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(Article begins on next page)

Performance prediction of radial ORC Turboexpanders

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Abstract

In this paper, a zero-dimensional model for the design of radial Turbo-Expanders for ORC applications is discussed, with special reference to the estimation of losses and efficiency; a comparison between different fluids (R134a, R1234yf, R236fa, R245fa, Cyclo-Hexane, N-Pentane) is presented and discussed, referring to a typical small-size application (50 kW). In the model, different methods for the design of radial turbines are screened, with special attention to the estimation of losses, for which correlations from literature are used. Real Equations Of State (EOS) are applied to the expansion process in place of the traditionally adopted Mach relationships for ideal gas, which is a significant advancement for modeling organic fluids in ORC, often operating near to critical conditions. The results show that the total to total efficiency of the designed machines range between 0.72 and 0.80, depending on the considered fluid. Generally, higher efficiency (1.5 – 2.5 % points) can be achieved adopting backswept-bladed rotors. The most significant losses come from the rotor secondary flows, due to the high curvature of blade profiles combined to the large pressure gradient. The best performing fluids are R236fa and R245fa, followed by R134a and R1234yf.

Finally, starting from the developed design tool, an off-design analysis of turbo-expanders is presented. Once the design data are available, the characteristic curves of the expander at variable temperature, pressure and fluid mass flowrate at the expander inlet for different values of the specific speed are built. It is thus possible to evaluate the performance of the radial expanders when working far from design point. This analysis, demonstrated for R134a, shows that the total to static

32 efficiency has a relatively modest sensitivity to the off design of the expansion ratio, especially at
33 corrected speed below the design value.

34

35 **Keywords**

36 Radial Turbine Design, Expansion Efficiency Losses, Off design, Micro ORC

37

38 **1. Introduction**

39 Organic Rankine Power Cycles (ORC) are becoming a leading technology for energy conversion,
40 with special reference to low size (< 100 kW) and low-temperature applications ($T < 150^{\circ}\text{C}$), where
41 the use of steam is not convenient. The thermodynamic properties of organic fluids make them very
42 interesting for small/medium size power plants (50 to 5000 kW or more, at present); applications of
43 ORC cycles range from heat recovery at gas turbine discharge [1, 2, 3] or internal combustion
44 engines [4, 5], to energy conversion from biomass [6], solar [7, 8, 9], and geothermal resources [10,
45 11, 12]. Due to the low working temperature, ORCs have typically low efficiency levels: for this
46 reason, the accurate design of the expander is a very important issue to avoid further appreciable
47 reduction of performance. For these applications and power range, radial (or mixed flow) turbines
48 are usually preferred to the axial ones, because they offer several advantages: a low degree of
49 reaction (thereby simplifying sealing), capability of dealing with large enthalpy drops with
50 relatively low peripheral speeds, possibility of adopting a single-stage design. On the whole, this
51 results in good performance and affordable price. The optimization of the thermodynamic cycle,
52 with special reference to fluid selection, has been studied widely in the last years. Fluid-dynamic
53 design of turbo-expanders can take advantage of the availability of modern CFD methods [....];
54 however, there is a need for preliminary design methods, and of modeling tools capable of
55 predicting the off-design performance (which is determined, for example, by the variation of the
56 resource for solar-driven EGS, or by variation of ambient temperature in geothermal or heat
57 recovery applications). In the field of low power output (i.e. up to 100 – 150 kW), radial expanders
58 are almost the only choice, with the eventual competition of screw expanders. Generally, literature
59 is rich of theoretical – experimental correlations for the estimation of losses in axial expanders,
60 whereas much less data are available for radial turbines [13 – 30]. Most data available refer to ideal
61 gas and make use of the Mach compressibility relations [13-20]. This may not be a satisfactory
62 approximation when dealing with ORC turbines, which operate near the saturation line or close to
63 the critical point. In the model hereafter proposed, correlations from literature are used [13-30], but
64 real Equations Of State (EOS) are applied to the expansion process (static and total variables) in

place of ideal gas relations. This is an important feature, allowing to preliminary design and performance prediction of turbo-expanders that work with real substances. The correlations can be refined progressively as more data on ORC expanders become available, either from field operation, or from specific test arrangements. The thermodynamic properties of the working fluids are calculated using the libraries of the EES software, which is the programming environment adopted in this work. Making use of the design model, a sensitivity analysis investigating the effects of the different design parameters on the expander performance is presented. Finally, an off-design model has been developed and some results are discussed, in order to assess the behavior and estimate the performance of the turbo-expanders when working out of nominal conditions. This often happens when the ORC high-temperature resource is a time-dependent energy source, like solar; but also, with seasonal change of condenser conditions, on account of the heat/mass transfer performance of the cooling system (condenser/cooling tower/air cooler).

77

78 **2. Fundamental design concepts and parameters for Radial-Inflow Turbines (IFR)**

Radial-inflow turbines have been less studied than the axial ones, and have been manufactured by a limited number of companies [31, 32, 33]. The fundamental design of radial-inflow turbines is presented and discussed in books and scientific papers [13-30], most of which are based on experimental work performed at NASA between 1965 and 1975 [16, 17, 18, 20, 30]. At that time radial turbines, working with ideal gases (air, helium), were designed and tested for aerospace applications. The high values of centrifugal stresses on rotor blades and the limits on materials performance and production technology led to the design of the ideal 90° IFR turbine: that is, a rotor having radial blades at inlet. Some years later (1983) NASA researchers [18] studied and developed a radial – inflow turbine with a more performing rotor design, characterized by a blade sweep angle β_2 at rotor inlet (IFG, figure 1). The typical velocity triangles are shown in figure 2; in both cases, the absolute velocity c_3 is assumed to result axial at rotor outlet, in order to guarantee good diffuser performance (figure 2). Generally, nominal design conditions are referred to zero-incidence at rotor inlet and zero-deviation at rotor outlet; however, in radial turbines, the best efficiency values are obtained when incidence is non-zero (figure 3): this is a consequence of the rotational flow, which displaces the tangential component of the relative velocity (w_u) in the opposite direction with respect to the peripheral rotor velocity. From the technical literature [13, 14] it appears that the best values of rotor inlet angle in the relative flow are between -20° and -40° , referred to the camber line direction (the positive sign is conventionally assumed toward the direction of peripheral velocity u).

98 In radial turbine design, some non – dimensional parameters help designers to select a geometry
 99 optimizing efficiency using a limited set of variables; the following parameters are recommended in
 100 the literature [13, 14, 17, 19, 20]:

$$101 \quad \frac{d_{3h}}{d_{3s}} = 0.40 \quad (1)$$

$$102 \quad \frac{d_{3s}}{d_2} = 0.70 \quad (2)$$

103

104 Another important parameter is the *isentropic velocity ratio* (u_2/c_s), which maximizes the
 105 efficiency in the range 0.69 – 0.71 [13, 14, 17, 19, 20]. c_s is the *spouting velocity*, defined as the
 106 velocity at which the kinetic energy of the flow is equal to the isentropic enthalpy drop from turbine
 107 inlet stagnation pressure p_{01} to the final exhaust pressure [14]. The definition of spouting velocity is
 108 different depending on (I) whether a diffuser is present or not downstream the turbine, see
 109 relationships (3) and (4, 5) respectively, and (II) if total (4) or static (5) conditions are considered at
 110 the turbine exit (figure 4):

$$111 \quad c_s = \sqrt{2(h_{01} - h_{4ss})} \quad (3)$$

$$112 \quad c_s = \sqrt{2(h_{01} - h_{03ss})} \quad (4)$$

$$113 \quad c_s = \sqrt{2(h_{01} - h_{3ss})} \quad (5)$$

114

115 **3. Design Guidelines**

116 **3.1 Input data and expected process output**

117 The input data consist in the expander rated power output, in the thermo-fluid dynamic variables
 118 determined by the thermodynamic cycle [36], and in a set of dimensional and non – dimensional
 119 parameters chosen by the designer (table 1). The outputs of the calculations are the basic geometry
 120 with the related velocity triangles and the efficiency of the designed expander. The model is also
 121 able to calculate the turbine losses and their relative share in the resulting inefficiency.

122

123 **3.2 Preliminary sizing**

124 The first step is the preliminary calculation of geometry, using the non – dimensional parameters
 125 listed in table 1, which determine also the isentropic nozzle and rotor enthalpy variations. The load
 126 and flow coefficients ($\Psi = \Delta h_0/u_2^2$ and $\Phi = c_{m2}/u_2$ respectively) are adjusted to calculate the

127 peripheral velocity (u_2), specific speed (n_s), speed of revolution (ω) and meridional component of
 128 absolute velocity at nozzle exit/rotor inlet (c_{m2}). These data are used for the evaluation of the mass
 129 flow rate \dot{m} :

$$130 \quad \dot{m} = \rho_2 c_{m2} b_2 d_2 \pi (1 - BK_2) \quad (6)$$

131 The meridional component of the absolute velocity at nozzle inlet (c_{m1}) - which is the same as
 132 absolute velocity (c_1) as the flow at the IGV nozzle inlet is assumed to be radial - is given by the
 133 application of mass balance in section 1 (figure 1):

$$134 \quad c_{m1} = \frac{\dot{m}}{\rho_1 \pi b_1 d_1 (1 - BK_1)} \quad (7)$$

135 Consequently, knowing the inlet enthalpy $h_1(p_1, T_1)$, it is possible to calculate the total enthalpy
 136 (h_{01}). The thermodynamic variables at point 2 ($p_2, T_2, h_2, s_2, \rho_2, Ma_2, Ma_{u2} = \frac{u_2}{v s_2}, Ma_{r2} = \frac{w_2}{v s_2}$) and
 137 the velocity triangles at rotor inlet are determined calculating first the nozzle isentropic expansion
 138 and then the real transformation using the nozzle loss coefficient ξ_N (figure 2):

$$139 \quad h_2 = h_{2s} + 0.5 \xi_N c_2^2 \quad (8)$$

140 Once the nozzle exit/rotor inlet conditions are known, the thermodynamic variables at point 3 (rotor
 141 exit/diffuser inlet, figure 1) are calculated by solving at first the rotor isentropic expansion and
 142 assuming that the difference between the absolute velocities related to isentropic and real expansion
 143 at that point is negligible. The relative velocity at rotor output can be calculated by the conservation
 144 of rothalpy [14], figure 2:

$$145 \quad i = h + 0.5 w^2 - 0.5 u^2 \quad (9)$$

146 The meridional component of the absolute velocity at point 3 is determined by the conservation of
 147 mass (figure 2):

$$148 \quad \dot{m} = \rho_3 c_{m3} \frac{d_{3s}^2 - d_{3h}^2}{4} \pi (1 - BK_3) \quad (10)$$

149 Finally, using the rotor loss coefficient and assuming an axial discharge at rotor outlet, the static
 150 enthalpy and velocity triangles at rotor outlet can be calculated:

$$151 \quad h_3 = h_{3s} + 0.5 \xi_R w_3^2 \quad (11)$$

152 The calculation of the real conditions at diffuser outlet is done by combining the total enthalpy
 153 balance and the definition of diffuser loss coefficient ξ_D .

155 3.3 Geometry of the stator (IGV)

156 The prediction of the angle of flow leaving the bladed nozzle of a radial turbine, discussed in the
 157 previous section, is a fundamental design topic. The next step is the calculation of the angles of
 158 blades, which are radial at inlet for no pre swirled IGVs. At outlet, while leaving the nozzle, the
 159 flow does not follow the vanes completely but it turns toward the meridional direction of an angle
 160 known as deviation, due to the combined effects of boundary layer growth (limited by the
 161 accelerating flow) and the subsequent abrupt expansion due to the trailing edge thickness. As a
 162 simple design approach, the actual angle of blades at nozzle outlet is calculated interpolating data
 163 from Hiatt and Johnston [13, 29], which have a rather linear behavior, approximated by the
 164 interpolating function $\alpha_{b2} = 0.884 \cdot \alpha_{b1} + 4.56$.

165 Another important parameter is the number of stator blades, which directly influences the losses.
 166 Increasing the number of blades leads to better flow guidance at the price of higher frictional losses.
 167 A general design approach to define the stator number of blades is that it should be a prime number
 168 compared to the rotor one. In addition to this basic criterion, the criterion of Zweifel on the optimal
 169 ratio between the chord and the blade spacing can be adopted [14; 23]. It suggests that, in order to
 170 minimize the losses, the ratio between the tangential load of an actual to that of an ideal blade (Ψ_T)
 171 should be about 0.80. Knowing the absolute flow angles at nozzle inlet and outlet, and after
 172 calculating the chord length, from the expression of blade pitch $Z \cdot s = \pi \cdot d_2$, the following equation
 173 (12) gives the optimum blade spacing, from which the number of blades can be easily determined:

$$174 \quad \Psi_T = 2 \left(\frac{s}{x} \right) \cos^2 \alpha_2 (\tan \alpha_1 + \tan \alpha_2) \quad (12).$$

176 3.4 Optimal incidence at rotor inlet and number of rotor blades

177 Calculating the number of rotor blades is a fundamental issue in the design of radial turbines,
 178 because it defines the basic structure of the machine and has a primary role in the estimation of
 179 losses. There are no absolute criteria allowing an univocal evaluation of number of blades; the
 180 design guidelines tend to avoid very low local velocities near the blade surface in the inlet region of
 181 the rotor, with a consequent tendency to early separation [13]. On the other hand, the adoption of a
 182 high number of blades is not so convenient, especially for small rotors: the blockage effects, the
 183 weight and inertia of the rotor become very high. Moreover, a large number of blades is also

184 responsible for a large wetted surface area, which increases the friction losses. In the present model,
 185 rather than using the formulation proposed by Jamieson [14, 15], which determines an exceedingly
 186 large number of rotor blades, the formulation proposed by

187 Glassman [16] is followed:

$$188 \quad Z_R = \frac{\pi}{30} (110 - \alpha_2) \tan \alpha_2 \quad (14).$$

189 Once the number of blades is determined, the optimum rotor incidence angle can be calculated. As
 190 previously discussed, better efficiency values are achieved when incidence is non-zero. Referring to
 191 the camber line direction, the best values of rotor inlet angle in relative flow are between 20° and
 192 40° counterclockwise.

193 Referring to IFR rotors (radial blades at rotor inlet), the following are the recommended
 194 correlations:

$$195 \quad \tan(\beta_{2opt}) = \left(\frac{2}{Z_R}\right) \frac{u_2}{c_{m2}} \quad (15), [14]$$

$$196 \quad \tan(\beta_{2opt}) = 1 - \frac{0.73\pi}{Z_R} \quad (16), [13]$$

$$197 \quad \tan(\beta_{2opt}) = \frac{-1.98 \tan(\alpha_2)}{Z_R \left(1 - \frac{1.98}{Z_R}\right)} \quad (17), [13]$$

198 The above equations (15 – 17) provide similar results, even though (16) has been obtained
 199 considering minimum Mach number conditions at the rotor inlet [13]. From these relationships, one
 200 can notice that the optimum rotor incidence angle depends on the number of rotor blades and on the
 201 kinematic conditions of flow.

