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Nonlinear dynamics of higher education participation under social influence and endogenous returns[☆]

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ABSTRACT

Decisions to pursue higher education are shaped by economic incentives, peer effects, and expectations about future returns. This paper develops a non-linear dynamic model of educational participation for study fields in which the motivations to enrol are heterogeneous and cannot be reduced to a single driver. Agents allocate resources between education and consumption while their preferences evolve endogenously. Followers are reinforced by aggregate participation, while Positional Agents respond to social distinctiveness and to an endogenous wage premium that declines with the supply of educated workers. We show that the interaction between imitation, positional concerns, and labour-market feedback can generate endogenous cycles and irregular dynamics, especially for intermediate mixtures of behavioural types. Allowing population shares to evolve adds a further source of instability through switching between behavioural rules. Finally, policy interventions are not necessarily stabilizing: counter-cyclical cost subsidies may discipline enrolment, whereas strong behavioural signalling can amplify imitative responses and generate overreaction.

1. Introduction

In recent years, several studies have highlighted that the decision to enrol in higher education cannot be fully explained by purely economic incentives. In the Italian case, despite relatively low tuition fees and the absence of formal barriers to access, university attainment remains among the lowest in Europe. As shown by Leoni (2022), this pattern reflects not only the limited wage premium associated with tertiary education (see also Viesti, 2018; Franzini and Raitano, 2019; Naticchioni et al., 2016), but also the crucial role of social influence in shaping educational choices. In her agent-based model of the Italian higher education system, Leoni models individuals' preferences for enrolling on three main components: economic motivation, the effort required to complete university studies, and — most importantly — social interaction within one's reference network. Young people may decide to enrol in university not only because of the expected monetary return, but also because their peers do so; this educational conformism generates patterns of collective behaviour that cannot be reduced to individual optimization. The importance of social and peer effects in educational decisions has been further confirmed by more recent empirical and experimental works: Pratschke and Abbiati (2023) model peer effects in Italian classrooms using a spatial autoregressive framework, showing that endogenous interactions significantly shape students' trajectories; Zárate (2023) find that peer centrality within social networks amplifies both social and academic skills; and Carlana and Corno (2025) highlight that social image concerns and same-gender interactions crucially affect the likelihood of pursuing higher education. Beyond the Italian case, a growing body of research in economics and social dynamics emphasizes how imitation, conformity, and social contagion can account for aggregate patterns in education and inequality (see, among others, Dimarco et al., 2024; Herstad and Shin, 2025; De Domenico et al., 2025; Chiba-Okabe and Plotkin, 2024), reinforcing the view that educational choices are socially interdependent and that local interactions can produce complex collective outcomes. A related strand of literature has interpreted peer effects in education through the lens of social signalling and social image concerns. In their seminal contribution, Austen-Smith and Fryer (2005) model educational investment as a two-audience signalling problem in which students simultaneously signal productivity to the labour market and social compatibility to their peer group. In this framework, educational effort may generate social costs

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[1] Beyond the educational context, Bursztyn et al. (2014) highlight more generally how social interactions affect individual decisions through both social learning and social utility channels.

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and induce underinvestment in education, giving rise to behavioural patterns consistent with the so-called *acting white* phenomenon — the perception that academic effort and educational success are associated with "white" social behaviour and therefore may generate social stigma within some peer groups — for which empirical evidence is provided by Fryer and Torelli (2010). More recently, Bursztyn et al. (2019) show experimentally that peer pressure may reduce participation in educational activities either because students wish to avoid signalling excessive effort or because they fear revealing low ability.[1]

The literature discussed above significantly enriches the standard Human Capital Theory framework (Becker, 1964) by acknowledging that educational choices are not merely individual investment decisions but are deeply ingrained in sociological and relational contexts. However, the intensity and nature of these social drivers are not uniform across the academic landscape.[2] A distinct strand of research suggests that the relative weight of economic incentives versus social influence varies significantly depending on the chosen field of study. For instance, Beffy et al. (2012) demonstrate that while students in fields such as Engineering or Economics respond more rationally to expected earnings and labour market signals, those choosing Humanities or Arts are largely inelastic to monetary returns. In these fields, choices might be predominantly driven by intrinsic preferences and stable cultural orientations (Alstadsæter, 2011), leading to enrolment patterns characterized by a structural stability that appears indifferent to labour market fluctuations or shifting social trends.

Considering Italy, for instance, the yearly survey conducted by Consorzio Interuniversitario AlmaLaurea (2025b) reports that the highest share of graduates driven primarily by cultural factors belongs to the fields of Humanities and Literature (57.9%), Art and Design (49.6%), Psychology (45.0%). The disciplinary fields with the highest share of graduates driven primarily by vocational factors are Industrial and Information Engineering (17.9%), Computer Science and ICT Technologies (15.4%), Economics and Business (15.2%). Physical Education, Political Science, Communication and Languages are instead among the fields with the largest share of graduates reporting no strong cultural or vocational motivations. A different survey that AlmaLaurea conducts on graduates occupational status reports that net monthly earnings one year after graduation vary considerably by field: the highest salaries are found in STEM and Healthcare, ranging between 1650 and 1780 euros. Conversely, graduates in fields such as Sport Sciences and Humanities report lower average returns, with monthly earnings remaining close to 1200 euros (Consorzio Interuniversitario AlmaLaurea, 2025a). Overall, in 2024, 46.7% of graduates in Italy state that both cultural and vocational components were highly important, while 13,7% state that neither of them were important in their educational choices (Consorzio Interuniversitario AlmaLaurea, 2025b).

Therefore, in disciplines where the professional signal is ambiguous or the vocational drive is less pronounced, students might be more susceptible to the "two-audience signalling problem" described by Austen-Smith and Fryer (2005). In these fields, the degree itself may act as a Positional Good (Hirsch, 1976), where its value is derived from its scarcity or its ability to signal social compatibility rather than specific productivity. This creates a fertile ground for the educational conformism noted by Leoni (2022): when clear economic or vocational anchors are absent or weak, peer imitation becomes a primary coordination mechanism. The combination of heterogeneous educational motivations might lead to higher fluctuations or cycles for certain disciplines, contrasting sharply with the linear stability of purely vocational or strictly professional tracks.

The present paper develops a theoretical and dynamic counterpart of this literature: we retain the idea that individual educational choices are shaped by both economic incentives and social influence, but we formalize this mechanism in a stylized analytical way by adopting a *bandwagon–snob* dynamic à la (Caravaggio and Sodini, 2020). In their model of consumption choices, Caravaggio and Sodini—building on the classical insights of Leibenstein (1950), Veblen (1899), and Simmel (1904), and further developed by Naimzada and Tramontana (2009)—show how imitation and distinction among social groups can endogenously generate cyclical and even chaotic collective patterns.

Transposing this intuition into the context of education, we represent the social influence component of Leoni (2022) through a *bandwagon mechanism*: individuals' inclination to enrol increases with the observed prevalence of enrolment among peers, while other groups may display countervailing tendencies towards distinction or nonconformity. Related theoretical approaches to individual schooling choices include dynamic, generational (OLG-style) models where borrowing constraints and indivisibilities can trap families in low-education equilibria and generate multiple steady states (Galor and Zeira, 1993; de la Croix and Doepke, 2003), and a public-economics strand that studies optimal policy in *static* equilibrium setups: partial-equilibrium household-production models that justify quantity controls like compulsory schooling (Balestrino et al., 2017), sectoral general-equilibrium school-choice models with endogenous tuition and peer composition (Epple et al., 2002), and economy-wide general-equilibrium models with endogenous skill prices under skill-biased technical change that characterize the optimal mix of linear income taxes and education subsidies (Jacobs and Thuemmel, 2023).

By contrast, in this paper we focus on a bandwagon–snob dynamic à la (Caravaggio and Sodini, 2020), offering an explicitly dynamic account of how the wage premium and peer pressure jointly shape aggregate enrolment. This approach provides a compact representation of social diffusion processes and allows us to study analytically how the interplay between economic incentives and social imitation may produce multiple equilibria, cyclical fluctuations, or abrupt shifts in aggregate participation—phenomena reminiscent of the endogenous instability discussed by Benhabib and Day (1981), who showed that even rational choices in a stationary environment can generate erratic, non-convergent trajectories when preferences evolve through experience.

Our contribution is twofold: first, we provide a formal theoretical analysis of how the interaction between expected economic returns to education (the *wage premium*) and social influence (*peer pressure*) can generate endogenous oscillations in aggregate educational cycles that emerge from internal feedback rather than exogenous shocks; second, we extend the scope of bandwagon–snob models to the domain of educational decisions, highlighting that the same socially interactive forces governing collective consumption behaviour may also underlie collective educational patterns-at least in certain disciplines, thereby building a conceptual bridge between the economics of education and the non-linear dynamics of social interaction.

The remainder of the paper is organized as follows: Section 2 presents a set of stylized facts regarding enrolment volatility in the Italian higher education system; Section 3 introduces the basic dynamic model, describing how individual educational choices depend jointly on economic motivations and social interactions; Section 4 presents the analytical and numerical exploration of the model with fixed population shares, focusing on the conditions under which imitation and distinction give rise to regular or irregular collective dynamics; Section 5 considers the more general case with endogenous shares; and Section 6 discusses policy interventions and their implications for stabilization. Section 7 concludes by outlining possible extensions and policy implications.

2. Stylized facts on enrolment volatility

To empirically test the theoretical hypothesis that higher education does not configure itself uniformly as a pure investment good or a linear

[2] Literature has shown that social and economic drivers are not uniform across gender as well. According to Zafar (2013), women's choices in education depend roughly twice as much as men on non-monetary drivers than monetary ones.

consumption good, but is heavily or, at least partially, influenced by endogenous social and imitative dynamics, we analyses the historical trajectories of university enrolments in Italy. The original dataset provided by the Italian Ministry of University and Research, comprises the annual flows of newly enrolled students (*immatricolati*) disaggregated by degree class (*classe di laurea*).

The data cleaning strategy and manipulation is illustrated in the Appendix. In summary, we excluded nationally restricted programmes such as Medicine and Surgery whose number of newly enrolled and its potential variations are established at governmental level. We consider a restricted time span (2011–2024) in order to make sure that all degree classes considered were existing in every year considered, but also to ensure homogeneity in classes coding after two main reforms that characterized the Italian university system in the 90s (see the Appendix for further details). We only consider enrolment into Bachelor's degrees or Single-Cycle degrees to account for the first entry in Higher Education and avoid potential bias towards strictly vocational preferences that might come at a later stage such as the Master's Degree.

In order to isolate social and emulative components (short-term fads or network effects) from long-term structural trends (driven, for instance, by structural technological shifts or demographic transitions), an OLS-based detrending technique was applied. For each degree class, the annual enrolment share (E_t) was linearly regressed on time (Year_t):

$$\mathrm{E}_t = \alpha + \beta(\mathrm{Year}_t) + \varepsilon_t$$

The resulting residuals (ε_t) represent the pure year-on-year deviation from the baseline historical trajectory. The standard deviation of these residuals (σ_ε) was then calculated to quantify the amplitude of the oscillations left unexplained by the linear trend.

Furthermore, because degree classes exhibit profound dimensional asymmetries, with some disciplines capturing systematically larger enrolment shares than others, a comparison based solely on the absolute standard deviation would be biased. Therefore, we computed the Detrended Coefficient of Variation ($CV_{\mathrm{Detrended}}$), normalizing the standard deviation of the residuals by the historical mean share of the discipline (μ):

$$CV_{\mathrm{Detrended}} = \left(\frac{\sigma_\varepsilon}{\mu}\right) \times 100$$

This relative metric serves as an indicator of volatility with respect to the average trend. A high value indicates a discipline that might be highly exposed to emotional or imitative shocks in student preferences. The complete ranking of all analysed disciplines, along with their respective historical mean shares, residual standard deviations, and final $CV_{\mathrm{Detrended}}$ scores, is compiled in Table 3. To graphically illustrate the polarization of behavioural regimes within the Italian higher education system, Fig. 1 maps the empirical trajectories of the three highest-volatility degree classes (top row) against the three lowest-volatility classes (bottom row). Within each panel, three visual layers offer a comprehensive diagnostic look. The solid line represents the actual trajectory of the enrolment share over time. The dashed line represents the linear regression trend, acting as the structural baseline of the discipline's evolution. The dashed ribbons indicate the standard deviation of residuals ($\pm\sigma_\varepsilon$) centred on the linear trend, mapping the field's structural turbulence area.

Two opposing behavioural regimes emerge from the empirical analysis, validating the hypothesized theoretical taxonomy. However, given the vastly different structural sizes of the analysed degree classes, this polarization must be evaluated through the lens of relative volatility (the detrended CV) rather than absolute enrolment share fluctuations. The degree classes displayed in the upper three panels represent highly volatile regimes. A visual inspection might obscure this fact due to the small absolute percentages on their Y-axes; for instance, Navigation Sciences (L-28) fluctuates within a narrow absolute band of approximately 0.075 percentage points. Yet, relative to its tiny baseline share (around 0.1%), these shifts represent massive year-to-year structural shocks, where the student cohort nearly doubles or halves in size, yielding an exceptionally high detrended CV of 24.6%. Sociologically and economically, this pattern is characteristic of hybrid or expressive goods. In these niche or less institutionally anchored disciplines, professional boundaries are fluid and information asymmetry is high; consequently, student choices are strongly driven by emulative dynamics, temporary 'herd effects', or fluctuating social perceptions. Conversely, the bottom three panels depict the opposite scenario: highly stable regimes. Here, large shifts must be evaluated against a massive baseline market share, such as that one of Economics and Business Management (L-18) of over 12%. Relative to their structural volume, the detrended variance in these fields is minimal, with all three classes consistently exhibiting a CV well below 4%. Their micro-fluctuations do not suffer from sudden behavioural bubbles or structural shocks; instead, they more consistently anchor themselves to steady, highly predictable linear trends driven by gradual macroeconomic and demographic shifts. This might describe the typical behaviour of pure investment goods tightly linked to stable labour markets, clear career trajectories, or historically defined professions, such as accountant or teacher.

The structural variability of the Detrended Coefficients of Variation, thus, supports the idea that the nature of university choice changes profoundly depending on the discipline, confirming that the amplitude of chaotic oscillations responds to endogenous and social logics intrinsic to the nature of the field of study itself.

3. The model

We consider an economy populated by a unit mass of heterogeneous agents who must allocate their available resources between educational investment and current consumption. The central idea of the model is that, particularly for certain fields of study, higher educational choices are not purely driven by individual economic incentives, but are also shaped by social interaction and behavioural heterogeneity. The population is composed of two behavioural types. The first group consists of Followers (F), who display conformist and imitative behaviour and whose educational choices tend to align with the prevailing social behaviour. The second group consists of Positional Agents (P), who instead display counter-adaptive or status-seeking behaviour and evaluate education also in terms of social distinction. Let $\lambda \in [0,1]$ denote the population share of Followers, while $(1-\lambda)$ denotes the share of Positional Agents.

In each period, agents choose how to allocate their income between educational participation and standard consumption. More precisely, each individual chooses two variables: the level of educational investment $e_{i,t}$ and the quantity of a generic consumption good $c_{i,t}$, with $i \in \{F, P\}$. The variable $e_{i,t}$ represents the intensity of educational participation. Rather than modelling education as a purely binary decision (e.g. enrol versus not enrol), we interpret it as a continuous choice capturing the overall amount of resources, effort, and commitment devoted to higher education. Higher values of $e_{i,t}$ therefore correspond to stronger educational engagement, including tuition expenditures, time devoted to study, and investment in human capital accumulation. Conversely, $c_{i,t}$ denotes the consumption of a composite private good summarizing all alternative uses of income outside education. It captures current material consumption and all immediate utility-generating expenditures that compete with educational investment for the agent's limited resources.

