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1 **IMPROVED BOUNDARY REGULARITY FOR A STOKES-LAMÉ**
2 **SYSTEM**

3 FRANCESCA BUCCI

ABSTRACT. This paper recalls a partial differential equations system, which is the linearization of a recognized fluid-elasticity interaction three-dimensional model. A collection of regularity results for the traces of the fluid variable on the interface between the body and the fluid is established, in the case a suitable boundary dissipation is present. These regularity estimates are geared toward ensuring the well-posedness of the Riccati equations which arise from the associated optimal boundary control problems on a finite as well as infinite time horizon. The theory of operator semigroups and interpolation provide the main tools.

4 1. INTRODUCTION

5 We consider a variant of a recognized partial differential equations (PDE) system
6 that arises in the modeling of the interactions of an elastic body fully immersed in a
7 fluid. A mathematical description of the linear PDE problem under consideration,
8 that is (1.1) below with $a_2 > 0$, is given in Section 1.2. The dynamics of the fluid
9 and the solid are described by the equations of Stokes flow in the variables u (fluid
10 velocity field) and p (pressure), and by a system of linear elasticity in the variable
11 w , respectively. In both the original fluid-structure interaction (FSI) problem –
12 namely, (1.1) with $a_2 = 0$ – and the present one, the interface Γ_s between the
13 fluid and the solid is assumed to be fixed. This characteristic finds a physical
14 justification in the fact that the motion of the solid is considered as entirely due
15 to infinitesimal displacements (fast, though). Instead, the two PDE systems differ
16 inasmuch as the absence/presence of a boundary dissipation term ($a_2 = 0$ vs $a_2 > 0$)
17 changes – from strong to uniform – the stability of the corresponding dynamics, as
18 proved by Avalos and Triggiani [8]. Another favourable consequence of the change
19 in the transmission boundary condition is the improved regularity of the normal
20 component of the elastic stress tensor on the interface.

21 Our aim in this paper is to pinpoint the resulting regularity of the boundary
22 traces of the fluid variable. The obtained results, collectively stated as Theorem 1.2
23 in Section 1.2 below, are central to solvability of the associated optimal control prob-
24 lems with quadratic functionals; see Section 3, in particular Remark 3.2 and the
25 subsequent verification. To be more specific, Theorem 1.2 attests the applicability
26 of the Linear-Quadratic (LQ) theory on an infinite time horizon devised by Ac-
27 quistapace, Lasiecka and this author in [3]. This theory, tailored to coupled PDE
28 systems which comprise both hyperbolic and parabolic components, with the latter
29 subject to boundary/interface control, finds in (1.1) a significant FSI illustration,
30 besides and beyond relevant physical interactions such as mechanical-thermal (and
31 acoustic-structure) ones, which provided its original motivation.

1 While the former (undamped) FSI occurs already in the 1969 monograph [34]
 2 by J.-L. Lions, it was brought to wider attention much later by Du *et al.* [21].
 3 The proof of its well-posedness in a natural functional setting has been given by
 4 Barbu *et al.* [9]. Intrinsic features of the coupled PDE problem are (i) the Neumann-
 5 type boundary condition (involving the pressure as well) which hinders the Leray
 6 projection to divergence-free spaces, and also (ii) the transmission condition on
 7 the interface. The latter raises a major technical issue, that is the apparent dis-
 8 crepancy between the trace regularity of the variable that describes the fluid flow
 9 and the displacement of the elastic solid. First in the mathematical literature, the
 10 study [9] – which pertains to the actual *nonlinear* PDE problem, comprising the
 11 Navier-Stokes equations – shows the existence of finite energy weak solutions whose
 12 definition incorporates the exceptional boundary regularity of the elastic variable
 13 on the interface, thus solving an open problem until then.

14 With the focus on optimal control problems with quadratic functionals associated
 15 with the linearized Stokes-Lamé system (1.1) (still with $a_2 = 0$), a set of trace
 16 regularity results for the fluid variables u and u_t have been established jointly with
 17 Lasiecka [13, 14]. The article [14] provides a slightly more general treatment. In
 18 addition, it gives the explicit proof of a preliminary result (Lemma 2.4 therein)
 19 that pinpoints the regularity of the normal stress $\sigma(w) \cdot \nu$ on the interface (more
 20 specifically of a component of it), when (u, w, w_t) solves the *coupled* PDE problem
 21 (1.1) with initial data in a scale of spaces Y_α , $\alpha \in [0, 1]$, and $u|_{\Gamma_s} \in H^\alpha((0, T) \times \Gamma_s)$.
 22 The proof of the aforesaid Lemma 2.4 combines the methods of microlocal analysis
 23 utilized by Barbu *et al.* in [9, Theorem 3.3] with interpolation, as done previously
 24 by Lasiecka and Tuffaha in [31]. In addition, it utilizes a well-known trace result for
 25 the solutions to the Cauchy-Dirichlet problem for wave equations with initial data
 26 in $H^1 \times L^2$ (*cf.* [28]). (An improved ‘hidden’ regularity result for the *uncoupled*
 27 Lamé system has been derived in [39] for the case of moving interface, flat though.)
 28 The boundary regularity estimates collected in [13, Theorem 2.10] – interesting by
 29 themselves – are central to the invocation of the theory of the LQ-problem on a
 30 finite time horizon initiated by Acquistapace, Lasiecka and this author in [1]. Since
 31 the dynamics of the original FSI is not uniformly stable, the optimal boundary
 32 control problem on an infinite time interval had remained outside the investigation.

33 In this paper we devote our attention to the FSI (1.1) with $a_2 > 0$. Given the
 34 change in the transmission condition which renders the overall dynamics uniformly
 35 stable (*cf.* [8]), the theory of the infinite time horizon LQ-problem developed by
 36 Acquistapace *et al.* in [3] is potentially suited to be used. The requirement that
 37 the uncontrolled PDE system is (uniformly) exponentially stable – a prerequisite
 38 therein – is fulfilled. We work within a semigroup framework which is largely
 39 consistent with the one in [13, 14]. The coupled PDE system, subject to a control
 40 force acting on the interface, is reformulated as an abstract system $y' = \mathcal{A}y + \mathcal{B}g$ in a
 41 suitable Banach space Y , with the free dynamics generator \mathcal{A} and control operator
 42 \mathcal{B} defined in Proposition 2.2. The regularity estimates (in time and space, of local
 43 and global nature, and of independent interest) attained in Theorem 1.2, once
 44 interpreted in functional-analytic terms – i.e. as regularity properties of the adjoint
 45 of the unbounded operator $e^{t\mathcal{A}}\mathcal{B}$ –, allow for the application of the LQ-theory
 46 of [3]. The implications of the (PDE) trace regularity results in well-posedness
 47 of the associated (operator) Riccati equations will be highlighted in Section 3.
 48 It is important to emphasize that the obtained regularity results do not hinge

1 upon smoothing observations. This means that we allow fairly general functionals,
 2 including the integral of the full quadratic energy of the physical system; see (2.2)
 3 in Section 2. (For the undamped case, this advance on the earlier study [31] was
 4 first achieved in [13].)

5 On the mathematical side, we note that not only the obtained regularity esti-
 6 mates – that constitute the core of this work – are improved over Sobolev trace
 7 regularity. Also, the one pertaining to the fluid acceleration (i.e. (1.7) of Theo-
 8 rem 1.2) is enhanced, when compared to the respective one sought and established
 9 in [13, Theorem 2.10]. In addition, the proofs can be made somewhat simpler.
 10 We recall that in the case of the original FSI microlocal analysis arguments were
 11 necessitated to disclose the sharp regularity results pertaining to the elastic (hy-
 12 perbolic) component; these results were crucially utilized to single out the trace
 13 results for the fluid (parabolic) component. In the present case, as a result of the
 14 presence of the boundary dissipation term, the theories of operator semigroups and
 15 of interpolation spaces alone will suffice.

16 **1.1. Some bibliographical comments.** As a complement to the first part of the
 17 Introduction, we furnish some (non-exhaustive) bibliographical notes. The topic
 18 broadly referred to as FSI is a prominent section of the research and application
 19 area known as multiphysics. The questions that FSI raise, and the challenges to
 20 be faced, span both the modeling and the mathematical analysis. Because of the
 21 complexity of the PDE systems describing FSI, discretization techniques assume
 22 a fundamental role. It goes far beyond the scopes of the present work to pro-
 23 vide a comprehensive overview of the many contributions to the research advances
 24 within this subject; a minimum information follows. A majority of the modeling
 25 and numerical works focus on aeroelastic and hemodynamic problems, consistently
 26 with the clear engineering and medical (cardiovascular) applications, respectively;
 27 *cf.* Quarteroni *et al.* [38], Wick and Richter [41, 40], along with their references.
 28 Contributions to the understanding and mathematical analysis of nonlinear FSI
 29 arising in aeroelasticity, with major focus on the well-posedness of the PDE models
 30 (besides the modeling itself) and the long-time behaviour of the flow-plate dynam-
 31 ics (in particular, attractors), were given by Chueshov¹. We mention explicitly his
 32 joint work with Ryzkova [17], and lastly with Lasiecka and Webster; the reader
 33 is referred to the review article [16] and its references. See also the recent (and
 34 very nice) chapter devoted to the mathematical theory of evolutionary fluid-flow
 35 structure interactions [33].