202 In the case of the IFG design (non- radial blades at rotor inlet), the procedure suggested by Meitner
 203 & Glassman [18] can be followed by calculating first the optimal value of the peripheral component
 204 of the absolute velocity:

205 (18), [18].

$$206 \quad c_{u2 opt} = \begin{cases} u_2 \left(\frac{[1 - \sqrt{\cos(\alpha_{b3})} / (Z_R)^{0.7}] \{1 - [(r_3/r_2 - \varepsilon_{lim}) / (1 - \varepsilon_{lim})]^3\}}{1 - \frac{tg(\alpha_{b3})}{tg(\alpha_2)}} \right) & \text{if } \frac{r_3}{r_2} > \varepsilon_{lim} \quad (18a), [18] \\ u_2 \left(\frac{[1 - \sqrt{\cos(\alpha_{b3})} / (Z_R)^{0.7}]}{1 - \frac{tg(\alpha_{b3})}{tg(\alpha_2)}} \right) & \text{if } \frac{r_3}{r_2} \leq \varepsilon_{lim} \quad (18b), [18] \end{cases}$$

207 The parameter ε_{lim} can be determined by:

208
$$\varepsilon_{lim} = \frac{1}{e^{8.16 \cos \alpha_{b3} / Z_b}} \quad (19), [18]$$

209 Once $c_{u2 \text{ opt}}$ has been calculated, it is possible to calculate the relative velocity and blade angle as
210 follows:

211
$$w_{u2 \text{ opt}} = c_{u2, \text{opt}} - u_2 \quad (20), [18]$$

212
$$\beta_{2 \text{ opt}} = \tan^{-1} \left(\frac{w_{u2, \text{opt}}}{c_{m2}} \right) \quad (21), [18].$$

213

214 **3.5 Expander Efficiency, Power Output, Degree of Reaction and Specific Speed**

215 The calculation of the expander efficiency, power output, design degree of reaction and specific
216 speed are following the guidelines in [37], which are briefly recalled for completeness.

217 The efficiency can be either referred to turbine discharge or including also the diffuser; in the first
218 case, the Total-to-Total and Total-to-Static efficiency are given by:

219
$$\eta_{tt} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}} \quad (22)$$

220
$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} \quad (23)$$

221 If the diffuser is included, the Total-to-Static efficiency becomes:

222
$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{4ss}} \quad (24)$$

223 The power output can be calculated by one of the equivalent three following equations:

224
$$W = \dot{m}(h_{01} - h_{03}) \quad (25)$$

225
$$W = \dot{m}(u_2 c_{u2} - u_3 c_{u3}) \quad (26)$$

226

227
$$W = \frac{1}{2} \dot{m}[(u_2^2 - u_3^2) + (c_2^2 - c_3^2) - (w_2^2 - w_3^2)] \quad (27)$$

228 The degree of reaction and the specific speed are given by:

$$R = \frac{h_2 - h_3}{h_1 - h_3} \quad (28)$$

$$n_s = \frac{NQ_3^{1/2}}{\Delta h_{0s}^{3/4}} \quad (29) [14]$$

where

$$\Delta h_{0s} = h_{01} - h_{03ss} \quad (30)$$

4. Calculation of losses

In order to evaluate the actual performance of a turbomachine, the contributions of different losses must be calculated. This calculation cannot be substituted by advanced CFD methods, because it provides vital information to the designer, about the process of loss buildup determining the final turbomachine performance. On the other hand, advanced CFD is very useful for cross-checking the overall results of the efficiency/loss model, and often to supplement data which would require very detailed (often impossible) measurements.

Generally, the first step of the design procedure considers consequently the effects of losses through appropriate dimensionless coefficients. The related efficiency drop $\Delta\eta$ is then subtracted from the isentropic value to calculate the actual efficiency:

$$\eta_{act} = \eta_s - \Delta\eta \quad (31), [13]$$

In this model, the overall loss of the turbine is obtained by the sum of several contributions, each one estimated through correlations which depend on kinematic and geometric parameters. The dimensionless loss coefficients are defined in several ways, and it is important to merge them to a common basis, in order to apply them within the same model and do some reliable comparisons with results from literature. Referring to the j^{th} loss:

$$\xi_j = \frac{h_j - h_{j,is}}{\frac{1}{2}V_j^2} \quad (32), [13]$$

$$\Delta q_j = \frac{h_{0j} - h_{0j,is}}{u_j^2} \quad (33), [13]$$

This is the approach followed in the present model: starting from non – dimensional loss coefficients obtained from the correlations, it calculates ξ and the related efficiency drop ($\Delta\eta$), for each kind of loss. It allows, in all kinds of expanders and operating conditions, to analyze the

255 distribution of losses and to investigate how do they affect the overall performance. The so built
256 model provides a reliable basis to improve the design of different kinds of rotors with several
257 possible working fluids.

258

259 **4.1 Stator losses**

260 The stator losses, which are generally lower than the rotor losses, have been often evaluated with
261 less accuracy in the literature [13, 24]. They are generally based on experimental data and make use
262 of equations for stationary ducts. Referring to the experimental tests of Hiett and Johnston, Benson
263 determined the stator loss coefficients ($\xi_N = 0.05 - 0.15$) [24], showing that they are very small
264 compared to the corresponding values in the rotor. For the estimation of stator losses it is possible
265 to apply Rodger's correlation [13]:

$$266 \quad \xi_N = \frac{0.05}{Re^{0.2}} \left[\frac{3 \operatorname{tg} \alpha_2}{s/x} + \frac{s \cos \alpha_2}{b_2} \right] \quad (35), [13]$$

267 where:

$$268 \quad Re = \frac{c_2 b_2}{\nu_2} \quad (36)$$

269

270 **4.2 Rotor losses**

271 The flow in the rotor of a radial turbine is subject to a rapid acceleration in the flow direction, and
272 to a turn both in the meridian plane and along the camberline. These effects give rise to a complex
273 pattern of secondary flows. The flow in the rotor of a radial turbine does not result into a high
274 growth of the boundary layer and separation, even though, due to the three dimensional behavior, it
275 develops a significant non uniformity of the total pressure inside the flow channel, which can lead
276 to generation of losses. The probability of this occurrence increases when the blade loading is
277 augmented. In the present model, the losses are divided in different contributions:

- 278 - Incidence loss;
- 279 - Skin friction loss;
- 280 - Tip clearance loss;
- 281 - Blade loading loss;
- 282 - Disk friction loss.

283

284 4.2.1 Rotor Incidence loss

285 In the actual working conditions of the expander, the incidence angle of the relative flow at rotor
286 inlet is rarely at the optimal value (equations 15 – 17). For this reason, the incidence loss appears.
287 The recommended models for the estimation of rotor incidence loss were developed at NASA [13,
288 14, 24]. The general approach is to assume that the kinetic energy associated with the variation of
289 the tangential component of the relative velocity with respect to the design value, which is the result
290 of the fluid – blade impact, is converted in internal energy of the fluid, which leads to an increase of
291 entropy. The detailed calculation procedure is reported in [14]. The incidence losses may also be
292 calculated using alternative approaches [13], obtained with the same conceptual assumptions and
293 therefore formally similar:

$$294 \quad \delta h_{0,i} = \frac{w_2^2 \sin^2(|\beta_2 - \beta_{2,opt}|)}{2} \quad (37), [13];$$

$$295 \quad \delta h_i = \frac{(w_2 \sin \beta_2 - w_2 \sin \beta_{2,opt})^2}{2} \quad (38), [13].$$

296 The above discussed methods provide similar results, consistent with the literature. Thus, any of
297 the two proposed correlations may be adopted leading to negligible differences.

298

299 4.2.2 Friction losses

300 Friction losses can be estimated referring to a rotor-equivalent duct working on the same flow rate
301 [26]:

$$302 \quad \xi_{R,f} = \frac{\lambda_R L_R^*}{D_R^*} \quad (39)$$

303 Details about the calculation of the characteristic diameter and length can be found in [24] or [26].
304 As for the stator, the friction factor can be determined using Moody's diagram. The relative
305 roughness and Reynolds number are estimated referring to the rotor-equivalent duct. Alternatively
306 to equation (39), the frictional losses may be calculated using the following expression:

$$307 \quad \Delta q_{R,f} = \frac{4\lambda_R [(w_2/V S_{01})^2 + (w_3/V S_{01})^2]}{4(D_R^*/L_R^*)(u_2/V S_{01})^2} \quad (40), [13].$$

308 or

309 This correlation tends, generally, to overestimate the friction losses by 70 – 80%.

310

311

312 **4.2.3 Tip clearance losses**

313 Tip clearance losses are due to the fluid leaking through the clearance gaps between the blade tips
314 and the shroud. With reference to the blade geometry, in radial turbines two different types of
315 clearances can be distinguished from the construction point of view: axial at inlet and radial at
316 outlet [30, 38]. However, there is not a net distinction between the two kinds of clearance, but a
317 gradual and continuous change (figure 5). Referring to studies performed at NASA [30] and more
318 recent CFD calculations [38], it may be affirmed that the contribution of radial clearance to the
319 overall loss is almost one order of magnitude higher than the axial one [13, 15, 30, 38].

320 Several different correlations have been proposed for tip clearance losses, some of which are
321 specific for radial inflow turbines [...] and others are derived from centrifugal compressors [...]. A
322 wide spread in the results can be produced using different models. Here, the model of Rodgers [13]
323 is proposed:

$$324 \quad \Delta q_{R,cl} = 0.4 \left(\frac{\varepsilon}{b_2} \right) \left(\frac{c_{u2}}{u_{t,le}} \right)^2 \quad (43)$$

325 Equations (43) provide low values of clearance losses, which result in poor agreement with
326 literature. Specifically, it happens when the values of axial clearance in equation are used. When the
327 values of radial clearance are adopted, higher agreement with literature results are achieved [30] for
328 equation (43).

329

330 **4.2.4 Blade loading loss (including secondary flow)**

331 Blade loading loss, including secondary flow, are caused by the high curvature of the profile and the
332 pressure gradient in the rotor vanes. They give the largest contribution to the reduction of the
333 expander efficiency. However, they are not extensively reported and discussed in literature, often
334 because they are threatened in combination with other losses using experimental coefficients [18, 24,
335 28]. The model here proposed evaluates the secondary flow losses through correlations, as functions
336 of kinematic and geometric parameters. For the calculation of blade loading losses, the following
337 correlation proposed by Rodgers can be used [19]:

$$\delta q_{R,bl} = 2 \frac{\left(\frac{c_{u2}}{u_{t,le}}\right)^2}{Z_R \frac{z}{r_2}} \quad (48)$$

Where z/r_2 is the ratio between the expander axial length and rotor inlet radius[19].

For the calculation of profile losses, another correlation proposed by Rodgers [40] and suitably revised by Whitfield [19] may be adopted:

$$\delta q_{R,p} = 0.5 \left(\frac{\frac{b_2}{r_2} + \frac{b_3}{r_2}}{1 - \left(\frac{r_3}{r_2}\right)^2} \right) \left(\frac{w_2^2 + w_3^2}{2VS_{01}^2} \right) \left(\frac{VS_{01}^2}{u_2^2} \right) \quad (49).$$

When the results achieved from equations (48) and (49) are compared with those of literature, one must face the problem of lack of sufficient data for this kind of losses. However, comparing the values of the overall loss coefficient and efficiency with those reported in the literature, it seems that the above described correlations provide fairly reliable results.

4.2.5 Disk friction losses

Disk friction losses are produced in the enclosure between the back disk side of the impeller and the case of the machine, where an amount of fluid can leak due to the pressure gradient and rotate around the rotor axis.. In the present model, the formulation of Whitfield [13] was adopted, which is based on the original model of Daily & Nece [...], in alternative to the model proposed by Benson which provides exceedingly large values with respect to the available test data:

$$\Delta q_{R,df} = \frac{0.25 \bar{\rho} u_{t,le} r_2^2 K_v}{\dot{m}} \quad (50)$$

where:

$$k_v = \begin{cases} \frac{\left[3.7 \left(\frac{\varepsilon_{ax}}{r_2} \right)^{0.1} \right]}{Re^{0.5}} & , Re < 3 \cdot 10^5 \\ \frac{\left[0.102 \left(\frac{\varepsilon_{ax}}{r_2} \right)^{0.1} \right]}{Re^{0.2}} & , Re > 3 \cdot 10^5 \end{cases} \quad (51)$$

$$Re = \frac{u_2 r_2}{\nu_2} \quad (52).$$

359 As a possible alternative, providing similar results (in the range of 1%), the correlations proposed
360 by NASA [14, 17, 18] can be recommended.

361

362

363 **4.3 Diffuser loss**

364 A diffuser is generally present in radial turboexpanders downstream of the rotor, in order to allow
365 the partial recovery of the large kinetic energy still available through controlled diffusion of the
366 fluid. The calculation of the diffuser loss follows the standard procedure described in [42].

367

368 **5. Results and parametric analysis (Design process)**

369 Making use of the above-described loss correlations, a parametric analysis has been run to assess
370 the behavior of losses against the main design parameters and input data:

371 *a. Blade height – inlet rotor diameter ratio (b_2/d_2)*

372 *b. Flow coefficient (Φ)*

373 *c. Load coefficient (Ψ)*

374 *d. Isentropic degree of reaction (R_s)*

375 The reference case is a 50 kW turboexpander operating with a saturated or superheated vapour ORC between
376 the upper/lower temperature levels of 147/95 °C (referred to R134a [37]).

377 **5.1 Blade height – inlet rotor diameter ratio (b_2/d_2)**

378 This parameter influences the power output, mass flow rate, efficiency, blade shape and
379 flow conditions, especially at rotor outlet (figures 6,7) for both radial (IFR) and backswept
380 (IFG) rotor geometries. The mass flowrate (and thus the power output) increases linearly
381 when b_2/d_2 is augmented. The absolute value is strongly dependent on the considered fluid:
382 for example, the cyclohexane flow rate is much lower than that of the other working fluids,
383 because of the larger specific enthalpy drop. Among the fluids here considered, R1234yf
384 shows the highest flowrate. Generally, with the reduction of density and velocity of fluids,
385 the required blade height ratio increases at fixed expander power output. It is also important
386 to remark that, for a fixed flowrate, expanders with backswept blades (IFG) require higher

b_2/d_2 to achieve the same flow rate as for the IFR design. Generally, the rotor outlet blade height increases with increasing b_2/d_2 (i.e. the ratio between hub and shroud diameters at rotor outlet, d_{3h}/d_{3s} decreases, figure 7). R134a and R1234yf show a particular trend, with remarkable differences between IFR and IFG designs (figure 7). This is due to the fact that the outlet rotor blade height is determined from the mass flowrate balance, with the constraint of axial flow. Consequently, the meridional component of the outlet rotor velocity increases with the reduction of blade height. Figure 7 reflects a widely different design geometry of expanders working with different fluids.)

a. **5.2 Flow coefficient (Φ)**

Φ defines the velocity triangle at impeller inlet. The flow coefficient is one of the main parameters in the design of turboexpanders, as it directly influences the mass flow rate, the performance (power output and efficiency), the geometry and the rotor number of blades. To give an idea of how Φ influences the flow, the variation of shape of the velocity triangle at rotor inlet at two different values of Φ is shown on figures 8 a) (radial rotor blades IFR) and 8 b) (backswept rotor blades IFG). Keeping constant the other non-dimensional design parameters, the meridional component of the inlet absolute velocity (c_{m2}) increases with increasing Φ , whereas the peripheral velocity remains almost unchanged, as it mainly depends on the load coefficient. This results in an increase of absolute and relative velocity at rotor inlet (c_2 and w_2 respectively) and in a reduction of the related flow angles (α_2 and β_2). The change in β_2 directly affects the incidence losses, whereas α_2 influences the number of rotor blades, according to equations (13) and (14): as α_2 decreases with increasing flow coefficient, the number of rotor blades is reduced, as shown on figure 9 which reports the optimized values of Z_R vs. Φ . It must be remarked that, given the small size of the investigated expanders, it is a good practice trying to reduce large numbers of blades which results from the application of Eq. 14 (Glassman theory), in order to reduce the blockage effects. In fact, it is still possible to achieve high efficiencies also with a number of rotor blades much lower than that proposed by Eq. 14. Within the considered field of flow coefficient (typical of radial turboexpanders, $0,08 < \Phi < 0,22$), no significant differences in the “optimal” number of blades was found for the different investigated fluids (R134a shows the lowest optimal number of blades, whereas CycloHexane shows the highest one, for both IFR and IFG geometries). From figure 10, it is evident that backswept bladed (IFG) expanders have a total to static efficiency (η_{ts}) 1.5 – 2 points higher than radial (IFR) ones, which is in agreement with [18]. Generally, η_{ts} increases with Φ (with the exception of

cyclohexane for IFG rotors). The highest values of η_{ts} are achieved by R134a and R1234yf, whereas the lowest ones are shown by R245fa and cyclohexane.

