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Tuning the Discrete Wavelet Transform for Power Smoothing of Wind Turbines

Tuning the Discrete Wavelet Transform for Power Smoothing of Wind Turbines

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Abstract. In the study, an extended sensitivity analysis is presented, which was aimed at properly tuning the parameters of an algorithm based on the Discrete Wavelet Transform (DWT) for use in power smoothing of utility-scale wind turbines coupled with batteries. More specifically, a twofold implementation is proposed, so that the proposed algorithm can operate efficiently both in real time as a control system and using historical data for the preliminary sizing of the storage system. In particular, this study addresses the correct setting of the main parameters of DTW, i.e. the level of decomposition and the mother wavelet family that generates the multi-resolution analysis (MRA). Based on real wind data of an onshore site, the following wavelet families have been analyzed: Daubechies, Coiflet, Symmlet, Biorthogonal and Reverse Biorthogonal. It is shown that, as the severe wind fluctuations that need to be smoothed are a quite sudden phenomena, in which usually the wind speed increases and then decreases quickly, all the wavelet families having a centered peak show good performance. On the other hand, it is highlighted that, once the correct choice of the mother wavelet is made, neither increasing the decomposition level nor making it adjustable in time, brings significant benefits. Finally, the discussed hypothesis has been assessed in combination with the proposed technique to extend the wavelet for online control using data mirroring, corroborating the suitability of the method for use in wind energy applications.

1. Introduction

Wind energy penetration is increasing constantly [1]. While this trend is key for achieving a more sustainable energy mix, a larger amount of intermittent power production poses significant challenges in terms of integration with the grid. Grid codes are increasingly strict in limiting admissible power gradients in order to avoid damages to the electrical system. In most of grid codes, this implies a requirement in terms of the maximum tolerable ramp rate, which is typically set as 10% of the rated power of the wind farm per minute [2]. To be compliant with this kind of limit, several approaches for mitigating fluctuations and smoothing the power profile fed to the grid have been developed; these techniques are usually referred to as "power smoothing" [3]. The most common power smoothing methods make use of a storage system (electrical, mechanical, etc.) to compensate power gradients. In particular, Lithium-ion (Li-Ion) batteries are a mature and reliable technology if a fast power smoothing response is needed; moreover, their specific power and functioning conditions are suitable for this kind of application [4]. Several power smoothing approaches using Battery Energy Storage System (BESS) have been proposed, as discussed extensively in the review by Kasem et al. [5].

The present study is aimed at further developing and tuning a power smoothing approach developed by the authors recently [6]. The proposed strategy makes use of the discrete wavelet transform (DWT) to tackle the power signal decomposition between those high frequency fluctuations that need to be smoothed out and the slower one that can be handled by the grid connection. The distinctive feature of the use of DWT consists in the ability of processing non-stationary signals using a multi-resolution analysis (MRA) [7]. Initial studies have shown that the DWT approach is extremely efficient both in "offline" and "online" mode in comparison with more conventional approaches like direct power smoothing or the moving average [6]. More specifically, the offline mode consists into the use of DWT on a power data set, either experimentally recorded or synthetized based on wind forecast, to define the behavior of the hybrid energy system for different storage size. In this phase a possible objective would be to define the better storage system design. The online mode instead refers to the real-time control of the power produced by one or more wind turbines in order to discriminate the components that need to be allocated in the battery from those that can be released into the grid.
The DWT method is not natively ready to be used for real-time control, since the wavelet form acts like a centered filter, thus intrinsically needs data in the future. To overcome this, a strategy has been recently proposed by the authors for a single case study and a single wavelet form and decomposition level. The present study moves from these preliminary results, presenting an extended sensitivity analysis on the main parameters that have to be set in the DWT for power smoothing of utility-scale wind turbines coupled with batteries.
The paper is organized as follows: Section 2 resumes the logic of the DWT-based power smoothing algorithm and describes the main settings. In Section 3, some relevant wavelet families are presented, which are then applied in Section 4 in a multivariate sensitivity analysis. Finally, some conclusions are drawn in Section 5.

2. Discrete Wavelet Transform for use in wind power smoothing

2.1. Power smoothing algorithm

The proposed power smoothing algorithm is schematically presented in Figure 1. As apparent, the DWT is applied on the wind power signal before using the BESS for storing or releasing power to the grid. The algorithm can work in real-time mode or in offline, and its main settings are reported in Table 1. For additional info, the readers can refer to [6].

