



Centralized and decentralized supply chains: Performance maps for comparing the cost-effectiveness of distribution network configurations

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ABSTRACT

Optimizing Distribution Networks (DNs) is crucial for retailers, impacting service levels and logistics costs. A key DN configuration decision is the stock deployment policy, which entails choosing between centralized, decentralized, and hybrid DNs for each Stock Keeping Unit (SKU). Choosing the stock deployment policy is complex due to many variables influencing the decision (e.g., number of customers served, SKU purchasing costs, customer demand, etc.). Moreover, this decision must be revisited whenever customer demands changes, which can be time-consuming when DN resilience is challenged by geopolitical changes, market trends, and disruptive events. Dimensional Analysis (DA), and particularly the Buckingham Theorem (BT), shows capabilities to support retailers in guiding and streamlining stock deployment decisions. After modeling the stock deployment problem in a mathematical form, BT can identify its influential variables, extract knowledge on how variables mutually interact when affecting the stock deployment performance, and aid informed decision-making on the most cost-effective policy. Accordingly, BT enables creating performance maps which compare the characteristics of different DNs and SKUs, then suggesting similar stock deployment decisions for similar (scaled) DNs and SKUs. Despite the potential utility of these performance maps, no prior study has explored BT's capabilities for stock deployment decisions. This paper bridges this gap by proposing BT to create supportive maps for multidimensional scaling, similarity analysis, and economic performance prediction across centralized, decentralized, and hybrid DNs. The resultant maps provide retailers with visual decision support tools for associating similar DNs and SKUs with optimal stock deployment policies, ultimately improving DN performance and resilience.

1. Introduction

Distribution Networks (DNs) typically consist of two-echelon supply chains (Daskin et al., 2002; Tapia-Ubeda et al., 2020). For instance, Yu et al. (2023) claim that companies engaged in last-mile delivery operations commonly act as two-echelon DNs. In the first echelon, one or more suppliers provide stocks to replenish a set of Distribution Centers (DCs). In the second echelon, retailers deliver

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stocks from DCs to customers, aiming to fulfill their demand at specific consumption points (Lehmann and Winkenbach, 2024). Configuring DNs is the activity of modeling the structure of DNs, which involves determining the size, number, and location of DCs, as well as planning where to temporarily store the stocks on their way to end customers (Alemamy et al., 2021). According to Abbasi et al. (2021), configuring DNs is a crucial element of retailers' profitability since it impacts supply chain performance in terms of customer service level and logistics costs (for procuring, storing, and distributing stocks).

Among the critical decisions affecting DN configuration, choosing the stock deployment policies has been emphasized as of primary importance, which entails choosing how to allocate stocks inside DCs, opting for antithetical strategies such as centralized, decentralized, or hybrid DNs (Yang et al., 2024b). In a decentralized DN, stocks are stored in multiple DCs, each meeting the demand of local customers. This stock deployment policy results in DN flexibility, responsiveness, and low outbound transportation costs (Wanke and Saliby, 2009). However, it leads to high holding costs since many DCs are managed, each needing to guarantee high service levels. Conversely, in a centralized DN, stocks are stored in a single central DC that serves all customers. This stock deployment policy entails low holding and ordering costs due to the well-known 'risk-pooling' effect (Cantini et al., 2022; Gioia and Minner, 2023). However, it determines low DN flexibility and responsiveness, as well as high transportation costs due to the average distance between the central DC and customers (Li et al., 2019). Finally, in hybrid DNs, an intermediate number of DCs is selected between one and one per local customer, achieving trade-off benefits between centralization and decentralization (Savadkoobi et al., 2018).