5.3 Load coefficient (Ψ)

Ψ is a fundamental parameter in the design of turboexpanders, because it deeply influences their performance (rotational speed, absolute and relative flow angles at nozzle outlet/rotor inlet, overall performance) and is, with the degree of reaction, one of the main non-dimensional parameters to define the different categories of rotors. Keeping constant the other non-dimensional design parameters (table 1), an increase in the load coefficient leads to a reduction of the meridional component of the absolute velocity at rotor inlet (see modification of velocity triangles in figure 11). Thus, the mass flow rate is reduced and its effect is added to the reduction of the expander total enthalpy drop, leading to an overall reduction of power output. Due to the reduction of the meridional and peripheral velocity at rotor inlet, the absolute angle α_2 increases, which implies a reduction of the relative velocity (w_2'). For this reason, the load coefficient has a large influence on the incidence angle and on the associated loss. Figure 11 shows how the optimized velocity triangle for IFR tends to that of IFG with increasing Ψ . The trend of the rotational speed vs. load coefficient is shown on figure 12. It is interesting to remark the difference in rotational speed with different working fluids, which is in turn related to the total enthalpy drop and to the rotor size. Specifically, the largest rotational speeds occur for R134a and R245fa, whereas those of CycloHexane are considerably lower. Generally, a backswept (IFG) design allows a lower rotational speed. The dependence of the nozzle outlet/rotor inlet absolute velocity angle (α_2) on the Load Coefficient Ψ is shown on figure 13. These expanders are characterized by large nozzle flow angles, which become even higher in case of backswept blades. The largest values of α_2 for rotors with radial blades (IFR) are reached by CycloHexane and by R1234yf and R245f in the case of an IFG design. In both IFR and IFG configurations, R134a shows the lowest values of α_2 . The trend of the relative velocity angles at rotor inlet (β_2) vs. Ψ is shown on figure 14. The highest absolute values are shown by cyclohexane and R1234yf for radial bladed rotors and R245fa and R1234yf for the backswept bladed ones. Anyway, when Ψ is within the range 1.05 – 1.15, β_2 values are at the same levels for the different fluids in the case of backswept bladed rotors (IFG).

5.4 Isentropic degree of reaction (R_s)

R_s is defined as the ratio between the static isentropic enthalpy drop through the rotor and that of the overall stage. It strongly affects the performance of the expander. When combined with the other main parameters Φ and Ψ , it completes the definition of the

design geometry. When the other design parameters are fixed, with increasing R_s the isentropic stator static enthalpy drop is reduced. For this reason, the pressure at the stator outlet is higher and the related fluid density is increased. With fixed stator outlet cross sectional area, given the relatively limited change in mass flow rate, the meridional component of the absolute velocity is reduced, which offsets the increase in static pressure. Keeping constant the flow coefficient, the rotational speed is reduced. The related velocity triangle is modified as shown in figure 15. The expander power output decreases with increasing R_s of an amount variable with the different investigated fluids, as shown in figure 16. R245fa is the least sensitive to R_s because it has the lowest flowrate level and thus the lowest reduction of power output. Moreover, IFR expanders with radial blades have higher R_s than those with backswept blades. The variation of R_s has strong effects on the rotor peripheral speed. Referring to figure 17, the remarkable difference in peripheral speed for the different investigated fluids and rotors can be noticed. Specifically, R245fa shows the highest values, whereas the lowest is shown by R1234yf, due to its low value of the total enthalpy drop. Finally, the backswept rotors (IFG) have a lower peripheral speed compared to the radial bladed ones. This is due to the higher load coefficient Ψ (with fixed total enthalpy drop) which characterizes the backswept geometry.

6. Interpretation of results - Design process

6.1 IFR vs IFG design

Moving from the radial (IFR) to backswept blades (IFG) configuration, the load coefficient increases. Thus, in the backswept (IFG) design the meridional component of velocity at rotor inlet is reduced to maintain the fixed flow coefficient. For this reason, in order to achieve the target 50 kW power output, the IFG design shows higher b_2/d_2 ratios. An additional consequence of the higher load coefficient of backswept bladed rotors is their lower peripheral velocity (and rotational speed) for a given rotor size. Finally, backswept machines show a lower degree of reaction than the corresponding radial bladed ones. As remarked in the parametric analysis, when the overall enthalpy drop (stator + rotor) is fixed, the reduction of the degree of reaction implies a lower stator outlet backpressure and, consequently, a lower fluid density in this section. As the outlet conditions are fixed, and because the mass flowrate undergoes only limited variations, the meridional component of the absolute velocity increases to counterbalance the reduction of fluid density. Thus,

486 in order to maintain the flow coefficient unchanged, the peripheral velocity at rotor inlet increases.
487 Finally, in order to keep the load coefficient constant, the total enthalpy drop of the expander is
488 increased and, consequently, the related power output. In this way, by tuning b_2/d_2 , the load
489 coefficient and the reaction degree, it is possible to move across the two different configurations.
490 Generally, backswept bladed expanders show 1.5 – 2% better efficiency levels than the
491 corresponding radial bladed ones.

492 **6.2 Different working fluids**

493 A specific interpretation of the results is needed when considering the important matter of expander
494 design with different working fluids. All those here considered are good candidates for the power
495 cycle specifications (power output, temperature levels). On the other hand, large differences in
496 kinematic, geometric and performance characteristics are found between the different fluids. In the
497 following figures, the behavior of R134a, R1234yf, R245fa and cyclohexane is extensively
498 reported. Anyhow, for sake of completeness, the analysis of different fluids has been extended to
499 R236fa and CycloPentane. The inlet diameter of the expander is in the range 80 – 110 mm. The
500 largest size were achieved for the hydrocarbons like cyclohexane and N-Pentane, due to their much
501 lower density, in spite of the larger specific isentropic enthalpy drop compared to HFCs. The
502 rotational speed is between 30000 and 50000 rpm, generally lower for backswept configurations
503 due to the lower peripheral velocity (see the generic shape of velocity triangles in figure 15).
504 Among the different fluids, the cyclohexane shows the lowest rotational speed, due to the much
505 higher diameter, in spite of the high peripheral speed. The specific speed is in the 0.055 – 0.1 range.
506 The highest value is shown by the cyclohexane because, in spite of the lowest rotational speed and
507 the highest stage enthalpy drop, they are largely counterbalanced by the very high values of
508 volumetric flowrate due to the lowest density. The flow at nozzle exit is generally supersonic ($0.9 <$
509 $M < 1.5$), with the exception of R1234yf due to the high values of blade height at section 2 (b_2 ,
510 table 3). The highest value of Mach is shown by CycloHexane, due to the combined effects of low
511 density and high peripheral speed. Generally, high nozzle exit angles are found ($77 - 83.5^\circ$), with
512 larger values for backswept rotor (IFG) design. When considering the flow within the rotor, a high
513 deflection level has to be remarked, which is in the range $40 - 90^\circ$, generally higher for backswept
514 configurations. Due to the shape of velocity triangles (see velocities and angles on table 3),
515 cyclohexane shows the highest deflection level for the IFG configurations and the lowest for the
516 IFR ones. The hub to tip diameter ratio at rotor exit (d_{3h}/d_{3s}) is in the 0.39 – 0.52 range and agrees
517 with literature data [14, 17]. At rotor exit, the adoption of a diffuser for the partial recuperation of
518 the kinetic energy may be important only for fluids like cyclohexane, which have high values of

absolute Mach number in this section. The total- to-total efficiency of the investigated expanders and fluids ranges between 0.72 and 0.80, generally higher for backswept configurations than for the corresponding radial ones. Generally, higher efficiencies are achieved with expanders having lower velocities and deflections. Finally, it is important to remark that, for the specific size here considered (50 kW), sub-atmospheric values of total pressure at rotor exit are not recommendable. Thus, CycloHexane and pentane are critical from this point of view, in spite of their interesting efficiency levels in ORC. On the basis of expander design and cycle performance, R245fa and R236fa represent thus the most interesting options.

6.3 Distribution of losses and efficiency

It is also important to analyze the distribution of the different losses through the expander, whose contribution to the overall reduction of efficiency ($\Delta\eta$) is shown in table 3. It is practically the same for the different investigated fluids and configurations. The contribution of the *stator losses* to the overall losses ranges between 2% of R134a and 12% of cyclohexane, both referred to radial geometry. Especially in IFR configurations, the stator losses have the highest incidence for hydrocarbons (mainly cyclohexane) and R245fa compared to the other investigated fluids. It is mainly due to the high velocity in the nozzle (table 3, see also the high values of M_2). Literature data [13] report that the *stator losses* generally range between 5 and 15% of the overall, which is in line with the results achieved with the here proposed model.

Under design conditions, the *incidence losses* are negligible, as the relative velocity angle at rotor inlet is the optimizing value calculated by the (15 – 17) and (21) relationships. Their contribution to the overall reduction of efficiency $\Delta\eta_{ts,i}$ ranges between 0.03 and 0.2%.

The *disk friction losses* give a contribution within the 2 – 4% to the total and have a reduced relative influence on the expander efficiency ($0.4 \% < \Delta\eta_{ts,v} < 0.8 \%$). The highest values are shown by the cyclohexane, mainly due to the large rotor diameter and peripheral velocity.

The *tip clearance losses*, here referred to an average 3% clearance fraction of blade height and to backswept configuration, give a relevant contribution on the total losses, ranging from 10% of R1234yf to 21% of CycloHexane. Generally, higher tip clearance losses are shown by fluids having higher ratio between radial clearance and inlet blade height (ε/b_2), in agreement with eq. (43). The related overall efficiency drop ($\Delta\eta_{ts,cl}$) is within the 2 - 6.5% range and agrees with literature data [30, 39].

549 The highest relative contribution to the losses (60 – 70%) is given by *secondary flows* in the rotor,
550 which are due to the high blade curvature and pressure gradient through the blade vanes. As
551 suggested by Rodgers [40], this contribution is shared between blade loading and profile curvature.
552 The former represent the highest contribution to overall losses, ranging from 27% to about 50% of
553 total. The highest values are found for R134a and 1234yf, which have the lowest number of blades
554 and thus the highest blade loading. For this reason, cyclohexane shows the lowest relative
555 contribution of blade loading losses, in agreement with equation (48). Their contribution to the
556 efficiency reduction ($\Delta\eta_{ts,cp}$) is variable between 8 and 13 %.

557 The contribution of profile losses to the overall turbine losses ranges from 12% of R134a to 27% of
558 R1234yf. Their share on the overall losses is generally higher for hydrocarbons, ranging from 20 to
559 26%. It is attributable to the combined effects of rotor geometry (i.e. higher d_3/d_2), fluid properties
560 (i.e. sound speed) and kinematic conditions (i.e. higher relative velocities w_2 and w_3), in agreement
561 with eq. (49). They provide a total to static efficiency drop ($\Delta\eta_{ts,p}$) variable from 3 to more than
562 8%..

563 The *friction losses in the rotor* also represent an important contribution to the overall efficiency
564 drop, reducing its value ($\Delta\eta_{ts,a}$) from 0.9 to about 2.3 percentage points. They are generally higher
565 in cases of turbines with larger wet surface like, for example, cyclohexane.

566 Finally, it is also important to consider the *kinetic energy loss at expander output*.. Even though a
567 diffuser has always been considered here, this loss is representative of the difference between total
568 to total (η_{tt}) and total to static (η_{ts}) efficiency of the expander. On the whole, these losses are not
569 negligible, as they can reach up to 12% of the total. They are capable to reduce the overall
570 efficiency ($\Delta\eta_{ts,v}$) from 0.39 to 1.15 %. It is thus possible to recommend that a diffuser is always
571 included in the design of these expanders.

572 The comparison of the results with those achieved by Rohlik [14, 17, 20] shows a substantial
573 agreement, even though they were referred to air. Specifically, the highest contributions to the
574 overall efficiency drop are given by blade loading and profile losses into the rotor. A similar
575 behavior is found also for the remaining losses, even though the stator and disk friction losses
576 calculated in this model give a generally lower contribution compared to [17]. On the contrary, the
577 tip clearance losses are generally higher than those proposed by Rohlik [17]. Anyway, they are on
578 line with those originally proposed by NASA [30] and successively confirmed by numerical
579 calculations [39]. The comparison of the total to static efficiency of the here designed expanders
580 with those achieved by Rohlik in the maximum efficiency curve [30] and with the experimental

581 results from similar machines [24], shows a good agreement regarding the design specific speed,
582 whereas lower values of the efficiency are achieved here, especially for radial bladed expanders
583 (figure 18). This is mainly due to the different size, geometry and working fluids considered in the
584 present investigation, as well as to the different relationships adopted for the calculation of losses.
585 Finally, the comparison of the results achieved in this work with those coming from experimental
586 campaigns of Benson [24] shows a complete agreement.

587

588 **7. Off design performance prediction of the radial turbo-expander**

589 The above discussed design procedure can be used to build the characteristic curves of the
590 expanders, which fundamental to predict their off- design behavior. When dealing with ORC
591 working partially or totally with not continuously available renewables, it often happens that they
592 work most of time under off design conditions. It is the case, for example, of solar power stations or
593 integrated geothermal – solar binary cycles, like those proposed in [10] and [36], where the variable
594 amount of available solar heat leads to variable massflowrate and/or thermodynamic conditions of
595 the produced organic steam at the turbine inlet. When the characteristic curves of the designed
596 expanders are known, they can be used to provide an estimate of their performance under variable
597 inlet conditions (off design) and - when possible- to adjust their rotational speed in order to
598 minimize the losses.

599 Specifically, in this chapter we analyze the buildup of characteristic curves with variable
600 temperature, pressure and fluid mass flowrate at the turbine inlet (the latter depends on pressure
601 drop through the expander), for different values of the corrected speed:

$$602 \quad N_c = \frac{N}{\sqrt{T_{01}}} \quad (55)$$

603 In the off design approach, the turbine geometry is specified, resulting from the design procedure
604 described in the previous sections. Specifically, the following geometric parameters are given as
605 inputs:

- 606 - Blade height
- 607 - Stator and rotor inlet/outlet diameters
- 608 - Passage section areas
- 609 - Number of rotor and stator blades
- 610 - Blade metal angles

611 Moreover, in the present case the rotor outlet total pressure is maintained at the design value,
612 because it is fixed by the conditions at the condenser to satisfy the cogeneration conditions.
613 Anyway, with the developed model it is possible to let it be variable, in order to take into account of
614 the variable conditions at the condenser due to the timely change of the environmental conditions.

615 The loss correlations under off design conditions are the same adopted in the expander design, as
616 well as those relating flow and metal angles. These allow the determination of the actual flow
617 angles given the blade metal angles and the size of the expander, both coming from the design
618 procedure.

619 In order to analyze the off-design behavior of the turbo expander, the inlet total pressure and
620 temperature are changed. Consequently, the mass flowrate is determined from the inlet – outlet
621 pressure drop. Another possible way of estimating the off-design behavior is to fix the expander
622 inlet mass flowrate and total temperature and calculate the total pressure. Thus, the performance
623 maps of the expander (i.e. characteristic curves) are achieved by building the related curves under
624 design conditions for different values of N_c . They may be directly adopted within the
625 thermodynamic code for the analysis of the ORC. .