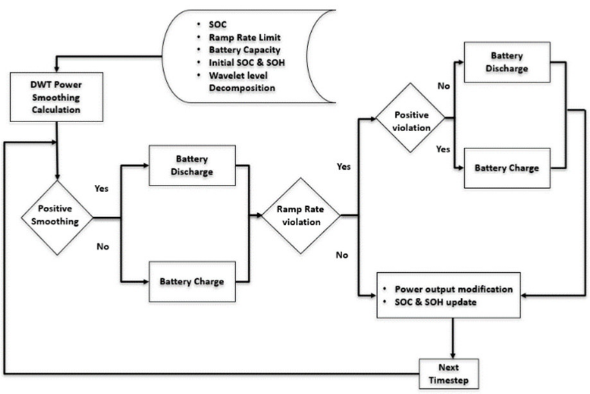

Figure 1. Schematic flowchart of the developed power smoothing algorithm [6].

DWT features:	▪ Mother Wavelet Family ▪ Smoothing decomposition level (ex. order = 1, 2, ...)
Ramp rate constraint:	▪ 10% P_{nom}
Technical storage parameters:	▪ C-rate $_{charge, max}$ = 1 & C-rate $_{discharge, max}$ = 2 ▪ SOC $_{max}$ = 0.95 & SOC $_{min}$ = 0.15 ▪ SOC $_{initial}$ = 0.5 ▪ SOH $_{initial}$ = 1 (New battery)

Table 1. Smoothing algorithm inputs.

The performance of the algorithm in terms of power smoothing is identified in terms of the Abatement Ratio (AR), defined in Eq. 1. The AR evaluates the percentage of ramp rate violations, which have been smoothed in contrast with an unsmoothed signal. It results equal to 1 when all the ramp-rate violations are under the maximum admissible ramp rate defined by the grid code. The definition of this parameter is then an indicator of the trade-off between smoothing capability and the cost of the BESS.

$$AR = 1 - \frac{\text{ramp violations in smoothed profile}}{\text{ramp violations without storage}} \quad (1)$$

2.2. Wavelet decomposition level

As discussed, the goal of the DWT is to decompose the wind power signal into high-frequency (D_i in Figure 2) and low-frequency (A_i in Figure 2) contents. Once a given level of decomposition is identified, the original wind power signal can then be obtained as a combination of the low and high frequency components, as in Eq. 2, where n is the selected level of decomposition.

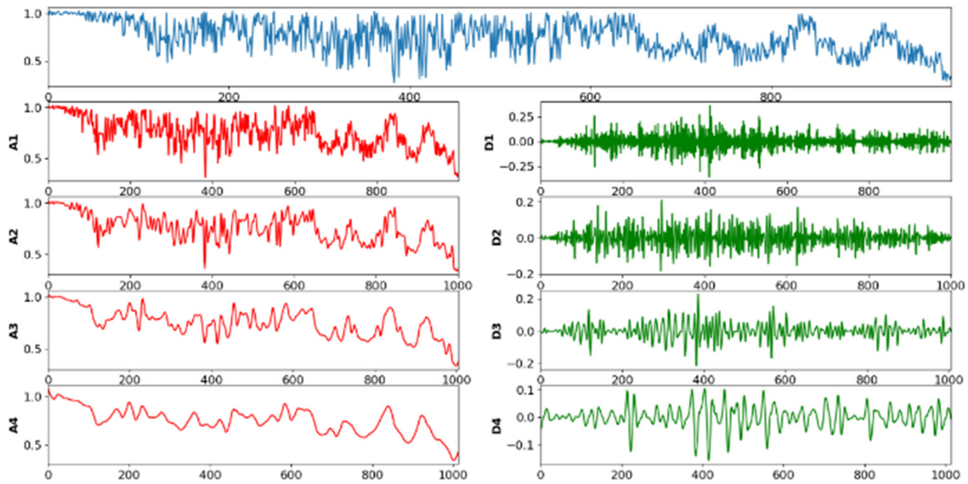

Figure 2. Signal decomposition for a sample dataset with the wavelet method.

$$P(t) = A_n(t) + \sum_{i=1}^{n} D_i(t) \quad (2)$$

It is then apparent that the optimization of the level of decomposition plays an important role to correctly define the power content to be split between the BESS and the grid. Previous experience showed that for the Daubechies family, often used in this kind of applications, moderate decomposition levels (2-3) are sufficient to guarantee a good smoothing of wind power values [6], as shown in Figure 3 and Figure 4.