36 Among the former studies of the motion of rigid bodies immersed in a fluid, we
 37 recall the ones by San Martín *et al.* [42] in 2D and by Feireisl [22] in 3D. Most FSI
 38 are free boundary problems. The local existence and uniqueness (and regularity)
 39 for a PDE system coupling the incompressible Navier-Stokes equations with the
 40 linear Kirchhoff material model for the description of an elastic solid immersed in a
 41 fluid, with *moving* interface, was first established by Coutand and Shkoller [18, 19].
 42 Subsequent progress in the study of the well-posedness (within this type of scenario)
 43 has been carried on by Kukavica and Tuffaha [23], Raymond and Vanninathan [39]
 44 and more recently by Ignatova *et al.* [25]; the reader is referred to [25] and its
 45 references. Advances in both the modeling and well-posedness of FSI are pursued
 46 by Canic and Muha (jointly as well as independently, and with different coauthors);

¹Igor Chueshov, passed away in 2016, will be sadly missed and forever remembered.

1 see, e.g., [37]. These include fluid-rigid body interactions that are described by
 2 the coupling of the Navier-Stokes equations with a system of ODE. A variety of
 3 scenarios for FSI are explored as well by Nečasová and coauthors, with novel choices
 4 and solutions devised; see e.g. [15].

5 Moving on to optimal control problems, we recall the work [31] by Lasiecka and
 6 Tuffaha, where it is first showed that for the control system $y' = \mathcal{A}y + \mathcal{B}g$ repre-
 7 senting the FSI (1.1) (with $a_2 = a_1 = 0$), subject to a boundary force g , a *singular*
 8 *estimate* for the norm of $\mathcal{O}e^{t\mathcal{A}}\mathcal{B}$ is valid, \mathcal{O} being a suitable *observation* operator.
 9 This property brings about the sought *feedback* form of the optimal control, pro-
 10 vided the minimization pertains to quadratic cost functionals in a restricted class.
 11 While the analysis in [31] includes the Bolza problem (i.e. allows a penalization of
 12 the state at a final time $T > 0$), it excludes the integral of the *full* quadratic energy
 13 of the system.

14 The recent paper by Bociu *et al.* [11] deals with an optimal control problem for
 15 a fluid-elasticity interaction with moving boundary, in the presence of a distributed
 16 control action (in feedback form). The existence of an optimal control is proved
 17 when the minimization specifically pertains to the vorticity inside the fluid. The
 18 reader is referred to [11] for an insight into the relevant technical issues that arise
 19 in the analysis and optimization of FSI with moving interface, as well as for a
 20 more complete literature review (including FSI comprising *compressible* fluids, left
 21 out of the discussion here for the sake of space). To the author's knowledge, a
 22 study of the optimal control problem (with general – quadratic or possibly non-
 23 quadratic – cost functionals) for the PDE system studied in [9] which couples the
 24 incompressible Navier-Stokes equations with the Lamé system (when subject to
 25 boundary actions), thereby deriving necessary conditions for an optimal *open-loop*
 26 control to exist, seems still lacking.

27 **1.2. The PDE problem. Main result.** Let us introduce the notation and then
 28 the coupled PDE system. The open, bounded and smooth domain representing
 29 the fluid-solid region is denoted by $\Omega \subset \mathbb{R}^d$, $d = 2, 3$; with Ω_f and Ω_s the (open
 30 and smooth) domains occupied by the fluid and the solid, respectively, then Ω is
 31 the interior of $\overline{\Omega}_f \cup \overline{\Omega}_s$. The interactions occur at an interface, say, $\Gamma_s = \partial\Omega_s$,
 32 which is assumed stationary. Finally, Γ_f is the outer boundary of Ω_f , namely
 33 $\Gamma_f = \partial\Omega_f \setminus \partial\Omega_s$. Next, the velocity field of the fluid is represented by a vector-
 34 valued function u , which satisfies the equations of Stokes flow in Ω_f ; the scalar
 35 function p represents, as usual, the pressure. The displacements of the solid region
 36 Ω_s are described by the variable w , which satisfies the Lamé system of dynamic
 37 elasticity. Thus, the PDE system in the unknown (u, w) is as follows,

$$\left\{ \begin{array}{ll} u_t - \operatorname{div} \epsilon(u) + \nabla p = 0 & \text{in } Q_f := (0, T) \times \Omega_f \\ \operatorname{div} u = 0 & \text{in } Q_f \\ w_{tt} - \operatorname{div} \sigma(w) + a_1 w = 0 & \text{in } Q_s := (0, T) \times \Omega_s \\ u = 0 & \text{on } \Sigma_f := (0, T) \times \Gamma_f \\ \epsilon(u) \cdot \nu = \sigma(w) \cdot \nu + p\nu & \text{on } \Sigma_s := (0, T) \times \Gamma_s \\ w_t - a_2 \sigma(w) \cdot \nu = u & \text{on } \Sigma_s \\ u(0, \cdot) = u_0 & \text{in } \Omega_f \\ w(0, \cdot) = w_0, \quad w_t(0, \cdot) = w_1 & \text{in } \Omega_s. \end{array} \right. \quad (1.1)$$

1 where σ and ϵ denote the elastic stress tensor and the strain tensor, respectively,
 2 that are

$$\epsilon_{ij}(w) = \frac{1}{2} \left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i} \right), \quad \sigma_{ij}(w) = \lambda \sum_{k=1}^3 \epsilon_{kk}(w) \delta_{ij} + 2\mu \epsilon_{ij}(w), \quad (1.2)$$

3 where λ, μ are the Lamé constants and δ_{ij} is the Kronecker symbol. We note that
 4 $\nu = \nu(x)$ is the outward unit normal for the fluid region Ω_f ; accordingly, it is
 5 pointing towards the interior of the solid region Ω_s .

The functional setup for the FSI (1.1) is consistent – in its basic elements –
 with the one of [9] (and [13, 14]). For the fluid component of the system one first
 introduces the space

$$\mathcal{H} := \{u \in [L^2(\Omega_f)]^d : \operatorname{div} u = 0, u \cdot \nu|_{\Gamma_f} = 0\}.$$

With $[H^1(\Omega_s)]^d \times [L^2(\Omega_s)]^d$ as the natural energy space for the Lamé system, the
 state (energy) space is then

$$Y := \mathcal{H} \times [H^1(\Omega_s)]^d \times [L^2(\Omega_s)]^d.$$

Next, we denote by V the space defined as follows:

$$V := \{v \in [H^1(\Omega_f)]^d : \operatorname{div} v = 0, v|_{\Gamma_f} = 0\}.$$

We follow the usual notation

$$(u, v)_f := \int_{\Omega_f} uv \, d\Omega_f, \quad (u, v)_s := \int_{\Omega_s} uv \, d\Omega_s, \quad \langle u, v \rangle := \int_{\Gamma_s} uv \, d\Gamma_s$$

for the various inner products. The space V is topologized with respect to the inner
 product given by

$$(u, v)_{1,f} := \int_{\Omega_f} \epsilon(u)\epsilon(v) \, d\Omega_f;$$

6 the corresponding (induced) norm $|\cdot|_{1,f}$ is equivalent to the usual $H^1(\Omega_f)$ norm,
 7 in view of Korn inequality and the Poincaré inequality.

8 **Remark 1.1.** In order to simplify the notation, we will write all the Sobolev spaces
 9 $[H^s(E)]^d$ (related to either function u, w) as $H^s(E)$. The norm $\|\cdot\|_{H^r(D)}$ in the
 10 Sobolev space $H^r(D)$ will be shortly denoted by $|\cdot|_{r,D}$ throughout the paper; $|\cdot|_{0,D}$
 11 will be simplified to $|\cdot|_D$.

The energy of the system at time t is (possibly neglecting a factor $\frac{1}{2}$)

$$|u(t)|_{\Omega_f}^2 + (\sigma(t), \epsilon(w(t)))_s + |w(t)|_{\Omega_s}^2 + |w_t(t)|_{\Omega_s}^2,$$

12 after setting $u(t) = u(t, \cdot)$, $w(t) = w(t, \cdot)$, etc.

13 We will prove a series of trace regularity results for the FSI (1.1) with $a_2 > 0$,
 14 whose respective statements are detailed in the following Theorem.

15 **Theorem 1.2.** *Given the (uncontrolled) PDE system (1.1) with $a_1, a_2 > 0$, let*
 16 *$y(t) = (u(t), w(t), w_t(t))$ be the semigroup solution corresponding to an initial state*
 17 *$y_0 = (u_0, w_0, w_1)$, in accordance with part (b) of Proposition 2.2. The following*
 18 *assertions pertain to the regularity of the boundary traces of the fluid velocity field*
 19 *u and acceleration u_t on the interface Γ_s .*

i) Assume $y_0 \in Y$. Then u can be represented as

$$u(t) = u_1(t) + u_2(t),$$

1 where the first component satisfies the (pointwise in time) estimate

$$|u_1(t)|_{\Gamma_s} \leq \frac{C e^{-\eta t}}{t^{1/4+\delta}} \|y_0\|_Y \quad \forall y_0 \in Y, \forall t > 0 \quad (1.3)$$

2 for some positive constants C, η and any positive $\delta < \frac{3}{4}$, while for any $T > 0$

$$u_2|_{\Gamma_s} \in L^p(0, T; L^2(\Gamma_s)) \quad \forall p, 1 \leq p < +\infty. \quad (1.4)$$

In particular, one has

$$u|_{\Gamma_s} \in L^{4-\sigma}(0, T; L^2(\Gamma_s)) + L^p(0, T; L^2(\Gamma_s)) \quad \forall p, 1 \leq p < +\infty,$$

3 for arbitrarily small σ .