626 **7.1 – performance behavior at off design expander pressure ratio**

627 The curves of power output, mass flowrate and efficiency vs. off design pressure ratio at different
628 values of corrected speed N_c are shown on figures 19, 20 and 21 respectively, each one reporting
629 the results for both radial and backswept configurations. For the sake of brevity, the off design
630 analysis is carried out for R134a only, without losing in generality, as the behavior is qualitatively
631 similar for all the investigated fluids. From figure 19, it is clear that the power output is reduced
632 when the expansion ratio falls below the design value. Moreover, it is important to notice that the
633 power output is reduced with reducing the corrected speed: this is the result of the lower enthalpy
634 drop, due to the lower rotational speed, which is also responsible for the reduced variation of
635 peripheral velocity (i.e. the effective component for momentum). Finally, it should be remarked that
636 the reduction of power output with respect to the nominal value is larger than the reduction of the
637 pressure ratio, when N_c is at the design value: for example, when the expansion ratio is at 90% of
638 the design value, the power output is reduced to about 88% of the design value. When the
639 expansion ratio is at 80% of design value, the power output is about 75% of the nominal. The
640 overall behavior is found for both IFG and IFR geometries.

641

642 7.2 – performance behavior at off design expander corrected mass flowrate

643 The other typical turbomachinery off design parameter is the corrected mass flowrate, defined as

$$644 \quad \dot{m}_c = \frac{\dot{m} \sqrt{T_{01}}}{p_{01}} \quad (56).$$

645 Starting from the values of the total inlet pressure and temperature, from the maps of figure 20 it is
646 possible to determine the mass flow rate with variable expansion ratio for the different values of
647 the ratio $N_c/(N_c)_{des}$. It can be noticed that the radial bladed rotors (IFR) are more influenced by
648 operation under off-design pressure ratio than the backswept ones (IFG).

649 Figure 21 shows the behavior of total to static efficiency (expressed by the off-design to design
650 ratio) as a function of the variable (off-design) expansion ratio. It is important to remark the
651 relatively low sensitivity to the expansion ratio for corrected speeds below the design value, due to
652 the typical accelerating behavior of fluids into the expanders, which allows to work into a relatively
653 wide range of incidence angles with only a limited increase of loss coefficients. Hence, the
654 reduction of N_c below the design value could be regarded as a way to reduce the efficiency drop of
655 the expander at reduced values of the expansion ratio. As it is seen, the investigated expanders show
656 a relatively limited sensitivity of the efficiency to off design expansion ratio, unless it is reduced to
657 very low values. It is interesting to notice the increasing efficiency (1.5 – 2%) at corrected speed
658 lower than the design value. To better understand this behavior, it is important to analyze the
659 variability of velocity triangles under the three different values of corrected speed (figure 22), Let
660 us consider a reduction of corrected speed N_c at fixed thermodynamic conditions of the inlet fluid.
661 As the inlet – outlet expander pressure drop is fixed, the flow rate remains practically unchanged.
662 The lower variation of peripheral velocity leads to a reduction of stage and nozzle enthalpy drop
663 (notice that $R \neq 0$). For this reason, the pressure at nozzle exit is higher, which leads to an increase of
664 fluid density. As the mass flow rate is constant, the meridional component of absolute velocity
665 must be reduced, thus the related velocity triangle height decreases. When N_c is reduced, at fixed
666 total inlet temperature (T_{01}), the rotational speed of the expander decreases largely, and thus also
667 the rotor peripheral velocity. The latter entails an increase of relative flow angle at rotor inlet (β_2) in
668 the same direction (see the modified red and green triangles of figure 24). Hence, the shape of the
669 velocity triangles is modified towards the backswept configuration. This modification affects the
670 loss coefficients of the expander. Specifically, a reduction of N_c down to 60 – 65% of the design
671 value leads to an increase of the incidence losses and a reduction of the stator, friction and
672 secondary flow losses, as reported on figure 25. This effect is due to the change of velocity profiles
673 within the rotor vanes. Finally, it is interesting to analyze the behavior of the ratio of the off-design

674 to design total to static efficiency ($\eta_{ts}/(\eta_{ts})_{des}$) vs the corrected speed ratio ($N_c/(N_c)_{des}$), shown on
675 figure 24 for both radial and backswept configurations, at fixed design inlet total pressure and
676 temperature. In both cases, η_{ts} is maximized at values of $N_c/(N_c)_{des}$ where the sum of losses is
677 minimum.

678

679 8. Conclusions

680 The paper describes the features and analyzes the results of a zero dimensional model for the design
681 of high efficiency small size ORC expanders. Two basic rotor blade geometries (radial IFR and
682 backswept IFG) and six different possible organic working fluids (R134a, R1234yf, R236fa,
683 R245fa, Cyclohexane, N-Pentane) have been analyzed and discussed. In all cases, the power output
684 has been fixed at about 50 kW. The reference thermodynamic data for the specific application are
685 taken from [36]. The relationships for the estimation of the expander losses, as well as the main
686 design parameters, have been collected by an extensive investigation of models and experimental
687 data available in literature for radial turbo-expanders. Generally, literature for radial expanders is
688 much less rich than for axial turbines and, often, data and models are derived from these last and
689 from centrifugal compressor applications, with limited adaptmets. Moreover, these relationships
690 and models are referred to ideal gases and Mach relationships, which is also the approach often
691 applied in many CFD calculation codes. In the present work, the model applies the most recent,
692 currently available, equations of state of the investigated real fluids expanding into the ORC
693 turbine.

694 For the investigated fluids, , the rotor diameters are in the 80 – 110 mm range and the rotational
695 speed is variable between 30000 and 50000 rpm (specific speed is always below 0.1). The only
696 exception is CycloHexane, which needs higher rotor diameters (190 – 200 mm) and a lower
697 rotational speed. The designed expanders are mostly supersonic and have high values of nozzle exit
698 angles ($\alpha_2=77 - 83.5^\circ$), whereas the deflection of flow in the rotor is between 40 and 90 °. The
699 outlet rotor hub to tip diameter ratio resulting from the developed calculation code is within the 0.39
700 – 0.52 range, which agrees with literature data. In order to partially recover the outlet kinetic energy
701 of flow, the adoption of a diffuser at rotor outlet is proposed, which is particularly recommendable
702 for the expansion of the cyclohexane, which has the highest value of Mach at rotor discharge.

703 The expected total-to-total efficiency of the designed units ranges between 0.72 and 0.80,
704 depending on the considered fluid and geometry configuration. The highest contribution to the

705 expander efficiency losses is given by the secondary flows within the rotor (blade loading and
706 profile curvature), due to the high curvature of blade profiles and the high pressure gradient.

707 The investigation, on the basis of overall thermodynamic, power plant and fluid dynamic features,
708 shows that the most suitable fluids are R236fa and R245fa, followed by R134a and R1234yf,
709 whereas the worst ones are CycloHexane and CycloPentane, which are further penalized by the
710 sub atmospheric pressure at the expander output.

711 Generally, backswept bladed rotors (IFG) show 1.5 to 2.5% higher efficiencies. Moreover, they
712 have larger values of the number of blades, load coefficient, nozzle exit angle and rotor deflection
713 angle than the corresponding radial bladed ones. The peripheral speed and the reaction degree of the
714 backswept configurations are instead lower. Generally, the results of the design and parametric
715 analysis are in agreement with literature data. The proposed calculation model has been successively
716 used to predict the off- design performance, through the construction of the expander characteristic
717 curves (corrected mass flowrate, power output and efficiency as functions of expansion ratio and
718 corrected speed N_c). These last can be directly introduced into the thermodynamic ORC calculation
719 code to evaluate the off design behavior of the expander under variable thermodynamic inlet
720 conditions (total temperature and pressure and mass flow rate for different values of corrected
721 speed), once the geometry of the expander is defined.

722 The calculation tool is open to improvements by the use of 2D – 3D CFD models and experimental
723 tests on existing rotors, which potentially could allow the improvement of the less validated
724 correlations, making it effective and reliable for the design and off-design analysis of radial turbo-
725 expanders for small-size ORC powerplants.

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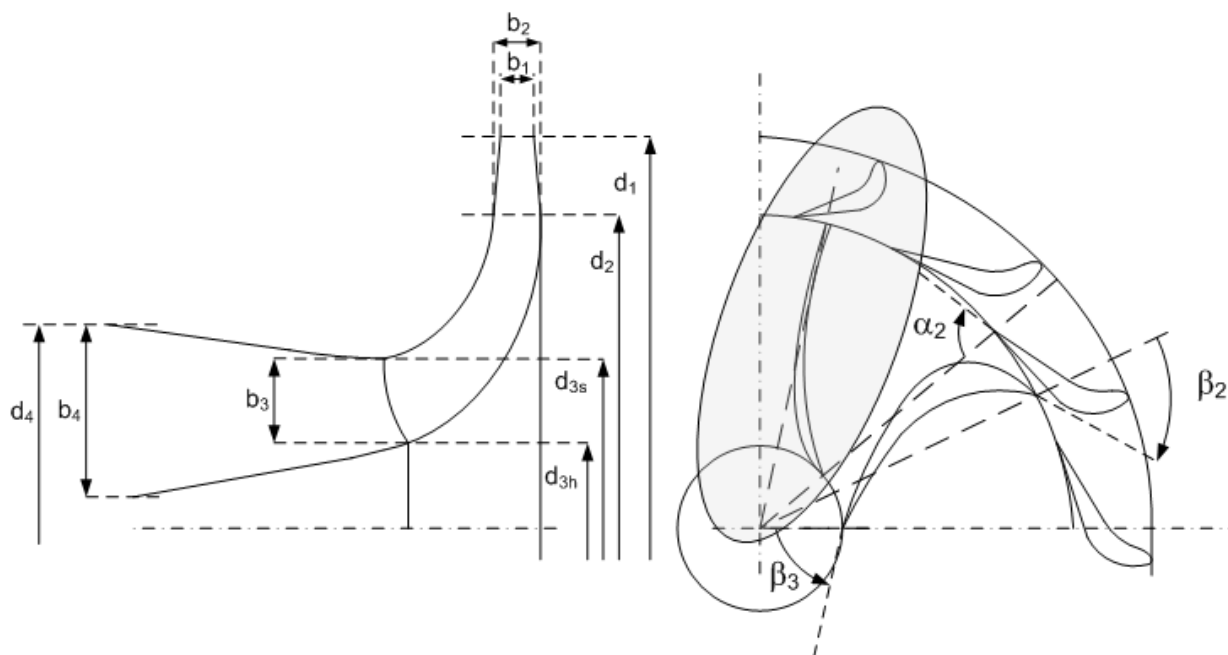
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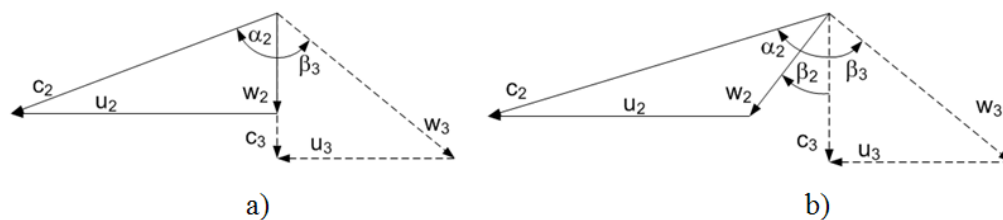


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750 *Figure 1 – General schematic of Radial inflow turbine: 90° IFR (shaded), and General rotor shape*
 751 *(IFG, unshaded).*

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756 *Figure 2 – Velocity triangles – nominal conditions; (a) 90° IFR (b) General (IFG)*

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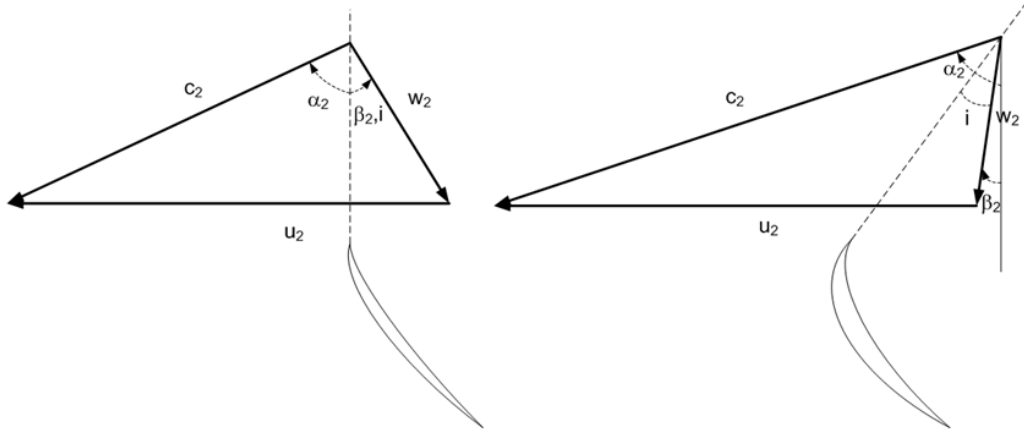


Figure 3 – Velocity triangles – optimal incidence conditions; (a) 90° IFR (b) IFG

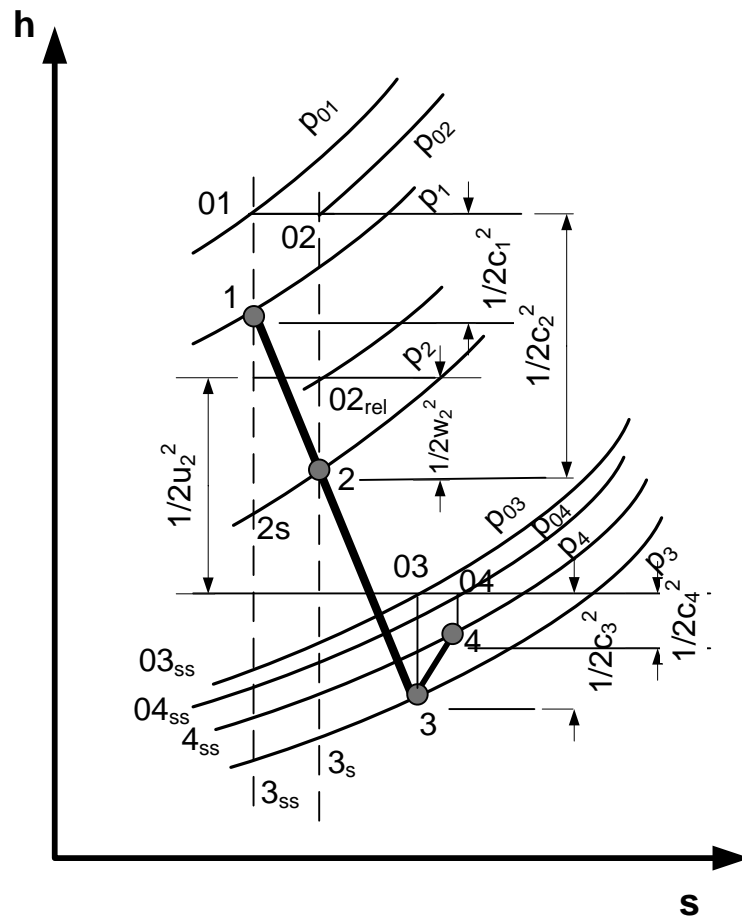


Figure 4: Enthalpy – Entropy representation of the expansion process [14].

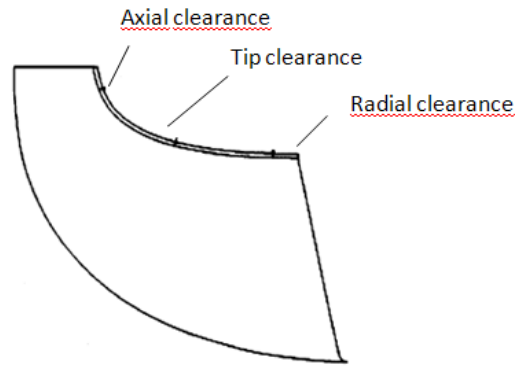


Figure 5: Tip clearance along the development of a rotor blade.