2.3. Wavelet family

It should be noted that the sensitivity analysis on the decomposition level shown in Figs. 3 and 4 strictly depends also on the shape of the mother wavelet chosen. To this end, the present study reports a detailed sensitivity analysis on the mother wavelet family. To help the reader comprehending the importance of this parameter, a brief mathematical dissertation follows. As reported by [8], the DWT has been introduced in order make practical and applicable the Continuous Wavelet Transform (CWT).

It is based on the use of wavelet functions, which are wave-like fluctuations of finite amplitude that have null average. The mother wavelet is translated along the time-varying signal and scaled to locally approximate it. This feature helps to better localize the signal in the time-frequency plane. In fact, long time intervals are selected where more precise low-frequency information is required, while short time intervals are used where more precise high-frequency information is required. In more analytical terms, the extension of the wavelet is governed by the scale factor a, while the movement along the time axis is described by the translation factor b. The element that is translated and scaled is defined as the "mother wavelet". Changing the a and b factors one can obtain an infinite number of wavelets $\psi_{a,b}(t)$, identified by parameters a and b and defined as follows:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\,\psi\left(\frac{t-b}{a}\right) \quad (3)$$

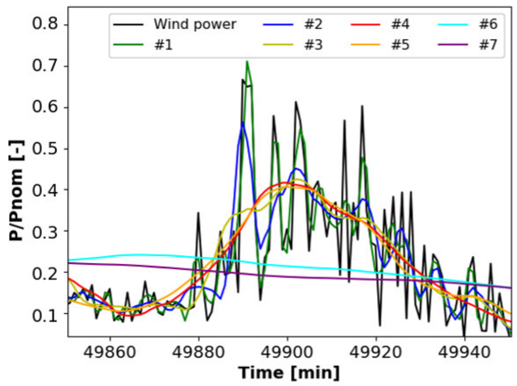

Figure 3. Wavelet power smoothing profiles as a function of decomposition level (Daubechies DWT – data from [6]).

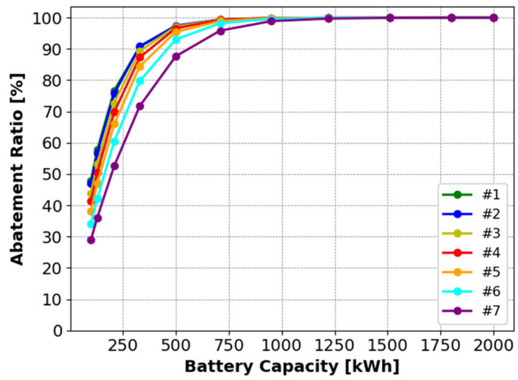

Figure 4. Abatement Ratio vs Battery Capacity: comparison for different wavelet decomposition levels (Daubechies DWT – data from [6]).

where the term $1/\sqrt{a}$ is used to normalize the wavelet. Consequently, the CWT and its inverse ICWT are defined as:

$$CWT_f(a,b) = \int_{-\infty}^{+\infty} f(t)\,\psi_{a,b}^{*}(t)dt \tag{4}$$

$$f(t) = \frac{1}{C_\psi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} CWT_f(a,b)_f\,\frac{1}{a^2}\psi_{a,b}(t\,)dadb \tag{5}$$

x where the superscript * denotes the complex conjugate, the term C_ψ^2 is the admissibility constant and its value depends on the chosen wavelet. The DWT has the same dependency to the mother wavelet and, in practice, it is implemented as a filter bank of high-pass and low-pass coefficients.

Therefore, to ensure accurate signal decomposition and reconstruction, an appropriate choice of the most suitable mother wavelet must be made. In particular, the best wavelet to perform the analysis depends on the type of signal and the phenomenon of interest [9].

In the relevant technical literature, a variety of discrete wavelet families are proposed [10]. The best-known are Haar, Daubechies, Coiflet, Symmlet, Biorthogonal and Reverse Biorthogonal. Wavelets have properties that govern their behavior which are mainly[11]:

1. the length of the support of the mother wavelet,
2. the number of vanishing moments,
3. the symmetry and/or the regularity,
4. the orthogonality or the biorthogonality.

Moreover, they are characterized by different low-pass and high-pass analysis and synthesis filters, as shown in Figure 5. The first ones are using for the signal decomposition process, whilst the second ones are applied in the reconstruction operation.

