second, in *Arabic numeral writing*, the child writes in Arabic format 36 or 48 spoken number words (three to six digits, the same as used for the numeral reading subtest) named by the experimenter. Accuracy is measured. Third, in *Arabic numeral repetition*, the child repeats 36 or 48 spoken number words of three to six digits (the same as used for the numeral reading and writing subtests). Accuracy is measured. Fourth, in *triplets*, for 14 or 22 trials the child chooses the largest number among a set of three Arabic numbers (one to six digits). Both accuracy and speed are measured. Fifth, in *insertions*, for 12 trials the child positions a number (one to five digits) in one of four possible positions among three other numbers. Both accuracy and speed are measured. Triplets and insertions are collapsed together in a combined index called "semantic coding." Sixth, in *simple calculation*, the child performs 16 multiplications (operands between one and nine), six additions, and six subtractions with results smaller than 10. A response is scored as correct only if it is given within a 2-s deadline. Seventh, in *complex calculation*, the child performs 10 additions and 10 subtractions with results above 10. A response is scored as correct only if it is given within a 15-s deadline. Eighth, in *counting*, the child counts aloud between 1 and 100 in ascending and then descending order. Both accuracy and speed are measured. Ninth, in *math tables*, the child recites aloud the four and seven times multiplication table. A response is scored as correct only if there is no hesitation longer than 2 s. Finally, in *complex written calculation*, the child performs 12 written calculations (four additions, four subtractions, and four multiplications) within 10 min. The child is given a sheet of paper with the mathematical operations to be performed. The numbers on which to operate comprise integer Arabic numerals. Accuracy is scored. The scores were then age-standardized to yield a total math score (the sum of the individual scores).

Reading ability

As an index of reading ability, we measured text reading accuracy, where children were asked to read standardized Italian texts quickly and accurately (Cornoldi & Colpo, 1995).

Data analysis

To quantify the relationships between variables, we used bivariate correlations (Pearson) and hierarchical regressions. All statistical assumptions were checked before reporting the results of the regressions. Only Weber fractions violated the assumption of normality of the residual, requiring logarithmic transform, which resolved the violation. Indeed, it is typical to represent Weber fractions on a logarithmic scale.

As data reduction analysis, we performed a principal component analysis with oblique rotation on the math scores extracted from the individual subtests. Principal component analysis revealed two factors with eigenvalues greater than 1, together explaining 56% of the variance (Table 1). We termed the first component "conceptual" because it collapses tasks mostly related to memory retrieval (e.g., math tables, number repetition) and automatized competences (e.g., simple calculation). The second component was termed "magnitude oriented" because it reflects competences related to number magnitude processing (e.g., complex calculation, digit magnitude comparison).

Results

Correlation between math and perceptual variables

Fig. 2 shows Pearson correlations between total math score and reading accuracy against the three perceptual tasks: number line, numerosity discrimination, and attention. Total score on math test is predicted by all three perceptual tasks. High scores on formal math test are associated with fewer errors on the number line task ($r = -.238$, $p = .05$), higher precision in numerosity comparison (low Weber fraction: $r = -.307$, $p = .01$), and higher attentional performance ($r = .403$, $p = .001$). On the other hand, none of the perceptual tasks is significantly correlated with reading ability. This shows that there is not simply some generic component driving the correlations (Table 2).

Correlation between math and perceptual variables after controlling for age, nonverbal IQ, gender, and reading accuracy

Using hierarchical regressions, we tested how much variance is explained by perceptual variables after partitioning out the effects of chronological age, general nonverbal intelligence, gender, and reading accuracy—which are important but nonspecific variables. The *F* change statistic indicates the partial effect of the perceptual variables on the total math score after controlling for the influence of the mentioned control variables.

Fig. 3 shows that controlling for chronological age does not eliminate any of the correlations. After partitioning out age, the proportion of variance R^2 explained by number line performance is 5.6% (F change$_{(1,65)} = 3.84$, $p = .054$), Weber fraction is 7.7% (F change$_{(1,65)} = 6.29$, $p = .015$), and attentional performance is 15% (F change$_{(1,65)} = 12.00$, $p = .001$). All remain highly significant.

Partitioning out both chronological age and general nonverbal intelligence together (Fig. 3) eliminated the correlation between number line and math (R^2 change = 3.5%, F change$_{(1,64)} = 2.41$, $p = .125$),

Table 1
Principal component analysis.