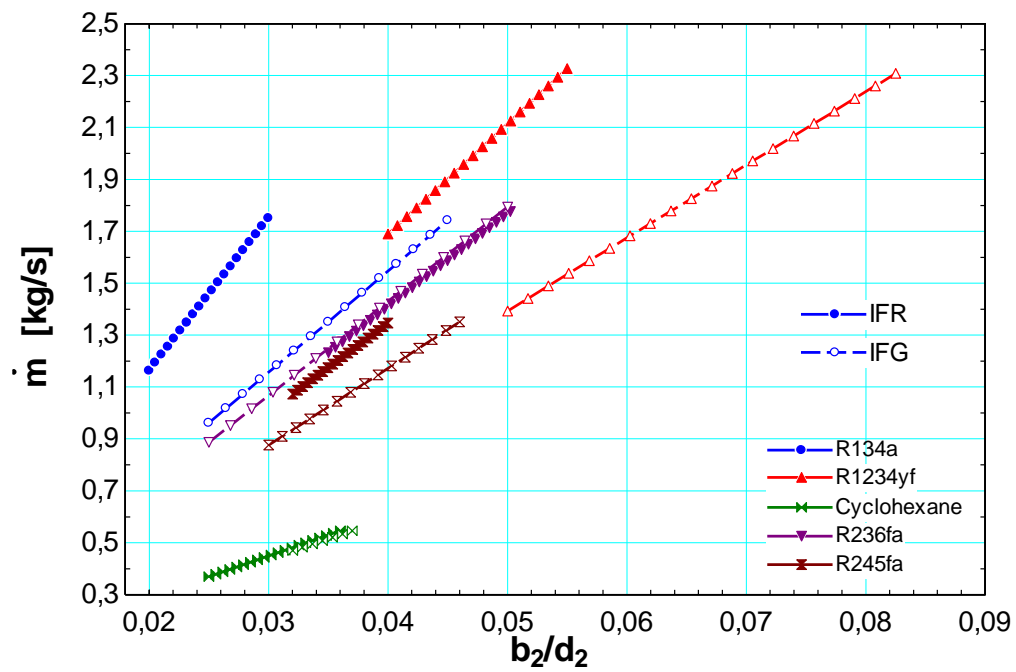


Figure 6: mass flow rate and ratio of rotor exit diameter at hub and shroud as a function of ratio between blade height and diameter at rotor inlet b_2/d_2 (IFR and IFG)

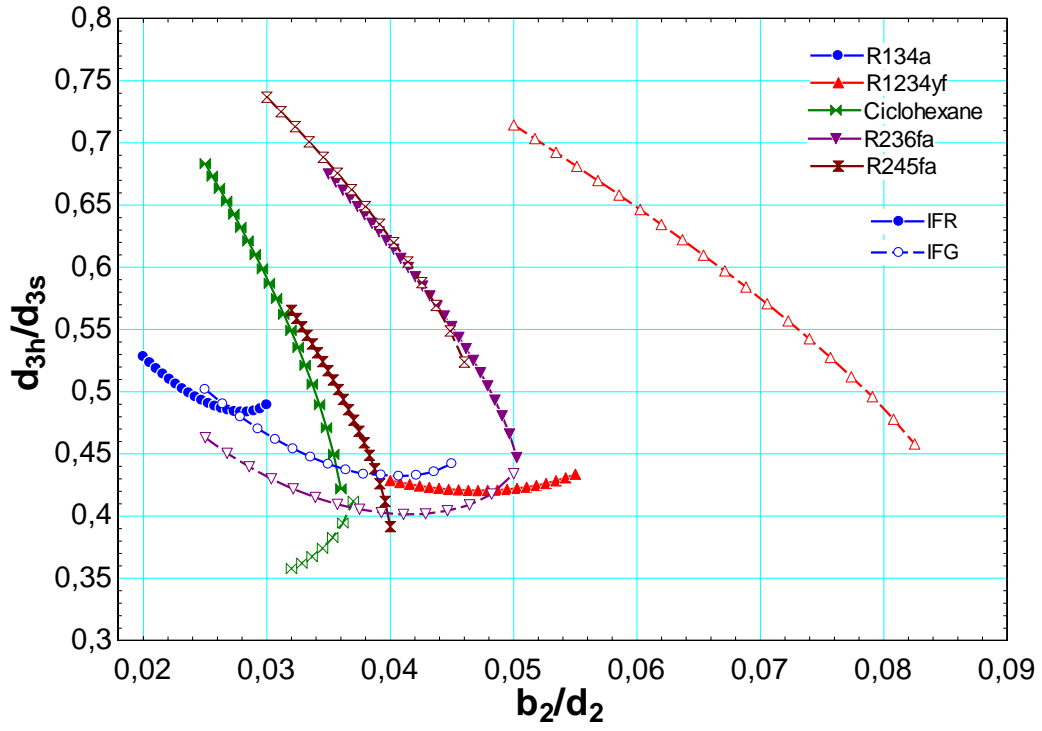


Figure 7: Mass flow rate and ratio of rotor exit diameter at hub and shroud as a function of ratio between blade height and diameter at rotor inlet b_2/d_2 (IFR and IFG)

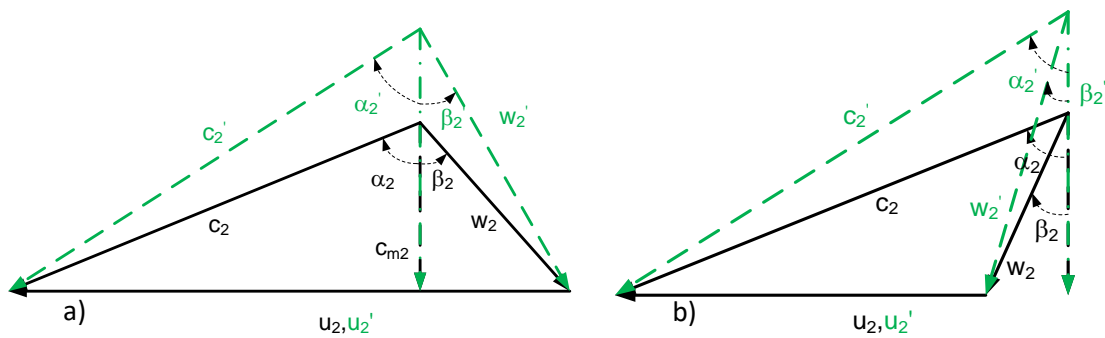


Figure 8: Variation of velocity triangles with increasing flow coefficient

(from solid black to dashed green, a) IFR, b) IFG)

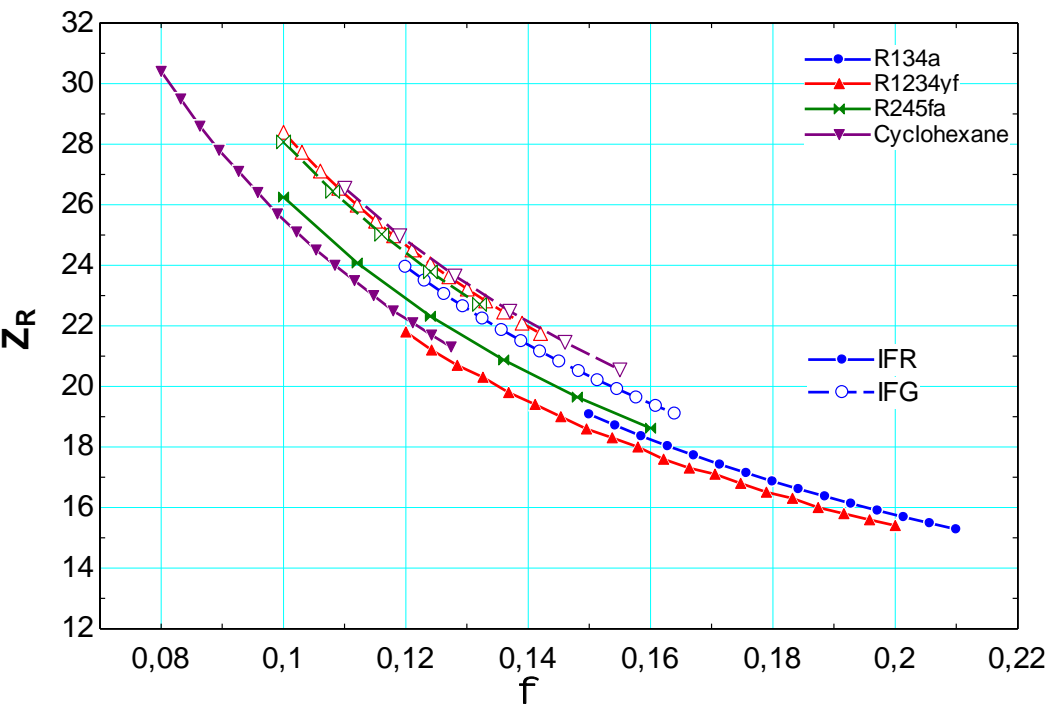


Figure 9: Number of rotor blades as a function of flow coefficient (IFR and IFG)

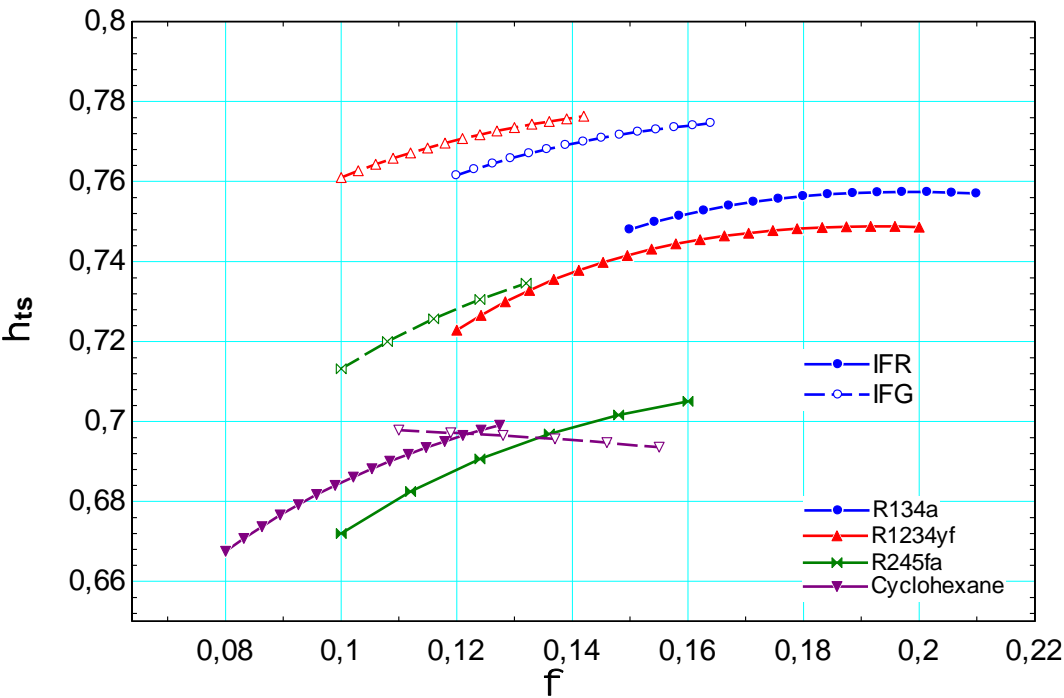


Figure 10 –Total-to-static efficiency as a function of flow coefficient (IFR and IFG)

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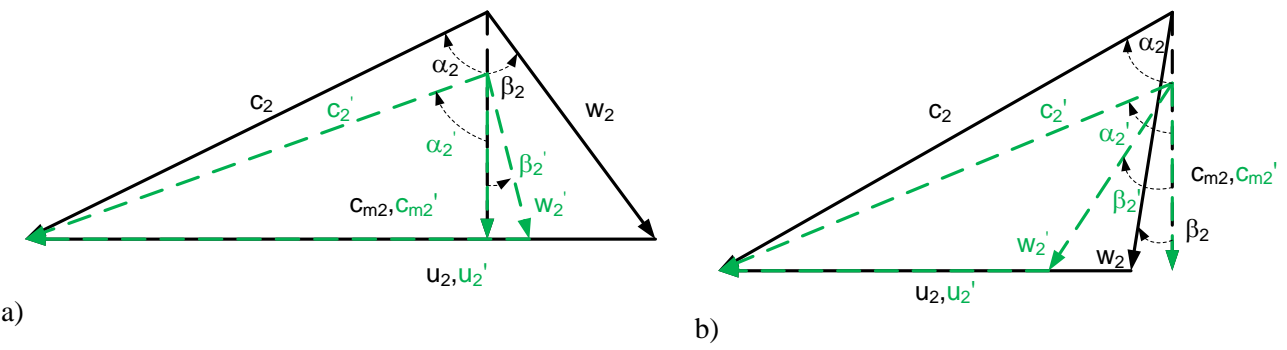
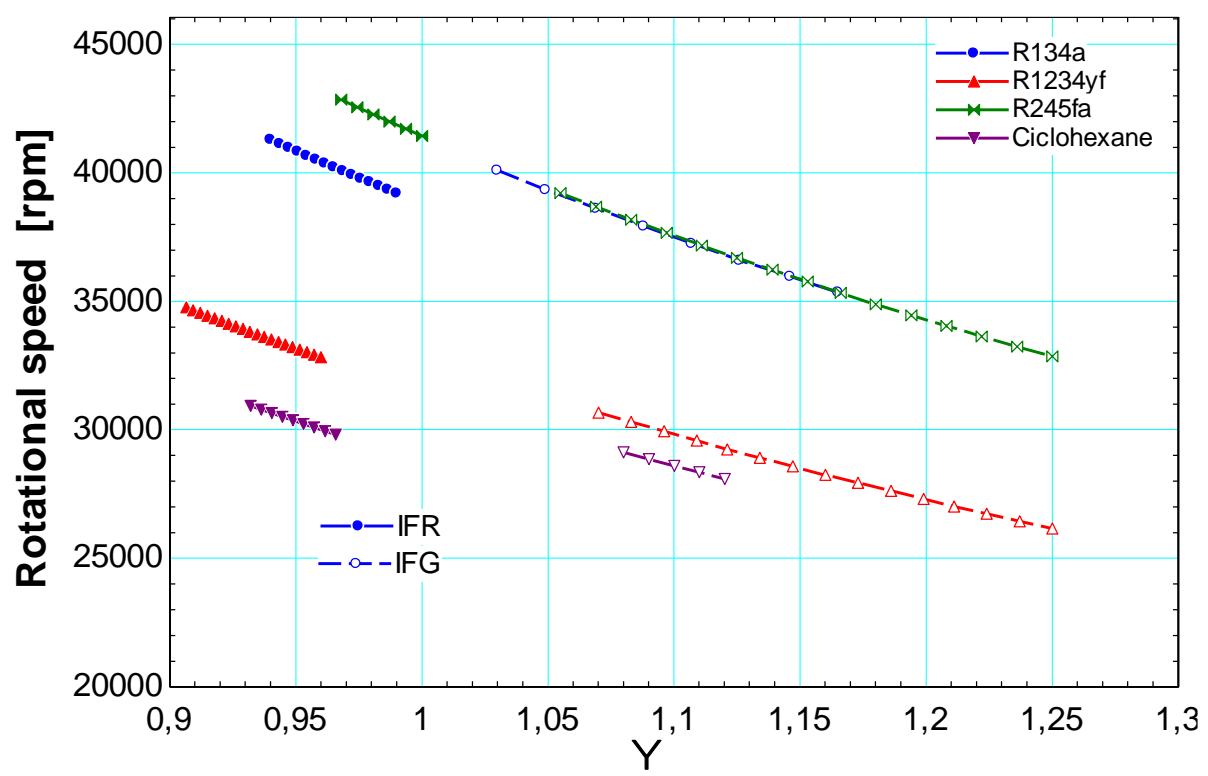