Agents are subject to the following budget constraint:

$$p_e e_{i,t} + p_c c_{i,t} = I, \tag{1}$$

where I denotes individual income, p_e is the unit cost of education (including tuition fees and other education-related costs), and p_c is the price of the consumption good. For simplicity, income and prices are assumed to be constant over time. Preferences of each agent type $i \in \{F, P\}$ are represented by the following Cobb–Douglas utility function:

$$U_{i,t}(e_{i,t}, c_{i,t}; \alpha_{i,t}, \beta_{i,t}) = \alpha_{i,t} \log e_{i,t} + \beta_{i,t} \log c_{i,t}, \qquad \alpha_{i,t}, \beta_{i,t} \in (0,1), \tag{2}$$

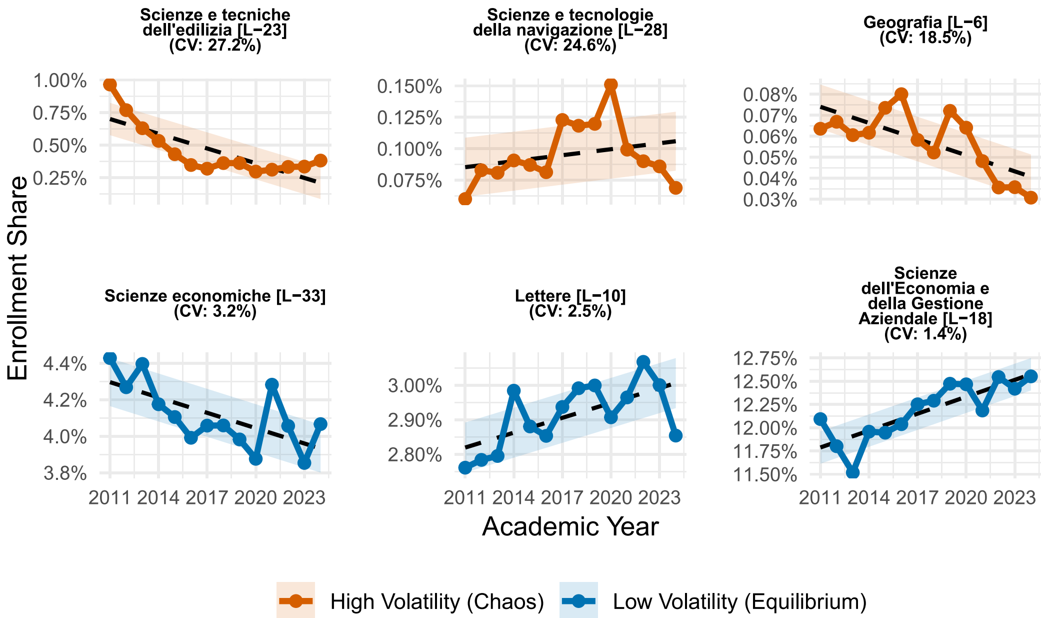

Fig. 1. Unstable Regimes vs. Equilibrium Regimes. The dashed line represents the linear trend. Ribbons indicate the standard deviation of residuals ($\pm\sigma_\varepsilon$) around the trend under a free Y-axis layout. Fields of study from left to right, from top to bottom: Building Sciences and Techniques; Navigation Sciences and Technologies; Geography; Economics; Literature and Humanities; Economics and Business Management.

The parameters $\alpha_{i,t}$ and $\beta_{i,t}$ weigh the endogenous intensity of preferences for education and consumption, respectively. Unlike standard Cobb–Douglas specifications, these coefficients are not constrained to sum to one and should therefore not be interpreted as expenditure shares. Instead, they capture the relative social and economic attractiveness of educational investment and consumption at each point in time.

Solving the constrained utility maximization problem we get:

$$e^*_{i,t} = \frac{\alpha_{i,t}}{\alpha_{i,t}+\beta_{i,t}} \cdot \frac{I}{p_e}; \qquad c^*_{i,t} = \frac{\beta_{i,t}}{\alpha_{i,t}+\beta_{i,t}} \cdot \frac{I}{p_c}, \qquad i \in \{F, P\}. \tag{3}$$

Following a strand of the theoretical literature on endogenous preferences and social interaction (see, among others, Benhabib and Day, 1981; Di Giovinazzo and Naimzada, 2015 and Caravaggio and Sodini, 2020), the key mechanism of the model is that agents' preference intensities evolve endogenously in response to aggregate social dynamics. More precisely, individuals observe past aggregate educational participation, aggregate consumption, and the wage premium associated with education, and adapt their preferences accordingly. Educational choices are therefore not determined solely by static economic incentives, but also by imitation, social influence, and positional concerns.

In this framework, the aggregate levels of education and consumption at time t are defined as:

$$E_t = \lambda\, e_{F,t} + (1-\lambda)\, e_{P,t}, \quad \text{where} \quad E_t \in \left[0, \frac{I}{p_e}\right], \tag{4}$$

$$C_t = \lambda\, c_{F,t} + (1-\lambda)\, c_{P,t}, \quad \text{where} \quad C_t \in \left[0, \frac{I}{p_c}\right]. \tag{5}$$

As drawn from the stylized facts discussed in Section 2, educational decisions are shaped by a heterogeneous mix of drivers; they depend not only on social interactions and consumption motives, but also on the expected economic returns associated with higher education. To capture this mechanism, we introduce an endogenous wage premium associated with educational attainment. We assume that the premium depends negatively on the aggregate level of educational participation. The underlying intuition is straightforward: when educated workers are relatively scarce, higher education provides a stronger labour-market advantage, whereas an expansion in the supply of educated labour progressively reduces the relative economic return to education. Formally, the wage premium is defined as

$$\Pi_t = \max\{\bar{\Pi} - \kappa E_t, 0\}, \tag{6}$$

where $\bar{\Pi} > 0$ denotes the maximum attainable wage premium, while $\kappa \geq 0$ measures the sensitivity of the wage premium to the aggregate supply of educated workers.[3]

The two agent types react differently to these aggregate social and economic signals. Followers are mainly driven by imitative behaviour: the attractiveness of education increases when educational participation becomes more widespread within society. Positional Agents, instead, evaluate education both as a social signal and as an economic investment. As educational participation expands and education becomes less socially distinctive, their incentive to invest in education declines; however, this negative effect may be partially offset by a sufficiently high wage premium. Following Caravaggio and Sodini (2020), we assume that preference intensities at time $t+1$ are determined by aggregate social and economic conditions observed at time t, according to the following behavioural specifications:

$$\begin{aligned} &\alpha_{F,t+1} = 1 - \exp\left(-\rho E_t - \rho_\pi \Pi_t\right), && \beta_{F,t+1} = 1 - \exp\left(-\rho C_t\right), \\ &\alpha_{P,t+1} = \exp\left(-\sigma E_t\right)\left[1 - \exp\left(-\sigma_\pi \Pi_t\right)\right], && \beta_{P,t+1} = \exp\left(-\sigma C_t\right). \end{aligned} \tag{7}$$

where $\rho, \rho_\pi, \sigma, \sigma_\pi > 0$ measure the sensitivity of agents' preferences to past aggregate educational participation, aggregate consumption, and the wage premium associated with education. More specifically, the parameters ρ and ρ_π capture the strength of the conformist behaviour displayed by Followers, while σ captures the counter-adaptive response of Positional Agents to aggregate educational participation and σ_π measures their sensitivity to the wage premium associated with education.

The exponential specifications adopted for the preference parameters can be interpreted as reduced-form representations of social interaction and status-based behavioural responses. Terms of the form $1 - \exp(-x)$ capture increasing but saturating effects of social exposure and economic incentives, reflecting the idea that the marginal impact

[3] The presence of the max operator introduces a regime-switching structure in the model. For $\bar{\Pi} - \kappa E_t > 0$, the system evolves in an interior smooth regime, while for $\bar{\Pi} - \kappa E_t \leq 0$ the wage premium is completely exhausted. Unless otherwise specified, the analytical derivations developed below refer to the interior regime.

of aggregate education and wage premium decreases as these variables become widespread. Conversely, decreasing exponential terms represent the erosion of positional attractiveness when a behaviour becomes socially diffused. In this framework, Followers are positively influenced by the social diffusion of education and by its associated economic return, whereas Positional Agents value education mainly when it remains simultaneously rewarding in economic terms and sufficiently exclusive from a social perspective. In addition, the exponential specifications ensure that all preference intensities remain bounded within the unit interval, preventing unbounded behavioural responses.

Since the aim of the analysis is to study educational choices, we can reduce the problem's dimensionality. The aggregate budget constraint creates a fixed linear trade-off between the two goods:

$$C_t = \frac{I}{p_c} - \frac{p_e}{p_c} E_t, \tag{8}$$

Hence, the dynamics of aggregate consumption C_t are entirely determined by the evolution of aggregate educational participation E_t. The aggregate system can therefore be reduced to a one-dimensional dynamical framework in which the entire evolution of the economy is fully characterized by the map governing E_t. Thus, by combining Eq. (3) and (4) we obtain:

$$E_{t+1} = \frac{I}{p_e}\left[\lambda \frac{\alpha_{F,t+1}}{\alpha_{F,t+1}+\beta_{F,t+1}} + (1-\lambda)\frac{\alpha_{P,t+1}}{\alpha_{P,t+1}+\beta_{P,t+1}}\right]. \tag{9}$$

By substituting the behavioural specifications in Eq. (7) into the aggregate educational demand function, the dynamics of aggregate educational participation can be represented by the following one-dimensional nonlinear map:

$$\begin{aligned}\Gamma :\ & E_{t+1} \\ = & \frac{I}{p_e}\cdot\left[\lambda \frac{1-\exp\left(-\rho E_t - \rho_\pi \Pi_t\right)}{2-\exp\left(-\rho E_t - \rho_\pi \Pi_t\right) - \exp\left(-\frac{\rho(I-p_e E_t)}{p_c}\right)}\right. \\ & \left. + (1-\lambda)\frac{\exp\left(-\sigma E_t\right)\cdot\left(1-\exp\left(-\sigma_\pi \Pi_t\right)\right)}{\exp\left(-\sigma E_t\right)\cdot\left(1-\exp\left(-\sigma_\pi \Pi_t\right)\right)+\exp\left(-\frac{\sigma(I-p_e E_t)}{p_c}\right)}\right].\end{aligned} \tag{10}$$

The map Γ summarizes the interaction between social imitation, positional incentives, and endogenous labour-market returns associated with education. Aggregate educational participation evolves through the combined effect of conformist behaviour, status-based responses, and the endogenous wage premium generated by the scarcity of educated workers. Moreover, since for each agent type $i \in \{F, P\}$ the ratio $\alpha_{i,t}/(\alpha_{i,t}+\beta_{i,t})$ belongs to the interval $(0,1)$, individual educational demand is necessarily bounded between zero and I/p_e. Consequently, aggregate educational participation also satisfies

$$0 < E_t < \frac{I}{p_e}.$$

It follows that the aggregate map defines a self-map on the compact interval $\Gamma : [0,\bar{E}] \to [0,\bar{E}]$, with $\bar{E} = I/p_e$. This property guarantees that the dynamics remain confined within the economically feasible domain and ensures that aggregate educational participation evolves within a bounded state space. At the same time, the interaction between imitative behaviour, positional concerns, and endogenous labour-market incentives may generate highly nonlinear and potentially non-monotonic dynamics, depending on parameter configurations. The qualitative properties of the map Γ, including the existence and stability of equilibria, the possible emergence of critical points and absorbing regions, and the associated long-run dynamic behaviour, will be analysed in the next section.

4. Dynamic trajectories with fixed population structure

In this section, we analyse the dynamic properties of the map Γ under the assumption of a fixed population composition, namely a constant share λ of Followers and $(1-\lambda)$ of Positional Agents. Under this assumption, the evolution of the economy is fully characterized by the one-dimensional nonlinear map Γ defined in Eq. (10).

We begin by establishing two basic properties of the system. First, we show that Γ admits at least one fixed point. Second, under suitable regularity conditions, we identify an invariant region that confines the long-run dynamics of the system.[4]

Lemma 1 (*Existence of Steady States*). *Consider the one-dimensional map*

$$E_{t+1} = \Gamma(E_t;\lambda), \qquad E_t \in D := [0,\bar{E}],$$

where, for every $\lambda \in [0,1]$, the map

$$\Gamma(\cdot;\lambda) : D \to D$$

is continuous. Then, for every $\lambda \in [0,1]$, the system admits at least one steady state $E^* \in D$, namely

$$E^* = \Gamma(E^*;\lambda).$$

Proposition 1 (*Uniqueness and Multiplicity of Steady States*). *Let*

$$\Gamma(E;\lambda) = \bar{E}\left[\lambda s_F(E) + (1-\lambda)s_P(E)\right], \qquad \lambda \in [0,1],$$

where

$$s_i(E) := \frac{\alpha_i(E)}{\alpha_i(E)+\beta_i(E)}, \qquad i \in \{F,P\}.$$

Assume that $\Gamma(\cdot;\lambda)$ is continuously differentiable on each smooth regime of D, and suppose that there exist two constants a_F and a_P such that

$$\bar{E}\, s'_F(E) \le a_F, \qquad \bar{E}\, s'_P(E) \le a_P$$

for every E in each smooth regime of D.

Then the steady state is unique whenever one of the following conditions holds:

$$\begin{cases} \lambda < \dfrac{1-a_P}{a_F-a_P}, & \text{if } a_F > a_P, \\ \lambda > \dfrac{1-a_P}{a_F-a_P}, & \text{if } a_F < a_P, \\ a_F = a_P < 1, & \text{for every } \lambda \in [0,1]. \end{cases}$$

Conversely, multiple steady states can arise only if

$$H(E;\lambda) := \Gamma(E;\lambda) - E$$

is non-monotone. In each smooth regime, changes in monotonicity can occur only at interior points satisfying

$$\bar{E}\left[\lambda s'_F(E) + (1-\lambda)s'_P(E)\right] = 1.$$

Moreover, if all steady states are such that $H_E(E^*;\lambda) \neq 0$, then non-uniqueness implies the existence of at least three steady states:

$$N(\lambda) > 1 \quad \Longrightarrow \quad N(\lambda) \ge 3.$$

The proposition highlights the role of population composition in shaping the structure of steady states. The parameter λ weights the relative importance of the two behavioural forces in the economy: conformist imitation, associated with Followers, and counter-adaptive positional behaviour, associated with Positional Agents. When the weighted marginal response of aggregate educational participation remains sufficiently moderate, the map crosses the 45-degree line only once and the steady state is unique. By contrast, multiple steady states can emerge only when the interaction between the two behavioural types generates a non-monotone response of $H(E;\lambda) = \Gamma(E;\lambda) - E$. This corresponds to a situation in which social imitation, status-based incentives, and the endogenous wage premium produce competing feedback effects. In such cases, different self-consistent levels of aggregate educational

[4] All proofs are available in the Appendix.

participation are possible, reflecting alternative coordination outcomes of the same socio-economic environment.

The previous results characterize the existence of steady states and provide conditions under which they are unique or multiple. To further clarify the economic mechanisms behind these equilibria, we now examine how an interior locally stable steady state responds to changes in the key parameters of the model. Although these comparative-static results are local in nature and do not provide a full characterization of the global dynamics, they help identify the channels through which population composition, wage-premium sensitivity, and behavioural reactivity affect long-run educational participation.