4 **ii)** Assume $y_0 \in \mathcal{D}(\mathcal{A}^\epsilon)$ for some $\epsilon \in (0, 1)$. Then, the regularity result in (1.4)
5 improves as follows:

$$u_2|_{\Gamma_s} \in H^{1+\epsilon/2, 1/2+\epsilon/4}(\Sigma_s), \quad (1.5)$$

6 which gives in particular

$$u_2|_{\Gamma_s} \in H^{1/2+\epsilon/4}(0, T; L^2(\Gamma_s)) \subset C([0, T], L^2(\Gamma_s)). \quad (1.6)$$

7 **iii)** Let now $y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta})$, with $\theta \in (0, \frac{1}{2})$ (and arbitrary $T > 0$, just like in i)).
8 Then, the following trace regularity result pertains to the fluid acceleration:

$$u_t|_{\Gamma_s} \in L^q(0, T; H^{1/2-\theta-\delta}(\Gamma_s)) \quad \text{for any } q < \frac{2}{2-\delta}, \quad (1.7)$$

9 continuously with respect to y_0 ; a fortiori,

$$u_t|_{\Gamma_s} \in L^q(0, T; L^2(\Gamma_s)) \quad \text{for any } q < \frac{4}{3+2\theta}. \quad (1.8)$$

10 **1.3. Outline of the paper.** The structure of the paper is outlined readily. The
11 above Subsection 1.2 introduced the reader to the FSI and to the obtained trace
12 regularity results, collected in Theorem 1.2.

13 Section 2 contains the proof of Theorem 1.2, that is our main result. It is pre-
14 ceded by the statement of two preliminary results. Proposition 2.2 illustrates the
15 abstract reformulation of the PDE problem, along with the definition of the oper-
16 ators which describe the (free and) controlled dynamics. Proposition 2.4 clarifies
17 the regularity of the normal component of the elastic stress tensor on the interface,
18 which constitutes a basic element for the the proof of Theorem 1.2.

19 In the last Section 3 we move on to the interpretation of several PDE estimates
20 established in Theorem 1.2, as suitable control-theoretic properties which are the
21 key to well-posedness of the Riccati equations that arise in the study of the opti-
22 mal control problems associated with the FSI. We first provide an overview of the
23 broader context, and at the end briefly discuss the technical details for the present
24 FSI.

1

2. TRACE REGULARITY RESULTS

2 **2.1. Preliminaries: function spaces, variational and semigroup formula-**
 3 **tion.** Insert into the PDE problem (1.1) a control ‘force’ $g = g(t, x)$ acting upon
 4 the interface Γ_s . Since throughout the paper the boundary dissipation term will
 5 occur in the FSI, i.e. $a_2 > 0$, for the sake of simplicity and without loss of gener-
 6 ality we set $a_2 = a_1 = 1$. This yields the following initial-boundary value problem
 7 (IBVP) with non-homogeneous boundary datum g :

$$\begin{cases} u_t - \operatorname{div} \epsilon(u) + \nabla p = 0 & \text{in } Q_f \\ \operatorname{div} u = 0 & \text{in } Q_f \\ w_{tt} - \operatorname{div} \sigma(w) + w = 0 & \text{in } Q_s \\ u = 0 & \text{on } \Sigma_f \\ \epsilon(u) \cdot \nu = \sigma(w) \cdot \nu + p\nu + g & \text{on } \Sigma_s \\ w_t - \sigma(w) \cdot \nu = u & \text{on } \Sigma_s \\ u(0, \cdot) = u_0 & \text{in } \Omega_f \\ w(0, \cdot) = w_0, \quad w_t(0, \cdot) = w_1 & \text{in } \Omega_s. \end{cases} \quad (2.1)$$

8 Then, we associate to (2.1) a *general* quadratic functional to be minimized overall
 9 $g \in L^2(0, T; L^2(\Gamma_s))$, such as the one involving the integral (on a time interval
 10 $(0, T)$, possibly with $T = +\infty$) of the full energy of the system, that is

$$\int_0^T \left(|u(t, \cdot)|_{\Omega_f}^2 + (\epsilon(w(t, \cdot)), \sigma(w(t, \cdot)))_s + |w(t, \cdot)|_{\Omega_s}^2 + |w_t(t, \cdot)|_{\Omega_s}^2 + |g(t, \cdot)|_{\Gamma_s}^2 \right) dt. \quad (2.2)$$

11 We recall that while the regularity analysis to be carried out – eventually culmi-
 12 nating in Theorem 1.2 – actually pertains the *uncontrolled* (or *free*) PDE problem,
 13 i.e. to (2.1) with $g \equiv 0$, the obtained trace regularity results render feasible the
 14 sought-after *closed-loop* form of the optimal control \hat{g} minimizing the cost func-
 15 tional (2.2).

16 To translate the boundary control system (2.1) into an abstract equation we
 17 follow the same avenue taken in [13, 14], whose starting point is the variational
 18 formulation of the PDE problem. The resulting abstract framework is suited to the
 19 computations to be carried out in order to attain the trace regularity estimates.
 20 The definition of *weak* solutions to the uncontrolled version of PDE system (2.1) is
 21 akin to the one introduced in [9].

Definition 2.1 (Weak solution). *Let $(u_0, w_0, w_1) \in Y$ and $T > 0$. A triple*

$$(u, w, w_t) \in C([0, T], \mathcal{H} \times H^1(\Omega_s) \times L^2(\Omega_s))$$

22 *is said a weak solution to the PDE system (2.1) if*

- 23 • $(u(0, \cdot), w(0, \cdot), w_t(0, \cdot)) = (u_0, w_0, w_1)$,
- 24 • $u \in L^2(0, T; V)$,
- 25 • $\sigma(w) \cdot \nu \in L^2(0, T; H^{-1/2}(\Gamma_s))$, $\frac{d}{dt} w|_{\Gamma_s} - \sigma(w) \cdot \nu|_{\Gamma_s} = u|_{\Gamma_s} \in L^2(0, T; H^{1/2}(\Gamma_s))$;
- 26 • *the following variational system holds a.e. in $(0, T)$:*

$$\begin{cases} \frac{d}{dt} (u, \phi)_f + (\epsilon(u), \epsilon(\phi))_f - \langle \sigma(w) \cdot \nu + g, \phi \rangle = 0 \\ \frac{d}{dt} (w_t, \psi)_s + (\sigma(w), \epsilon(\psi))_s - \langle \sigma(w) \cdot \nu, \psi \rangle + (w, \psi)_s = 0, \end{cases} \quad (2.3)$$

27 *for all test functions $\phi \in V$ and $\psi \in H^1(\Omega_s)$.*

1 For the sake of completeness and the reader's convenience we observe that the
 2 given definition is justified as follows. Multiply the equation for the fluid variable
 3 by a given test function $\phi \in V$, thereby obtaining

$$(u_t, \phi)_f - (\operatorname{div} \epsilon(u), \phi)_f + (\nabla p, \phi)_f = 0. \quad (2.4)$$

4 Integration by parts (via the Green formulas) in the summand in the middle yields
 5 (omitting the symbols $d\Omega_f$, $d\Omega_s$, $d\Gamma_s$, $d\Gamma_f$)

$$\begin{aligned} (\operatorname{div} \epsilon(u), \phi)_f - (\nabla p, \phi)_f &= \int_{\Omega_f} \phi \operatorname{div} \epsilon(u) - \int_{\Omega_f} \nabla p \phi \\ &= \int_{\partial\Omega_f} \phi \epsilon(u) \cdot \nu - \int_{\Omega_f} \epsilon(u) \epsilon(\phi) - \int_{\Omega_f} (\operatorname{div}(p\phi) - p \operatorname{div} \phi) \\ &= \int_{\Gamma_s} \phi \epsilon(u) \cdot \nu - \int_{\Omega_f} \epsilon(u) \epsilon(\phi) - \int_{\partial\Omega_f} \phi p \nu - \int_{\Omega_f} p \operatorname{div} \phi \\ &= \int_{\Gamma_s} (\sigma(w) \cdot \nu + p\nu + g)\phi - \int_{\Omega_f} \epsilon(u) \epsilon(\phi) - \int_{\Gamma_s} \phi p \nu - \int_{\Gamma_f} \phi p \nu \\ &= \int_{\Gamma_s} (\sigma(w) \cdot \nu + g)\phi - \int_{\Omega_f} \epsilon(u) \epsilon(\phi) = -[(\epsilon(u), \epsilon(\phi))_f - \langle \sigma(w) \cdot \nu + g, \phi \rangle]. \end{aligned}$$

6 In the above computations the distinct boundary conditions (BC)

$$\begin{cases} u = 0 & \text{on } \Gamma_f \text{ (non-slip BC)} \\ \epsilon(u) \cdot \nu = \sigma(w) \cdot \nu + p\nu + g & \text{on } \Gamma_s \text{ (transmission BC)}, \end{cases}$$

7 have been taken into account, whilst $\operatorname{div} \phi = 0$ and $\phi|_{\Gamma_f} = 0$, because $\phi \in V$. Thus,
 8 the reformulation of (2.4) which embodies the BC reads as

$$\frac{d}{dt}(u, \phi)_f + (\epsilon(u), \epsilon(\phi))_f - \langle \sigma(w) \cdot \nu + g, \phi \rangle = 0, \quad (2.5)$$

9 that is nothing but the first one of (2.3). The computations pertaining to the
 10 equation for the solid variable w are even more straightforward.