Figure 11 – Variation of velocity triangles at rotor inlet with increasing load coefficient (from black solid to green dashed) a): IFR; b) IFG

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Figure 12: Speed of revolution vs. load coefficient (IFR and IFG)

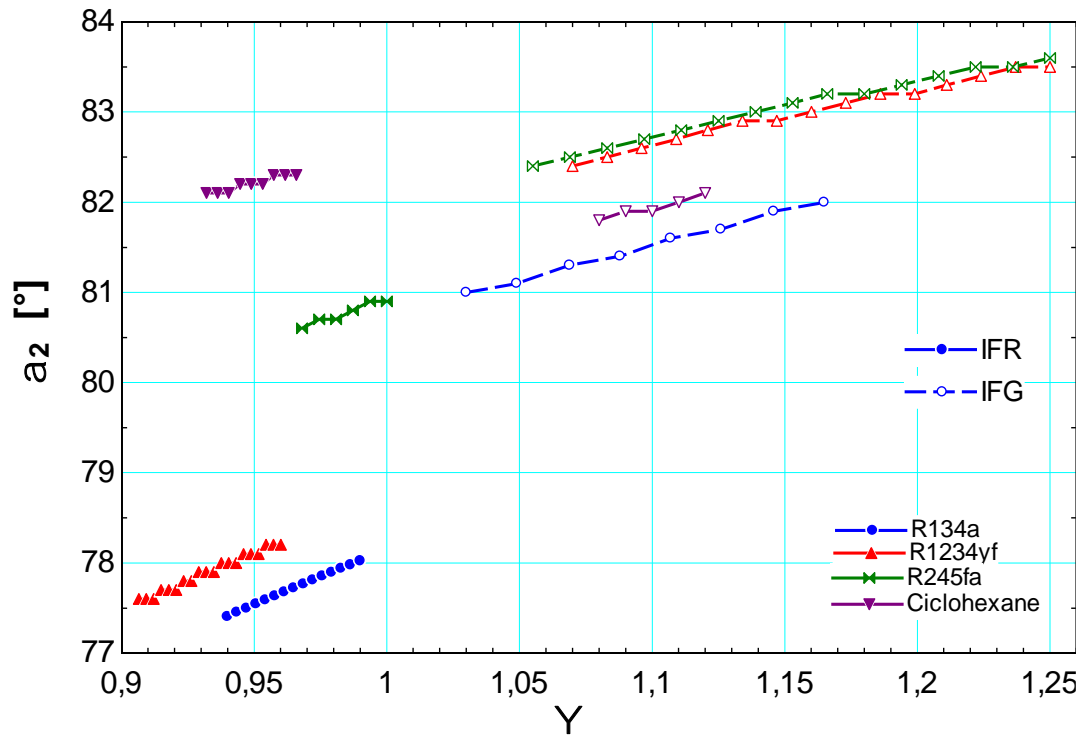


Figure 13 –Angle of absolute flow velocity (α_2) at nozzle outlet/rotor inlet vs. load coefficient (IFR and IFG)

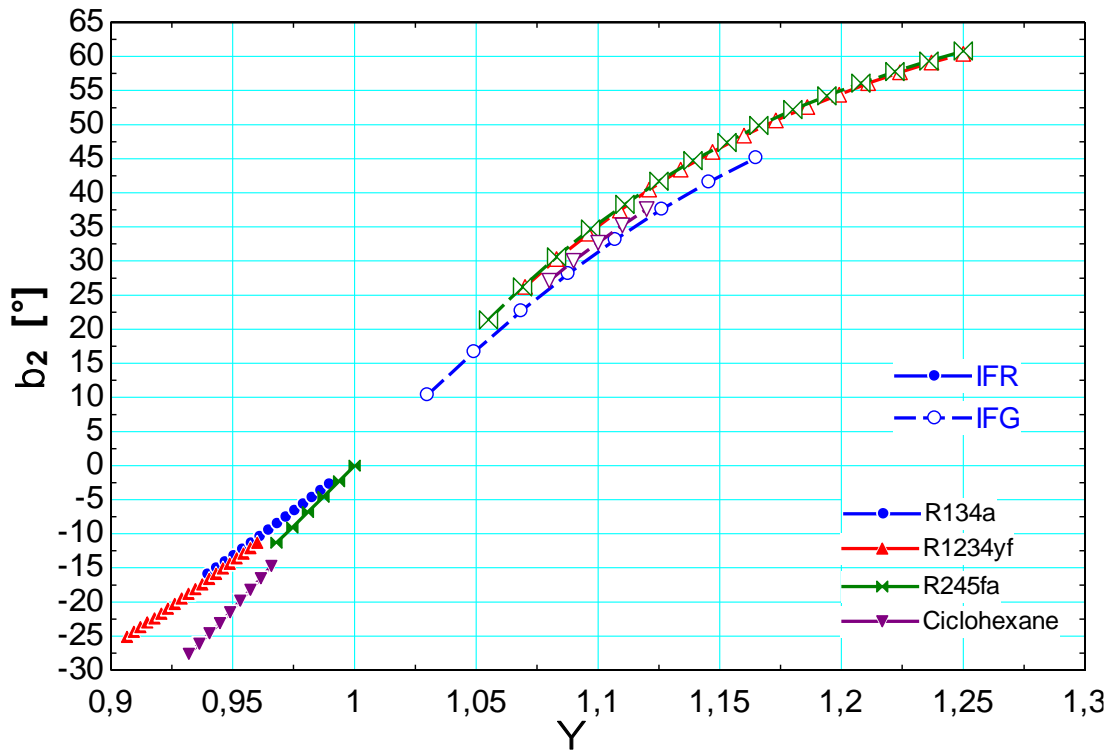


Figure 14 –Angle of relative flow velocity (β_2) at nozzle outlet/rotor inlet vs. load coefficient (IFR and IFG)

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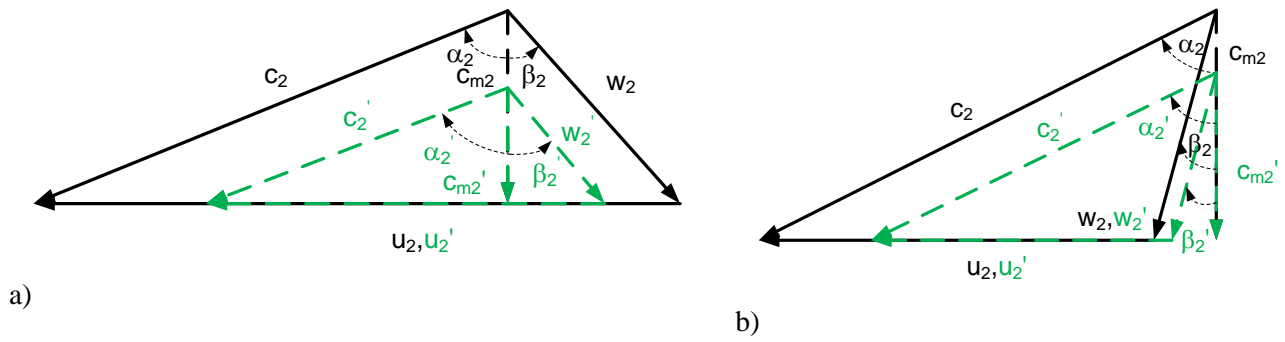
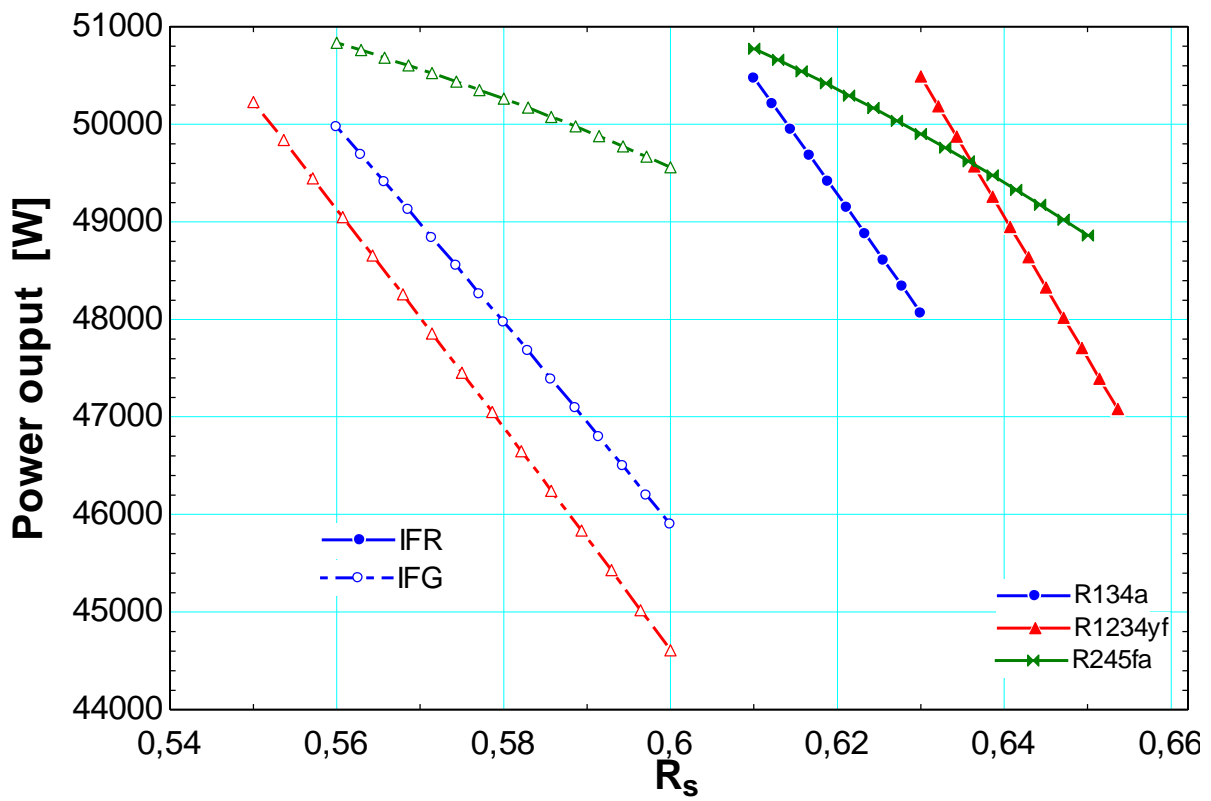


Figure 15: Variation of velocity triangles with increasing in isentropic degree of reaction R_s (from black to green, a) IFR, b) IFG)

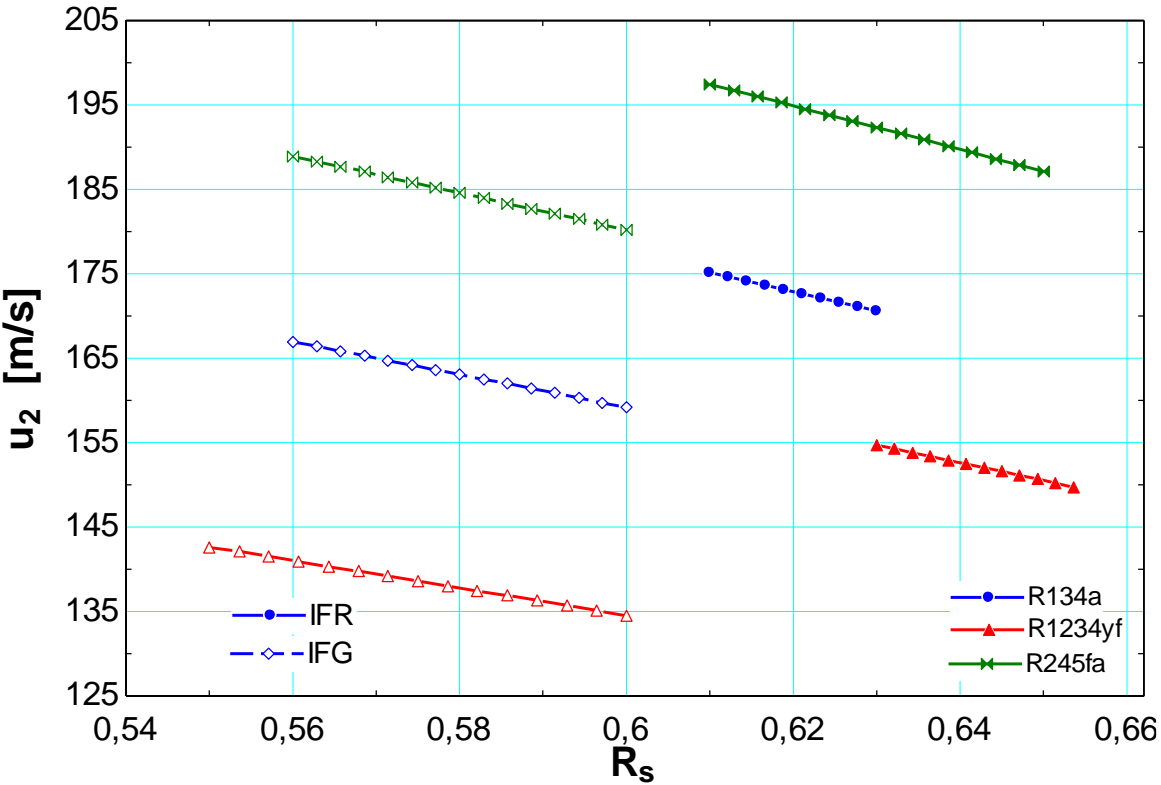
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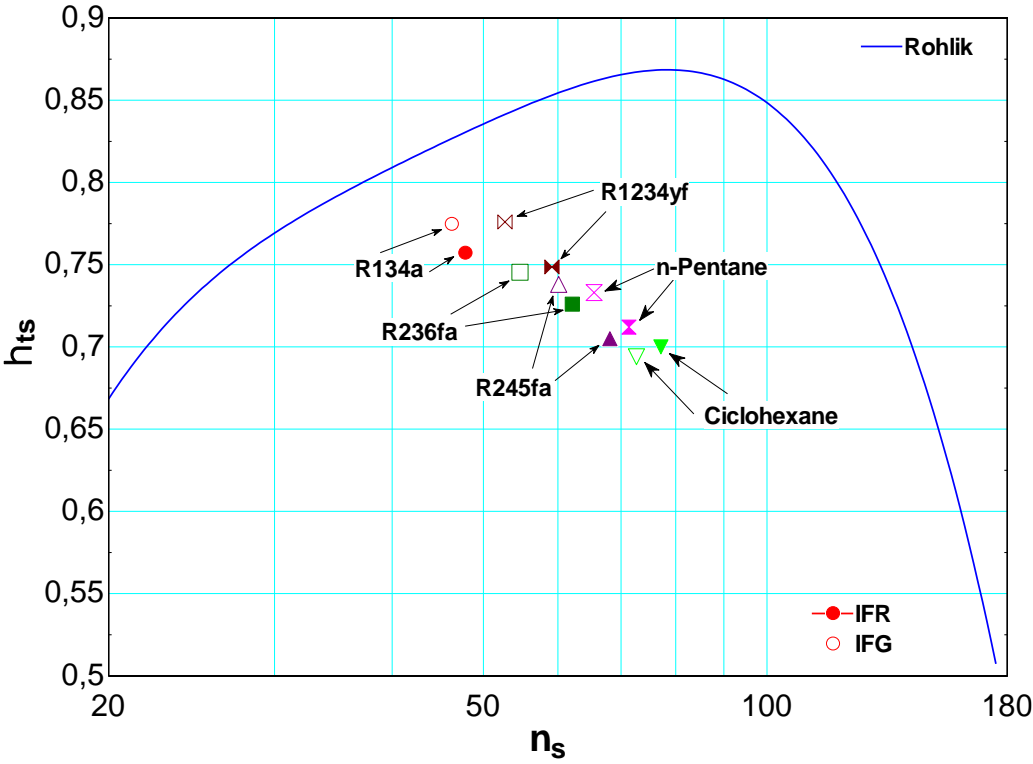
Figure 16: Expander Power output vs. isentropic degree of reaction (IFR and IFG)