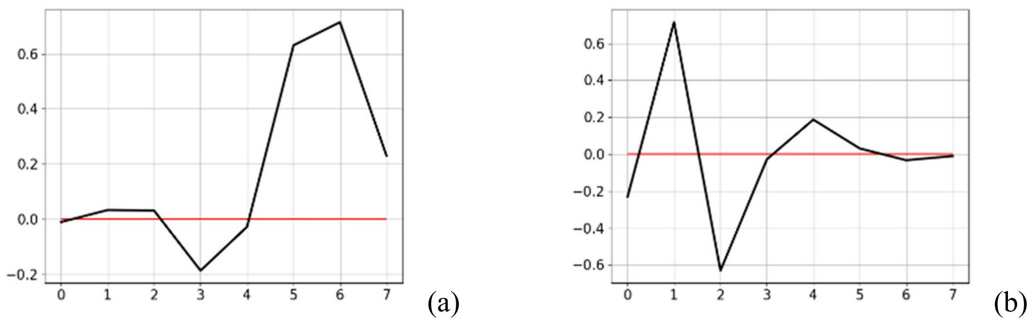

Figure 5. Low-pass (a) and high-pass (b) filters for 'db4' wavelet family.

Having an orthogonal wavelet means that its related wavelet transform preserves energy. In general, no orthogonal wavelets with compact support result symmetric and they have a nonlinear phase associated filter. Moreover, a discrete mother wavelet is characterized by a certain number of building coefficients, called vanishing moments. A wavelet, which has a number N of vanishing moments, is defined orthogonal to polynomials of degree N-1. In fact, the number of vanishing moments have a weak connection with the oscillation of the wavelet. This means that higher oscillations can be linked with a greater number of vanishing moments. This parameter is fundamental for defining wavelet families, so that even the names for many wavelets are derived from the number of vanishing moments. For example, db4 is the Daubechies wavelet with four vanishing moments and sym5 is the Symlet with five vanishing moments. For Coiflets wavelets, the links with vanishing moments is proportional to a factor 2: coif4 is the Coiflets with eight vanishing moments. Similarly, biorthogonal wavelet names result from the number of vanishing moments of the analysis wavelet and synthesis wavelet, which build them. For instance, bior2.8 is the biorthogonal wavelet with two vanishing moments in the synthesis wavelet and eight vanishing moments in the analysis wavelet. The number of vanishing moments also influences the support of a wavelet. In addition, regularity is a measure of smoothness, i.e. it is related to the number of continuous derivatives a function has. The regularity property is required to identify a sudden fluctuation in the signal. In particular, the vanishing moments of the wavelet family limit the max number of continuous derivates: N vanishing moments ensure maximum N-1 derivates. This means that a quite smooth signal with low gradients is better fit by a more regular wavelet.
In the study, all these cited wavelet families have been analysed. They can be divided in two groups according to the orthogonality property: orthogonal families and the biorthogonal families.
The first ones are strongly epitomized by the Daubechies wavelet family, which represents a set of orthogonal mother wavelets characterized by a maximal number of vanishing moments for some given length of the support. The db1 is also called Haar wavelet, and it was the first mother wavelet proposed and has the shortest support among all orthogonal wavelets. It has a step shape and is not able to approximate smooth functions because it has only one vanishing moment. But, at the same time, it is conceptually simple and fast and is suitable for detecting time-localized information. Daubechies least asymmetric mother wavelets are also known as Symmlets. They are compact, orthogonal, continuous, but they are not perfectly symmetric. Their construction is very similar to the construction of Daubechies wavelets, but the symmetry of Symmlets is stronger. Another Daubechies-derived wavelet family is the Coiflets. It is designed to be more symmetrical than Daubechies mother wavelet and to have a support of size 3N - 1 instead of 2N - 1 (like in the case of Daubechies mother wavelets) [12].
The Biorthogonal wavelets, instead, exhibit the property of linear phase, which is recommend for signal and image reconstruction. They are designed combining two different wavelets, one for decomposition and the other for reconstruction. In this way, they have additional degrees of freedom than orthogonal wavelets, for example the possibility of constructing symmetric wavelet functions [13]. Reverse biorthogonal wavelets family is obtained from biorthogonal wavelet pairs. Moreover, both biorthogonal and reverse biorthogonal wavelet families are compactly supported biorthogonal spline wavelets.
The properties described above are important for the time-frequency localization of wavelets. However, another important aspect to take into account is related to the shape of the signal to be analyzed [14]. In particular, the aim of the analysis presented in the following section is to demonstrate a possible correlation between these properties and the DWT application for power smoothing of wind power signals.