Proposition 2 (*Local Comparative Statics*)**.** *Let $E^* \in (0, \bar{E})$ be a locally stable steady state of the map, with*

$$E^* = \Gamma(E^*; \lambda), \qquad 1 - \Gamma_E(E^*; \lambda) \neq 0.$$

Let

$$C^* := C(E^*), \qquad \Pi^* := \Pi(E^*), \qquad E^\dagger := \frac{I}{p_e + p_c}.$$

Then the following comparative-static results hold. First,

$$\frac{dE^*}{d\lambda} = \frac{\bar{E}\left[s_F(E^*) - s_P(E^*)\right]}{1 - \Gamma_E(E^*; \lambda)}.$$

Hence,

$$s_F(E^*) > s_P(E^*) \Rightarrow \frac{dE^*}{d\lambda} > 0, \qquad s_F(E^*) < s_P(E^*) \Rightarrow \frac{dE^*}{d\lambda} < 0,$$

while $s_F(E^*) = s_P(E^*)$ implies $dE^*/d\lambda = 0$. Second, if $\Pi^* > 0$, then

$$\frac{dE^*}{d\kappa} < 0, \qquad \frac{dE^*}{d\rho_\pi} > 0.$$

Third, the effect of the imitative reactivity parameter ρ satisfies

$$\operatorname{sign}\left(\frac{dE^*}{d\rho}\right) = \operatorname{sign}\left[E^*\left(e^{\rho C^*} - 1\right) - C^*\left(e^{\rho E^* + \rho_\pi \Pi^*} - 1\right)\right].$$

In particular,

$$E^*\left(e^{\rho C^*} - 1\right) > C^*\left(e^{\rho E^* + \rho_\pi \Pi^*} - 1\right) \Rightarrow \frac{dE^*}{d\rho} > 0,$$

whereas the opposite inequality implies $dE^*/d\rho < 0$. Moreover,

$$E^* \geq E^\dagger \Rightarrow \frac{dE^*}{d\rho} \leq 0,$$

with strict inequality whenever $\rho_\pi \Pi^* > 0$. If $\rho_\pi = 0$, this reduces to

$$\operatorname{sign}\left(\frac{dE^*}{d\rho}\right) = \operatorname{sign}\left(C^* - E^*\right).$$

Finally, the effect of the positional reactivity parameter satisfies

$$E^* < E^\dagger \Rightarrow \frac{dE^*}{d\sigma} > 0, \qquad E^* > E^\dagger \Rightarrow \frac{dE^*}{d\sigma} < 0.$$

The comparative-static results identify two types of effects. Some are unambiguous. A higher κ reduces steady-state educational participation because the wage premium declines more sharply with the supply of educated workers. By contrast, a higher ρ_π increases educational participation whenever the premium is positive, since Followers become more responsive to the economic return to education. Other effects are conditional. The impact of λ depends on whether, at the steady state, Followers or Positional Agents display the higher educational share. Similarly, the effect of positional reactivity σ depends on whether education is relatively scarce or widespread compared with aggregate consumption. If $E^* < E^\dagger$, stronger positional reactivity raises educational participation; if $E^* > E^\dagger$, it lowers it. The effect of ρ admits the same simple threshold interpretation only when $\rho_\pi = 0$; otherwise, the wage-premium component alters the balance between imitation and economic incentives.

While these comparative-static results describe how a locally stable equilibrium reacts to changes in key parameters, they do not furnish any analytical characterization of the system's dynamic behaviour away from equilibrium. Because the nonlinear structure of the map makes a complete closed-form stability analysis difficult, we focus instead on a global boundedness property of trajectories. The following proposition identifies conditions under which all orbits eventually enter a compact absorbing interval.

Proposition 3 (*Absorbing Interval*)**.** *Let $D = [0, \bar{E}]$, and let $\Gamma : D \to D$ be continuous. Assume that there exists a unique $E_c \in (0, \bar{E})$ such that*

$$\Gamma_E(E; \lambda) > 0 \text{ for } E < E_c, \qquad \Gamma_E(E; \lambda) < 0 \text{ for } E > E_c,$$

and define

$$E_{\max} := \Gamma(E_c), \qquad E_{\min} := \Gamma(E_{\max}), \qquad J_\Gamma := [E_{\min}, E_{\max}].$$

If

$$\Gamma([0, E_{\max}]) \subseteq J_\Gamma,$$

then J_Γ is a forward invariant absorbing interval. In particular,

$$\Gamma(J_\Gamma) \subseteq J_\Gamma$$

and

$$\Gamma^2(D) \subseteq J_\Gamma.$$

Therefore, for every $E_0 \in D$, there exists $n \leq 2$ such that

$$E_t \in J_\Gamma \qquad \text{for all } t \geq n.$$

The result indicates that when the dynamical system is self-mapped and unimodal, all trajectories remain confined within a bounded region of the feasible interval. Whether they converge to an equilibrium point, a stable periodic orbit, or a chaotic attractor, they are all contained within a specific subset of the plane known as the *absorbing region* (or *trapping region*).

In the following sections we will analyse the behaviour of the map in two different scenarios. The first one where only positional agents care for the wage premium while in the second one both the types care for it.

4.1. The case $\rho_\pi = 0$: followers without wage-premium sensitivity

In this subsection, we consider the benchmark case in which Followers react only to social imitation and do not directly respond to the wage premium. Formally, this amounts to setting $\rho_\pi = 0$, while Positional Agents may still take the wage premium into account through the parameter σ_π. This specification isolates the dynamic tension between a purely imitative force, generated by Followers, and the counter-adaptive behaviour of Positional Agents, whose educational preferences depend both on social distinctiveness and economic returns.

Although the previous results provide analytical insights into existence, possible multiplicity, and boundedness, the nonlinear structure of Γ prevents a complete closed-form classification of stability regions in the parameter space. We therefore complement the analytical analysis with numerical simulations, aimed at describing the long-run behaviour of the system under different behavioural and population configurations. The numerical investigation focuses on two main dimensions. The first concerns the population composition, captured by λ, and explores how the relative weight of the two behavioural types affects aggregate stability. The second concerns the behavioural reactivity of the two groups, namely the interplay between the imitative response of Followers, governed by ρ, and the counter-adaptive positional response of Positional Agents, governed by σ_E, σ_π, and the wage-premium sensitivity κ.

Unless otherwise stated, the numerical simulations are performed using the benchmark calibration reported in Table 1. Since the model is theoretical and several behavioural parameters have no direct empirical counterpart, the numerical analysis should not be interpreted as a calibration exercise aimed at reproducing a specific degree class or cohort.

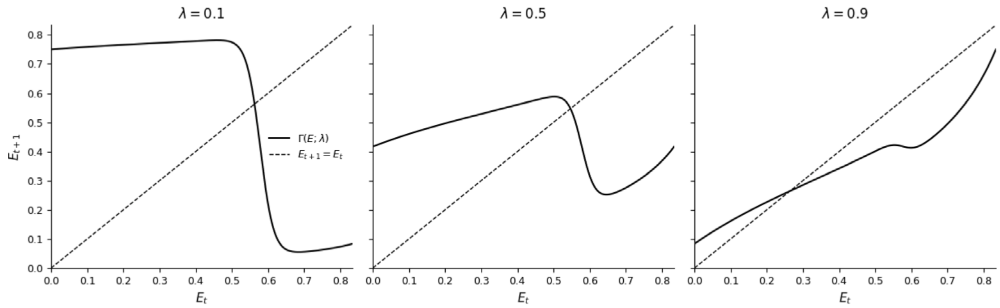

Fig. 2. Aggregate map $\Gamma(E;\lambda)$ and 45-degree line for different population compositions. Intersections with the diagonal identify steady states, while the local slope around each intersection provides information on local stability.

Table 1
Benchmark calibration used in the numerical simulations.

I	p_e	p_c	ρ	σ	σ_π	$\bar{\Pi}$	κ	λ	E_0
1	1.2	0.53	0.98	16.5	5	1	0.3	0.5	0.3

The benchmark parameter set is instead used as a controlled numerical laboratory: it keeps the dynamics within the feasible domain, represents a heterogeneous environment in which imitative and positional forces coexist, and places the system in a region where stable, periodic, and irregular regimes can all emerge. Some parameters are varied in specific exercises; the values reported in Table 1 are therefore used whenever the corresponding parameter is kept fixed.

After fixing the benchmark calibration, all one-dimensional simulations start from $E_0 = 0.3$, with λ denoting the fixed population share of Followers. In each bifurcation diagram, the parameter on the horizontal axis is varied over the range shown in the figure, while all remaining parameters are kept at their benchmark values, unless explicitly stated otherwise.

Before turning to bifurcation diagrams, we first inspect the shape of the aggregate map for different population compositions. This preliminary exercise is useful because the parameter λ does not simply scale aggregate educational participation; rather, it changes the relative weight of two distinct behavioural forces: the imitative response of Followers and the counter-adaptive response of Positional Agents.

Fig. 2 provides a first graphical intuition on the role of population composition. For low values of λ, the map is mainly driven by Positional Agents and displays a marked counter-adaptive response: as education becomes more widespread, its attractiveness as a distinctive signal declines. For high values of λ, instead, Followers dominate and the map is shaped primarily by imitative behaviour. The intermediate case combines these two forces and produces a more nonlinear map, with both increasing and decreasing regions. This suggests that heterogeneous populations, in which imitation and positional concerns coexist, are the most likely to generate complex dynamic outcomes.

Having shown that the population composition affects the geometry of the aggregate map, we now examine how this translates into long-run dynamics. Fig. 3 reports the bifurcation diagram obtained by varying the share of Followers, λ, while keeping the remaining parameters fixed. This exercise is designed to assess whether instability is mainly associated with homogeneous behavioural populations or with intermediate population compositions, where imitative and positional forces coexist.

Fig. 3 shows that population composition plays a central role in shaping the asymptotic dynamics. For low values of λ, the population is dominated by Positional Agents and the system exhibits a period-2 cycle, with persistent switching between high and low levels of aggregate educational participation. This reflects the counter-adaptive nature of positional behaviour: when education is scarce, it is attractive as a distinctive signal; when it becomes widespread, its positional value declines and participation falls. As λ increases, the imitative component associated with Followers becomes progressively more relevant. In the intermediate range of population compositions, imitation and positional concerns coexist with comparable strength. Their interaction generates a richer dynamic structure, including higher-period cycles and irregular fluctuations. This is the region in which the map displays the strongest folding behaviour and the long-run dynamics become more complex. For high values of λ, the imitative force dominates. The system then moves away from the high-low switching generated by positional behaviour and tends towards more regular dynamics. Overall, the diagram suggests that different forms of instability are associated with different population structures: positional dominance generates persistent alternation, whereas mixed populations generate more complex fluctuations through the interaction between imitation and status-seeking behaviour.

We next compare the dynamic role of the two behavioural reactivity parameters, as shown in Panels (a) and (b) of Fig. 4.

Fig. 4 shows that the two behavioural reactivity parameters play markedly different dynamic roles. In panel (a), increasing ρ progressively regularizes the dynamics. Starting from an initially irregular regime, the system undergoes a sequence of period-halving transitions and settles into a stable period-two cycle. Thus, stronger imitative reactivity does not amplify complexity in this parameter set; rather, it disciplines the dynamics by reinforcing a more regular alternation in aggregate educational participation. Panel (b) displays the opposite pattern. As σ increases, Positional Agents become more sensitive to the diffusion of education and to the loss of its distinctive value. This strengthens the counter-adaptive mechanism and generates a period-doubling cascade, eventually leading to persistent irregular fluctuations. Hence, while imitation tends to regularize the long-run dynamics, stronger positional reactivity is the main source of destabilization. From a socio-economic perspective, the contrast reflects the different nature of the two mechanisms. Imitation tends to coordinate educational choices around recurrent patterns, whereas positional concerns generate a counter-adaptive feedback: education is attractive when scarce, but loses status value as it becomes widespread. This feedback is the main source of the nonlinear amplification observed in panel (b).

We now examine the role of the wage-premium feedback in the benchmark case $\rho_\pi = 0$. Since Followers do not directly react to the wage premium, changes in κ affect the dynamics through the response of Positional Agents. The parameter κ measures how rapidly the wage premium declines as aggregate educational participation increases. Hence, it captures the strength of the labour-market feedback generated by the supply of educated workers. A distinctive feature of this channel is the presence of a switching threshold. Since

$$\Pi_t = \max\{\bar{\Pi} - \kappa E_t, 0\},$$

the wage premium is positive whenever

$$E_t < E_\Pi(\kappa) := \frac{\bar{\Pi}}{\kappa},$$

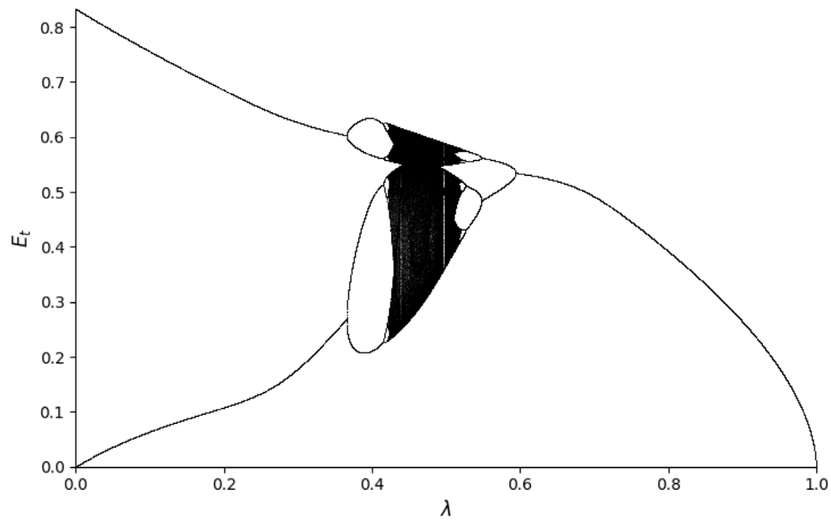

Fig. 3. Bifurcation diagram with respect to the population share of Followers, λ.

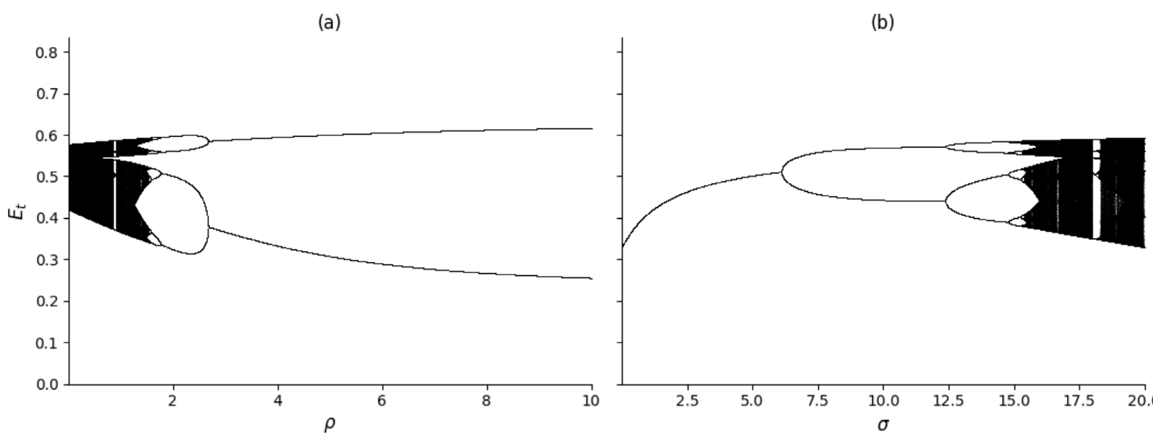

Fig. 4. Bifurcation diagrams with respect to behavioural reactivity parameters. Panel (a) varies the imitative reactivity of Followers, ρ, fixing $\sigma = 16.5$. Panel (b) varies the positional reactivity of Positional Agents, σ, fixing $\rho = 0.98$.

and is exhausted whenever

$$E_t \geq E_\Pi(\kappa).$$

Thus, $E_\Pi(\kappa)$ separates the positive-premium regime from the saturated-premium regime. As κ increases, this threshold moves downward, making it more likely that the long-run orbit crosses the boundary at which the wage premium collapses. The exercise below studies how this moving threshold affects the asymptotic dynamics of educational participation.

Fig. 5 highlights the non-smooth nature of the wage-premium feedback. For low values of κ, the switching threshold $E_\Pi(\kappa) = \bar{\Pi}/\kappa$ lies above the attractor, so that the wage premium remains positive along the simulated orbit. In this range, the system displays irregular fluctuations generated within the positive-premium regime. As κ increases, the threshold moves downward and begins to intersect the region visited by the long-run trajectory. The closure of the irregular regime and the subsequent emergence of low-period cycles are therefore consistent with a border-collision-type mechanism induced by the saturation of the wage premium.[5] The time series reported in Fig. 6 clarify the adjustment process behind this transition. Around the second change in periodicity, the wage premium is zero in most periods and reappears only when educational participation falls sufficiently. After the transition, this recovery becomes more regularly synchronized with the troughs of E_t, indicating that the attractor has been reorganized around recurrent episodes of premium exhaustion and recovery. Thus, a stronger wage-premium sensitivity does not simply dampen fluctuations; it changes the timing of the feedback between educational participation and economic returns. To further support this interpretation, Table 2 reports the detected number of asymptotic branches and the frequency with which the premium is exhausted for selected values of κ.

Table 2
Asymptotic branches and premium saturation along the κ-bifurcation diagram.