Four challenges arise when choosing the optimal stock deployment policy among centralized, decentralized, and hybrid DNs. First, retailers usually manage thousands of Stock Keeping Units (SKUs) and the optimal stock deployment alternative must be associated with each SKU. In fact, as reported by Mangiaracina et al. (2015), DN configuration choices depend on several contextual factors, including the type of product and its demand characteristics. The second challenge is that customer demand is not stationary but varies over time due to changes in market trends, geopolitical disruptions, or catastrophic events (e.g., pandemics) that may result in inflation phenomena, changes in exports and imports of products, etc. (Leoni et al., 2024; Liu et al., 2024a). To manage demand fluctuations, stock deployment cannot be optimized once (i.e., during the first DN configuration) (Amorim et al., 2012). Rather, it must be continuously reviewed, performing fine-tuning activities to ensure that logistics activities remain aligned with evolving customer needs, ultimately ensuring DN resilience to the next disruption (Eldem et al., 2022; Raaymann and Spinler, 2024). As the third challenge, when determining the optimal stock deployment policy for each SKU, the optimal choice between centralized, decentralized, and hybrid DNs must be identified depending on many influential variables (Biuki et al., 2020). Just a few examples of influential variables are: the number of customers served, their geographic location, their average demand, the number and location of DCs, and the unit costs associated with holding inventories (holding cost), issuing replenishment orders (ordering cost), distributing stocks to customers (transportation cost), and accepting backorders (backorder cost) (Cantini et al., 2024a). These influential variables must be taken into account simultaneously since they mutually interact while affecting DN characteristics. Finally, as the fourth challenge, different combinations of the values assumed by the influential variables result in a different stock deployment performance. Therefore, among the possible combinations, retailers must identify the optimal one that reduces logistic costs while ensuring high service levels.

Because of the former challenges, optimizing the stock deployment policies is a daunting task that requires adopting structured methodologies to be accomplished (Chopra, 2019). According to Biuki et al. (2020) and Firoozi et al. (2013), existing methodologies to optimize stock deployment policies propose mathematical programming models to maximize the performance of specific DNs. However, their results are case-specific, while few studies have been provided to directly compare the performance of centralized, decentralized, and hybrid DNs. Therefore, companies miss general guidelines and clear tools to analyze the big picture and understand when it is more convenient to adopt centralized, decentralized, or hybrid DNs. In this context, according to Rayleigh's authoritative statement reported by Lira (2013), Dimensional Analysis (DA) emerges as a useful methodology to support DN configuration. After expressing a general problem (in this case, the stock deployment choice) in a mathematical form, DA checks the meaningfulness of its governing equations. Hence, as reported by Zare et al. (2023), DA enables the identification of the influential variables affecting the problem, extracting in-depth knowledge and managerial insights on how they mutually interact when influencing the outcomes (namely, the performance of centralized, decentralized, and hybrid DNs). Based on this, DA has the potential to support retailers in predicting the performance of different stock deployment policies as a function of the identified influential variables (Vignaux, 2001). Accordingly, DA facilitates informed decisions about the optimal alternative to select for each SKU among centralized, decentralized, and hybrid DNs (Gálfi et al., 2022).

According to Jonsson (2024), the highest achievement of DA is the Buckingham Theorem (BT), also known as the π -theorem or Vaschy theorem. The BT states that any equation modeling a physical problem can be reorganized and simplified using a set of dimensionless variables (referred to as 'dimensionless groups') (Girard, 2024). Particularly, the number of influential variables originally used to describe the problem can be reduced by the number of independent fundamental physical dimensions (e.g., length, mass, time, etc.) characterizing the investigated equation. Therefore, the starting problem, characterized by n (dimensional) influential variables described by k fundamental dimensions, can be simplified as a function of $(n-k)$ dimensionless groups (Brunetkin et al., 2018). Given this background, BT has the potential to simplify stock deployment decisions in two ways. First, by identifying a reduced set of dimensionless groups that govern the outcomes of a problem and by conveying the basics of DA, the BT provides a richer knowledge of the studied problem, highlighting the relationships between its influential variables and their impact on stock deployment performance. Second, and most important, the BT enables similitude and scaling considerations to be performed (Gambini and Vellini, 2021). For instance, different DNs with different stock deployment policies characterized by the same values of dimensionless groups (albeit with different influential variables) can be generalized to reach similar performance, allowing for comparing their behavior (Girard, 2024; Oshiro and Alves, 2024). Consequently, to support stock deployment decisions, performance maps can be built like those traditionally used for turbomachinery (Bicchi et al., 2023; Prasad, 2024), in which the performance of several scaled

centralized, decentralized, and hybrid DNs is shown. By referring to scaled DNs, companies can put themselves in similarity and make strategic choices on their stock deployment policies.

Despite the potential usefulness for retailers, the application of the BT in the field of DN configuration (and, more broadly, in supply chain and operations management) has been reported as unusual within the literature (Miragliotta, 2011; Naddor, 1966). To fill this gap, this paper aims to investigate for the first time the use of the BT to achieve generalized dimensionless performance maps to compare the economic performance of different stock deployment policies. As a result, ten parameters are identified to predict the performance of centralized, decentralized, and hybrid DNs, selecting the optimal (cost-effective) alternative by means of multidimensional scaling analysis and similitude.