3. Results and discussion

The sensitivity analyses have been implemented using Anaconda distribution of Python 3 and PyWavelets library, which is an open-source wavelet transform software for Python [15]. One of the main points of strength of the present study is the possibility of basing the analysis on 4-years real wind turbine data collected in the field. In particular, measured data have a 10-min timescale, which has been artificially downscaled to 1 minute timescale following the approach presented in [6]. Based on that previous experience, the considered BESS capacity is 330 kWh, with the storage parameters reported in Table 1.

3.1. Sensitivity analysis on the level of decomposition

The easiest approach for determining the optimal decomposition level is to select the unique one providing the highest AR for the selected battery capacity. However, modern mathematical methods could enable even a time-variable modulation of the decomposition level. To this end, a more detailed data analysis has been made. In detail, time data have been classified in terms of the percentage of ramp rates greater than the grid code target. In Figure 6, this number is plotted vs. the months of the year. Upon examination of the figure, it is apparent how winter and summer months are characterized by fluctuations higher than others. Moreover, Figure 7 confirms how the central hours of the day are typically featured by stronger wind power variability with a peak at 3 p.m. [16].

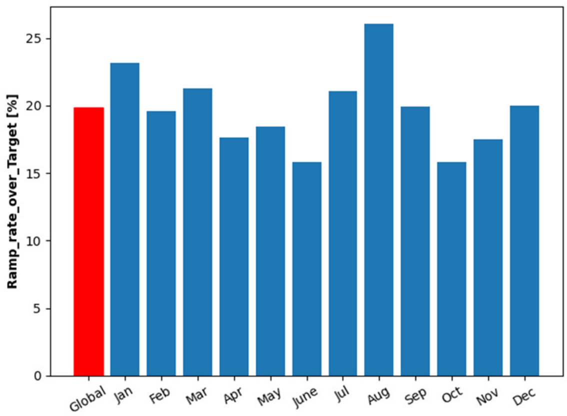

Figure 6. Monthly ramp rate characterization.

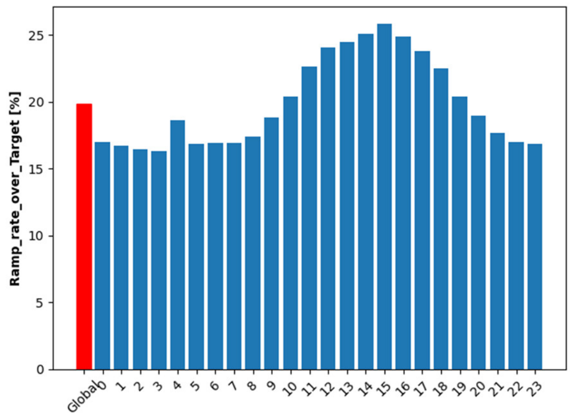

Figure 7. Hourly ramp rate characterization.

In order to check if variable settings in different conditions could bring to an optimization of the power smoothing efficiency, data have been first divided into high ramp rate months (January, March, July, August and December) and low ramp rate months (the others) and a sensitivity analysis varying the level of decomposition has been carried out. For this analysis, the db4 mother wavelet has been used. Results are shown in Table 2. The same procedure has been adopted splitting the data between high ramp rate hours (from 10.00 to 20.00) and low ramp rate hours. Table 3 reports these results.

Upon a combined analysis of the results, it is apparent that, even though the same level of AR ca ben achieved with different parameters' combinations, overall a variable level of decomposition, provided that a suitable mother wavelet is selected, does not bring significant benefits. Also, the analysis confirmed that level 2, highlighted by the analysis of Figure 4, is that ensuring the best performance.

		High ramp rate months				
		1	*2*	*3*	*4*	*5*
Low ramp rate months	*1*	0.8488	0.8694	0.8697	0.8574	0.8236
	2	0.8638	**0.8848**	0.8846	0.8724	0.8386
	3	0.8639	0.8845	0.8847	0.8725	0.8387
	4	0.8548	0.8754	0.8757	0.8634	0.8296
	5	0.8270	0.8477	0.8479	0.8356	0.8018

Table 2. AR varying monthly the level of decomposition.

		High ramp rate hours				
		1	*2*	*3*	*4*	*5*
Low ramp rate hours	*1*	0.8488	0.8614	0.8615	0.8543	0.8284
	2	0.8718	**0.8848**	0.8844	0.8772	0.8514
	3	0.87215	0.8846	0.8847	0.8776	0.8517
	4	0.8579	0.8705	0.8705	0.8634	0.8375
	5	0.8222	0.8349	0.8349	0.82769	0.8018

Table 3. AR varying hourly the level of decomposition.