11 We proceed as in [13, 14], which in turn followed [31]. We introduce the fluid
 12 dynamics operator $A: V \rightarrow V'$ defined by

$$(Au, \phi) = -(\epsilon(u), \epsilon(\phi)) \quad \forall \phi \in V, \quad (2.6)$$

13 as well as the (Neumann) map $N: L^2(\Gamma_s) \rightarrow \mathcal{H}$ defined as follows,

$$Ng = h \iff (\epsilon(h), \epsilon(\phi)) = \langle g, \phi \rangle \quad \forall \phi \in V. \quad (2.7)$$

14 These operators are the key elements for the semigroup formulation of the IBVP
 15 (2.1). We recall from [31, Section 4] that A defined by (2.6) may be considered
 16 as acting on \mathcal{H} , with domain $\mathcal{D}(A) := \{u \in V : |(\epsilon(u), \epsilon(\phi))| \leq C|\phi|_{\mathcal{H}}\}$. Thus,
 17 $A: \mathcal{D}(A) \subset \mathcal{H} \rightarrow \mathcal{H}$ is a self-adjoint, negative operator and therefore is the infinitesimal
 18 generator of an *analytic* semigroup e^{At} , $t \geq 0$, on \mathcal{H} . Therefore, the fractional
 19 powers of $-A$ are well defined; to simplify the notation, we shall denote them by
 20 A^α (rather than by $(-A)^\alpha$) throughout.

On the basis of the respective definitions (2.6) and (2.7) of the operators A and N , we see that

$$\langle \sigma(w) \cdot \nu + g, \phi \rangle = (\epsilon(N(\sigma(w) \cdot \nu + g)), \epsilon(\phi))_f = -(AN(\sigma(w) \cdot \nu + g), \phi),$$

1 which inserted in (2.5) yields

$$(u_t, \phi) = (Au, \phi)_f - (AN(\sigma(w) \cdot \nu + g), \phi)_f.$$

2 The principal facts concerning the abstract setup for problem (2.1), including the
3 semigroup well-posedness for the uncontrolled problem, are collected in the follow-
4 ing Proposition.

5 **Proposition 2.2.** (a) Let $Y = \mathcal{H} \times H^1(\Omega_s) \times L^2(\Omega_s)$ and $U := L^2(\Gamma_s)$. The
6 initial-boundary value problem (2.1) can be recast as the abstract Cauchy problem

$$\begin{cases} y' = \mathcal{A}y + \mathcal{B}g \\ y(0) = y_0 \in Y, \end{cases} \quad (2.8)$$

7 where the linear (dynamics and control, respectively) operators \mathcal{A} and \mathcal{B} are defined
8 as follows:

$$\left\{ \begin{array}{l} \mathcal{D}(\mathcal{A}) = \{(u, w, z)^T \in Y : u \in V, \operatorname{div} \sigma(w) \in L^2(\Omega_s), z \in H^1(\Omega_s), \\ \quad u - N\sigma(w) \cdot \nu \in \mathcal{D}(A), [z - \sigma(w) \cdot \nu]|_{\Gamma_s} = u|_{\Gamma_s}\} \\ \mathcal{A} \begin{pmatrix} u \\ w \\ z \end{pmatrix} = \begin{pmatrix} A(u - N\sigma(w) \cdot \nu) \\ z \\ \operatorname{div} \sigma(w) - w \end{pmatrix}, \end{array} \right. \quad (2.9)$$

$$\mathcal{B}g = \begin{pmatrix} -ANg \\ 0 \\ 0 \end{pmatrix}. \quad (2.10)$$

9 (b) The operator $\mathcal{A}: \mathcal{D}(\mathcal{A}) \subset Y \rightarrow Y$ is the infinitesimal generator of a strongly
10 continuous semigroup $\{e^{t\mathcal{A}}\}_{t \geq 0}$ of contractions in Y , while $\mathcal{B} \in \mathcal{L}(U, [\mathcal{D}(\mathcal{A}^*)]')$.
11 (The equation in (2.8) is initially understood in $[\mathcal{D}(\mathcal{A}^*)]'$.) In addition, the semi-
12 group $e^{t\mathcal{A}}$ is exponentially stable, namely, there exist constants $C, \omega > 0$ such that
13 $\|e^{t\mathcal{A}}\|_{\mathcal{L}(Y)} \leq Ce^{-\omega t}$, $t \geq 0$.

14 *Proof.* (a) The reformulation of the initial-boundary value problem (2.1) as the
15 Cauchy problem (2.8), with \mathcal{A} and \mathcal{B} defined by (2.9) and (2.10), respectively, comes
16 along with its variational formulation and the discussion preceding the Proposition's
17 statement.

18 (b) That \mathcal{A} is the infinitesimal generator of a strongly continuous semigroup of
19 contractions $\{e^{t\mathcal{A}}\}_{t \geq 0}$ in Y is proved following – *mutatis mutandis* – the proof of
20 Proposition 3.1 in [9]. To confirm that $\mathcal{B} \in \mathcal{L}(U, [\mathcal{D}(\mathcal{A}^*)]')$, it suffices to compute
21 $\mathcal{A}^{-1}\mathcal{B}$. It is readily verified that $\mathcal{A}^{-1}\mathcal{B}g = (-Ng, 0, 0)^T$ for $g \in U$, showing $\mathcal{A}^{-1}\mathcal{B} \in$
22 $\mathcal{L}(U, Y)$, which is equivalent to $\mathcal{B} \in \mathcal{L}(U, [\mathcal{D}(\mathcal{A}^*)]')$. Finally, uniform (exponential)
23 stability of the free dynamics generator was shown in [8, Theorem 1.2]. \square

24 **Remarks 2.3.** We recall that the stability analysis for the uncontrolled FSI (2.1)
25 pursued in [8] is based on a different semigroup setup (devised by the same authors
26 in [7]), whose free dynamics generator arises after finding that the pressure p solves
27 a suitable elliptic boundary problem and hence can be represented via proper Green
28 maps. (This idea has proved very effective also in the analysis of very different FSI,

1 such as the one studied in [4, 5].) As it will appear clearer in the next subsection,
 2 multipliers/energy methods allow to attain a dissipation relation – viz. (1.24) in
 3 [8] – which readily yields the trace regularity estimate $\sigma(w) \cdot \nu \in L^2(0, T; L^2(\Gamma))$
 4 for the hyperbolic component of the damped FSI; see (2.11) below. We note that
 5 this was not the case for the undamped FSI, i.e. (1.1) with $a_2 = a_1 = 0$. Indeed,
 6 the respective regularity $\sigma(w) \cdot \nu \in L^2(0, T; H^{-1/2}(\Gamma))$ for the normal component
 7 of the stress tensor on the interface is not intrinsic to – and not disclosed by – the
 8 semigroup framework of [7]. It was *embedded in the very definition of weak solutions*
 9 to the FSI and shown to hold true in [9], utilizing both nonlinear semigroup theory
 10 and microlocal analysis arguments.

11 **2.2. Boundary regularity of the normal stresses.** In this subsection we render
 12 explicit the regularity of the normal component of the stress tensor on the interface
 13 Γ_s , for strong and strict solutions of the *uncontrolled* FSI (2.1), as well as for
 14 solutions corresponding to initial data in intermediate spaces between the state
 15 space Y and the domain of the (free dynamics) generator. The distinct trace
 16 regularity results constitute fundamental elements for the subsequent analysis of
 17 the boundary regularity of the fluid variable u and u_t .

18 **Proposition 2.4.** *Consider the FSI (2.1) with $g \equiv 0$. Then, the following state-*
 19 *ments are valid. (i) For initial data $(u_0, w_0, w_1) =: y_0$ which belong to the energy*
 20 *space Y , the corresponding semigroup (weak) solutions $y(t) := (u(t), w(t), w_t(t)) =$*
 21 *$e^{\mathcal{A}t}y_0$ are such that*

$$\sigma(w) \cdot \nu \in L^2(0, T; L^2(\Gamma_s)) \quad \forall T > 0. \quad (2.11)$$

22 (ii) *For initial data (u_0, w_0, z_0) which belong to the domain $\mathcal{D}(\mathcal{A})$ of the dynamics*
 23 *generator, the corresponding solutions $(u(t), w(t), w_t(t))$ are such that*

$$\sigma(w) \cdot \nu \in C([0, T], H^{1/2}(\Gamma_s)) \cap H^1(0, T; L^2(\Gamma_s)) \quad \forall T > 0. \quad (2.12)$$

24 (iii) *Let $0 < \epsilon < 1$. For initial data (u_0, w_0, z_0) which belong to $\mathcal{D}(\mathcal{A}^\epsilon)$, the corre-*
 25 *sponding semigroup solutions $(u(t), w(t), w_t(t))$ satisfy*

$$\sigma(w) \cdot \nu \in L^2(0, T; H^{\epsilon/2}(\Gamma_s)) \cap H^\epsilon(0, T; L^2(\Gamma_s)) \quad \forall T > 0. \quad (2.13)$$

Proof. Assume $y_0 = (u_0, w_0, w_1) \in Y$. The first statement follows from a dissipation
 identity, viz. formula (1.24) in [8]. The said equality – which can be proved by using
 multipliers methods – clarifies in particular that for any s, t with $0 \leq s \leq t$ it holds

$$\int_s^t |\sigma(w(\tau, \cdot)) \cdot \nu|_{\Gamma_s}^2 d\tau \leq C \|y_0\|_Y^2;$$

26 taking $s = 0$ and $t = T > 0$ we obtain (2.11).