κ	$E_\Pi(\kappa)$	Detected branches	Frequency $\Pi_t = 0$
1.5	0.6667	>100	0.000
2.0	0.5000	>100	0.295
3.0	0.3333	2	0.500
5.5	0.1818	2	0.500
6.2	0.1613	5	0.600
6.5	0.1538	3	0.667
9.0	0.1111	3	0.667

The evidence in Table 2 confirms the mechanism suggested by the figures. When the premium is never exhausted, the orbit displays a large number of asymptotic branches. Once the trajectory starts visiting

[5] We use the expression “border-collision-type mechanism” in a descriptive sense. A complete bifurcation analysis of the associated piecewise-smooth map is beyond the scope of the present paper.

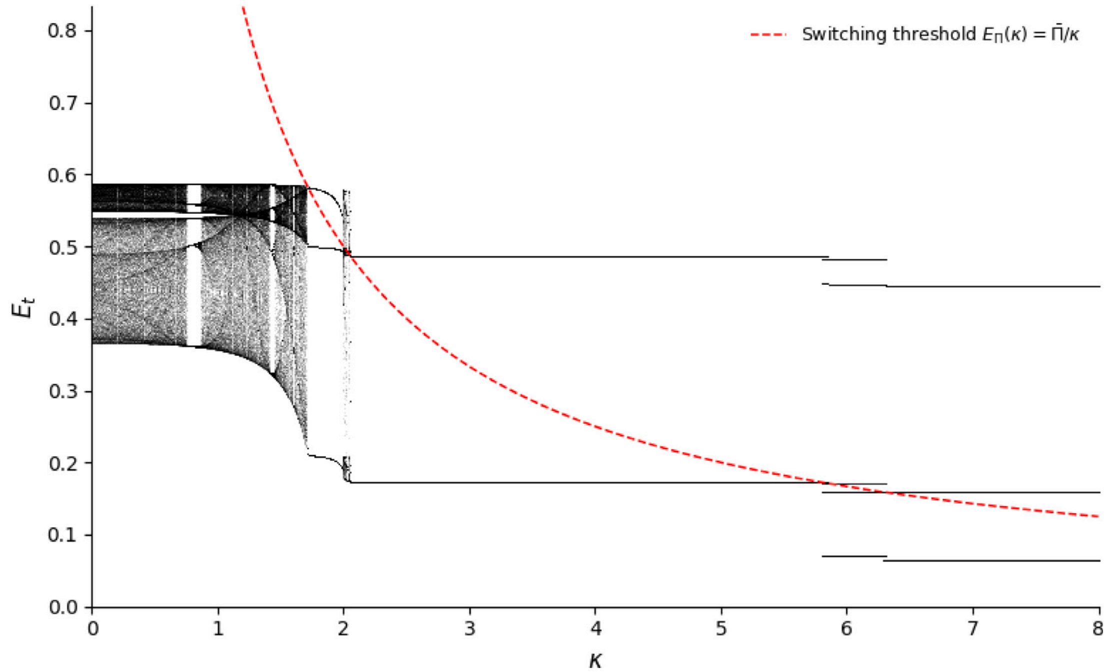

Fig. 5. Bifurcation diagram with respect to the wage-premium sensitivity parameter κ. The dashed curve reports the switching threshold $E_{\Pi}(\kappa) = \bar{\Pi}/\kappa$, which separates the positive-premium regime from the saturated-premium regime. The figure shows how changes in κ alter the long-run dynamics as the threshold moves through the region visited by the orbit.

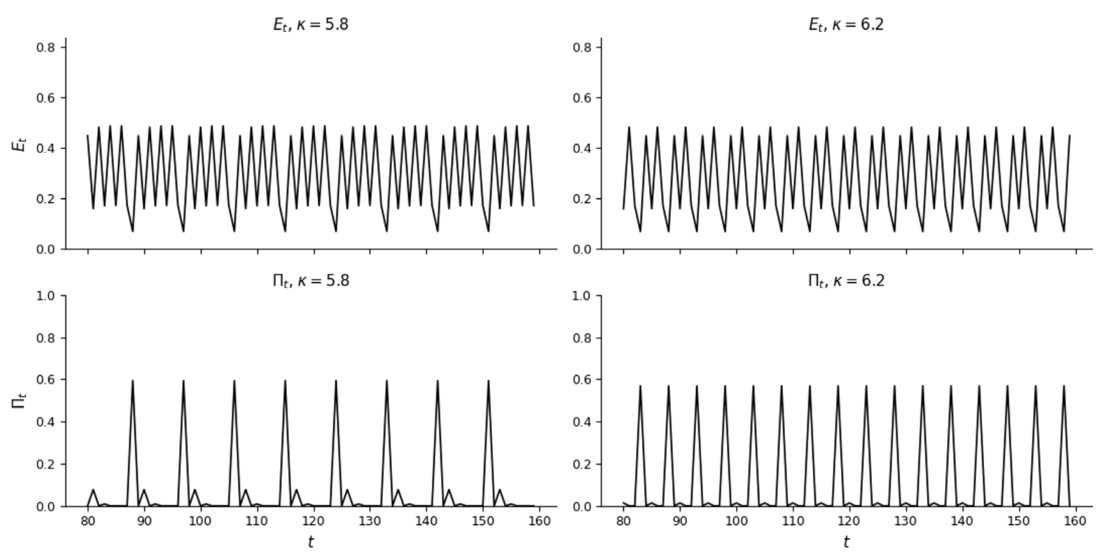

Fig. 6. Time series of aggregate educational participation and wage premium around the transition in the κ-bifurcation diagram. The left panels refer to $\kappa = 5.8$, while the right panels refer to $\kappa = 6.2$. The comparison shows how the change in the periodic structure is associated with a different synchronization between educational participation and the intermittent recovery of the wage premium.

the saturated-premium regime, the irregular attractor collapses into a period-two cycle. For larger values of κ, the threshold crosses additional points of the periodic orbit, and the detected periodicity changes again, eventually leading to a period-three regime. The increasing frequency of observations with $\Pi_t = 0$ shows that the labour-market feedback progressively imposes a stronger switching structure on the dynamics.

The benchmark case $\rho_\pi = 0$ shows that complex aggregate dynamics can arise even when Followers do not directly react to the wage premium. In this setting, instability is mainly driven by the interaction between imitation and positional counter-adaptation, with richer fluctuations emerging for intermediate population compositions.

Behavioural reactivity and wage-premium feedback shape these fluctuations in different ways. Stronger imitative reactivity tends to regularize the dynamics, whereas stronger positional reactivity amplifies counter-adaptive responses and may generate persistent irregular fluctuations. The wage-premium channel, operating only through Positional Agents in this benchmark, can further reorganize the attractor through the switching boundary between positive and exhausted returns to education. The next subsection asks whether these dynamics change when Followers also take the wage premium into account, that is, when $\rho_\pi > 0$.

4.2. Followers with wage-premium sensitivity

We now extend the benchmark case by allowing Followers to react directly to the wage premium, that is, by considering $\rho_\pi > 0$. In this

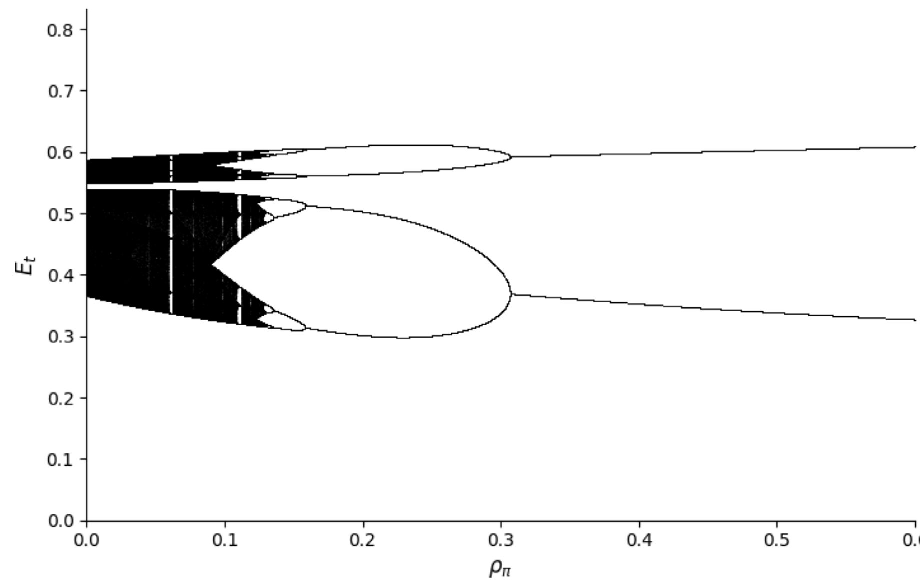

Fig. 7. Bifurcation diagram with respect to Followers' sensitivity to the wage premium, ρ_π. Increasing ρ_π progressively replaces the irregular regime observed for low wage-premium sensitivity with a stable period-two cycle.

specification, Followers are no longer driven only by social imitation, but also by the expected economic return associated with education. This allows us to assess whether the wage-premium channel amplifies the imitative mechanism or instead disciplines it by anchoring educational choices to labour-market incentives.

Fig. 7 reports the bifurcation diagram obtained by varying ρ_π, while keeping the remaining parameters fixed. The exercise is meant to isolate the role of Followers' sensitivity to the wage premium in shaping the long-run dynamics of aggregate educational participation.

Fig. 7 shows that increasing Followers' sensitivity to the wage premium has a regularizing effect on the aggregate dynamics. For low values of ρ_π, the system remains close to the benchmark case in which Followers are mainly driven by imitation, and the long-run dynamics display irregular fluctuations. As ρ_π increases, the economic return to education becomes more relevant also for Followers, and the irregular regime is progressively replaced by a stable period-two cycle. This result suggests that the wage-premium channel disciplines the purely imitative component of the model. When Followers react only to the diffusion of education, aggregate choices are more exposed to self-reinforcing social dynamics. When they also respond to the economic return associated with education, their behaviour becomes anchored to a labour-market signal. The outcome is not convergence to a fixed point, but a more regular alternation between high and low levels of educational participation.

The geometric mechanism underlying this transition is illustrated in Fig. 8, which reports the map and the corresponding cobweb diagrams for two representative values of ρ_π, one in the irregular region and one in the period-two region.

Fig. 8 reinforces the evidence from the bifurcation diagram and provides a clear illustration of Proposition 3. For low values of ρ_π, the orbit does not converge to a finite-period cycle but remains confined within the absorbing interval highlighted in the figure. For higher values of ρ_π, the same map generates a regular dynamics: the iterates converge to the two poles of a stable period-two cycle. Thus, increasing Followers' sensitivity to the wage premium does not eliminate endogenous oscillations, but it changes their structure. The dynamics move from bounded irregular motion inside an absorbing region to a regular alternation between two recurrent levels of educational participation. From a socio-economic perspective, the key point is that the wage premium changes the nature of the imitative force. When ρ_π is sufficiently high, Followers no longer react only to the social diffusion of education, but also to its economic return. This additional channel anchors imitation to labour-market incentives and reduces the dispersion of the long-run dynamics. The system still oscillates, but the oscillation becomes structured and predictable, taking the form of a stable alternation between two educational participation levels.

5. Dynamic trajectory with endogenous population structure

The results obtained in the previous section are based on the assumption of a fixed population composition. The share of Followers, λ, and the share of Positional Agents, $1-\lambda$, are taken as constant, so that the aggregate dynamics are entirely governed by the one-dimensional map for E_t. This assumption is useful to isolate the effects of imitation, positional behaviour, and wage-premium feedback. However, it rules out an additional channel that may be relevant in socio-economic environments: individuals may revise not only their educational choices, but also the behavioural rule they follow. We therefore extend the model by endogenizing the population share of Followers. Let $\lambda_t \in [0,1]$ denote the share of Followers at time t, while $1-\lambda_t$ denotes the share of Positional Agents. The aggregate state of the economy is now

$$(E_t,\lambda_t) \in D := \left[0,\bar{E}\right] \times [0,1], \qquad \bar{E} = \frac{I}{p_e}.$$

In this framework, aggregate educational participation and population composition coevolve: educational choices depend on the current distribution of behavioural types, while the evolution of λ_t depends on the relative performance of the two behavioural rules. As before, aggregate educational participation evolves according to

$$E_{t+1} = \Gamma(E_t;\lambda_t) = \bar{E}\left[\lambda_t s_F(E_t) + (1-\lambda_t)s_P(E_t)\right],$$

where

$$s_i(E_t) := \frac{\alpha_{i,t+1}}{\alpha_{i,t+1}+\beta_{i,t+1}}, \qquad i \in \{F,P\}.$$

Following the evolutionary discrete-choice approach adopted by Naimzada and Pireddu (2018), the population share evolves according to a discrete-choice switching rule based on the indirect utilities obtained by the two types:

$$\lambda_{t+1} = V(E_t,\lambda_t) = \frac{\lambda_t \exp\left(\mu U_{F,t}\right)}{\lambda_t \exp\left(\mu U_{F,t}\right) + (1-\lambda_t)\exp\left(\mu U_{P,t}\right)}. \tag{11}$$

The parameter $\mu \geq 0$ measures the intensity of behavioural switching. When $\mu = 0$, the population composition is preserved and $\lambda_{t+1} =$

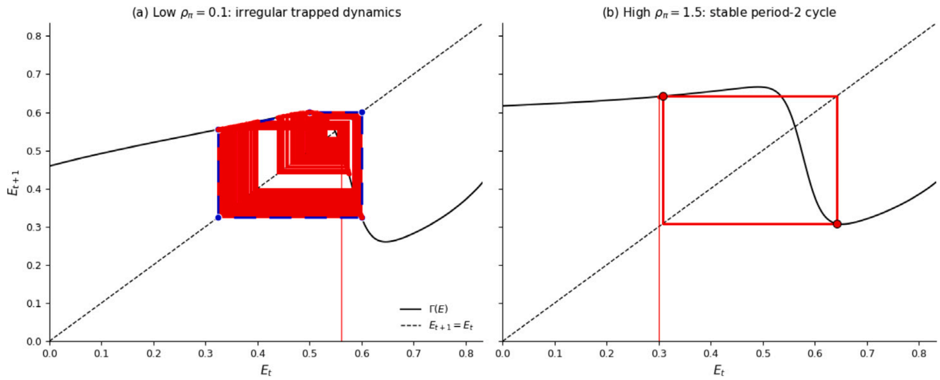

Fig. 8. Geometric illustration of the effect of Followers' wage-premium sensitivity. Panel (a) refers to a low value of ρ_π, for which the orbit displays irregular dynamics while remaining confined within the absorbing interval highlighted by the blue dot lines. This interval is determined by the local maximum of the one-dimensional map and bounds the region visited by the iterates. Panel (b) refers to a higher value of ρ_π, for which the orbit converges to a stable period-two cycle. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

λ_t. For $\mu > 0$, agents are more likely to adopt the behavioural rule that delivered the higher indirect utility in the previous period. As μ increases, the population composition becomes more sensitive to utility differences between Followers and Positional Agents. The resulting two-dimensional map is

$$\Phi : \begin{cases} E_{t+1} = \Gamma(E_t; \lambda_t), \\ \lambda_{t+1} = V(E_t, \lambda_t), \end{cases} \qquad (E_t, \lambda_t) \in D. \tag{12}$$

By construction, $\Gamma(E_t; \lambda_t) \in [0, \bar{E}]$ and $V(E_t, \lambda_t) \in [0, 1]$, so that D is forward invariant. The map Φ is continuous on D and smooth on each regime of the wage-premium function; the only possible loss of smoothness occurs at the switching frontier $\Pi_t = 0$, where the premium becomes exhausted.

To study the local properties of the two-dimensional system, in Proposition 4 we first characterize its interior steady states. This provides the analytical basis for the numerical exploration developed below. For each type $i \in \{F, P\}$, let $U_i(E)$ denote the indirect utility obtained after substituting the optimal demands into the utility function, namely

$$\begin{aligned} U_i(E) =& \alpha_i(E) \left[\log\left(\frac{I}{p_e}\right) + \log\left(\frac{\alpha_i(E)}{\alpha_i(E) + \beta_i(E)}\right)\right] + \\ & \beta_i(E) \left[\log\left(\frac{I}{p_c}\right) + \log\left(\frac{\beta_i(E)}{\alpha_i(E) + \beta_i(E)}\right)\right]. \end{aligned}$$

We define the relative indirect utility of Followers as

$$\Delta U(E) := U_F(E) - U_P(E). \tag{13}$$

Since preference intensities depend on aggregate education, $\Delta U(E)$ measures how E affects the relative attractiveness of the two behavioural rules.