3.2. Sensitivity analysis on the wavelet family

Moving from the results of Section 3.1, comparative analyses between the different wavelet families have been carried out both in offline (Section 3.2.1) and online mode (Section 3.2.2).

3.2.1 Offline analysis

Figure 8 compares the performance in terms of Abatement Ratio of the different wavelet forms. Upon examination of the plot, it is apparent that the majority of wavelet families is suitable for wind power smoothing, since they all exhibit about the same performance. Discrepancies are only seen for families Bior 3.1 and RBior 3.1, which result in a significantly lower AR. In order to understand this behavior, it is relevant to analysis the wind power signal. Those intense wind fluctuations that need to be smoothed are indeed often due to sudden phenomena. In those periods, the wind speed increases and/or decreases quickly (so that the ramp rate is violated), but it often undergoes an opposite trend quite soon. In detail, analyzing the wind power signal, it has been highlighted that 60% of the violations of Figure 6 are characterized by a "triangular" trend as shown in Figure 9.

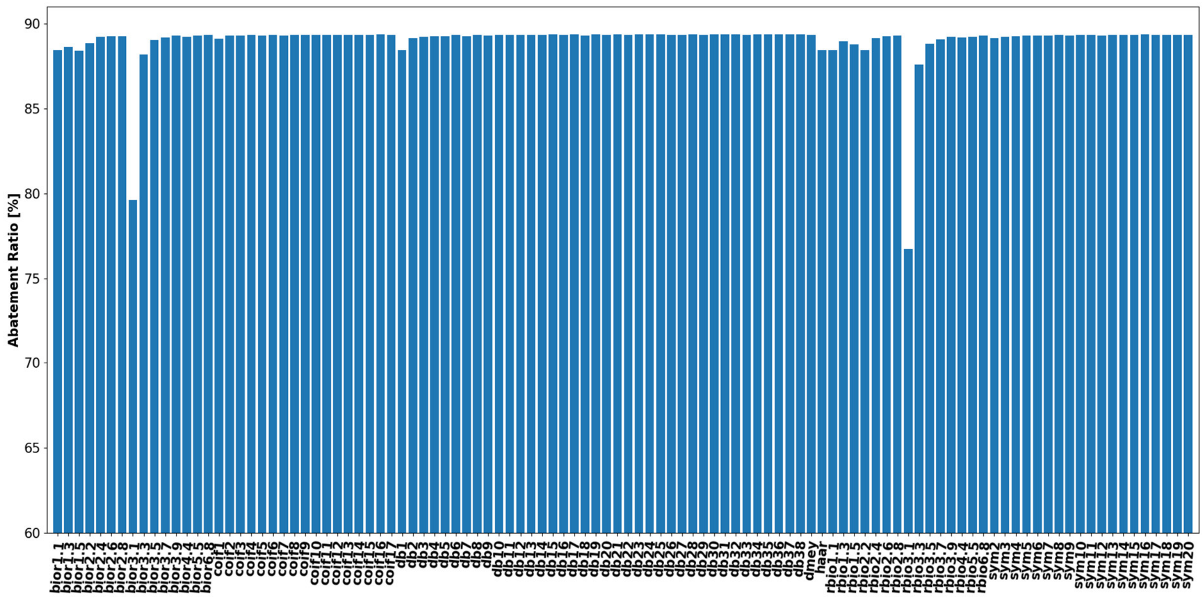

Figure 8. Wavelet families performance in offline mode.

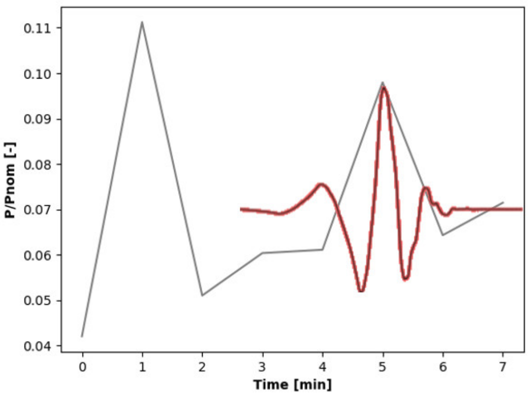

Figure 9. Fitting of 'db4' wavelet family with a typical wind fluctuation.