The remainder of this paper is as follows. Section 2 overviews the applications of the BT proposed by the literature. Section 3 describes the materials leveraged to develop this study, including the BT (Section 3.1) and the adopted mathematical formulation for the stock deployment problem (Section 3.2). Moreover, it describes the method followed to apply the BT in stock deployment decisions (Section 3.3), achieving the aforementioned ten parameters and performance maps. Section 4 explains how to leverage the performance maps to support stock deployment decisions in three application contexts (Sections 4.1, 4.2, and 4.3, respectively). Section 5 tests the achieved results on a case study. Finally, Section 6 offers some conclusions.

2. Background literature on BT applications

DA is a research methodology aimed at deducing in-depth information about a certain phenomenon relying on the postulate that this phenomenon can be described through a dimensionally homogeneous equation among certain variables (Naddor, 1966; Rayleigh, 1899). The BT is a well-known expression of DA, which was first presented in 1914 (Buckingham, 1914). Concerning the applications of the BT in research fields, it has been primarily adopted in the field of physics, where two traditional contributions are those by Van Driest (1946) and Langhaar (1951). Van Driest (1946) refined the BT statement while Langhaar (1951) demonstrated its validity by proposing the mathematical procedure that we use nowadays to identify dimensionless groups. After the first application in physics, the use of the BT was extended to the field of engineering thanks to Quade (1961) and Huntley (1967). Quade (1961), followed by Siano (1985), delved into the mathematical basis of the BT, analyzing the concept of dimension and providing a formal definition. Huntley (1967) highlighted the mechanism by which certain dimensions can influence a phenomenon independently of each other. Therefore, he explained why, for example, in some mechanical engineering problems, the length and cross-section of a metal bar can be treated as if they have two independent fundamental dimensions (i.e., the longitudinal length and the transverse width, respectively). A comprehensive review of the BT's applications proposed in the literature of the 19th and 20th centuries is provided by Taylor (1974). The latter provided indications of the mistakes not to make when using the BT. Moreover, he firstly discussed how the applications of the BT represent an intimate moment for researchers to synthesize and enhance their understanding of a phenomenon.

Before Taylor (1974), applications of the BT were reported to be restricted (Naddor, 1966). Subsequently, the interest of researchers in BT's applications has strongly increased. Many studies have been investigating the capabilities of the BT in a wide variety of research fields, of which just a few examples are: thermo-fluid dynamics (Kholi et al., 2023), medicine and biology (Valentini et al., 2017), statistics (Vignaux, 2001), ecology and environmental engineering (Zelenáková et al., 2016), chemistry and materials science (Zong et al., 2021), energetic engineering (Fathima et al., 2022), civil and construction engineering (Lira, 2013), mechanical engineering (Okano and Mochizuki, 2021), and mechatronic engineering (Ewing, 1999). Whatever the application field, the BT allows for achieving one or more of the following objectives (Girard, 2024): i) empowering the formulation of a model and assisting the memory in reconstituting forgotten formulae, as exemplified by Ho (2024); ii) correcting algebraic errors by underlining the lack of dimensional homogeneity of equations, thus ensuring dimensional consistency, as shown by Yang et al. (2024a); iii) providing a conversion factor for changing from one system of units to another, as proposed by Therrien et al. finally, vi) obtaining partial solutions of problems too

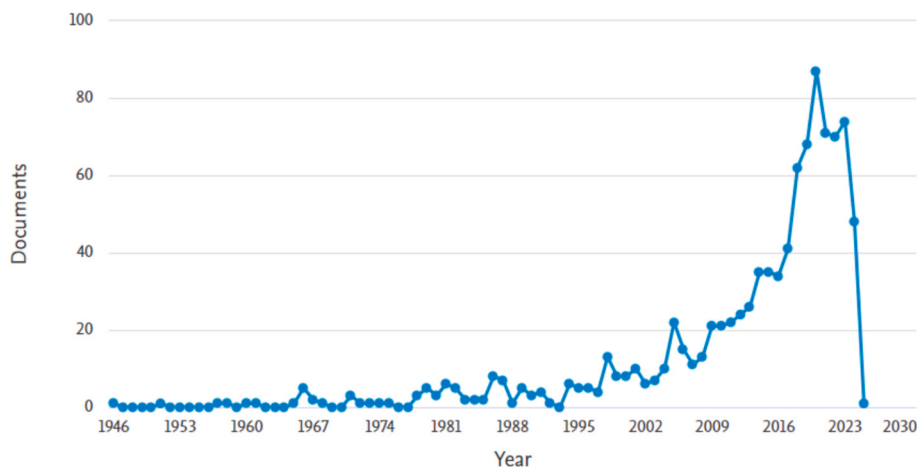


Fig. 1. Temporal distribution of publications concerning the BT.