Proposition 4 (*Interior Steady States*)**.** *Let $\mu > 0$. The two-dimensional system admits an interior steady state*

$$(E^*, \lambda^*) \in (0, \bar{E}) \times (0, 1)$$

if and only if there exists $E^* \in (0, \bar{E})$ such that

$$\Delta U(E^*) = 0, \qquad s_F(E^*) \neq s_P(E^*),$$

and

$$\lambda^* = \frac{\frac{E^*}{\bar{E}} - s_P(E^*)}{s_F(E^*) - s_P(E^*)} \in (0, 1).$$

Equivalently, the admissibility condition can be written as

$$\frac{E^*}{\bar{E}} \in \left(\min\{s_F(E^*), s_P(E^*)\}, \max\{s_F(E^*), s_P(E^*)\}\right).$$

This characterization also identifies a simple mechanism through which interior coexistence may break down: if the relative utility difference has a constant sign over the feasible domain, the switching rule selects one behavioural type and drives the system towards the boundary.

Remark 1 (*Loss of Interior Steady States*)**.** Let

$$M(\rho_\pi) := \max_{E \in (0, \bar{E})} \Delta U(E; \rho_\pi).$$

If, for some ρ_π, one has

$$M(\rho_\pi) < 0,$$

then the system admits no interior steady state. Moreover, since

$$\lambda_{t+1} = \frac{\lambda_t e^{\mu \Delta U(E_t; \rho_\pi)}}{1 - \lambda_t + \lambda_t e^{\mu \Delta U(E_t; \rho_\pi)}},$$

it follows that, for every $\lambda_t \in (0, 1)$,

$$\lambda_{t+1} < \lambda_t.$$

Hence the population composition moves monotonically towards the boundary

$$\lambda = 0.$$

Having characterized the interior steady states, we now analyse their local stability and isolate the role of the switching intensity parameter μ.

Proposition 5 (*Local Stability and Switching Intensity*)**.** *Let $(E^*, \lambda^*) \in (0, \bar{E}) \times (0, 1)$ be an interior steady state belonging to a smooth regime of the map. Define*

$$a := \Gamma_E(E^*; \lambda^*)$$

and

$$\chi := \Gamma_\lambda(E^*; \lambda^*)\, \lambda^*(1 - \lambda^*)\, \Delta U_E(E^*).$$

Assume that

$$-1 < a < 1 \qquad \text{and} \qquad \chi < 0.$$

Then the steady state is locally asymptotically stable if and only if

$$0 < \mu < \mu_c, \qquad \mu_c := \frac{1 - a}{-\chi}.$$

For $\mu > \mu_c$, the steady state is locally unstable.

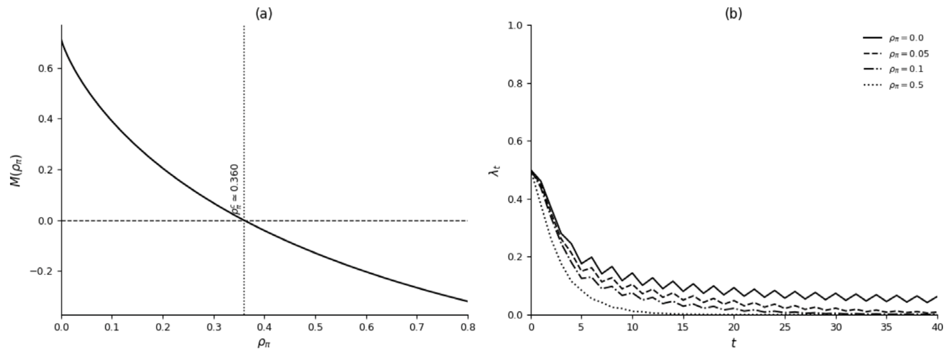

Fig. 9. Loss of interior coexistence as Followers become sensitive to the wage premium. Panel (a) reports $M(\rho_\pi) = \max_{E\in(0,\bar{E})} \Delta U(E;\rho_\pi)$. When $M(\rho_\pi) < 0$, the relative indirect utility of Followers is lower than that of Positional Agents over the whole feasible range of aggregate education; hence, by Remark 1, the two-dimensional system admits no interior steady state and the switching rule drives the population composition towards the boundary $\lambda = 0$. Panel (b) shows the evolution of λ_t for selected values of ρ_π.

Proposition 5 shows that endogenous population adjustment introduces an additional feedback channel into the system. The term χ combines two effects: the impact of population composition on aggregate education, measured by Γ_λ, and the impact of aggregate education on the relative attractiveness of the two behavioural rules, measured by ΔU_E. Local stability requires this feedback to be negative. If the adjustment reinforces the initial movement in aggregate education, the steady state becomes unstable. The parameter μ controls the intensity of this feedback. For low values of μ, behavioural switching is gradual and the steady state may remain stable. When μ exceeds the critical threshold μ_c, the population composition reacts too strongly to utility differences, generating overshooting and destabilizing the fixed point.

The numerical exercises for the two-dimensional system use the same structural benchmark calibration as in Table 1. The state variable is now (E_t, λ_t), and trajectories are initialized at $(E_0, \lambda_0) = (0.3, 0.5)$. The parameter μ, absent from the one-dimensional model, measures the intensity of behavioural switching. Unless μ is the parameter under investigation, we set $\mu = 1$. When μ is varied, this is explicitly indicated in the corresponding figure and caption.

We first illustrate the mechanism identified by Proposition 4 and Remark 1. The existence of an interior steady state requires the relative indirect utility of the two behavioural rules to vanish for some admissible level of aggregate education. Hence, the sign of

$$M(\rho_\pi) := \max_{E\in(0,\bar{E})} \Delta U(E;\rho_\pi) \tag{14}$$

provides a useful diagnostic for the loss of interior coexistence. If $M(\rho_\pi) < 0$, the Followers' rule never yields a higher indirect utility than the Positional rule over the feasible state space, and the switching mechanism drives the population share of Followers towards the boundary.

Fig. 9 shows that increasing ρ_π can destroy interior coexistence between the two behavioural rules. For low values of ρ_π, the function $\Delta U(E;\rho_\pi)$ crosses zero within the feasible range of aggregate education, so an interior steady state may exist. As ρ_π increases, $M(\rho_\pi)$ eventually becomes negative. Beyond this threshold, the condition for an interior steady state fails because Followers deliver a lower indirect utility than Positional Agents for every admissible value of E. The right panel displays the corresponding population dynamics. Higher values of ρ_π accelerate the decline of λ_t, and once $M(\rho_\pi) < 0$ the switching mechanism pushes the system towards the boundary $\lambda = 0$. Thus, when population composition is endogenous, behavioural heterogeneity is not automatically preserved. Coexistence between types must be sustained by relative utility indifference; otherwise, the model selects the behavioural rule that yields the higher indirect utility.

We now turn to the role of the switching intensity parameter μ. The previous result shows that, when an interior steady state exists, its local stability depends on the strength of the feedback generated by the endogenous adjustment of population composition. Hence, we fix a parameter configuration for which interior coexistence is admissible and study how the long-run dynamics change as μ increases. Fig. 10 reports the bifurcation diagrams of aggregate educational participation and population composition with respect to μ. For low values of the switching intensity, the adjustment of λ_t is sufficiently gradual and the system remains close to the stable interior configuration. In this region, behavioural composition acts as a slow-moving state variable and does not generate large aggregate fluctuations. As μ increases, the population share becomes more reactive to relative utility differences. The switching mechanism then amplifies the feedback between educational participation and behavioural composition: small changes in the relative performance of the two rules induce larger movements in λ_t, which in turn affect aggregate educational participation in the following period. Beyond a critical range, this feedback destabilizes the interior steady state and gives rise to persistent fluctuations in both E_t and λ_t. The two panels show that instability is not confined to aggregate education. Once population composition is endogenous, fluctuations in E_t are accompanied by fluctuations in the behavioural structure of the population. Periods in which one rule becomes temporarily more attractive are followed by shifts in λ_t; these shifts then modify the aggregate weight of imitation and positional behaviour, feeding back into subsequent educational participation. Thus, the willingness to switch does not merely affect the speed of adjustment: when sufficiently strong, it becomes a source of endogenous instability.

We finally inspect the dynamics that arise once the endogenous switching mechanism drives the population composition towards the boundary. This exercise complements the bifurcation analysis with respect to μ by showing that the loss of interior coexistence does not necessarily stabilize aggregate educational participation. When the share of Followers approaches zero, the long-run dynamics are governed by the boundary map associated with Positional Agents.

Fig. 11 shows that the selection of the boundary $\lambda = 0$ does not remove endogenous fluctuations. As the share of Followers vanishes, the dynamics become effectively governed by Positional Agents. Panel (a) shows that λ_t rapidly converges to zero, whereas E_t continues to alternate between very low and very high levels of educational participation. Panel (b) explains the mechanism. The boundary map is strongly counter-adaptive: when education is scarce, Positional Agents find it attractive because it remains socially distinctive, and the map sends the system towards high participation. Once education becomes widespread, its positional value collapses and the following-period participation falls sharply. The cobweb therefore converges to an extreme period-two cycle. From a socio-economic viewpoint, this result clarifies that endogenous population adjustment may replace heterogeneous coexistence with a monomorphic regime without restoring stability. The economy becomes dominated by Positional Agents, but their counter-adaptive response continues to generate persistent educational cycles.

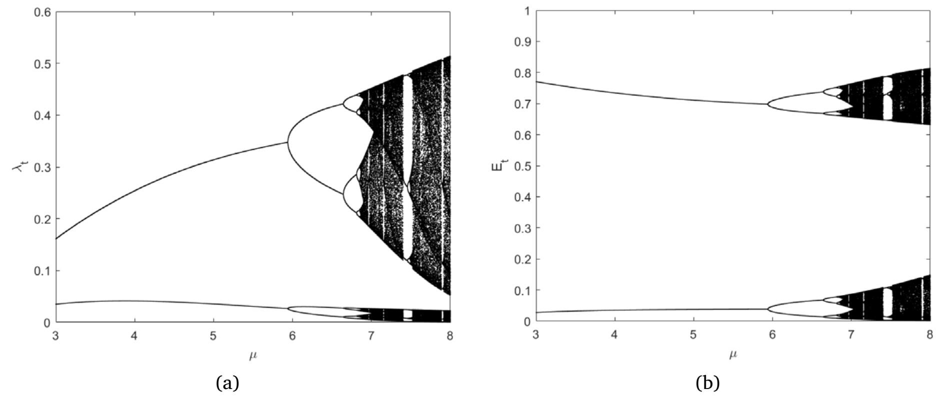

Fig. 10. Bifurcation diagrams with respect to the switching intensity μ. Panel (a) reports the long-run values of aggregate educational participation E_t, while Panel (b) reports the corresponding long-run values of the population share of Followers λ_t. The remaining parameters are fixed as in the benchmark calibration, except for those explicitly indicated.

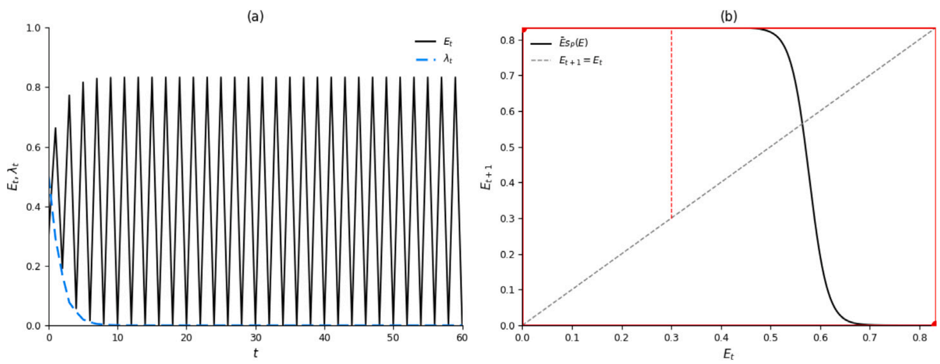

Fig. 11. Boundary dynamics after the loss of interior coexistence. Panel (a) reports the time series of aggregate educational participation E_t and the population share of Followers λ_t. The share of Followers rapidly converges towards the boundary $\lambda = 0$, while aggregate educational participation settles into persistent high-low oscillations. Panel (b) plots the induced boundary map $E_{t+1} = \bar{E}s_P(E_t)$, together with the 45-degree line and the associated cobweb dynamics.

Thus, the disappearance of behavioural heterogeneity is not necessarily stabilizing; it may simply shift instability from an interior interaction between types to a boundary dynamics driven by status concerns alone.

6. Policy interventions and educational dynamics

The previous sections show that aggregate educational participation may display persistent fluctuations as a result of endogenous social and economic feedbacks. In particular, imitation, positional behaviour, and the wage-premium channel may jointly generate cycles or irregular dynamics even in the absence of exogenous shocks. From a policy perspective, these fluctuations can be interpreted as a coordination problem. Individual agents react to social and economic signals generated by past aggregate choices, but they do not internalize how their current educational decisions affect future wage premia, social attractiveness, and institutional planning.

This interpretation is consistent with the empirical evidence discussed in Section 2, where enrolment volatility is measured as the residual variation of degree-class enrolment shares around their trend. In this perspective, the policy issue is not simply whether aggregate participation should be maximized, but whether excessive deviations from a reference trajectory generate planning costs for students, universities, and the labour market. Large endogenous swings in enrolment may complicate capacity planning, distort labour-market signals, and amplify mismatch between educational supply and future demand for skills.

We therefore introduce a parsimonious policy extension of the one-dimensional model. The aim is not to solve a full optimal policy problem, but to study how policy interventions may alter the endogenous dynamics generated by decentralized decisions. The policy maker is assumed to target a reference level of aggregate educational participation, denoted by E^{tar}. This target may be interpreted as a trend-consistent, capacity-consistent, or socially desirable level of enrolment.

A key feature of the extension is that policy may operate through two distinct channels. The first is a cost-stabilization channel: tuition fees, scholarships, subsidies, or other forms of financial aid affect the net unit cost of education. The second is a behavioural signal channel: information campaigns, orientation programmes, public access incentives, or institutional messages may affect the perceived attractiveness of education, especially when enrolment is below the target. Hence, policy is not assumed to be unambiguously stabilizing. It may discipline decentralized dynamics through the cost channel, but it may also amplify social or imitative responses through the behavioural channel.

Formally, the net cost of education depends on the deviation of aggregate participation from the target according to

$$p_e(E_t) = p_e + \phi\left(E_t - E^{tar}\right), \qquad \phi \geq 0. \tag{15}$$

The parameter ϕ measures the intensity of the cost-response channel. When $E_t < E^{tar}$, the net cost of education is reduced, supporting participation. When $E_t > E^{tar}$, the net cost increases, dampening further enrolment. We assume that

$$p_e - \phi E^{tar} > 0,$$

so that the net cost of education remains positive for all $E \geq 0$. Assuming

$$\tilde{p}_e := p_e - \phi E^{tar} > 0,$$

the policy rule can be written as

$$p_e(E_t) = \tilde{p}_e + \phi E_t.$$

The second channel captures the fact that policy may also act as a public signal. Let

$$G(E_t) := \max\{E^{tar} - E_t, 0\} \tag{16}$$

denote the intensity of the policy stimulus when aggregate educational participation is below the target. We assume that this stimulus increases the perceived attractiveness of education for Followers according to

$$\alpha_F^{pol}(E_t) = 1 - \exp\left[-\rho E_t - \rho_\pi \Pi(E_t) - \eta G(E_t)\right], \qquad \eta \geq 0, \tag{17}$$

where η measures the strength of the behavioural policy-signal channel. When $\eta = 0$, policy only affects the budget constraint. When $\eta > 0$, policy also modifies the perceived attractiveness of education when enrolment is below target.