For the second assertion, we assume $y_0 \in \mathcal{D}(\mathcal{A})$ and observe that semigroup theory
 implies $y = (u, w, w_t) \in C([0, T], \mathcal{D}(\mathcal{A}))$. By the very definition of the domain of
 the free dynamics generator \mathcal{A} in (2.9), we know that

$$u \in C([0, T], V) \subset C([0, T], H^1(\Omega_f)), \quad w_t \in C([0, T], H^1(\Omega_s)).$$

27 Sobolev trace theory then obtains $u|_{\Gamma_s}, w_t|_{\Gamma_s} \in C([0, T], H^{1/2}(\Gamma_s))$. Thus, using
 28 once more the definition of $\mathcal{D}(\mathcal{A})$ and specifically the interface condition, we see
 29 that

$$\sigma(w) \cdot \nu|_{\Gamma_s} = [w_t - u]|_{\Gamma_s} = w_t|_{\Gamma_s} - u|_{\Gamma_s} \in C([0, T], H^{1/2}(\Gamma_s)). \quad (2.14)$$

1 In addition, still with $y_0 \in \mathcal{D}(\mathcal{A})$, since the generator commutes with the semigroup,
 2 $\sigma(w_t) \cdot \nu$ possesses the very same regularity than $\sigma(w) \cdot \nu$ with $y_0 \in Y$; namely,
 3 $\sigma(w_t) \cdot \nu \in L^2(0, T; L^2(\Gamma_s))$. The latter membership, combined with (2.14), confirms
 4 (2.12).

5 The third statement, namely, (2.13), follows from (2.11) and (2.12) via interpola-
 6 tion. \square

7 Thus, while moving towards the proof of our main result, we record a few regu-
 8 larity results pertaining to the mild solution

$$z(t) = \int_0^t e^{(t-s)A} f(s) ds \quad (2.15)$$

to the Cauchy problem

$$\begin{cases} z' = Az + f \\ z(0) = 0 \end{cases}$$

9 with L^2 (in time) affine term, in the case $\{e^{tA}\}_{t \geq 0}$ is an analytic semigroup on a
 10 Hilbert space Y . One has

$$f \in L^2(0, T; Y) \implies \begin{cases} z \in L^2(0, T; \mathcal{D}(A)), \\ z \in C([0, T], (\mathcal{D}(A), Y)_{\frac{1}{2}}), \\ z \in C([0, T], \mathcal{D}((-A)^{1/2-\sigma})), \quad 0 < \sigma < \frac{1}{2}. \end{cases} \quad (2.16)$$

11 **Remark 2.5.** We note that the first statement is an instance of the parabolic
 12 regularity which dates back to de Simon [20]; it constitutes by now a basic result of
 13 the theory of analytic operator semigroups. A neat proof of this implication is given
 14 by Lasiecka in [26, Appendix A]. Instead, the second statement is a consequence
 15 of an “intermediate derivative theorem” by Lions and Magenes [35, Theorem 2.3,
 16 p. 15]. As a general reference on analytic semigroup and optimal regularity for
 17 parabolic problems, *cf.* Lunardi’s monograph [36].

2.3. Proof of Theorem 1.2 (Main result). *i)* Assume $y_0 = (u_0, w_0, w_1) \in Y$.
 By Proposition 2.2 we know that the fluid variable $u(t)$ is the mild solution of the
 equation $u' = Au - AN\sigma(w) \cdot \nu$; therefore, it is given by the formula

$$u(t) = e^{tA}u_0 - A \int_0^t e^{(t-s)A} N\sigma(w(s)) \cdot \nu ds.$$

Apply the Dirichlet trace operator to $u(t)$, recalling that it coincides with $-N^*A$
 (*cf.* [31, Proposition 4.3], also recorded as Lemma A.1 in [13]), to find

$$\begin{aligned} u(t)|_{\Gamma_s} &= -N^*Au(t) = -N^*Ae^{tA}u_0 + N^*A \int_0^t e^{(t-s)A} AN\sigma(w(s)) \cdot \nu ds \\ &=: T_1(t) + T_2(t). \end{aligned}$$

18 For the first summand we readily obtain

$$|T_1(t)|_{\Gamma_s} \leq \|N^*A^{3/4-\delta}\| \|A^{1/4+\delta}e^{At}u_0\| \leq C \frac{e^{-\eta t}}{t^{1/4+\delta}} |u_0|, \quad (2.17)$$

19 valid for all $t > 0$ and suitable constants $C, \eta > 0$. (In the last inequality it has
 20 been used that the Stokes semigroup is analytic, as well as that the solutions to the
 21 uncoupled Stokes flow decay exponentially.)

1 The analysis of the summand $T_2(t)$ simplifies over the one carried out in [13,
 2 Proof of Theorem 2.10] in the undamped case ($a_2 = 0$). This is because the im-
 3 proved regularity of the hyperbolic component of the system brought about by the
 4 presence of the feedback stabilizer – and singled out in Proposition 2.4 – overcomes
 5 the further splitting required by the application of the trickier Lemma A.2 in [13].
 6 Thus, it will suffice to follow the computations performed on the term u_{22} therein
 7 to achieve the sought regularity estimate; see [13, p. 228]. Below we repeat the
 8 argument for the reader's convenience and the sake of completeness.

Rewrite $T_2(t)$ as follows:

$$\begin{aligned} T_2(t) &= N^* A \int_0^t e^{A(t-s)} AN\sigma(w(s)) \cdot \nu ds \\ &= N^* A^{3/4-\delta} \int_0^t A^{1/2+2\delta} e^{A(t-s)} A^{3/4-\delta} N\sigma(w(s)) \cdot \nu ds. \end{aligned}$$

Because

$$A^{3/4-\delta} N \in \mathcal{L}(L^2(\Gamma_s), L^2(\Omega_f)), \quad N^* A^{3/4-\delta} \in \mathcal{L}(L^2(\Omega_f), L^2(\Gamma_s)),$$

the boundary regularity (in time and space) of $T_2(t)$ on Γ_s is determined by the one possessed by the convolution term

$$\int_0^t k(t-s) h(s) ds,$$

specifically with

$$k(s) := A^{1/2+2\delta} e^{As}, \quad h(s) := A^{3/4-\delta} N\sigma(w(s)) \cdot \nu.$$

9 Recall now the basic asymptotic estimate (in a right neighbourhood of $s = 0$)

$$\|k(s)\| = \|A^{1/2+2\delta} e^{As}\| \sim \frac{C}{s^{1/2+2\delta}},$$

10 together with the basic regularity (2.11) of $\sigma(w) \cdot \nu$, that is (2.11) of Proposition 2.4,
 11 to infer

$$k \in L^{2-\sigma}(0, T; L^2(\Omega_f)), \quad h \in L^2(0, T; L^2(\Omega_f)),$$

where $\sigma > 0$ can be taken arbitrarily small. By virtue of the Young inequality, it follows

$$k * h \in L^r(0, T; L^2(\Omega_f)), \quad \frac{1}{r} = \frac{1}{2-\sigma} + \frac{1}{2} - 1,$$

12 which establishes $k * h \in L^r(0, T; L^2(\Omega_f))$ for any finite $r \geq 1$. In the present case
 13 then, we find

$$T_2 \in L^r(0, T; L^2(\Gamma_s)) \quad \forall r \geq 1. \quad (2.18)$$

(Note that (2.18) cannot be extended to $r = +\infty$, as $\sigma > 0$.) We conclude that the sought decomposition $u(t) = u_1(t) + u_2(t)$ is indeed simply the one with

$$u_1(t) := e^{At} u_0, \quad u_2(t) := -A \int_0^t e^{A(t-s)} N\sigma(w(s)) \cdot \nu ds,$$

14 with (2.17) and (2.18) confirming (1.3) and (1.4), respectively.

ii) Assume now $y_0 \in \mathcal{D}(\mathcal{A}^\epsilon)$ for some positive $\epsilon < 1$. From Proposition 2.4 we know that (2.13) holds. Exploiting the improved space regularity $\sigma(w) \cdot \nu \in L^2(0, T; H^{\epsilon/2}(\Gamma_s))$, we suitably rewrite u_2 as follows:

$$\begin{aligned} u_2(t) &= -A \int_0^t e^{A(t-s)} N \sigma(w(s)) \cdot \nu \, ds \\ &= -A^{1/4+\delta} A^{-\epsilon/4} \int_0^t e^{A(t-s)} \underbrace{A^{\epsilon/4} [A^{3/4-\delta} N] \sigma(w(s)) \cdot \nu}_{f(s)} \, ds. \end{aligned}$$

1 Apply next the trace operator $-N^*A$ to find

$$u_2(t)|_{\Gamma_s} = [N^*A^{3/4-\delta}]A^{1/2+2\delta}A^{-\epsilon/4} \int_0^t e^{A(t-s)} f(s) \, ds, \quad (2.19)$$

2 where $f \in L^2(0, T; L^2(\Omega_f))$. On the basis of (2.19), we appeal to the latter estimate
3 in (2.16) to deduce that $u_2|_{\Gamma_s} \in C([0, T], L^2(\Gamma_s))$ i.e. (1.6) holds true (δ must be
4 suitably chosen: any $0 < \delta < \epsilon/8$ will work).