Measure	Rotate component matrix	
	Factor 1 (44%) Conceptual	Factor 2 (12%) Magnitude oriented
Number repetition	.928	
Number writing	.853	
Written calculation	.482	
Simple addition and subtraction	.704	
Tables	.610	
Simple multiplication	.521	
Counting		.844
Complex addition and subtraction		.797
Semantic coding		.692
Number reading		.456

Note. Proportion of variance is accounted for by each factor and loading of each measure on factor.

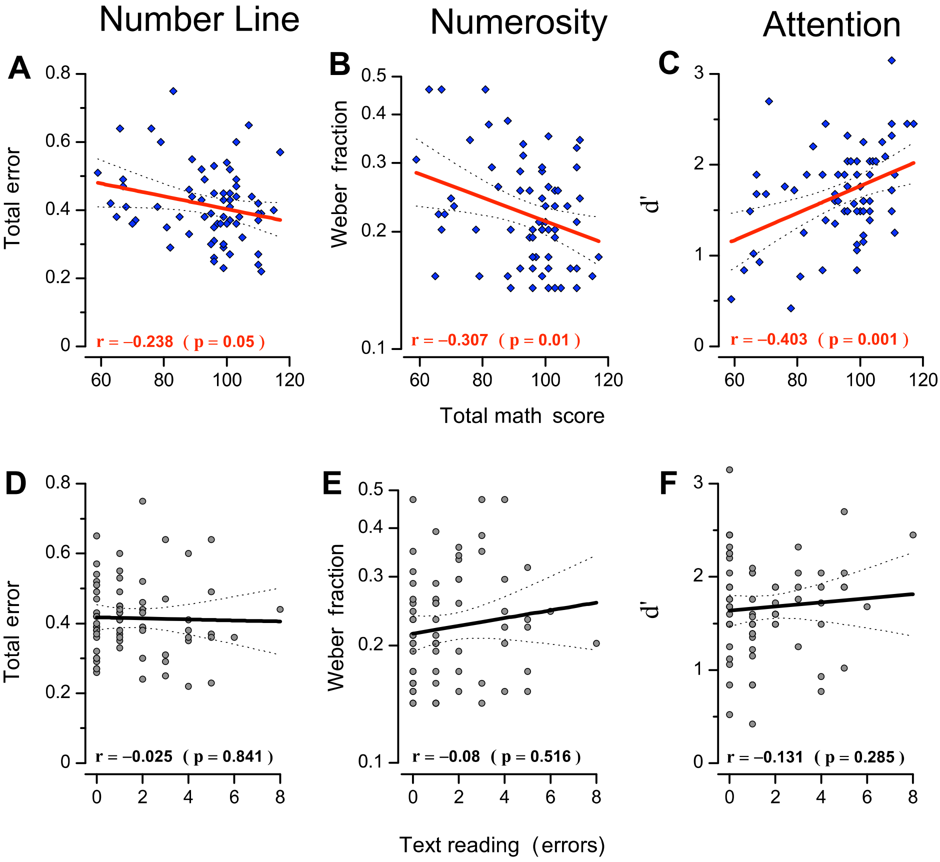

Fig. 2. Bivariate Pearson correlations among variables. Psychophysical measures predicted math abilities (A–C) but not reading abilities (D–F). Total math score is the sum of the individual scores on math subtests, whereas reading reflects the number of errors during text reading.

Table 2
Correlation matrix.

	1	2	3	4	5	6	7	8	9
1. Raven's matrices	1	**0.309***	0.037	**0.464****	−0.187	**−0.320****	0.177	**0.348****	**0.390****
2. Math score		1	**−0.291***	**0.403****	**−0.238***	**−0.307***	**0.884****	**0.816****	0.085
3. Reading errors			1	0.080	−0.025	0.131	**−0.355****	−0.082	0.152
4. Attention				1	−0.050	**−0.375****	**0.269***	**0.392****	0.209
5. Number line (total errors)					1	0.033	−0.083	**−0.351****	−0.027
6. Weber fraction						1	−0.151	**−0.371****	**−0.350****
7. Conceptual factor							1	**0.473****	−0.043
8. Magnitude factor								1	**0.235***
9. Age (months)									1

Note. Bivariate Pearson correlation coefficients are shown. Significant correlations highlighted in bold.
* $p \leqslant .05$.
** $p < .01$.

but numerosity comparison precision (R^2 change = 5.3%, F change$_{(1,64)}$ = 4.28, $p = .043$) and attentional performance (R^2 change = 7.5%, F change$_{(1,64)}$ = 6.78, $p = .011$) remained statistically significant.