The remaining preference components are defined as in the benchmark model, except that aggregate consumption is now affected by the state-dependent education cost. In particular, the wage premium remains $\Pi(E) = \max\{\bar{\Pi} - \kappa E, 0\}$ while aggregate consumption is obtained from the policy-adjusted aggregate budget constraint $p_e(E)E + p_c C = I$. Therefore,

$$C^{pol}(E) = \frac{I - p_e(E)E}{p_c} = \frac{I - \tilde{p}_e E - \phi E^2}{p_c}.$$

Accordingly,

$$\beta_F^{pol}(E) = 1 - \exp\left[-\rho C^{pol}(E)\right],$$

whereas the preference intensities of Positional Agents remain

$$\alpha_P^{pol}(E) = \exp(-\sigma E)\left[1 - \exp\left(-\sigma_\pi \Pi(E)\right)\right],$$
$$\beta_P^{pol}(E) = \exp\left[-\sigma C^{pol}(E)\right].$$

Given these policy-adjusted preference intensities, the educational share of type $i \in \{F, P\}$ is

$$s_i^{pol}(E) = \frac{\alpha_i^{pol}(E)}{\alpha_i^{pol}(E) + \beta_i^{pol}(E)}.$$

The individual problem is unchanged except for the state-dependent education cost in the budget constraint,

$$p_e(E_t)e_{i,t} + p_c c_{i,t} = I.$$

Thus optimal educational demand is

$$e_{i,t}^* = s_i^{pol}(E_t)\frac{I}{p_e(E_t)}.$$

The policy-augmented aggregate map is therefore

$$E_{t+1} = \Gamma^{pol}(E_t; \lambda) = \frac{I}{p_e(E_t)}\left[\lambda s_F^{pol}(E_t) + (1-\lambda)s_P^{pol}(E_t)\right]. \tag{18}$$

Equivalently, defining

$$S^{pol}(E) := \lambda s_F^{pol}(E) + (1-\lambda)s_P^{pol}(E),$$

we can write

$$\Gamma^{pol}(E) = \frac{I}{\tilde{p}_e + \phi E} S^{pol}(E). \tag{19}$$

The feasible state space is affected by the state-dependent education cost. Since $C^{pol}(E) \geq 0$, admissible levels of aggregate educational participation satisfy

$$I - \tilde{p}_e E - \phi E^2 \geq 0.$$

For $\phi > 0$, the upper bound of the feasible interval is

$$\bar{E}_\phi = \frac{-\tilde{p}_e + \sqrt{\tilde{p}_e^2 + 4\phi I}}{2\phi},$$

while for $\phi = 0$ it reduces to

$$\bar{E}_0 = \frac{I}{p_e}.$$

Hence, the policy-augmented map is defined on

$$D_\phi = [0, \bar{E}_\phi].$$

The benchmark model is recovered when $\phi = 0$ and $\eta = 0$. When $\phi > 0$ and $\eta = 0$, policy operates only through the cost-stabilization channel. In this case, it introduces a counter-cyclical institutional feedback: enrolment below the target reduces the cost of education, while enrolment above the target increases it. When $\eta > 0$, however, policy also acts as a behavioural signal that may strengthen the response of Followers when enrolment is below target. The overall effect of policy is therefore ambiguous. If the cost channel dominates, policy may regularize the dynamics; if the behavioural signal channel dominates, policy may amplify imitative responses and generate wider or more complex fluctuations.

The numerical exercises in this section use the same structural benchmark calibration reported in Table 1. The policy extension introduces three additional parameters: the target level E^{tar}, the cost-response intensity ϕ, and the behavioural signal intensity η. Unless otherwise stated, we set $E^{tar} = 0.3$. The values assumed by η and ϕ are either reported in the figure captions or varied along the horizontal axis in the bifurcation diagrams.

We now examine numerically how the policy feedback modifies the geometry and the long-run dynamics of the benchmark map. The purpose of the exercise is not to identify an optimal policy intensity, but to clarify the dynamic consequences of the two policy channels introduced above. We proceed in two steps. First, we set $\eta = 0$ and isolate the cost-stabilization channel. This allows us to study how a counter-cyclical net-cost rule changes the benchmark dynamics when policy does not directly affect perceived educational attractiveness. Second, we allow $\eta > 0$ and examine whether the behavioural signal channel may offset or amplify the stabilizing effect of the cost channel. Fig. 12 provides a first geometric comparison between the benchmark map and the policy-augmented map in the case $\eta = 0$.

Panel (a) shows that the policy wedge does not simply move the aggregate map upward or downward. Rather, it acts as a tilting mechanism. For low levels of current educational participation, the net cost of education is reduced and the policy-augmented map lies above the benchmark map. This means that, when enrolment is below the reference level, the policy supports future participation. For sufficiently high values of E_t, instead, the net cost rises and the policy map is shifted downward relative to the benchmark. In this region, the policy dampens future participation and counteracts excessive expansion. Mathematically, this effect comes from the denominator of the policy-augmented map defined in Eq. (19). Since $\eta = 0$, the behavioural signal channel is switched off and the policy affects the map only through the state-dependent education cost. The behavioural component is therefore the same aggregation of Followers and Positional Agents as in the benchmark model, but it is filtered through the counter-cyclical cost rule. Hence, the policy does not remove imitation, positional concerns, or the wage-premium feedback. It modifies the way these behavioural forces are translated into aggregate educational demand. The policy feedback is therefore institutional rather than behavioural. Panel (b) shows the dynamic implication of this geometric change. The cobweb does not converge to the target or to a fixed point. Instead, the orbit converges to a stable period-two cycle, with one pole below the target and one pole above it. This result is important because it clarifies the meaning of stabilization in the present framework. The counter-cyclical policy does not necessarily eliminate fluctuations in aggregate

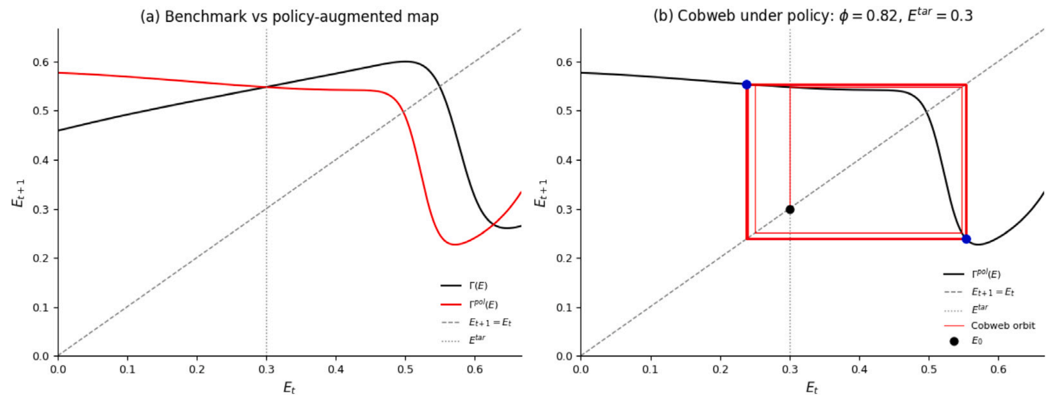

Fig. 12. Geometry of the policy-augmented map under the cost-stabilization channel. Panel (a) compares the benchmark aggregate map $\Gamma(E)$ with the policy-augmented map $\Gamma^{pol}(E)$ for $\eta = 0$, obtained under the counter-cyclical education-cost rule $p_e(E_t) = p_e + \phi(E_t - E^{tar})$. The policy does not shift the map uniformly; instead, it tilts the aggregate response by supporting participation below the target and dampening it above the target. Panel (b) reports the cobweb dynamics of the policy map showing convergence to a stable period-two cycle around the target. In both panels, $\phi = 0.82$.

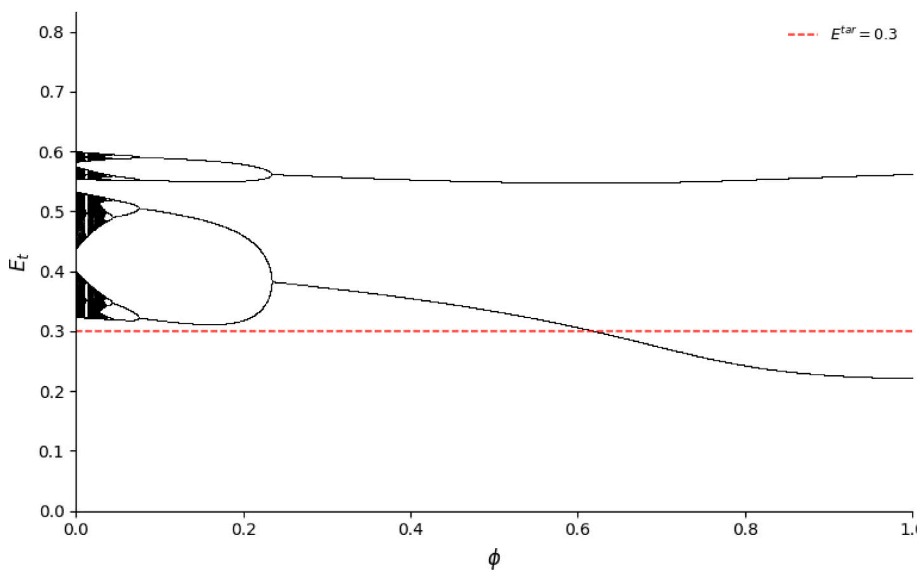

Fig. 13. Bifurcation diagram with respect to the cost-response intensity ϕ under the policy-augmented map with $\eta = 0$. The case $\phi = 0$ corresponds to the benchmark model without policy intervention. The horizontal dashed line denotes the target level $E^{tar} = 0.3$. As ϕ increases, the irregular benchmark dynamics are progressively replaced by a stable period-two cycle around the target.

educational participation. Rather, it regularizes the dynamics: it replaces irregular movements with a predictable high-low alternation around the target. From a socio-economic viewpoint, this distinction is crucial. If enrolment volatility creates planning problems for universities, students, and labour-market institutions, then predictability is itself valuable. A stable cycle is not equivalent to full equilibrium stabilization, but it is less disruptive than dispersed irregular fluctuations. The policy rule disciplines decentralized educational dynamics by introducing an institutional anchor, while still preserving the endogenous oscillatory nature of the system. In this sense, the policy improves the informational environment not by suppressing all variation, but by transforming unpredictable volatility into a structured and recurrent adjustment process.

The geometric mechanism described in Fig. 12 can be studied more systematically by varying the intensity of the cost-response channel. We therefore keep the behavioural signal channel switched off, $\eta = 0$, and construct a bifurcation diagram with respect to ϕ. This allows us to isolate the effect of the counter-cyclical net-cost rule on the long-run dynamics of aggregate educational participation.

Fig. 13 confirms the regularizing effect of the cost-stabilization channel. When $\phi = 0$, the model coincides with the benchmark case and displays the same irregular fluctuations generated by imitation, positional behaviour, and the wage-premium feedback. For small positive values of ϕ, the intervention is still too weak to remove the irregular component of the attractor, although the long-run set already starts to reorganize. As ϕ increases, irregular fluctuations disappear and the system converges to a stable period-two cycle. Thus, the counter-cyclical cost rule does not restore convergence to a fixed point. Rather, it regularizes the dynamics by transforming dispersed fluctuations into a predictable high-low alternation around the target. The endogenous forces behind educational cycles remain active, but their transmission is filtered through the policy rule: below target, the lower cost supports enrolment; above target, the higher cost dampens further expansion. In this weak dynamic sense, the policy is stabilizing because it reduces unpredictability without eliminating oscillations. If enrolment volatility creates planning problems for students, universities, and labour-market institutions, such regularization can itself be interpreted as a policy improvement over irregular dynamics.

We finally examine the case in which the policy acts through both channels. In the previous exercises, the behavioural policy-signal parameter was either switched off or kept at moderate levels, so that the cost-response channel remained the dominant mechanism. We now ask

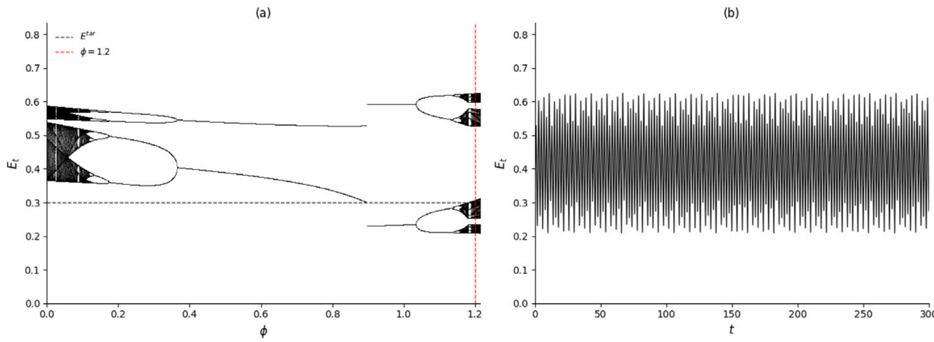

Fig. 14. Policy-induced re-destabilization under a strong behavioural signal. Panel (a) reports the bifurcation diagram of the policy-augmented map with respect to the cost-response intensity ϕ for $\eta = 4$. The horizontal dashed line denotes the target level $E^{tar} = 0.3$, while the vertical dashed line marks $\phi = 1.2$. After an intermediate region of regular low-period dynamics, the diagram displays a new area with many asymptotic branches, indicating the re-emergence of high-period or irregular dynamics. Panel (b) shows the corresponding time series for $\eta = 4$ and $\phi = 1.2$, confirming the presence of persistent complex fluctuations in aggregate educational participation.

whether a sufficiently strong policy signal may overturn this result and re-destabilize the dynamics. Fig. 14 addresses this issue by considering a high value of the behavioural amplification parameter, $\eta = 4$. Panel (a) reports the bifurcation diagram with respect to the cost-response intensity ϕ, while panel (b) shows a representative trajectory for $\phi = 1.2$.

Fig. 14 shows that, when the behavioural policy-signal channel is sufficiently strong, the policy effect becomes ambiguous. For relatively low values of ϕ, the cost channel still regularizes the dynamics and selects a low-period oscillation. As ϕ increases, however, the period-two cycle undergoes a qualitative reorganization. The transition occurs when the lower branch of the cycle reaches the target E^{tar}, which is the switching frontier of the behavioural signal $G(E) = \max\{E^{tar} - E, 0\}$. Below the target, the signal is active and increases the perceived attractiveness of education for Followers; above the target, the signal vanishes. Hence, when the low pole of the cycle collides with E^{tar}, one point of the orbit moves from the signal-active regime to the signal-inactive regime. This produces a border-collision-type reorganization of the policy-induced cycle. This mechanism is similar in spirit to the regime-switching phenomena already observed in the benchmark model, although the source of the switching is different. In the benchmark case, the reorganization of the attractor is linked to the wage-premium channel; here, it is generated by the policy rule itself. The target E^{tar} introduces a kink in the behavioural response: the map remains continuous, but its local slope changes when the orbit crosses the target. As a result, the low-period cycle selected by the cost channel can break down and give rise to high-period or irregular dynamics. Panel (b) shows the corresponding trajectory for $\eta = 4$ and $\phi = 1.2$. The system no longer converges to a simple period-two cycle, but displays persistent irregular fluctuations. The reason is that below-target enrolment triggers both a lower cost of education and a strong behavioural encouragement. This produces an amplified rebound. Once participation becomes high, the signal is withdrawn and the cost channel becomes restrictive, generating a sharp correction. The repeated interaction between behavioural amplification and cost correction may therefore reintroduce aggregate unpredictability.

The policy implication is that intervention is not unambiguously stabilizing. A cost-based rule may regularize decentralized dynamics, but a strong behavioural signal can amplify imitative responses and generate policy-induced overreaction. Hence, financial incentives and information-based policies should be jointly calibrated: reducing costs may discipline enrolment dynamics, whereas excessive signalling may recreate the fluctuations that the policy is intended to mitigate.

7. Conclusion

In this article, we have analysed the emergence of endogenous fluctuations and complex phenomena in a discrete-time dynamic model of educational choice where aggregate enrolment is shaped by the interaction between economic incentives, social influence, and behavioural heterogeneity. As shown for the Italian case (Section 2) and supported by Beffy et al. (2012) among others, enrolment volatility differs substantially across fields and may reflect more than smooth demographic or labour-market trends.

The model formalizes this mechanism through two behavioural types. Followers capture the pro-cyclical component of social imitation, since the attractiveness of education increases when educational participation becomes more widespread. Positional Agents introduce a counter-adaptive force, because the social distinctiveness of education declines as participation expands. At the same time, educational participation affects an endogenous wage premium: when educated labour is relatively scarce, the return to education is high, whereas a larger educated cohort progressively reduces its relative advantage. Aggregate dynamics therefore arise from the joint evolution of social attractiveness, positional incentives, and labour-market feedback.