complex for other mathematical analyses, as suggested by Huntley (1967). Recent examples of studies targeting these objectives are as follows. Najafzadeh and Sheikhpour (2024) and Liu et al. (2024b) use the BT prior to the application of artificial intelligence to reduce the number of significant variables in the training dataset and improve prediction accuracy. Whereas Islam et al. (2024) and Estrada-Díaz et al. (2024) use the BT to support additive manufacturing companies in optimizing production parameters and predicting manufacturing performance.

Nowadays, BT's applications have become so widespread that software and computer packages have been created to rapidly implement DAs in programming environments like Python (Karam and Saad, 2021). Moreover, the literature interest towards BT has grown to such an extent that a dedicated research strand has focused on expanding the use of BT into new research fields. For instance, applications of the BT have been proposed in the domain of sport (Blondeau, 2021), geology (Tian et al., 2018; Yu et al., 2018), hermeneutics (Juárez, 2020), and nutrition sciences (Velasquez et al., 2018). Evidence of the recent proliferation of applications of the BT appears by searching this string on Scopus: *TITLE-ABS-KEY ("Buckingham theorem" OR "pi-theorem" OR "pi theorem" OR "Vaschy theorem" OR " π -theorem" OR " π theorem") AND (LIMIT-TO (LANGUAGE, "English")) AND (LIMIT-TO (PUBSTAGE, "final"))*. Performing this query on 10th October 2024 yielded 962 documents, of which 43.6 % were published within the last 5 years (Fig. 1).

Despite the strong interest of literature towards applications of the BT, Miragliotta (2011) emphasized that only a few studies have investigated applications in the field of supply chain and operations management. This gap persists nowadays, as evidenced by the 35 sources that, according to Scopus, cited Miragliotta's work. As a matter of fact, among those sources, only two pertain to the area of supply chain and operations management (Awatade and Suhane, 2023; Naim and Gosling, 2023). However, Naim and Gosling (2023) merely mention Miragliotta's work to assert the importance of designing resilient supply chains using system approaches, without proposing any applications of the BT. Whereas Awatade and Suhane (2023) do utilize the BT, but in the very specific context of the hot rolling process of steel sheets, where they aim to establish a relationship between production rate and the variables roller diameter, the distance between rollers, fillet thickness, pressure applied on roller, roller speed, and raw material Young's modulus.

By consulting the title, abstract, and content of the 962 documents that emerged in the literature through the above search query, we could summarize the existing literature applications of the BT and, more broadly, on DA in supply chain and operations management. Naddor (1966) and Vignaux (2001) investigated inventory and queuing systems, pointing out that they provide a rich source of problems for DA applications (being typically characterized by thousands of fundamental dimensions and variables). In inventory systems, DA was applied in a warehouse to explain the meaning of the Wilson lot-size formula. In queuing systems, DA helped to understand the basics of queuing theory. Sivazlian (1971) adopted DA to identify dimensionless groups and reduce the number of variables to plot when presenting the results of an (s, S) inventory policy with gamma-distributed demand. Similarly, Silver (1983) determined dimensionless groups to plot indifference curves in a continuous review stochastic (s, S) system. Notably, indifference curves can be considered the first attempt to build performance maps of similar systems (in this case inventories), comparing their behavior based on scaling considerations. Braglia and Gabbriellini (2000) employed the BT to identify dimensionless groups that assist in selecting appropriate industrial robots when willing to consider multiple conflicting performance attributes. Miragliotta (2011) applied the BT to support strategic design decisions in flexible manufacturing systems. More recently, Awatade and Suhane (2023) proposed the aforementioned application in steel production. Concerning supply chains, Daganzo (1987) applied DA in DNs where the number of DCs was already defined and the location of suppliers and customers was known. Here, DA was used to optimize transportation activities by planning delivery intervals, shipment sizes, and the number of stops. Finally, Pérez-Domínguez et al. (2018) used DA to select suppliers and streamline multi-criteria vendor rating decisions.

Given this background, to the best of the authors' knowledge, no study has been provided to exploit the capabilities of DA (and in particular of the BT) to support stock deployment decisions. This gap is addressed in the present paper, where the BT is proposed with a twofold purpose: i) provide a richer knowledge of this decision-making problem by highlighting the relationships between its influential variables; ii) achieve performance maps of scaled models, which allow suggesting similar stock deployment policies for similar DNs, and reviewing stock deployment decisions over time to ensure DN resilience against evolving customer needs.