The obtained trace regularity result (1.6) for the component u_2 can be actually made more precise. Indeed, return to (2.13) which gives *a fortiori* $\sigma(w) \cdot \nu \in H^{\epsilon/2}(\Sigma_s)$. By way of the optimal parabolic regularity, $u_2 \in H^{(\epsilon+3)/2, (\epsilon+3)/4}(Q_f)$ follows; thus, standard Sobolev trace theory yields

$$u_2|_{\Gamma_s} = -N^*A u_2 \in H^{\epsilon/2+1, \epsilon/4+1/2}(\Sigma_s) \subset H^{1/2+\epsilon/4}(0, T; L^2(\Gamma_s)),$$

5 that is precisely the stronger (1.5).

6 *iii)* Take now $y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta})$, $\theta \in (0, 1)$. On the one hand, the proof of the trace
7 regularity result (1.8) pertaining to the fluid acceleration u_t can be streamlined, in
8 comparison with [13, Proof of Theorem 2.10, (iii)]. (This is again a consequence of
9 the fact that [14, Lemma 2.4] is here replaced by Proposition 2.4.) On the other
10 hand, we seek to disclose the improved (*space*) regularity (1.7), even though for the
11 purpose of invoking the theory in [1, 3] the membership $L^q(0, T; L^2(\Gamma_s))$ in (1.8)
12 would suffice. A shared tool for both proofs are interpolation result which also in-
13 volve the dual of Sobolev spaces with fractional exponents (*cf.* [35, Theorem 12.5]).

We start from the expression

$$u_t(t) = \underbrace{Ae^{At}u_0}_{V_1(t)} + \underbrace{\frac{d}{dt} \int_0^t e^{A(t-s)} AN \sigma(w(s)) \cdot \nu \, ds}_{V_2(t)},$$

which is meant in the sense of distributions. Assuming initially $y_0 \in Y$, we compute

$$V_2(t) = AN \sigma(w(t)) \cdot \nu + A \int_0^t e^{A(t-s)} AN \sigma(w(s)) \cdot \nu \, ds =: V_{2a}(t) + V_{2b}(t),$$

and pinpoint the regularity of either summand by rewriting and splitting as follows:

$$V_{2a}(t) = A^{1/4+\delta_1} [A^{3/4-\delta_1} N] \sigma(w(t)) \cdot \nu \in L^2(0, T; [\mathcal{D}(A^{1/4+\delta_1})]'),$$

$$\begin{aligned} V_{2b}(t) &= A \int_0^t e^{A(t-s)} AN \sigma(w(s)) \cdot \nu \, ds \\ &= A^{1/4+\delta_1} \left(A \int_0^t e^{A(t-s)} [A^{3/4-\delta_1} N] \sigma(w(s)) \cdot \nu \, ds \right) \in L^2(0, T; [\mathcal{D}(A^{1/4+\delta_1})]'). \end{aligned}$$

1 To infer the latter membership we simply used the first (basic) analytic estimate
2 in (2.16). Thus, the regularity of V_{2a} and V_{2b} combine to bring about

$$y_0 \in Y \implies V_2 \in L^2(0, T; [H^{1/2+2\delta_1}(\Omega_f)]'), \quad 0 < \delta_1 < \frac{1}{2}, \quad (2.20)$$

3 where we used the identification $H^s(\Omega_f) \equiv \mathcal{D}(A^{s/2})$ valid for $s < 3/2$. (We note
4 that consistently with the improved regularity of the normal component of the
5 elastic tress tensor (*cf.* Proposition 2.4), (2.20) is a better regularity result than
6 (3.18) in [13].)

When $y_0 \in \mathcal{D}(\mathcal{A})$, the summand $V_2(t)$ is more conveniently rewritten in a different
fashion:

$$V_2(t) = V_{2a}(t) + V_{2b}(t) = \underbrace{e^{At} AN \sigma(w(0)) \cdot \nu}_{V_{21}(t)} + \underbrace{\int_0^t e^{A(t-s)} AN \sigma(w_t(s)) \cdot \nu ds}_{V_{22}(t)},$$

7 where by Proposition 2.4 and specifically in view of (2.12) we know that

$$\sigma(w(0)) \cdot \nu \in H^{1/2}(\Gamma_s), \quad \sigma(w_t) \cdot \nu \in L^2(0, T; L^2(\Gamma_s)). \quad (2.21)$$

The regularity result on the right of (2.21) suggests that we rewrite the summand
 V_{22} as follows:

$$V_{22}(t) = A^{1/4+\delta_2} \int_0^t e^{(t-s)A} [A^{3/4-\delta_2} N] \sigma(w_t(s)) \cdot \nu ds$$

(valid for arbitrary $\delta_2 \in (0, 3/4)$). Then, the first analytic estimate in (2.16) estab-
lishes

$$A^{3/4-\delta_2} V_{22} \in L^2(0, T; L^2(\Omega_f)),$$

8 that is

$$V_{22} \in L^2(0, T; \mathcal{D}(A^{3/4-\delta_2})) \equiv L^2(0, T; H^{3/2-2\delta_2}(\Omega_f)). \quad (2.22)$$

As for V_{21} , the regularity result on the left of (2.21) reveals that

$$V_{21}(t) = A^{\delta_2/2} e^{At} \chi, \quad \chi := A^{1/4} [A^{3/4-\delta_2/2} N] \sigma(w(0)) \cdot \nu \in L^2(\Omega_f).$$

which enables us to ascertain that

$$A^{3/4-\delta_2} V_{21} = A^{3/4-\delta_2/2} e^{At} \chi \in L^{q_2}(0, T; L^2(\Omega_f)) \quad \text{provided } q_2(3/4 - \delta_2/2) < 1;$$

9 equivalently,

$$V_{21} \in L^{q_2}(0, T; H^{3/2-2\delta_2}(\Omega_f)) \quad \text{for any } q_2 < \frac{4}{3-2\delta_2}. \quad (2.23)$$

10 We resume then (2.22) and observe that the upper bound $4/(3-2\delta_2)$ for the
11 summability exponent q_2 is not greater than 2 as long as $\delta_2 \leq 1/2$; at the same time,
12 δ_2 must be taken arbitrarily close to 0 in order to attain the best space regularity.
13 Therefore, we limit $\delta_2 \leq 1/2$ and conclude from (2.23) and (2.22) (and still with
14 $y_0 \in \mathcal{D}(\mathcal{A})$) that

$$y_0 \in \mathcal{D}(\mathcal{A}) \implies V_2 \in L^{q_2}(0, T; H^{3/2-2\delta_2}(\Omega_f)) \quad \text{for any } q_2 < \frac{4}{3-2\delta_2} \quad (2.24)$$

15 holds true, for any given $\delta_2 \leq 1/2$.

A fundamental tool is now interpolation. On the basis of (2.20) and (2.24) valid for $y_0 \in Y$ and $y_0 \in \mathcal{D}(\mathcal{A})$, respectively, we utilize [35, Theorem 12.5] to establish

$$y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta}) \implies V_2 \in L^{q_2}(0, T; W) \quad \text{for any } q_2 < \frac{4}{3-2\delta_2},$$

where W is the Sobolev space

$$W = (H^{3/2-2\delta_2}(\Omega_f), [H^{1/2+2\delta_1}(\Omega_f)]')_{1-\theta, 2} = H^s(\Omega_f),$$

- 1 with $s = (3/2 - 2\delta_2)(1 - \theta) - \theta(1/2 + 2\delta_1)$, which simplifies to $s = 3/2 - 2(\theta + \delta)$
 2 by setting $\delta = \delta_1 = \delta_2$. By standard trace theory, we conclude that

$$y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta}) \implies V_2|_{\Gamma_s} \in L^{q_2}(0, T; H^{1-2\theta-2\delta}(\Gamma_s)), \quad \text{for any } q_2 < \frac{4}{3-2\delta}, \quad (2.25)$$

- 3 provided $s \geq 1/2$, which means

$$\delta + \theta \leq \frac{1}{2}. \quad (2.26)$$

- 4 We note that (2.26) forces $\theta < 1/2$ (as well as $\delta < 1/2$).