The results show that this interaction can generate rich endogenous dynamics even in the absence of exogenous shocks. With fixed population shares, the education map may converge to a steady state, settle into regular cycles, or display high-period and irregular fluctuations depending on behavioural reactivity and on the composition of the population. The most complex outcomes arise for intermediate mixes of Followers and Positional Agents, where neither imitation nor distinction fully dominates. This confirms that heterogeneity is not merely a descriptive feature of the population, but a key source of aggregate instability.

The extension with endogenous population shares reinforces this conclusion. When individuals can switch between behavioural rules according to relative payoffs, the composition of the population becomes itself a dynamic variable. Instability may then originate not only from the education map, but also from the feedback between aggregate participation and the evolution of behavioural types. The willingness to switch is central: low values preserve the existing composition, while high values may generate abrupt reallocations across types and reinforce oscillatory or irregular dynamics.

The policy analysis developed in Section 6 adds a further qualification. Counter-cyclical financial incentives can help discipline enrolment dynamics by reducing education costs when participation is below target and increasing them when participation is above target. However, intervention is not unambiguously stabilizing. If policy also acts as a strong behavioural signal, it may amplify imitative responses and generate policy-induced overreaction. Financial incentives and information-based policies should therefore be jointly calibrated, since excessive signalling may recreate the very fluctuations that policy is intended to mitigate.

Overall, the paper suggests that educational systems may be intrinsically prone to coordination problems when economic returns and peer effects interact. Persistent fluctuations in enrolment can complicate planning for universities, households, and labour markets, because they affect both the demand for educational capacity and the expected return to human capital investment. Future research could extend the framework by introducing discipline-specific labour-market returns, richer network structures, or empirical estimation of the behavioural parameters governing imitation, positional concerns, and switching between behavioural rules.

CRediT authorship contribution statement

Andrea Caravaggio: Writing – original draft, Visualization, Methodology, Formal analysis, Conceptualization. **Marco Catola:** Writing – original draft, Visualization, Methodology, Formal analysis, Conceptualization. **Silvia Leoni:** Writing – original draft, Visualization, Methodology, Formal analysis, Conceptualization.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work the authors used ChatGPT, Gemini Pro, DeepSeek in order to code and proofread the manuscript. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Sample selection and data cleaning strategy

The empirical analysis conducted in Section 2 is based on the dataset of *Immatricolati per classe di laurea*, i.e. newly enrolled students disaggregated by degree class. We retrieved the data from the Data portal of Higher Education of the Italian Ministry of University and Research (MUR) at https://dati-ustat.mur.gov.it/dataset/immatricolati.

The implementation of the statistical framework required three preliminary methodological interventions, which were essential to guarantee the purity of the estimates and eliminate exogenous distortions.

First, it is necessary to exclude the nationally restricted programmes (*Numero Chiuso*). Degree classes such as Medicine and Surgery (LM-41), Architecture (LM-4cu/L-17), and Healthcare Professions (L/SNT1-4) are strictly regulated by administrative constraints and ministerial caps. In these fields, enrolment trends do not reflect the free interplay of student preferences, but rather centralized political decisions. Consequently, these programmes were excluded both from individual analysis and from the computation of the aggregate enrolment pool (the denominator used to calculate market shares). This choice preserves the reference universe as a free market of preferences, where fluctuations depend exclusively on behavioural factors.

Second, the Italian university system underwent a profound regulatory transition caused by the shift from D.M. 509/1999 to D.M. 270/2004. In the Italian legislative framework, a *Decreto Ministeriale* (D.M.) is a decree issued by a Ministry. In the context of higher education, D.M. 509/1999 officially implemented the European “Bologna Process” in Italy, replacing the old undivided 4-to-6-year degree programmes (*Vecchio Ordinamento*) with a sequential “3+2” structure comprising a 3-year Bachelor’s equivalent (*Laurea Triennale*) and a 2-year Master’s equivalent (*Laurea Specialistica*). Subsequently, D.M. 270/2004 revised this framework to fix structural redundancies, reclassifying the degree categories and renaming the second-cycle degree as *Laurea Magistrale*. It is the administrative re-coding and consolidation resulting from this latter transition that necessitates truncating the dataset to post-2011. The bureaucratic effects of recoding, merging, and restructuring these courses were heavily concentrated up until 2011. Including the years prior to 2011 would introduce artificial noise resulting from the closure and splitting of old degree codes. By setting the starting point at 2011, the analysis focuses on a fully stabilized regulatory environment. Furthermore, we applied strict regular expression filtering to purge any residual legacy codes or non-standard administrative tracks (e.g., military academy courses or special teacher-training pathways), retaining exclusively standard Reform 270 taxonomies.

Finally, to ensure a homogeneous behavioural cohort, the sample is strictly limited to First-Cycle Degrees (*Lauree Triennali*) and Single-Cycle Master’s Degrees (*Lauree Magistrali a Ciclo Unico*) with open enrolment, specifically Law (*Giurisprudenza* – LMG/01). Consecutive Second-Cycle Degrees (*Lauree Magistrali*) are deliberately excluded to capture high school graduates entering the university system for the first time.

Table 3 displays all Degree Classes considered ranked by their Detrended Coefficient of Variation computed as illustrated in Section 2.

Appendix B. Proofs of lemmas and propositions

Proof of Lemma 1. Fix $\lambda \in [0,1]$ and define

$$H(E;\lambda) := \Gamma(E;\lambda) - E.$$

Since $\Gamma(\cdot;\lambda)$ is continuous on D, $H(\cdot;\lambda)$ is continuous on D. Moreover, since $\Gamma(D;\lambda) \subseteq D$, one has

$$H(0;\lambda) = \Gamma(0;\lambda) \geq 0,$$

and

$$H(\bar{E};\lambda) = \Gamma(\bar{E};\lambda) - \bar{E} \leq 0.$$

By the Intermediate Value Theorem, there exists at least one $E^* \in D$ such that

$$H(E^*;\lambda) = 0.$$

Hence,

$$\Gamma(E^*;\lambda) = E^*,$$

which proves the claim. □

Proof of Proposition 1. For fixed $\lambda \in [0,1]$, define

$$H(E;\lambda) := \Gamma(E;\lambda) - E.$$

Then $E^* \in D$ is a steady state if and only if $H(E^*;\lambda) = 0$. On each smooth regime of D, differentiating

$$\Gamma(E;\lambda) = \bar{E}\left[\lambda s_F(E) + (1-\lambda)s_P(E)\right]$$

with respect to E gives

$$\Gamma_E(E;\lambda) = \bar{E}\left[\lambda s'_F(E) + (1-\lambda)s'_P(E)\right],$$

and therefore

$$H_E(E;\lambda) = \bar{E}\left[\lambda s'_F(E) + (1-\lambda)s'_P(E)\right] - 1.$$

By assumption,

$$\bar{E}\, s'_F(E) \leq a_F, \qquad \bar{E}\, s'_P(E) \leq a_P,$$

Table 3
Complete ranking of Detrended Coefficients of Variation (CV) across degree classes (2011–2024).

Degree class name	Code	Mean share	Volatility (Resid SD)	CV %
Building Sciences and Techniques *Scienze e tecniche dell'edilizia*	L-23	0.46%	0.0012	27.16
Navigation Sciences and Technologies *Scienze e tecnologie della navigazione*	L-28	0.1%	0.0002	24.56
Geography *Geografia*	L-6	0.06%	0.0001	18.47
Legal Services Sciences *Scienze dei Servizi Giuridici*	L-14	1.99%	0.0035	17.74
Conservation and Restoration of Cultural Heritage *Diagnostica per la conservazione dei beni culturali*	L-43	0.07%	0.0001	16.35
Pharmaceutical Sciences and Technologies *Scienze e tecnologie farmaceutiche*	L-29	0.71%	0.0012	16.23
Civil and Environmental Engineering *Ingegneria Civile e Ambientale*	L-7	2.02%	0.0029	14.21
Linguistic Mediation *Mediazione linguistica*	L-12	3.07%	0.0043	13.86
Biotechnology *Biotecnologie*	L-2	2.25%	0.0030	13.34
Spatial, Urban, Environmental and Landscape Planning *Scienze della pianificazione territoriale, urbanistica, paesaggistica e ambientale*	L-21	0.19%	0.0002	13.05
Social Sciences for Cooperation, Development and Peace *Scienze sociali per la cooperazione, lo sviluppo e la pace*	L-37	0.22%	0.0003	12.99
Environmental and Natural Sciences and Technologies *Scienze e tecnologie per l'ambiente e la natura*	L-32	1.16%	0.0015	12.87
Agricultural and Forestry Sciences and Technologies *Scienze e tecnologie agrarie e forestali*	L-25	1.31%	0.0015	11.72
Law (Five-year single cycle) *Magistrali in giurisprudenza*	LMG/01	7.25%	0.0078	10.78
Statistics *Statistica*	L-41	0.45%	0.0005	10.26
Geological Sciences *Scienze geologiche*	L-34	0.42%	0.0004	10.23
Degree class name	Ministerial code	Mean market share	Volatility (Resid SD)	Volatility coefficient (CV %)
Psychological Sciences and Techniques *Scienze e tecniche psicologiche*	L-24	3.82%	0.0038	10.00
Chemical Sciences and Technologies *Scienze e tecnologie chimiche*	L-27	1.45%	0.0014	9.36
Food Sciences and Technologies *Scienze e tecnologie alimentari*	L-26	1.13%	0.0010	9.27
Administration and Organization Sciences *Scienze dell'amministrazione e dell'organizzazione*	L-16	0.75%	0.0007	9.22
Mathematical Sciences *Scienze matematiche*	L-35	1.08%	0.0010	9.03
Tourism Sciences *Scienze del turismo*	L-15	0.87%	0.0008	8.78
Education and Training Sciences *Scienze dell'educazione e della formazione*	L-19	4.39%	0.0038	8.76
Modern Languages and Cultures *Lingue e culture moderne*	L-11	4.39%	0.0038	8.57
Physical Sciences and Technologies *Scienze e tecnologie fisiche*	L-30	1.31%	0.0010	7.74
Fine Arts, Music, Performing Arts and Fashion Studies *Discipline delle arti figurative, della musica, dello spettacolo e della moda*	L-3	1.17%	0.0009	7.42
Animal Science and Production Technologies *Scienze zootecniche e tecnologie delle produzioni animali*	L-38	0.64%	0.0004	6.79
Social Work *Servizio sociale*	L-39	1.27%	0.0008	6.60

(*continued on next page*)

Table 3 (*continued*).

History *Storia*	L-42	0.72%	0.0005	6.52
Sociology *Sociologia*	L-40	1.05%	0.0007	6.48
Biological Sciences *Scienze biologiche*	L-13	3.49%	0.0021	6.00
Cultural Heritage *Beni culturali*	L-1	1.7%	0.0010	5.99
Communication Sciences *Scienze della comunicazione*	L-20	3.7%	0.0020	5.30
Sports and Exercise Sciences *Scienze delle attività motorie e sportive*	L-22	3.59%	0.0017	4.86
Industrial Engineering *Ingegneria Industriale*	L-9	8.4%	0.0039	4.69
Computer Science and Technologies *Scienze e tecnologie informatiche*	L-31	2.67%	0.0012	4.54
Political Science and International Relations *Scienze politiche e delle relazioni internazionali*	L-36	3.46%	0.0015	4.43
Information Engineering *Ingegneria dell'Informazione*	L-8	5.72%	0.0025	4.41
Industrial Design *Disegno industriale*	L-4	1.04%	0.0004	4.29
Philosophy *Filosofia*	L-5	1.26%	0.0005	3.74
Economics *Scienze economiche*	L-33	4.11%	0.0013	3.20
Humanities *Lettere*	L-10	2.91%	0.0007	2.49
Economics and Business Management *Scienze dell'Economia e della Gestione Aziendale*	L-18	12.18%	0.0017	1.43

hence

$$\Gamma_E(E;\lambda) \le \lambda a_F + (1-\lambda)a_P = a_P + \lambda(a_F - a_P).$$

Therefore, in each of the three cases stated in the proposition,

$$a_P + \lambda(a_F - a_P) < 1.$$

Indeed, if $a_F > a_P$, this follows from

$$\lambda < \frac{1-a_P}{a_F - a_P};$$

if $a_F < a_P$, it follows from

$$\lambda > \frac{1-a_P}{a_F - a_P},$$

because the denominator is negative; and if $a_F = a_P < 1$, the inequality holds for every $\lambda \in [0,1]$.

Thus,

$$\Gamma_E(E;\lambda) < 1$$

on each smooth regime, and consequently

$$H_E(E;\lambda) < 0.$$

Hence $H(\cdot;\lambda)$ is strictly decreasing on the smooth regimes of D. Since H is continuous on D, it can have at most one zero. Existence of at least one zero follows from Lemma 1; therefore, the steady state is unique.

We now consider multiplicity. Multiple isolated steady states are multiple isolated zeros of $H(\cdot;\lambda)$. Such multiplicity can occur only if H is not monotone on D. On each smooth regime, changes in monotonicity can occur only at interior critical points satisfying

$$H_E(E;\lambda) = 0.$$

Using the expression above for H_E, this is equivalent to

$$\bar{E}\left[\lambda s'_F(E) + (1-\lambda)s'_P(E)\right] = 1.$$

Finally, suppose that all steady states are simple, namely

$$H_E(E^*;\lambda) \neq 0$$

whenever $H(E^*;\lambda) = 0$. Then every steady state is a transversal crossing of the horizontal axis. Since, by the self-map property,

$$H(0;\lambda) \ge 0, \qquad H(\bar{E};\lambda) \le 0,$$

the number of transversal crossings is generically odd. Hence, if the steady state is not unique, there must be at least three distinct steady states:

$$N(\lambda) > 1 \quad \Longrightarrow \quad N(\lambda) \ge 3.$$

This completes the proof. $\square$

Proof of Proposition 2. Let

$$F(E;\theta) := \Gamma(E;\theta) - E,$$

where θ denotes a generic parameter. Since E^* is a regular steady state,

$$F(E^*;\theta) = 0, \qquad F_E(E^*;\theta) = \Gamma_E(E^*;\theta) - 1 \neq 0.$$

By the Implicit Function Theorem,

$$\frac{dE^*}{d\theta} = -\frac{F_\theta(E^*;\theta)}{F_E(E^*;\theta)} = \frac{\Gamma_\theta(E^*;\theta)}{1-\Gamma_E(E^*;\theta)}.$$

Moreover, local stability implies

$$|\Gamma_E(E^*;\lambda)| < 1,$$

and therefore

$$1 - \Gamma_E(E^*;\lambda) > 0.$$

Hence, the sign of each comparative-static effect is determined by the sign of the corresponding partial derivative $\Gamma_\theta(E^*;\theta)$.

We first consider the population composition parameter. Since

$$\Gamma(E;\lambda) = \bar{E}\left[\lambda s_F(E) + (1-\lambda)s_P(E)\right],$$

we obtain

$$\Gamma_\lambda(E;\lambda) = \bar{E}\left[s_F(E) - s_P(E)\right].$$

Therefore,

$$\frac{dE^*}{d\lambda} = \frac{\bar{E}\left[s_F(E^*) - s_P(E^*)\right]}{1 - \Gamma_E(E^*;\lambda)}.$$

Since the denominator is positive, the sign of $dE^*/d\lambda$ is the sign of $s_F(E^*) - s_P(E^*)$. This proves the first set of results.