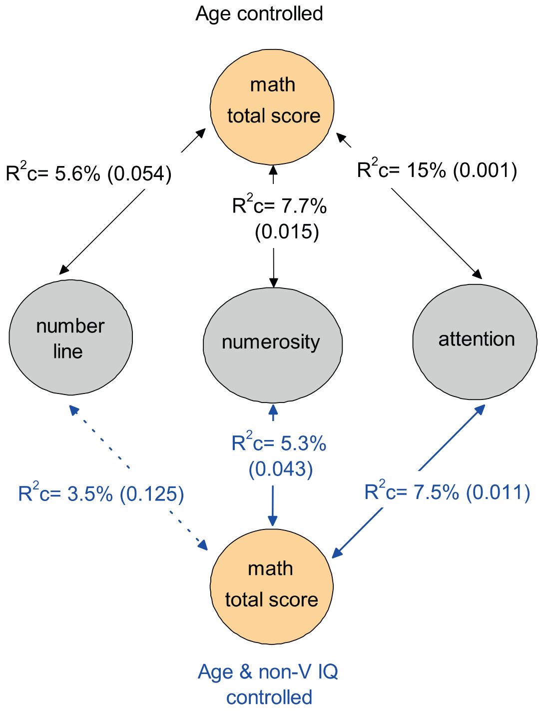

Fig. 3. Partitioning out the influence of chronological age (black arrows), all of the psychophysical measures still explain a significant proportion of variance of the math abilities. Controlling for both chronological age and nonverbal intelligence (blue arrows) only eliminated the correlation between number line and math. R^2c, R^2 change; non-V, nonverbal. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

Partitioning out age, nonverbal intelligence, and gender, the pattern of results does not change. The correlation between number line and math remains not statistically significant (R^2 change = 2.7%, F change$_{(1,64)}$ = 1.88, p = .17), whereas numerosity comparison precision (R^2 change = 5.1%, F change$_{(1,64)}$ = 4.11, p = .04) as well as attention still explained a statistically significant portion of the math score variance (R^2 change = 7.8%, F change$_{(1,64)}$ = 6.90, p = .01).

Finally, we simultaneously controlled for age, nonverbal intelligence, gender, and reading accuracy. After controlling for all of these variables, attentional performance remain correlated with math score (R^2 change = 9.7%, F change$_{(1,64)}$ = 8.80, p = .004), whereas the correlations with numerosity comparison precision (R^2 change = 3.3%, F change$_{(1,64)}$ = 2.61, p = .11) and number line (R^2 change = 3.5%, F change$_{(1,64)}$ = 2.48, p = .12) dropped below significance.

Considering the three perceptual variables together as math predictors, they explain a significant portion of variance of math scores (R^2 change = 14.5%, F change$_{(3,60)}$ = 4.37, p = .008) even after simultaneously controlling for the influence of age, nonverbal intelligence, gender, and reading accuracy.

Correlation between math components and perceptual variables

Hierarchical regression analysis on factor scores (Fig. 4) reveal that after partitioning out the influence of chronological age, attention still explains a significant proportion of variance of the conceptual component (R^2 change = 7.8%, F change$_{(1,65)}$ = 5.74, p = .019) as well as of the magnitude-oriented component (R^2 change = 12.4%, F change$_{(1,65)}$ = 9.69, p = .003). Number line and numerosity comparison performance are related to the magnitude-oriented component (R^2 change = 12.5%, F change$_{(1,65)}$ = 9.33, p = .003, and R^2 change = 8.8%, F change$_{(1,65)}$ = 7.28, p = .009, for number line