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Figure 17: Expander peripheral velocity at rotor inlet vs. isentropic degree of reaction (IFR and IFG)



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Figure 18: comparison of total to static efficiency of the here designed models with the results of Rohlik [30] (IFR and IFG)

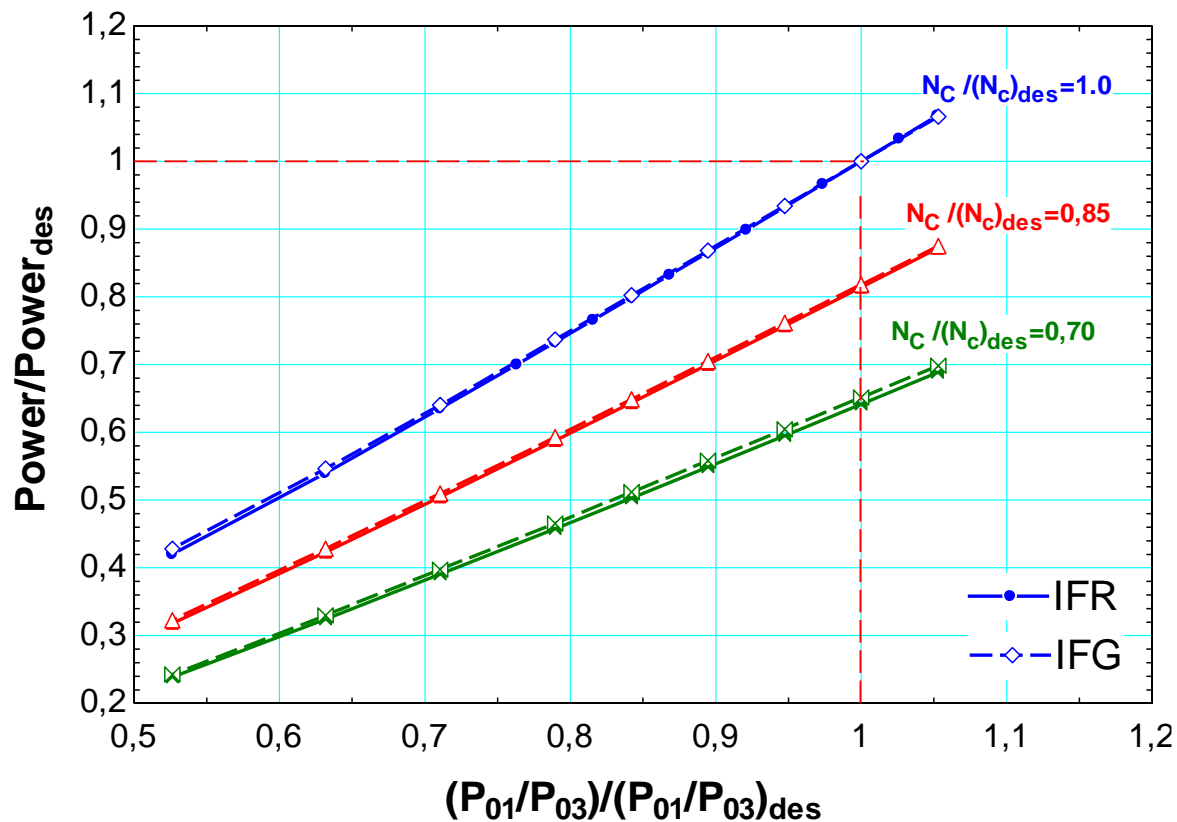


Figure 19 – Power output ratio – expansion ratio characteristic curve of the expander (Referred to R134a)

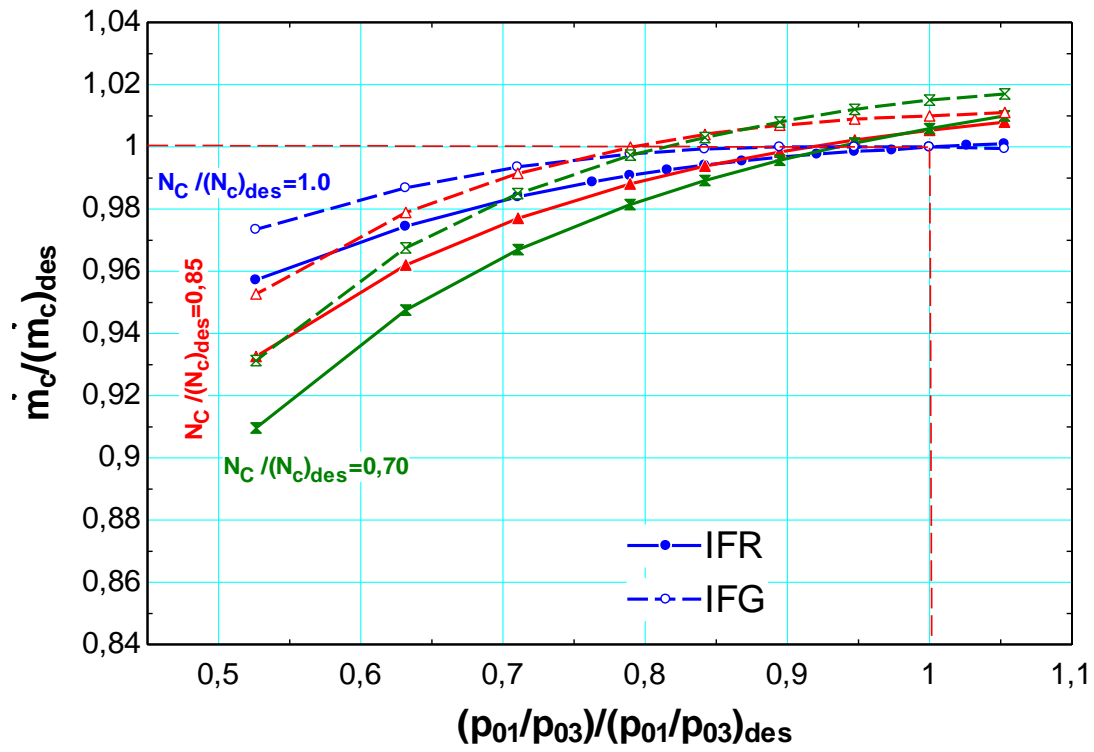


Figure 20 – Corrected flow rate – expansion ratio characteristic curves of the expander (Referred to R134a)

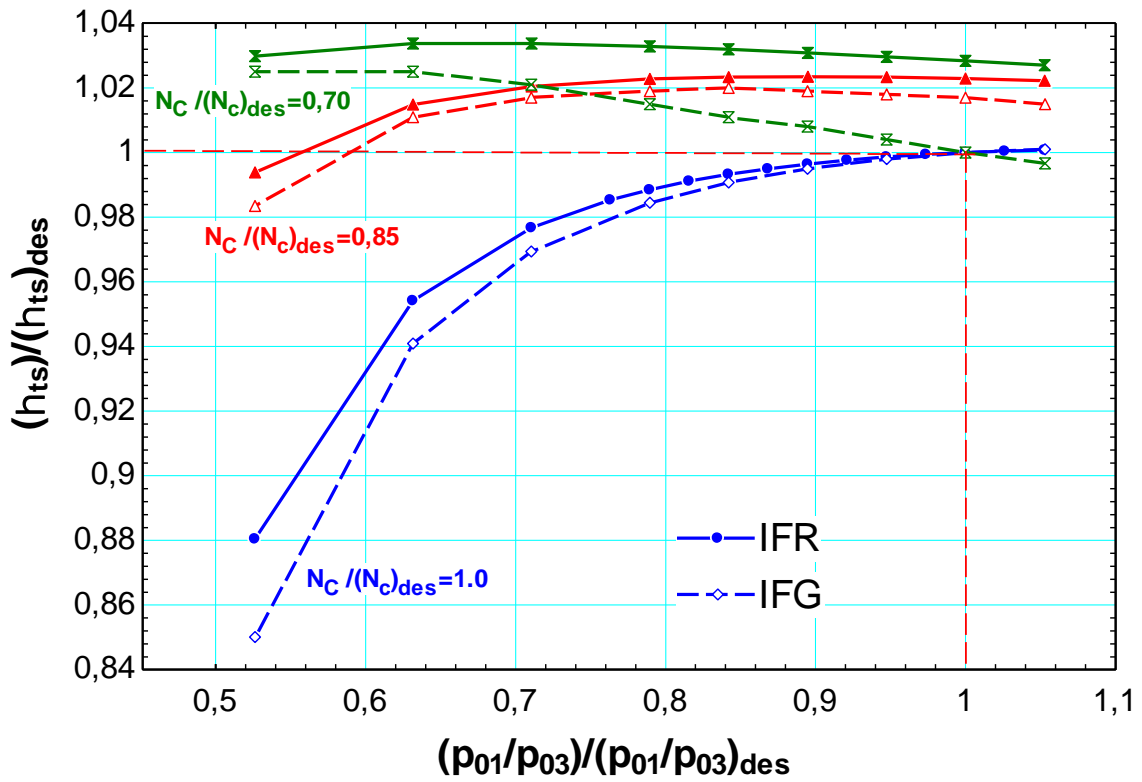


Figure 21 – Total to static – expansion ratio characteristic curves of the expander (Referred to R134a)

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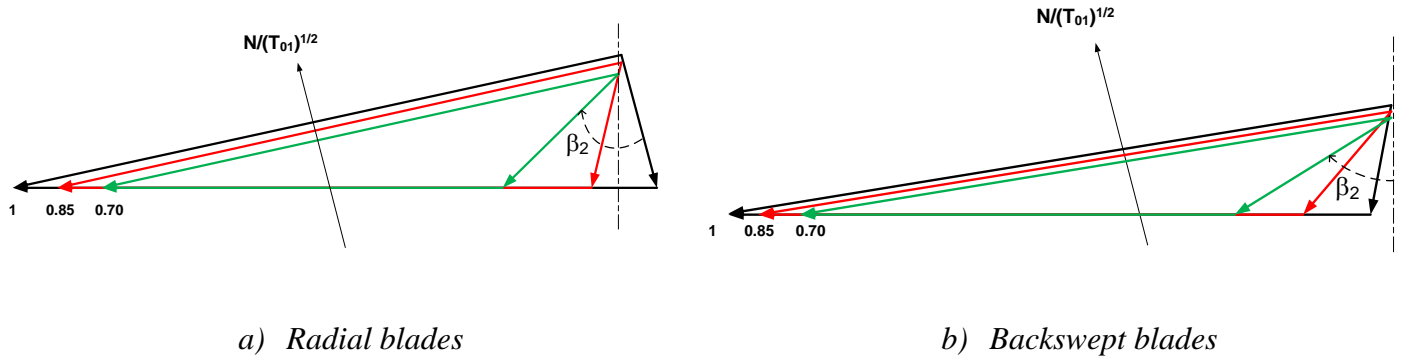


Figure 22 – Velocity triangles at rotor inlet at variable corrected speed

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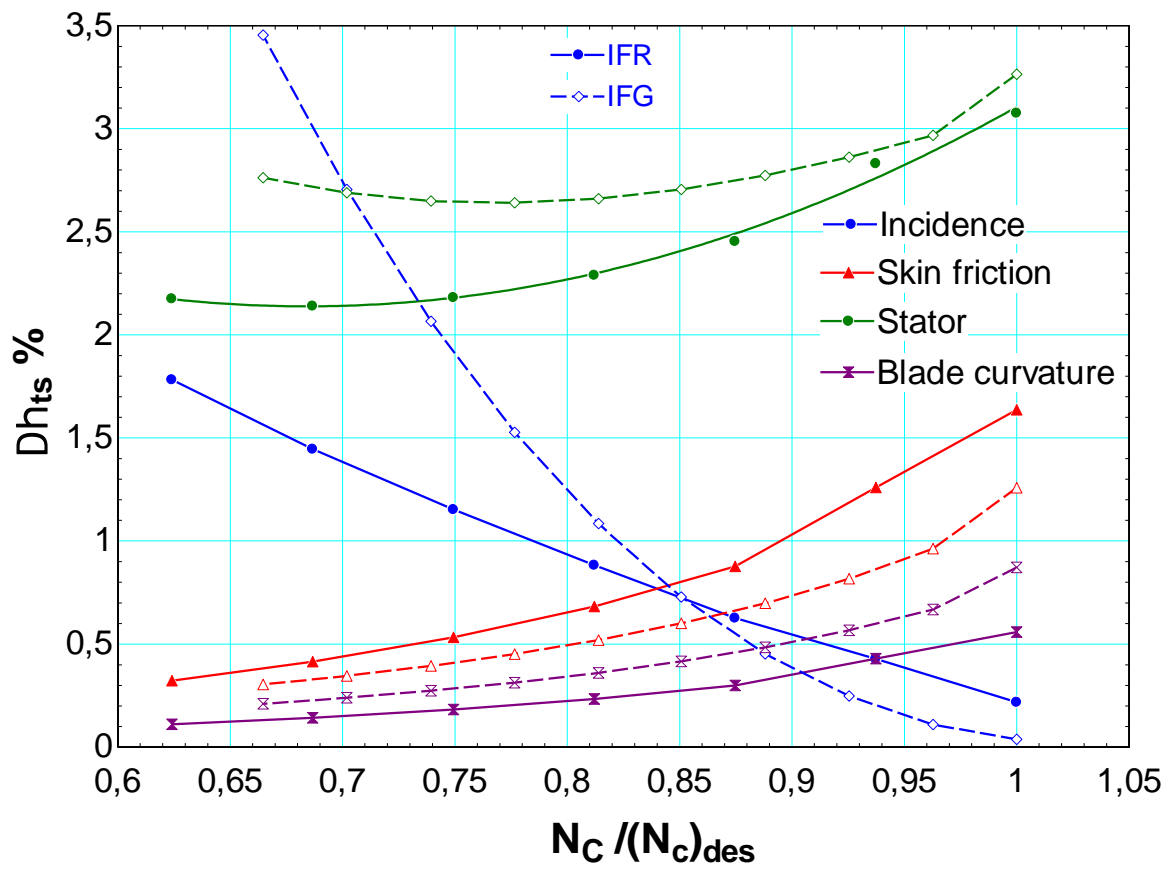


Figure 23 – behavior of total to static efficiency losses ($\Delta\eta_{ts}$) with variable N_c under off design conditions

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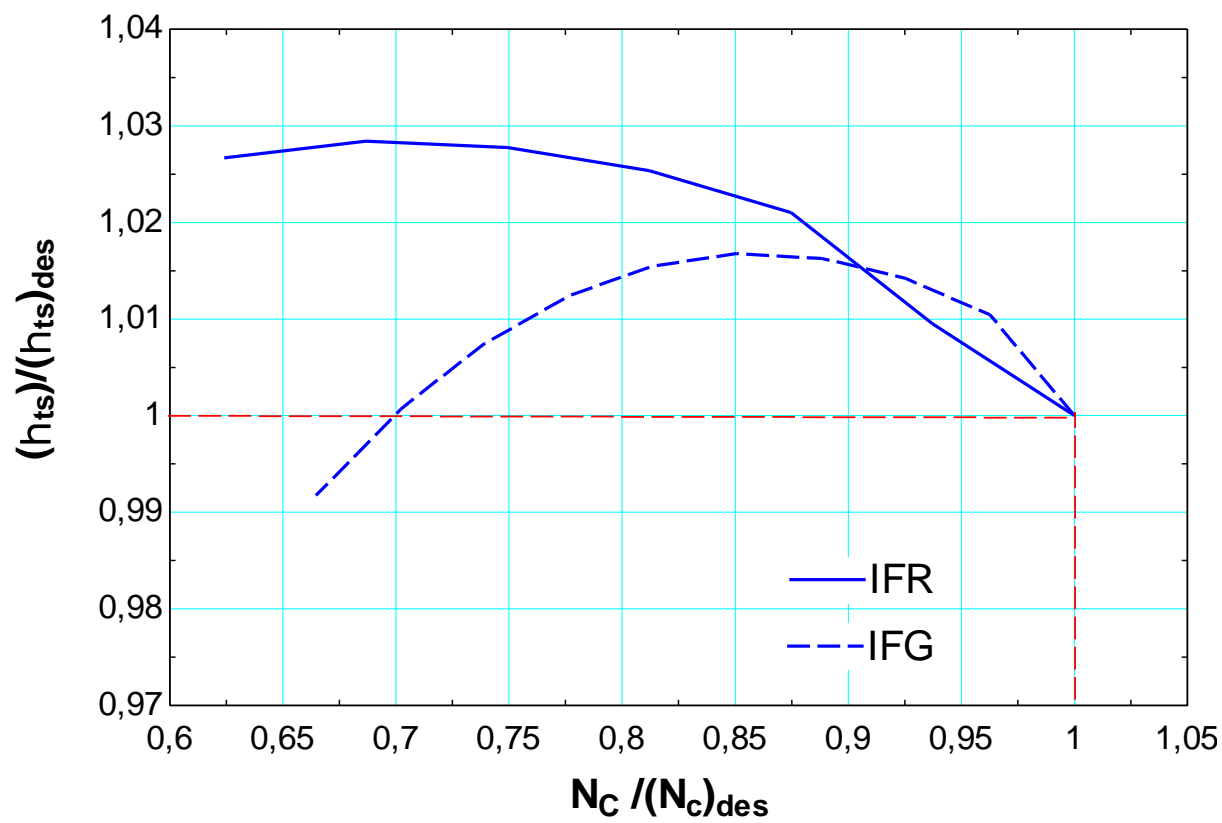


Figure 24 – Total-to-static efficiency vs N_c under off design conditions

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List of tables

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Table 1: Input data and typical range of values.