The analysis of V_1 is simpler. Since $y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta})$, for the first component one has

$$u_0 \in (H^1(\Omega_f), L^2(\Omega_f))_\theta = H^{1-\theta}(\Omega_f) \equiv \mathcal{D}(A^{(1-\theta)/2}),$$

where the identification is justified by the fact that $1 - \theta < 3/2$. We now proceed differently than in [13, Proof of Theorem 2.10, (iii)]. Aiming to pinpoint a better (space) regularity of V_1 , we rewrite $V_1(t) = Ae^{At}u_0 = A^{(1+\theta)/2}e^{At}[A^{(1-\theta)/2}u_0]$ to find

$$A^{(1-\theta)/2-\sigma}V_1(t) = A^{1-\sigma}e^{At}[A^{(1-\theta)/2}u_0],$$

- 5 which shows

$$y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta}) \implies V_1 \in L^{q_1}(0, T; \mathcal{D}(A^{(1-\theta)/2-2\sigma})) = L^{q_1}(0, T; H^{1-\theta-2\sigma}(\Omega_f)) \quad (2.27)$$

- 6 for any $q_1 < \frac{1}{1-\sigma}$. Only subsequently we invoke trace theory: if $1 - \theta - 2\sigma \geq 1/2$,
 7 i.e.

$$2\sigma + \theta \leq \frac{1}{2}, \quad (2.28)$$

- 8 the interior regularity (2.27) implies

$$y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta}) \implies V_1|_{\Gamma_s} \in L^{q_1}(0, T; H^{1/2-\theta-2\sigma}(\Gamma_s)) \quad \text{for any } q_1 < \frac{1}{1-\sigma}. \quad (2.29)$$

- 9 Since the role of δ and σ is similar, and consistently with the constraints (2.26) and
 10 (2.28), we set $\sigma = \delta/2$ in (2.29), which becomes

$$y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta}) \implies V_1|_{\Gamma_s} \in L^{q_1}(0, T; H^{1/2-\theta-\delta}(\Gamma_s)) \quad \text{for any } q_1 < \frac{2}{2-\delta}. \quad (2.30)$$

A comparison between the Sobolev exponents in (2.30) and (2.25) shows

$$\frac{1}{2} - \theta - \delta \leq 1 - 2\theta - 2\delta$$

owing to (2.26); in addition, readily $\frac{2}{2-\delta} < \frac{4}{3-2\delta}$. Therefore, we conclude that

$$y_0 \in \mathcal{D}(\mathcal{A}^{1-\theta}) \implies V|_{\Gamma_s} \in L^q(0, T; H^{1/2-\theta-\delta}(\Gamma_s)) \quad \text{for any } q < \frac{2}{2-\delta},$$

- 11 which is nothing but (1.7). Finally, by taking $\delta = 1/2 - \theta$ we see that (1.7) yields
 12 (1.8), which ends the proof. \square

3. APPLICATION TO OPTIMAL BOUNDARY CONTROL

Regularity estimates are a critical part of most results in control theory for PDE. In this section we show how the trace regularity results provided by Theorem 1.2 combine to bring about the well-posedness of both differential and algebraic Riccati equations (DRE and ARE, respectively) corresponding to the quadratic optimal control problems on a finite and infinite time horizon associated with the PDE problem (2.1) (including, in particular, problem (2.1)-(2.2) with $0 < T \leq +\infty$), thus confirming their *full* solvability. We will ascertain that the Assumptions 1.1 and 1.4 in [3] – pertaining to the (dynamics, control, observation) operators involved – are fulfilled. An account of this technical issue, which will be kept brief, is given in Subsection 3.2, thereby concluding the article. Rather, we find it worth giving insight into the prerequisite role of certain trace regularity results within the LQ-problem for hyperbolic PDE subject to boundary control, as well as for composite systems of hyperbolic-parabolic PDE (that is the case of the present work), along with some historical comments. This is the content of the next subsection.

3.1. The broader context. Let Y and U be two separable Hilbert spaces, the *state* and *control* space, respectively. Consider a linear control system such as the one in (2.8), under the following basic assumptions.

- The closed linear operator $\mathcal{A}: \mathcal{D}(\mathcal{A}) \subset Y \rightarrow Y$ is the infinitesimal generator of a strongly continuous semigroup $e^{t\mathcal{A}}$ on Y , $t \geq 0$;
- $\mathcal{B} \in \mathcal{L}(U, [\mathcal{D}(\mathcal{A}^*)]')$.

Thus, given $y_0 \in Y$, the Cauchy problem (2.8) possesses a unique *mild* solution given by

$$y(t) = e^{t\mathcal{A}}y_0 + \int_0^t e^{(t-s)\mathcal{A}}\mathcal{B}g(s) ds, \quad (3.1)$$

initially understood in the extrapolation space $[\mathcal{D}(\mathcal{A}^*)]'$; see [30, §0. 3, p. 6, and Remark 7.1.2, p. 646]. To fully understand the regularity of the state (3.1) one needs to pinpoint the one of the operator

$$L: g(\cdot) \longrightarrow (Lg)(t) := \int_0^t e^{(t-s)\mathcal{A}}\mathcal{B}g(s) ds,$$

that is the (so called “input-to-state”) mapping which associates to any control function $g \in L^2(0, T; U)$ the solution to the Cauchy problem (2.8) with $y_0 = 0$. We note that the regularity properties of L and its adjoint L^* are, in turn, fully determined by the regularity (in time and space) of the kernel $e^{t\mathcal{A}}\mathcal{B}$ and its adjoint. Pinpointing it may be a challenging task in the case of interest, where $\mathcal{B} \notin \mathcal{L}(U, Y)$. It is here that the study of the parabolic and the hyperbolic cases diverge, and where a glimpse of the possibility for systems of coupled parabolic-hyperbolic PDE emerges.

The property of being *analytic* possessed by the semigroup $e^{t\mathcal{A}}$, combined with the structure of the operator \mathcal{B} (stemming from the modeling of boundary control forces) results in $Lg \in L^2(0, T; Y)$ via the first one of (2.16), thereby obtaining $y \in L^2(0, T; Y)$ for *any* state – while $y \in C([0, T], Y)$ might be false, depending on the specific BC in place. Furthermore, the very same property gives rise to well-posed Riccati equations with bounded gains. This scenario pertains to parabolic (and parabolic-like) PDE; see [29] and [10]. Instead, the fact that

- i) the semigroup $e^{t\mathcal{A}}$ that drives the free dynamics is *not* analytic, while

1 ii) the control operator \mathcal{B} is still *unbounded* (it is necessarily so because of the
2 control action exerted from the boundary),

3 renders the hyperbolic case trickier.

4 To the state equation we associate the quadratic functional

$$J(u) = \int_0^T (\|\mathcal{R}y(t)\|_Z^2 + \|g(t)\|_U^2) dt, \quad (3.2)$$

5 where Z is a third separable Hilbert space – the so called observation space (possibly,
6 $Z \equiv Y$). A priori, the *observation* operator \mathcal{R} simply satisfies

$$\mathcal{R} \in \mathcal{L}(Y, Z). \quad (3.3)$$

7 The formulation of the optimal control problem under study is the usual one.

8 **Problem 3.1 (The optimal control problem).** *Given $y_0 \in Y$, seek a control*
9 *function \hat{g} which minimizes the cost functional (3.2) overall $g \in L^2(0, T; U)$, where*
10 *$y(\cdot) = y(\cdot; y_0, g)$ is the solution to (2.8) corresponding to $g(\cdot)$.*

11 We note however that by “solving” Problem 3.1 it is meant that certain principal
12 facts should hold, besides the existence of a unique optimal pair $(\hat{g}(\cdot, s; y_0), \hat{y}(\cdot, s; y_0))$,
13 a property which simply follows via classical variational arguments. Namely,

- 14 – that the optimal control $\hat{g}(t)$ admits a (pointwise in time) *feedback* repre-
15 sentation, in terms of the optimal state $\hat{y}(t)$;
- 16 – that the optimal cost operator P ($P(t)$, when $T < +\infty$) solves the corre-
17 sponding algebraic (differential, respectively) Riccati equation, that is

$$(Px, Az)_Y + (Ax, Pz)_Y - (\mathcal{B}^*Px, \mathcal{B}^*Pz)_U + (Rx, Rz)_Z = 0 \quad (3.4)$$

for any $x, z \in \mathcal{D}(\mathcal{A})$;

18 thus, the issue of well-posedness of the ARE (DRE) arises, requiring

- 19 – that a meaning is given to the gain operator \mathcal{B}^*P ($\mathcal{B}^*P(t)$) either on the
20 state space Y (possibly by means of extensions), or – which will be the
21 case, here – as a *bounded* operator on a suitable dense subset of Y .

It is by now well known that the latter property might be lacking, in view of the
aforesaid (intrinsic) features of the control system under examination. Now, the
key role played by the kernel $e^{t\mathcal{A}}\mathcal{B}$ (or, possibly – which is weaker –, by $Re^{t\mathcal{A}}\mathcal{B}$)
becomes even more evident upon recalling that the optimal cost operator P is
explicitly defined in terms of the optimal state $\hat{y}(t) = \Phi(t)y_0$, $0 \leq t < +\infty$, via the
formula

$$Py_0 := \int_0^\infty e^{t\mathcal{A}^*} R^* R \Phi(t) y_0 dt \quad x \in Y.$$

22 It dates back to the beginning of the eighties the discovery that in the case of
23 linear hyperbolic equations – with the wave equation as a paradigm – the following
24 (*admissibility*) estimate

$$\exists C_T > 0: \int_0^T \|\mathcal{B}^* e^{t\mathcal{A}^*} y_0\|_U^2 dt \leq C_T \|y_0\|_Y^2 \quad \forall y_0 \in Y, \quad (3.5)$$

25 is the one to be ascertained. Indeed, it implies $L \in \mathcal{L}(L^2(0, T; U), C([0, T], Y))$,
26 so that any state belongs to $C([0, T], Y)$. In addition, it enables to show a well-
27 posedness result for the corresponding DRE, provided \mathcal{R} is smoothing in an appro-
28 priate sense. The PDE interpretation of the abstract condition (3.5) is intriguing:

1 if the Cauchy problem (2.8) represents the Cauchy-Dirichlet problem

$$\begin{cases} w_{tt} = \Delta w & \text{in } Q := (0, T) \times \Omega \\ w(0, \cdot) = w_0, w_t(0, \cdot) = w_1 & \text{in } \Omega \\ w(t, x) = g & \text{on } \Sigma := (0, T) \times \Gamma \end{cases} \quad (3.6)$$

2 for a single, linear wave equation (where $y(t) = (w(t, \cdot), w_t(t, \cdot))$, $Y = L^2(\Omega) \times$
 3 $H^{-1}(\Omega)$ and $U = L^2(\Gamma)$), then the abstract condition (3.5) is equivalent to the
 4 trace regularity result

$$\exists C_T > 0: \quad \int_0^T \int_{\Gamma} \left| \frac{\partial w}{\partial \nu}(t, x) \right|^2 d\sigma dt \leq C_T \int_{\Omega} (|\nabla w_0|^2 + |w_1|^2) dx, \quad (3.7)$$

5 valid for the solution to the IBVP (3.6) with initial data $(w_0, w_1) \in H_0^1(\Omega) \times L^2(\Omega)$
 6 and (trivial) boundary datum $g \equiv 0$ (*cf.* [28]). The proof of the boundary regularity
 7 results more often rely on energy methods, along with an appropriate choice of
 8 multipliers which are suited to the specific PDE problem, or they may require
 9 pseudodifferential methods. For a concise introduction (yet pretty comprehensive
 10 one, till the eighties) to the LQ-problem for either parabolic (and parabolic-like) or
 11 hyperbolic equations with boundary or point control, the reader is referred to [29].
 12 For a thorough treatise and bibliography, see [10] and [30].