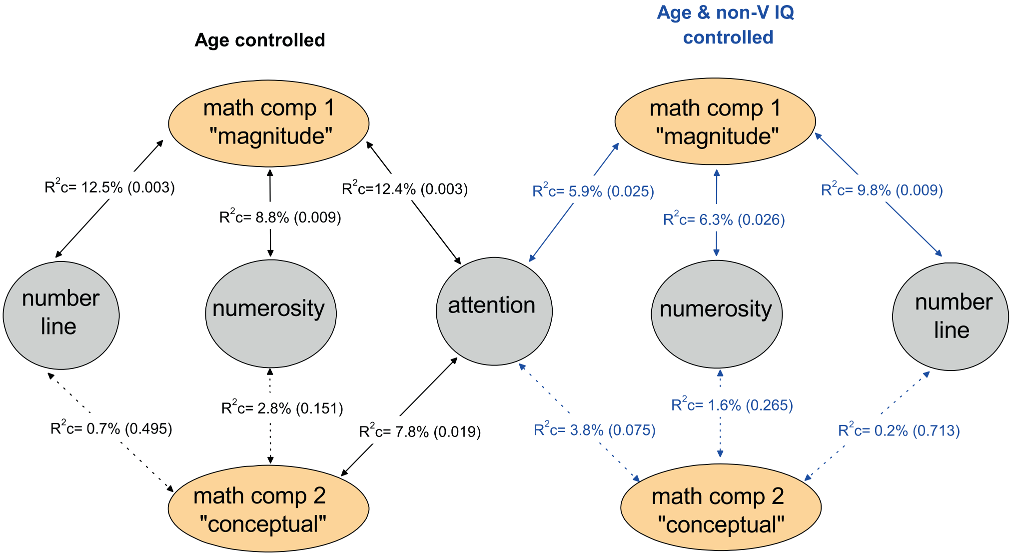

Fig. 4. Partitioning out the influence of chronological age (black arrows) or chronological age plus nonverbal intelligence (blue arrows), all of the psychophysical measures still explain a significant proportion of variance of the magnitude-oriented component. After controlling for chronological age, the conceptual component remained related only with attention. Importantly, partitioning out the influence of age plus nonverbal intelligence, none of the psychophysical measures correlated with the conceptual component (dotted blue arrows). non-V, nonverbal; comp, component; R^{2}c, R^2 change. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

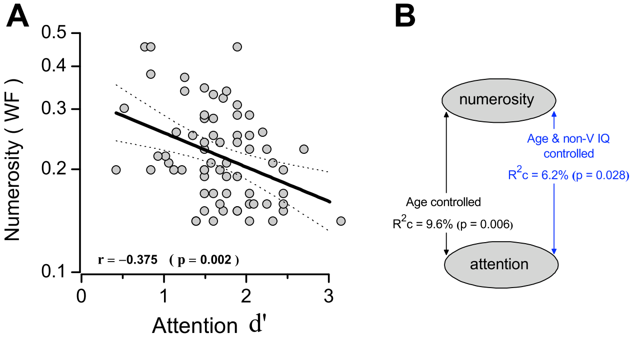

Fig. 5. (A) Numerosity comparison precision (Weber fraction) strongly correlated with attentional performance. (B) Controlling for the influence of chronological age alone (black arrow) or together with nonverbal intelligence (blue arrow) does not eliminate the correlation. WF, Weber fraction; non-V, nonverbal; R^{2}c, R^2 change. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

and numerosity, respectively), but are not related to the conceptual component (R^2 change = 0.7%, F change$_{(1,65)}$ = 0.47, p = .495, and R^2 change = 2.8%, F change$_{(1,65)}$ = 2.11, p = .151, for number line and numerosity, respectively).

More interesting, after partitioning out the influence of both age and general nonverbal intelligence, all three perceptual tasks correlate with the magnitude-oriented factor (R^2 change = 6.3%, F change$_{(1,64)}$ = 5.20, p = .026; R^2 change = 9.8%, F change$_{(1,64)}$ = 7.21, p = .009; and R^2 change = 5.9%, F change$_{(1,64)}$ = 5.24, p = .025, for numerosity, number line, and attention, respectively) but not with the conceptual component (R^2 change = 1.6%, F change$_{(1,64)}$ = 1.26, p = .265; R^2 change = 0.2%,

F change$_{(1,64)}$ = 0.137, p = .713; and R^2 change = 3.8%, F change$_{(1,64)}$ = 3.28, p = .075, for numerosity, number line, and attention, respectively).

Correlation between perceptual variables

We also performed preliminary bivariate correlations between perceptual variables. Total error on number line does not correlate with Weber fraction (r = .033, p = .788) or d' (r = −.050, p = .684). However, numerosity comparison precision (Weber fraction) strongly correlates with attentional performance (r = −.375, p = .002) even after controlling for chronological age (R^2 change = 9.6%, F change$_{(1,64)}$ = 7.94, p = .006) or for chronological age and nonverbal intelligence (R^2 change = 6.2%, F change$_{(1,64)}$ = 5.07, p = .028 (Fig. 5A and B).