We now turn to the wage-premium sensitivity κ. In the interior regime,

$$\Pi(E) = \bar{\Pi} - \kappa E > 0, \qquad \frac{\partial \Pi}{\partial \kappa} = -E < 0.$$

Both type-specific educational shares are weakly increasing in the wage premium. Indeed, for Followers,

$$\alpha_F(E) = 1 - e^{-\rho E - \rho_\pi \Pi(E)}, \qquad \beta_F(E) = 1 - e^{-\rho C(E)},$$

so that

$$\frac{\partial \alpha_F}{\partial \Pi} = \rho_\pi e^{-\rho E - \rho_\pi \Pi(E)} \geq 0, \qquad \frac{\partial s_F}{\partial \alpha_F} = \frac{\beta_F}{(\alpha_F + \beta_F)^2} > 0.$$

For Positional Agents,

$$\alpha_P(E) = e^{-\sigma E}\left(1 - e^{-\sigma_\pi \Pi(E)}\right), \qquad \beta_P(E) = e^{-\sigma C(E)},$$

and therefore

$$\frac{\partial \alpha_P}{\partial \Pi} = e^{-\sigma E}\sigma_\pi e^{-\sigma_\pi \Pi(E)} > 0, \qquad \frac{\partial s_P}{\partial \alpha_P} = \frac{\beta_P}{(\alpha_P + \beta_P)^2} > 0.$$

It follows that

$$s_{F,\kappa}(E) \leq 0, \qquad s_{P,\kappa}(E) < 0$$

for $E > 0$, with the first inequality being strict whenever $\rho_\pi > 0$. Since

$$\Gamma_\kappa(E;\lambda) = \bar{E}\left[\lambda s_{F,\kappa}(E) + (1-\lambda)s_{P,\kappa}(E)\right],$$

and $\lambda \in (0,1)$, we have

$$\Gamma_\kappa(E^*;\lambda) < 0.$$

Thus,

$$\frac{dE^*}{d\kappa} < 0.$$

Next, consider the sensitivity of Followers to the wage premium, ρ_π. Holding E fixed,

$$\frac{\partial \alpha_F}{\partial \rho_\pi} = \Pi(E)e^{-\rho E - \rho_\pi \Pi(E)}.$$

If $\Pi(E^*) > 0$, then

$$\frac{\partial \alpha_F}{\partial \rho_\pi} > 0,$$

and therefore

$$s_{F,\rho_\pi}(E^*) > 0.$$

Since

$$\Gamma_{\rho_\pi}(E;\lambda) = \bar{E}\lambda s_{F,\rho_\pi}(E),$$

we obtain

$$\Gamma_{\rho_\pi}(E^*;\lambda) > 0,$$

and hence

$$\frac{dE^*}{d\rho_\pi} > 0.$$

We now analyse the imitative reactivity parameter ρ. For Followers,

$$\alpha_F(E) = 1 - e^{-\rho E - \rho_\pi \Pi(E)}, \qquad \beta_F(E) = 1 - e^{-\rho C(E)}.$$

Hence

$$s_F(E) = \frac{1 - e^{-\rho E - \rho_\pi \Pi(E)}}{2 - e^{-\rho E - \rho_\pi \Pi(E)} - e^{-\rho C(E)}}.$$

Differentiating with respect to ρ, holding E fixed, yields

$$s_{F,\rho}(E) = \frac{Ee^{-\rho E - \rho_\pi \Pi(E)}\left(1 - e^{-\rho C(E)}\right) - C(E)e^{-\rho C(E)}\left(1 - e^{-\rho E - \rho_\pi \Pi(E)}\right)}{(\alpha_F(E) + \beta_F(E))^2}.$$

The denominator is positive. The numerator can be written as

$$e^{-\rho(E+C(E)) - \rho_\pi \Pi(E)}\left[E\left(e^{\rho C(E)} - 1\right) - C(E)\left(e^{\rho E + \rho_\pi \Pi(E)} - 1\right)\right].$$

Therefore,

$$\operatorname{sign}\left(s_{F,\rho}(E)\right) = \operatorname{sign}\left[E\left(e^{\rho C(E)} - 1\right) - C(E)\left(e^{\rho E + \rho_\pi \Pi(E)} - 1\right)\right].$$

Since

$$\Gamma_\rho(E;\lambda) = \bar{E}\lambda s_{F,\rho}(E),$$

and the denominator in the implicit derivative is positive, we obtain

$$\operatorname{sign}\left(\frac{dE^*}{d\rho}\right) = \operatorname{sign}\left[E^*\left(e^{\rho C^*} - 1\right) - C^*\left(e^{\rho E^* + \rho_\pi \Pi^*} - 1\right)\right].$$

This proves the general sign condition for $dE^*/d\rho$.

Moreover, if $E^* \geq E^\dagger$, then $E^* \geq C^*$. Since the function

$$z \mapsto \frac{e^{\rho z} - 1}{z}$$

is strictly increasing for $z > 0$, one has

$$E^*\left(e^{\rho C^*} - 1\right) \leq C^*\left(e^{\rho E^*} - 1\right).$$

Since

$$e^{\rho E^* + \rho_\pi \Pi^*} \geq e^{\rho E^*},$$

it follows that

$$E^*\left(e^{\rho C^*} - 1\right) \leq C^*\left(e^{\rho E^* + \rho_\pi \Pi^*} - 1\right),$$

with strict inequality whenever $\rho_\pi \Pi^* > 0$. Hence,

$$E^* \geq E^\dagger \Rightarrow \frac{dE^*}{d\rho} \leq 0,$$

with strict inequality whenever $\rho_\pi \Pi^* > 0$.

In the special case $\rho_\pi = 0$, the sign condition reduces to

$$\operatorname{sign}\left(\frac{dE^*}{d\rho}\right) = \operatorname{sign}\left(C^* - E^*\right),$$

again by the monotonicity of $z \mapsto (e^{\rho z} - 1)/z$.

Finally, consider the positional reactivity parameter σ, keeping σ_π fixed. Write

$$\alpha_P(E) = e^{-\sigma E}Q(E), \qquad Q(E) := 1 - e^{-\sigma_\pi \Pi(E)}, \qquad \beta_P(E) = e^{-\sigma C(E)}.$$

Then

$$\frac{\alpha_P(E)}{\beta_P(E)} = Q(E)e^{\sigma(C(E) - E)}.$$

Since $Q(E)$ does not depend on σ, differentiation of

$$s_P(E) = \frac{\alpha_P(E)}{\alpha_P(E) + \beta_P(E)}$$

gives

$$s_{P,\sigma}(E) = s_P(E)\big(1 - s_P(E)\big)\left(C(E) - E\right).$$

Thus,

$$\operatorname{sign}\left(s_{P,\sigma}(E)\right) = \operatorname{sign}\left(C(E) - E\right).$$

Since

$$\Gamma_\sigma(E;\lambda) = \bar{E}(1-\lambda)s_{P,\sigma}(E),$$

we obtain

$$\operatorname{sign}\left(\frac{dE^*}{d\sigma}\right) = \operatorname{sign}\left(C^* - E^*\right).$$

Using

$$C(E) = \frac{I - p_e E}{p_c},$$

the condition $C(E) > E$ is equivalent to

$$E < \frac{I}{p_e + p_c} = E^{\dagger},$$

whereas $C(E) < E$ is equivalent to $E > E^{\dagger}$. Hence

$$E^* < E^{\dagger} \Rightarrow \frac{dE^*}{d\sigma} > 0, \qquad E^* > E^{\dagger} \Rightarrow \frac{dE^*}{d\sigma} < 0.$$

This completes the proof. □

Proof of Proposition 3. Since $\Gamma : D \to D$ is continuous and satisfies

$$\Gamma_E(E;\lambda) > 0 \quad \text{for } E < E_c, \qquad \Gamma_E(E;\lambda) < 0 \quad \text{for } E > E_c,$$

the point E_c is the unique critical point of the map and

$$E_{\max} := \Gamma(E_c)$$

is the global maximum of Γ on D. Hence,

$$\Gamma(E) \leq E_{\max} \qquad \text{for all } E \in D.$$

Since $\Gamma(D) \subseteq D = [0, \bar{E}]$, it follows that

$$\Gamma(D) \subseteq [0, E_{\max}].$$

By definition,

$$E_{\min} := \Gamma(E_{\max}), \qquad J_\Gamma := [E_{\min}, E_{\max}].$$

Moreover, since $E_{\max}$ is the global maximum of Γ, one has

$$E_{\min} = \Gamma(E_{\max}) \leq E_{\max},$$

so J_Γ is a well-defined interval.

We first prove forward invariance. Since $J_\Gamma \subseteq [0, E_{\max}]$, the additional assumption

$$\Gamma([0, E_{\max}]) \subseteq J_\Gamma$$

implies immediately that

$$\Gamma(J_\Gamma) \subseteq \Gamma([0, E_{\max}]) \subseteq J_\Gamma.$$

Thus J_Γ is forward invariant. We now prove absorption. From the global maximality of $E_{\max}$, we have

$$\Gamma(D) \subseteq [0, E_{\max}].$$

Applying Γ once more and using again the assumption

$$\Gamma([0, E_{\max}]) \subseteq J_\Gamma,$$

we obtain

$$\Gamma^2(D) = \Gamma(\Gamma(D)) \subseteq \Gamma([0, E_{\max}]) \subseteq J_\Gamma.$$

Therefore, every trajectory starting from $E_0 \in D$ enters J_Γ after at most two iterations. Finally, since J_Γ is forward invariant, once a trajectory enters J_Γ, all subsequent iterates remain in J_Γ. Hence, for every $E_0 \in D$, there exists $n \leq 2$ such that

$$E_t \in J_\Gamma \qquad \text{for all } t \geq n.$$

This proves that J_Γ is a forward invariant absorbing interval. □

Proof of Proposition 4. Let $(E^*, \lambda^*) \in (0, \bar{E}) \times (0, 1)$ be an interior steady state of the two-dimensional system. Then

$$E^* = \Gamma(E^*; \lambda^*), \qquad \lambda^* = V(E^*, \lambda^*).$$

We first consider the second condition. By definition of the switching rule,

$$V(E, \lambda) = \frac{\lambda \exp\left(\mu U_F(E)\right)}{\lambda \exp\left(\mu U_F(E)\right) + (1-\lambda)\exp\left(\mu U_P(E)\right)}.$$

Hence, at an interior steady state,

$$\lambda^* = \frac{\lambda^* \exp\left(\mu U_F(E^*)\right)}{\lambda^* \exp\left(\mu U_F(E^*)\right) + (1-\lambda^*)\exp\left(\mu U_P(E^*)\right)}.$$

Since $\lambda^* \in (0,1)$, we can divide by $\lambda^* > 0$ and obtain

$$\lambda^* \exp\left(\mu U_F(E^*)\right) + (1-\lambda^*)\exp\left(\mu U_P(E^*)\right) = \exp\left(\mu U_F(E^*)\right).$$

Rearranging gives

$$(1-\lambda^*)\exp\left(\mu U_P(E^*)\right) = (1-\lambda^*)\exp\left(\mu U_F(E^*)\right).$$

Since $1 - \lambda^* > 0$, this implies

$$\exp\left(\mu U_P(E^*)\right) = \exp\left(\mu U_F(E^*)\right).$$

Because $\mu > 0$, the exponential function is injective and therefore

$$U_F(E^*) = U_P(E^*),$$

or equivalently,

$$\Delta U(E^*) = 0.$$

We now turn to the first steady-state condition. Since

$$\Gamma(E; \lambda) = \bar{E}\left[\lambda s_F(E) + (1-\lambda)s_P(E)\right],$$

the condition $E^* = \Gamma(E^*; \lambda^*)$ becomes

$$E^* = \bar{E}\left[\lambda^* s_F(E^*) + (1-\lambda^*)s_P(E^*)\right].$$

Dividing by $\bar{E} > 0$, we obtain

$$\frac{E^*}{\bar{E}} = s_P(E^*) + \lambda^*\left[s_F(E^*) - s_P(E^*)\right].$$

If $s_F(E^*) \neq s_P(E^*)$, this yields

$$\lambda^* = \frac{\frac{E^*}{\bar{E}} - s_P(E^*)}{s_F(E^*) - s_P(E^*)}.$$

Since the steady state is interior, $\lambda^* \in (0,1)$, which gives the admissibility condition stated in the proposition.

Moreover, the condition

$$\lambda^* = \frac{\frac{E^*}{\bar{E}} - s_P(E^*)}{s_F(E^*) - s_P(E^*)} \in (0,1)$$

is equivalent to requiring that $E^*/\bar{E}$ lies strictly between $s_F(E^*)$ and $s_P(E^*)$, namely

$$\frac{E^*}{\bar{E}} \in \left(\min\{s_F(E^*), s_P(E^*)\}, \max\{s_F(E^*), s_P(E^*)\}\right).$$

Conversely, suppose that there exists $E^* \in (0, \bar{E})$ such that

$$\Delta U(E^*) = 0, \qquad s_F(E^*) \neq s_P(E^*),$$

and define

$$\lambda^* = \frac{\frac{E^*}{\bar{E}} - s_P(E^*)}{s_F(E^*) - s_P(E^*)} \in (0,1).$$

By construction,

$$\frac{E^*}{\bar{E}} = s_P(E^*) + \lambda^*\left[s_F(E^*) - s_P(E^*)\right],$$

and therefore

$$E^* = \bar{E}\left[\lambda^* s_F(E^*) + (1-\lambda^*)s_P(E^*)\right] = \Gamma(E^*; \lambda^*).$$

Moreover, since $\Delta U(E^*) = 0$, we have

$$U_F(E^*) = U_P(E^*).$$

It follows that

$$V(E^*, \lambda^*) = \frac{\lambda^* \exp\left(\mu U_F(E^*)\right)}{\lambda^* \exp\left(\mu U_F(E^*)\right) + (1-\lambda^*)\exp\left(\mu U_F(E^*)\right)} = \lambda^*.$$

Thus,

$$(E^*, \lambda^*)$$

is an interior steady state of the two-dimensional system. This completes the proof. □

Proof of Proposition 5. Let (E^*, λ^*) be an interior steady state in a smooth regime. The Jacobian of

$$\Phi(E,\lambda) = \big(\Gamma(E;\lambda), V(E,\lambda)\big)$$

is

$$J_\Phi(E^*,\lambda^*) = \begin{pmatrix} \Gamma_E & \Gamma_\lambda \\ V_E & V_\lambda \end{pmatrix}_{(E^*,\lambda^*)}.$$

Using

$$\Delta U(E) := U_F(E) - U_P(E),$$

the switching rule can be written as

$$V(E,\lambda) = \frac{\lambda e^{\mu \Delta U(E)}}{1-\lambda+\lambda e^{\mu\Delta U(E)}}.$$

By Proposition 4, an interior steady state satisfies

$$\Delta U(E^*) = 0,$$

and hence $e^{\mu \Delta U(E^*)} = 1$. Direct differentiation of V gives

$$V_\lambda(E,\lambda) = \frac{e^{\mu\Delta U(E)}}{\big(1-\lambda+\lambda e^{\mu\Delta U(E)}\big)^2},$$

and

$$V_E(E,\lambda) = \mu V(E,\lambda)\big(1-V(E,\lambda)\big)\Delta U_E(E).$$

Therefore, at the interior steady state,

$$V_\lambda(E^*,\lambda^*) = 1, \qquad V_E(E^*,\lambda^*) = \mu\lambda^*(1-\lambda^*)\Delta U_E(E^*).$$

Thus,

$$J_\Phi(E^*,\lambda^*) = \begin{pmatrix} a & \Gamma_\lambda(E^*;\lambda^*) \\ \mu\lambda^*(1-\lambda^*)\Delta U_E(E^*) & 1 \end{pmatrix},$$

where $a := \Gamma_E(E^*;\lambda^*)$. Hence

$$\operatorname{tr} J_\Phi = 1 + a, \qquad \det J_\Phi = a - \mu\chi,$$

with

$$\chi := \Gamma_\lambda(E^*;\lambda^*)\lambda^*(1-\lambda^*)\Delta U_E(E^*).$$

For a two-dimensional discrete-time map, local asymptotic stability is equivalent to the Schur conditions

$$1 - \operatorname{tr} J_\Phi + \det J_\Phi > 0, \qquad 1 + \operatorname{tr} J_\Phi + \det J_\Phi > 0, \qquad 1 - \det J_\Phi > 0.$$

Substituting the above trace and determinant, these conditions become

$$-\mu\chi > 0, \qquad 2(1+a) - \mu\chi > 0, \qquad 1 - a + \mu\chi > 0.$$

Under the assumptions $-1 < a < 1$ and $\chi < 0$, the first two inequalities are automatically satisfied for every $\mu > 0$. The last one is equivalent to

$$\mu < \frac{1-a}{-\chi}.$$

Defining

$$\mu_c := \frac{1-a}{-\chi},$$

we obtain local asymptotic stability for $0 < \mu < \mu_c$. For $\mu > \mu_c$, the third Schur condition is violated, and the steady state is locally unstable. □

Data availability

Data will be made available on request.

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