Considering a target ramp rate of 10% imposed by the grid, 75% of total ramp rate violations occurs in concurrence of an up and down trend of the wind power profile. Under these preconditions, it can be hypothesized that the centered-peak families all have similar performance when applied to a wind power signal, provided that the correct level of decomposition is ensured (Figure 10). This is also due to the orthogonal and symmetry properties. In particular, the orthogonal families have all good performance as they derived from the Daubechies family. The Biorthogonal wavelet, instead, have variable outcomes depending on their specific design criteria. For example, the Bior3.1 family is not effective because its shape features a discontinuity in correspondence to the center of the window, not corresponding to the attended physical trend of wind power.

3.2.2 Online analysis
The conclusions drawn from the offline analysis have been then validated also in online operation. As discussed, the DWT is not readily applicable to this end since it is based on a centered filtering, thus leading to border distortion [17]. In fact, the power value that needs to be managed at each timestep is exactly at the edge of the wind power series, resulting in an inaccuracy in signal reconstruction and a consequent low power smoothing performance.

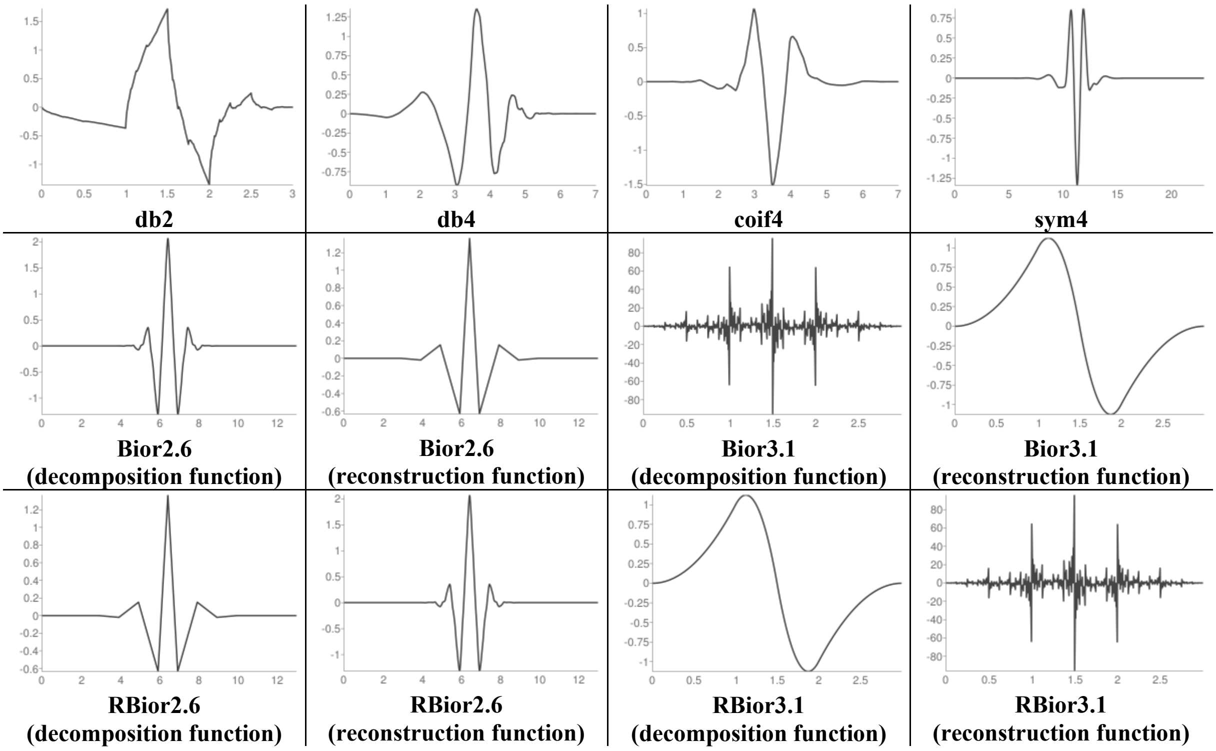

Figure 10. Wavelet families.