Discussion

We investigated the relationship between formal math skills and three psychophysical measures of numerosity and visual attention in a sample of typically developing children between 8 and 11 years of age. Our results replicate those showing a correlation among number acuity, number-to-space mapping, and math skills (Mazzocco et al., 2011; Piazza et al., 2010; Steele, Karmiloff-Smith, Cornish, & Scerif, 2012).

Interestingly, individual differences in formal math skills are also highly correlated with visual sustained attention. This correlation remains highly significant even after controlling for chronological age, nonverbal intelligence, gender, and text reading accuracy. As reported in other studies, we found that number acuity and number-to-space mapping are not two "generic performance" predictors because they do not correlate with children's reading ability. This is also the case with visual sustained attention.

These results are closely in line with those recently reported by Steele et al. (2012). They measured attentional performance in preschool children, with several tasks involving the capacity to sustain attention for a prolonged period of time (Go/No-Go task) to visually search targets among distracters and to benefit from a target presented in a position spatially congruent with the response. Correlating attentional performance with math and reading skills measured 1 year later, they found that sustained attention and visual search predict numeracy but not literacy. Our results show that the correlation between attention and formal mathematics also extends to ages later than preschool, namely from 8 to 11 years. During this age period, both formal numerical cognition and the ability to estimate and manipulate numerical quantities are evolving and are characterized by large interindividual variability.

Formal math aggregates many different skills. We asked which math components are most closely correlated with visual attention, number acuity, and the ability to map number onto space. Some evidence (Piazza et al., 2010) shows that number acuity of dyscalculic children correlates with errors in semantic tasks—in tests involving number magnitude processing—but not with other number-related competences such as simple calculation and transcoding. We performed a simple principal component analysis on the different math subtests and revealed two factors: one more related to semantic processing and the other more related to conceptual and memory-based math competence. Interestingly, the three psychophysical tasks—after controlling for chronological age and nonverbal intelligence—correlate with the magnitude-oriented factor but not with the conceptual factor, again pointing to a specific role of numerosity, number line, and attention in the apprehension of magnitude-related number processing. Finally, we showed that sustained visual attention strongly correlates with number acuity.

As others have suggested, understanding the interplay between attention and learning could be important both theoretically and operationally (see Scerif, 2010, for a review). Recent studies have shown that, along with deficits in numerical processing, people suffering from dyscalculia also have deficits in attention (Ashkenazi & Henik, 2010; Askenazi & Henik, 2010). However, the same research group found that attentional training, through the use of video games, does not improve arithmetic processing in a group of dyscalculic university students (Ashkenazi & Henik, 2012). Other evidence comes from studies showing dysfunctional subitizing in dyscalculics (Ashkenazi, Mark-Zigdon, &

Henik, 2012; Koontz & Berch, 1996). Importantly, our recent works (Anobile, Turi et al., 2012; Burr et al., 2010) as well as others' recent works (Olivers & Watson, 2008; Railo, Koivisto, Revonsuo, & Hannula, 2008; Vetter, Butterworth, & Bahrami, 2010) demonstrate that subitizing is strongly attentional dependent.

Several questions remain open, including why sustained visual attention correlated with math but not with reading accuracy. Previous studies (Franceschini et al., 2012, 2013) have reported a correlation between reading skills and attention. However, those studies did not measure sustained attention but rather measured transient attention using visual search and Posner-like paradigms. On the other hand, Steele et al. (2012) measured sustained attention with a continuous performance test and reported similar results to ours. They advanced the intuitive hypothesis that reading could be more related to sustained auditory attention than it is to visual attention. Given the evidence that attentional resources for visual tracking are independent of auditory resources (Arrighi, Lunardi, & Burr, 2011), this suggestion seems quite plausible. It would be interesting to test auditory attention to see whether it correlates with reading ability.

Few studies have measured attentional performance in people with dyscalculia, but the little information available agrees with the idea that difficulties in acquisition of math skills may be related to attentional deficits (Ashkenazi & Henik, 2010; Askenazi & Henik, 2010). This evidence is beginning to call into question the domain-specific nature of this disorder. Although our research did not involve participants with dyscalculia, it adds weight to this idea and highlights how, at least during typical development, math and attention are strongly linked. Given the correlational nature of our study, we cannot assert a causal role of attention on acquisition of arithmetic skills. However, the fact that visual sustained attention continues to be correlated with math skills even after the influence of IQ, age, gender, and reading accuracy has been eliminated strongly supports the hypothesis that effective learning of mathematical skills could depend on proper functioning of the visual sustained attentional system.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jecp.2013.06.006.

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