13 3.1.1. *Composite systems of PDE.* By the end of the nineties a theory of the LQ-
 14 problem ensuring a bounded gain operator and well-posed Riccati equations – and
 15 hence akin to the one pertaining to parabolic-like PDE – has been devised, for
 16 abstract control systems which yield the (so called) *singular estimate*

$$\|e^{tA}\mathcal{B}\|_{\mathcal{L}(U, Y)} \sim \frac{C}{t^\gamma} \quad (3.8)$$

17 in a right neighbourhood of $t = 0$ (for some $\gamma \in (0, 1)$, and $C > 0$), even in
 18 the absence of analyticity of the semigroup e^{tA} ; see [27], and [32] for the Bolza
 19 problem. A breakthrough in this direction came from the study [6] of an optimal
 20 control problem for an acoustic-structure interaction PDE system.

21 The project carried out and accomplished in the studies [1, 3] (with a few ad-
 22 ditions and refinements that are forthcoming) provides a framework for optimal
 23 control problems with quadratic functionals for a wider class of PDE systems
 24 which comprise both hyperbolic and parabolic components, with the latter sub-
 25 jected to boundary/interface control. This class has proven to be sufficiently gen-
 26 eral to encompass widely different PDE models such as certain thermoelastic sys-
 27 tems, acoustic-structure and fluid-structure interactions (*cf.* [2], [12], [13, 14]). The
 28 achievements of [1, 3] include

- 29 • the feedback synthesis of the optimal control, along with
- 30 • well-posed (differential, first, and algebraic, next) Riccati equations,

31 with a *gain* operator which is bounded on a dense subset of the state space. This
 32 theory allows non-smoothing observations \mathcal{R} . The aforesaid conclusions, whose
 33 relevance is well-understood within systems theory, raise highly technical issues in
 34 the context of composite PDE systems, both on a functional-analytic level (see in
 35 particular [3, Section 5]) and on the ‘PDE level’, when it comes to the verification
 36 of the control-theoretic properties that characterize the couple (A, B) (these are
 37 Assumptions 1.4 in [3]). When dealing with systems of coupled parabolic-hyperbolic

1 PDE one needs to disclose and exploit the regularity of the hyperbolic traces in order
 2 to eventually show the sought regularity of the parabolic ones – a more challenging
 3 task especially when the coupling occurs either on a sub-boundary of the domain
 4 or at an interface, like in the case of the FSI under consideration here.

3.2. Optimal boundary control of the present FSI. For the linearization of the original FSI studied by Barbu *et al.* (i.e. for (1.1) with $a_2 = a_1 = 0$), subject to a control force acting on the interface, the study of Lasiecka and Tuffaha [31] revealed that the sought estimate (3.8) does not hold, unless the observation operator is suitably smoothing. On the other hand, the weaker estimate

$$\|\mathcal{O}e^{t\mathcal{A}}\mathcal{B}\|_{\mathcal{L}(U,Z)} \sim \frac{C}{t^\gamma}$$

5 is established (with suitable \mathcal{O} and $\gamma \in (0, 1)$), which is nothing but a singular
 6 estimate in a weaker topology than the one of the energy space, and which intro-
 7 duces a limitation on the cost functionals allowed. But then the “revisited singular
 8 estimate” theory in [32] applies, with the plus of the Bolza problem being feasible.
 9 (We used above the usual notation $e^{t\mathcal{A}}$ and \mathcal{B} for the semigroup that describes the
 10 overall free dynamics and the control operator.)

11 As explained in the Introduction, with the focus on the optimal control problem
 12 on a finite time horizon for the very same linear FSI, the analysis pursued in [13, 14]
 13 succeeded in removing the constraints on the cost functionals, by showing that the
 14 FSI falls in the more general class and set-up devised in [1]. The present work
 15 continues and completes the PDE analysis of [13], by exploring the infinite (besides
 16 finite) time horizon case. Given the stability properties of the uncontrolled FSI,
 17 we were necessarily led to study a variant of the original linearization, that is (1.1)
 18 with $a_2 > 0$ (and $a_1 > 0$, for the sake of simplicity).

19 **Remark 3.2.** All the boundary regularity results contained in Theorem 1.2 can be
 20 interpreted as respective regularity properties of the operators $\mathcal{B}^*e^{t\mathcal{A}^*}$ or $\mathcal{B}^*e^{t\mathcal{A}^*}\mathcal{A}^{*\epsilon}$,
 21 showing that the requirements of [3, Assumptions 1.1 and 1.4] are met. This is
 22 accomplished making use of pretty much the same arguments that were used in the
 23 end of [13, Section 4] for the undamped FSI. Once again, the observation operator \mathcal{R}
 24 is allowed to be the identity, just like in the undamped case. Thus in particular, the
 25 cost functional (2.2) (that is integral of the full quadratic energy of the system) fits
 26 into the picture. The major steps of the needed verification are briefly illustrated
 27 below, thereby concluding the article.

Validity of the Assumptions 1.1 and 1.4 in [3], as a consequence of Theorem 1.2.
 Let us start from the basic Assumptions 1.1. Just observe that these are found
 among the statements of Proposition 2.2, with the property of exponential stability
 of the semigroup $e^{t\mathcal{A}}$ proved in [8]. To check the Assumptions 1.4, the *incipit* is as
 follows: in view of (2.10) and [31, Proposition 4.3]

$$\mathcal{B}^*e^{t\mathcal{A}^*}y_0 = -N^*A\hat{u}(t) = \hat{u}(t)|_{\Gamma_s},$$

having denoted by $\hat{u}(t)$ the first component of the solution $\hat{y}(t) = (\hat{u}(t), \hat{w}(t)\hat{w}_t(t))$
 to the *uncontrolled* Cauchy problem

$$\begin{cases} \hat{y}' = \mathcal{A}^*\hat{y} \\ \hat{y}(0) = y_0. \end{cases}$$

1 Thus, the sought decomposition of the operator $\mathcal{B}^* e^{t\mathcal{A}^*}$, along with the regularity
 2 estimates in Assumption 1.4(i)-(ii) in [3] are confirmed by (1.3) (with $\gamma = 1/4 + \delta$)
 3 and (1.4) of Theorem 1.2, respectively.

4 Next, (1.6) establishes Assumption 1.4(iii), initially with arbitrary $\epsilon \in (0, 1)$.
 5 By following the argument and computations in the very end of [13, Section 4], and
 6 given the obtained regularity (1.8) for the fluid acceleration, we see that Assump-
 7 tion 1.4(iii) is satisfied, with $\epsilon = \theta < 1/2$, provided

$$\mathcal{R}^* \mathcal{R} \in \mathcal{L}(\mathcal{D}(\mathcal{A}^\epsilon), \mathcal{D}(\mathcal{A}^{*\epsilon})). \quad (3.9)$$

8 We note that the latter is not a requirement that the observation operator should
 9 possess an appropriate smoothing property; in particular, (3.9) holds true when
 10 $\mathcal{R} \equiv I$. As already explained in [13, Remark 2.8], this is because $\mathcal{D}(\mathcal{A}^\epsilon) \equiv \mathcal{D}(\mathcal{A}^{*\epsilon})$
 11 for $\epsilon > 0$ sufficiently small; and in fact, in view of Theorem 1.2, ϵ can be taken
 12 *arbitrarily* small. (Recall that the equivalence between $\mathcal{D}(\mathcal{A}^\epsilon)$ and $\mathcal{D}(\mathcal{A}^{*\epsilon})$ for $\epsilon > 0$
 13 sufficiently small is a consequence of $\mathcal{D}(\mathcal{A}) = \mathcal{D}(\mathcal{A}^*)$, as the domains of fractional
 14 powers of the generator \mathcal{A} (\mathcal{A}^*) are intermediate spaces between $\mathcal{D}(\mathcal{A})$ ($\mathcal{D}(\mathcal{A}^{*\epsilon})$)
 15 and Y .)

16 In view of the above, all the statements of [3, Theorem 1.5] – pertaining to the
 17 infinite time horizon case – are valid. In addition, since the said Assumptions 1.1
 18 and 1.4 in [3] contain the ones needed to solve the optimal control problem on a
 19 finite time interval, that are Hypotesis 2.1 and 2.2. in [1], *a fortiori* [1, Theorem 2.3]
 20 applies as well. Both differential and algebraic Riccati equations are solvable, and
 21 the synthesis of the (unique) optimal control is guaranteed.

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