To overcome this issue, the authors have proposed the use of a symmetric extension method as shown in [18]. This simple approach provides efficient performance with a low computational cost. Based on the fact that it makes use of synthetic data creation in the future to remove border distortion, however, the method is even more sensitive to the selection of the wavelet shape used. A dedicated sensitivity analysis has been then carried out considering five different wavelet families out of all those investigated in Figure 9. The results of this analysis are shown in Table 4. Upon examination of the results, it is apparent that the findings of the offline sensitivity analysis are confirmed also in real-time application, with the only exception of the bior3.1 as discussed in the previous section. Moreover, it is also worth noticing that the overall value of AR with the proposed method for data extension is still very high and in line with that obtained for the real data set in offline mode. This corroborates the suitability of the proposed method for data extension. In this view, a final sensitivity analysis was devoted to mirroring of data. In particular, the "symmetric" window originally proposed by [18] (eq. 5) has been compared with a "smooth" window (eq. 6) and a "periodic" window (eq. 7). The impact of the different extension methods is also more clearly visualized in Figure 11.

$$W_i = W_i + W_i^{sym} = \{x(i - lw + 1), \dots, x(i)\} + \{x(i), \dots, x(i - lw + 1)\} \tag{6}$$

$$W_i = W_i + W_i^{smooth} = \{x(i - lw + 1), \dots, x(i)\} + \{x(i), \dots, x(i)\} \tag{7}$$

$$W_i = W_i + W_i^{per} = \{x(i - lw + 1), \dots, x(i)\} + \{x(i - lw + 1), \dots, x(i)\} \tag{8}$$

lw is the length of the extension window. For the simulations it is selected equal to 24, as in [6]. Methods have been compared using the db4 family. The results are shown in Figure 11. The conclusions that can be drawn from this latter analysis are twofold. First, it is apparent that the selected method for data extensions does not significantly alter the smoothing performance of the DWT. While it is key to remove border distortion, in fact it does not impact on the filtering effect on the signal. Notwithstanding the above, the choice of a symmetric window improves the overall performance of the DWT, in turn confirming the assumptions made on the recurring trends of wind power signals.

Mother Wavelet Family	AR
db4	88.34
coif4	87.79
sym4	87.12
bior2.6	88.44
bior3.1	76.51

Table 4. Sensitivity analysis of wavelet families in online mode: AR as a function of the selected wavelet family.

Window extension	AR
Symmetric	88.34
Smoothing	88.05
Periodic	88.24

Table 3. Sensitivity analysis on window extension criteria.

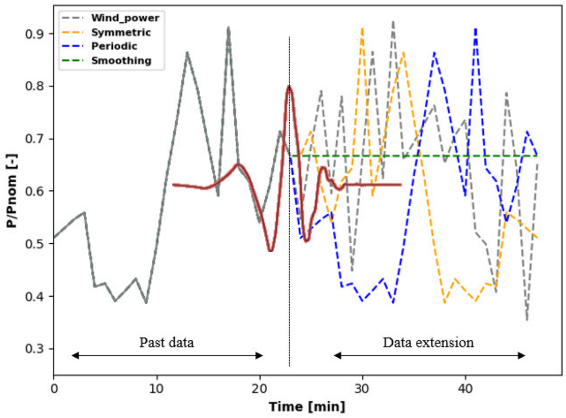

Figure 11. Types of window extension vs ‘db4’ wavelet family shape.

4. Conclusions

The scope of the present study was to tune properly the parameters of an algorithm based on the Discrete Wavelet Transform (DWT) for use in power smoothing of utility-scale wind turbines coupled with batteries. Based on different sensitivity analyses, interesting outcomes have been found, which can be resumed as follows:

1. Provided that a suitable wavelet family is chosen, moderate decomposition levels (2-3) are those ensuring the best trade-off between smoothing effectiveness and computational cost.
2. For a given site (i.e. a wind profile), a variable decomposition level as a function of the season and/or of the daily hours seems not to bring significant benefits in terms of increase of Abatement Ratio.
3. Different wavelet families have been compared, including: Daubechies, Coiflet, Symmlet, Biorthogonal and Reverse Biorthogonal. All the wavelet families having a centered peak show good performance. This is due to the fact that the severe wind fluctuations that need to be smoothed are quite sudden phenomena, in which the wind speed increases and then decreases. In particular, orthogonal and symmetric wavelets fit well the wind power signal. The only discrepancies were found for those wavelet forms having a discontinuity at the center of the filtering window, which are therefore not recommended for wind energy applications.

4. The discussed results were confirmed also in case the DWT is used for real-time control, i.e. when synthetic data in the future are needed to avoid border distortion. In this case, the use of the data mirroring techniques seems to provide good performance when applied to wind data series.

In conclusion, the study corroborated the suitability of the Discrete Wavelet Transform for power smoothing techniques applied to wind turbines' output.

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