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N.	Variable	Typical values
0	Rated Power Output, kW	Variable, typically 5 to 500 kW
1	Fluidname	R134a, R1234yf, R245fa, R236fa, Cyclo-Hexane, N-Pentane
2	Total inlet pressure p_{01} [kPa]	Variable, typically 500 to 4000
3	Total inlet Temperature T_{01} [°C]	Variable, typically 100 - 200
4	Isentropic enthalpy drop Δh_{ss} [kJ/kg]	Variable, typically 28 to 130
5	Work coefficient Ψ	0.90-1.10
6	Flow coefficient Φ	0.13-0.21
7	Isentropic degree of reaction R_s	0.55-0.63
8	Rotor inlet diameter d_2 [m]	0.08-0.195
9	Nozzle geometry ratio d_1/d_2	1.30-1.80
10	Rotor geometry ratio d_3/d_2	0.45 – 0.60
11	Diffuser geometry ratio d_4/d_3	1.4 – 1.6
12	Diffuser length – diameter ratio L_d/d_3	1.5 – 2. 5
13	Rotor aspect ratio b_2/d_2	0.03 – 0.08
14	Nozzle height ratio b_1/b_2	0.8 – 1

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Fluid Name	R134a		Cyclohexane		N-Pentane		R245fa		R1234yf		R236fa	
Rotor geometry	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR
Rated power output [kW]	50	50	50	50	50	50	50	50	50	50	50	50
Total inlet pressure p_{01} [bar]	38	38	5	5	10	10	31	31	31	31	30	30
Total inlet Temperature T_{01} [°C]	147	147	147	147	147	147	147	147	147	147	147	147
Isentropic enthalpy drop Δh_{ss} [kJ/kg]	38	37	131	132	105	102	54	51	30	28	38	38
Work coefficient Ψ	0.94	1.03	0.93	1.08	0.95	1.05	0.97	1.06	0.91	1.07	0.95	1.04
Flow coefficient Φ	0.21	0.16	0.13	0.16	0.19	0.17	0.16	0.14	0.20	0.14	0.15	0.15
Isentropic degree of reaction R_s	0.61	0.56	0.60	0.57	0.61	0.55	0.61	0.55	0.63	0.55	0.61	0.55
Nozzle geometry ratio d_1/d_2	1.42	1.45	1.73	1.43	1.76	1.74	1.75	1.50	1.42	1.32	1.80	1.72
Rotor geometry ratio d_3/d_2	0.46	0.47	0.55	0.62	0.51	0.54	0.54	0.54	0.56	0.48	0.52	0.55
Diffuser geometry ratio d_4/d_3	1.40	1.40	1.60	1.60	1.40	1.40	1.50	1.50	1.60	1.60	1.50	1.50

Diffuser length – diameter ratio L_d/d_3	2.0	1.5	2.5	2.5	2.0	2.0	2.0	2.0	2.5	2.5	2.0	2.0
Rotor aspect ratio b_2/d_2	0.030	0.045	0.036	0.037	0.042	0.055	0.040	0.046	0.055	0.082	0.049	0.050
Nozzle height ratio b_1/b_2	1.0	1.0	0.8	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

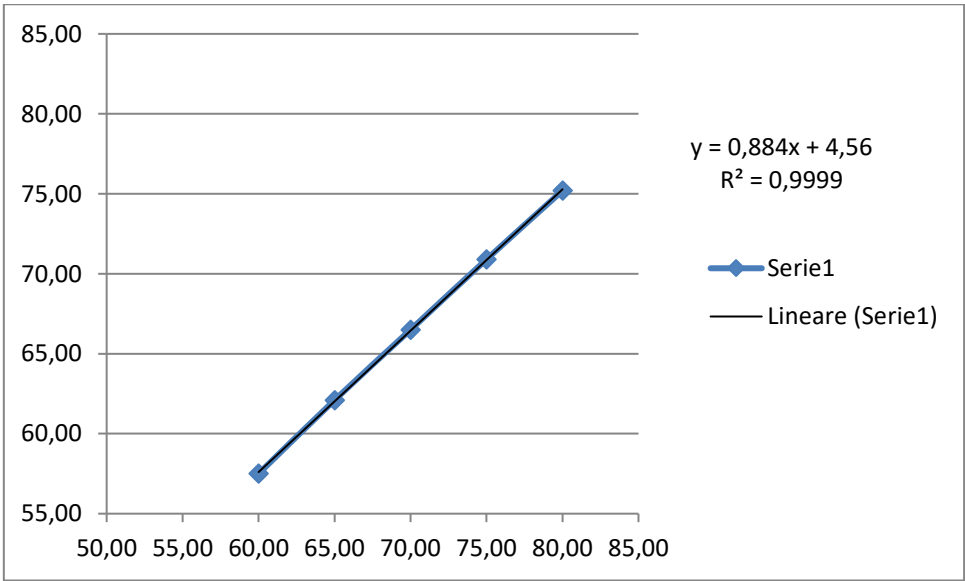
860

861 *Table 2: Nozzle setting angle as a function of absolute flow angle at nozzle outlet*

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α_2	α_{b2}
57.5	60
62.1	65
66.5	70
70.9	75
75.2	80

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870 *Table 3 – design results of the ORC expander for different working fluids (IFR and IFG)*

Fluid	R134a		Cyclo hexane		N-Pentane		R245fa		R1234yf		R236fa	
	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG	IFR	IFG
d₁ [m]	0.115	0.115	0.335	0.273	0.188	0.189	0.154	0.138	0.121	0.117	0.157	0.160
d₃ [m]	0.037	0.037	0.106	0.118	0.054	0.059	0.047	0.052	0.047	0.043	0.045	0.050
d₄ [m]	0.052	0.052	0.170	0.189	0.076	0.082	0.071	0.077	0.076	0.068	0.068	0.075
b₁ [m]	0.002	0.004	0.006	0.006	0.005	0.006	0.004	0.004	0.005	0.007	0.004	0.005
b₂ [m]	0.002	0.004	0.007	0.007	0.005	0.006	0.004	0.004	0.005	0.007	0.004	0.005
b₃ [m]	0.013	0.014	0.043	0.050	0.024	0.023	0.021	0.016	0.019	0.016	0.016	0.020
p₀₄ [bar]	9.230	9.550	0.149	0.137	0.891	0.930	1.812	2.100	8.109	8.520	3.460	3.450
Δh_{0,studio}[kJ/kg]	28.800	28.700	91.700	91.600	74.700	74.700	37.700	37.600	21.700	21.700	27.600	27.900
ṁ [kg/s]	1.750	1.749	0.546	0.546	0.672	0.677	1.346	1.349	2.327	2.308	1.815	1.793
u₂ [m/s]	175.1	166.9	313.7	291.2	280.5	266.8	197.4	188.9	154.7	142.6	170.5	164.0
Rpm	41296	40097	30932	29115	50064	46746	42845	39211	34762	30673	37418	33672
α₂ [°]	77.4	81.0	82.1	81.9	78.7	81.1	80.6	82.4	77.6	82.4	81.0	81.8
β₂ [°]	-15.9	10.4	-27.6	27.1	-14.7	16.9	-11.3	21.4	-25.1	26.2	-18.4	14.9
β₃ [°]	-63.9	-65.7	-57.9	-63.3	-56.0	-59.0	-60.8	-60.1	-70.4	-59.5	-59.6	-66.7
δβ_R [°]	48.0	76.1	30.3	90.4	41.3	75.9	49.5	81.5	45.3	85.7	41.2	81.6
M₂	1.047	1.077	1.500	1.626	1.321	1.374	1.441	1.493	0.921	0.982	1.276	1.325
M₃	0.240	0.211	0.563	0.470	0.461	0.412	0.412	0.423	0.190	0.249	0.375	0.277
M₄	0.120	0.106	0.191	0.166	0.216	0.197	0.171	0.174	0.073	0.095	0.157	0.119
M_{r2}	0.237	0.172	0.235	0.261	0.268	0.223	0.240	0.211	0.219	0.144	0.210	0.196
M_{r3}	0.537	0.514	1.059	1.048	0.823	0.802	0.843	0.848	0.566	0.490	0.741	0.696
M_{u2}	1.087	1.032	1.602	1.490	1,363	1,292	1.469	1.403	0.992	0.910	1.327	1.261
M_{u3}	0.480	0.468	0.897	0.936	0.682	0.688	0.736	0.735	0.533	0.422	0.639	0.639
N_s	0.059	0.057	0.095	0.09	0.089	0.080	0.084	0.074	0.073	0.065	0.077	0.067
R	0.52	0.48	0.55	0.47	0.53	0.49	0.52	0.49	0.54	0.47	0.54	0.48
d_{3h}/d_{3s}	0.490	0.443	0.422	0.412	0.393	0.490	0.391	0.520	0.434	0.457	0.479	0.434
Z_B	15	18	21	21	16	18	16	19	15	19	14	18
Δη_{ts,N} (%)	0.55	0.87	3.75	2.17	1.785	2.37	1.753	1.87	0.692	0.85	1.625	2.02
Δη_{ts,i}(%)	0.22	0.03	0.03	0.19	0.21	0.08	0.27	0.11	0.05	0.20	0.26	0.06
Δη_{ts,cl}(%)	4.59	3.90	5.00	6.48	4.48	2.80	5.00	3.70	3.33	2.22	3.20	4.00
Δη_{ts,f}(%)	1.62	1.25	2.05	1.43	1.80	1.28	2.32	1.30	1.36	0.89	1.32	1.25
Δη_{ts,ke}(%)	1.65	1.33	3.29	2.22	3.31	2.78	2.42	2.78	1.25	2.31	2.68	1.50
Δη_{ts,bl}(%)	11.9	11.27	8.1	9.12	10.8	11.09	11.0	10.63	11.2	11.20	12.77	10.90
Δη_{ts,p}(%)	3.06	3.26	6.49	8.14	5.83	5.80	6.02	5.16	6.79	4.25	4.90	5.03
Δη_{ts,drf}(%)	0.72	0.59	1.15	0.81	0.65	0.53	0.67	0.64	0.46	0.39	0.61	0.57
η_{tt}	0.773	0.788	0.733	0.716	0.744	0.760	0.730	0.766	0.761	0.800	0.753	0.76
η_{ts}	0.757	0.775	0.700	0.693	0.711	0.732	0.705	0.738	0.748	0.780	0.726	0.745
η_{ts,eff}	0.7689	0.784	0.727	0.711	0.736	0.752	0.723	0.760	0.758	0.795	0.747	0.757

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873 **Nomenclature**

874 b blade height [m]

875 *BK* blockage factor

876 C absolute velocity [m/s]

877 c_s spouting velocity [m/s]

878	d	diameter [m]
879	D^*	characteristic diameter [m]
880	h	enthalpy [J/kg]
881	I	rothalpy [J/kg]
882	L^*	characteristic length [m]
883	M	Mach number
884	\dot{m}	mass flow rate [kg/s]
885	N	rotational speed [rpm]
886	N_c	corrected rotational speed [rpm K ^{1/2}]
887	$N_s = \frac{N/60 \cdot Q_3^{1/2}}{\Delta h_{0s}^{3/4}}$	specific speed
888	p	pressure [Pa]
889	q	loss coefficient
890	Q	volumetric flow rate [m ³ /s]
891	r	radius [m]
892	Re	Reynolds number
893	R	Degree of reaction
894	s	blade spacing [m]
895	T	temperature [K]
896	u	peripheral velocity [m/s]
897	w	relative velocity [m/s]
898	\bar{w}	average relative velocity [m/s]
899	VS	sound speed [m/s]
900	V	Velocity (general) [m/s]

901	x	chord [m]
902	z	axial length of rotor [m]
903	Z	number of blades
904		
905	<u>Greeks</u>	
906	α	absolute angle (from radial direction, positive with u) [°]
907	α_b	actual angle of blades at nozzle outlet
908	β	relative angle (from radial direction, positive with u) [°]
909	Δ, δ	variation
910	ε	clearance[percentage of the blade height]
911	ε_{ax}	axial disk clearance [m]
912	ξ	loss coefficient
913	η	efficiency
914	λ	frictional factor
915	μ	dynamic viscosity [Kg/s-m]
916	ν	kinematic viscosity [m ² /s]
917	ρ	density [kg/m ³]
918	$\overline{\rho}$	average density [kg/m ³]
919	Φ	flow coefficient
920	Ψ	load coefficient
921	Ψ_T	Zweifel's coefficient (ratio between the ideal and real tangential load of a blade)
922	ω	speed of revolution [rad/s]
923		

924 Subscripts

925	0	total value (stagnation)
926	1, 2, 3, 4	referred to sections 1, 2, 3, 4 (figure 4)
927	act	actual
928	bl	blade loading
929	cl	clearance
930	D	diffuser
931	df	disk friction
932	f	friction
933	h	hub
934	i	incidence
935	id	ideal
936	ke	kinetic energy
937	le	leading edge
938	m	meridional (flow rate component)
939	N	nozzle
940	opt	optimal
941	p	profile
942	R	rotor
943	r	relative, radial (referring to clearance)
944	s	isentropic (Nozzle or rotor, h-s diagram)
945	s	shroud (referred to diameter)
946	ss	double isentropic (Nozzle + Rotor, h-s diagram)
947	t	tip

948 te trailing edge

949 u peripheral

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951 Acronyms

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953 IFR turbine with Radial blades at rotor inlet (Radial Inlet Flow)

954 IFG turbine with General rotor shape at inlet (General Inlet Flow)

955 ORC Organic Rankine Cycle

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