Although the focus of this paper remains on stock deployment problems, it is worth noting that the literature background reviewed here reveals broader gaps in the application of the BT within the wider field of supply chain and operations management. Specifically, despite conducting a comprehensive Scopus query aimed at retrieving any study mentioning BT (without restricting the field of application), only four papers were identified in the domain of warehouse management (Naddor, 1966; Silver, 1983; Sivazlian, 1971; Vignaux, 2001). These works primarily focus on inventory policies, while overlooking other critical decisions such as optimal stock placement on shelves or the selection of appropriate storage systems (whether automated or not). Additionally, only two studies were found in the area of manufacturing systems design (Braglia and Gabbriellini, 2000; Miragliotta, 2011), which addressed technology-related decisions such as whether or not to invest in robotics, yet neglected other key aspects like plant capacity planning, workforce allocation, or production scheduling. Several studies were found to explore the use of BT for optimizing production parameters in both conventional and additive manufacturing (Awatade and Suhane, 2023; Estrada-Díaz et al., 2024; Islam et al., 2024), although this remains an area with significant room for further research, given the continuous evolution of production technologies. Finally, two contributions were found explicitly addressing supply chains (Daganzo, 1987; Pérez-Domínguez et al., 2018), with a focus on vendor rating and transportation routing decisions, leaving substantial opportunities for future work on other DN choices (beyond stock deployment) such as customer segmentation, facility layout, or DN configuration and geographical location under uncertainty. In this sense, as further elaborated in the conclusions, the present study may serve as a foundational research effort, from which a wide range of future investigations and BT-based performance maps for supporting informed decisions can be developed.

3. Materials and method

3.1. Buckingham Theorem

According to Corrsin (1951), the BT states that if there exists a unique relation among n variables (A) which are described by k physical dimensions (e.g., mass, length, time, etc.), as in Equation (1):

$$F(A_1, A_2, \dots, A_n) = 0 \quad (1)$$

Then, this relation can be rearranged into an equivalent equation (Equation (2)) involving a set of $(n-k)$ dimensionless groups (π), which are derived as products of powers of the original variables. In other words, the number of independent influential variables (namely, the problem's inherent 'degrees of freedom') may be reduced by the number k (Cong, 2024).

$$\Phi(\pi_1, \pi_2, \dots, \pi_{n-k}) = 0 \quad (2)$$

The BT is applicable under the following conditions (Miragliotta, 2011): (i) the starting relation (Equation (1)) is physically meaningful and dimensionally homogeneous, namely, its analytical form does not depend on the selected system of measurement units (e.g., centimeters or inches); (ii) the starting influential variables (A) are independent and represent a complete set, namely all variables relevant to represent the phenomenon are included; (iii) each variable can be quantitatively measured, and has a physical "dimension" expressed as a product of powers of linear fundamental quantities (e.g., length, mass, time, etc.); (iv) the values of influential variables in the original problem are positive, which prevents obtaining dimensionless groups with imaginary values. Under these conditions, according to Buckingham (1914), the validity of the BT can be demonstrated a posteriori, by assuming the existence of dimensionless groups and deducing the sets of simultaneous equations among pertinent exponents.

3.2. Stock deployment problem formulation

According to the literature (Abbasi et al., 2021; Tapia-Ubeda et al., 2020), the problem of choosing stock deployment policies can be framed as an optimization problem, where the total costs of different centralized, decentralized, and hybrid DNs are compared, and individual SKUs are associated with the most cost-effective alternative which also respects pre-established service levels. Following (Gregersen and Hansen, 2018) and (Cantini et al., 2024b), different stock deployment policies (i) can be distinguished using a parameter called 'degree of centralization' (Deg_i). Deg_i ranges between 0 and 1, being 0 in the case of decentralization, 1 in centralization, and a number between 0 and 1 in hybrid DNs. As per Equation (3), Deg_i depends on the ratio between the number of DCs (DC_i) able to fulfil customer demand, and the number of customers to be served (N). Notably, this work considers only three stock deployment policies ($i = 1, 2, 3$), including a centralized, a decentralized, and a hybrid DN. However, this work can be easily extended to any other alternative of Deg_i since the mathematical equations are general enough. Fig. 2 schematically represents the three stock deployment policies (and the corresponding DNs) herein investigated, referring to an example company with six customers.

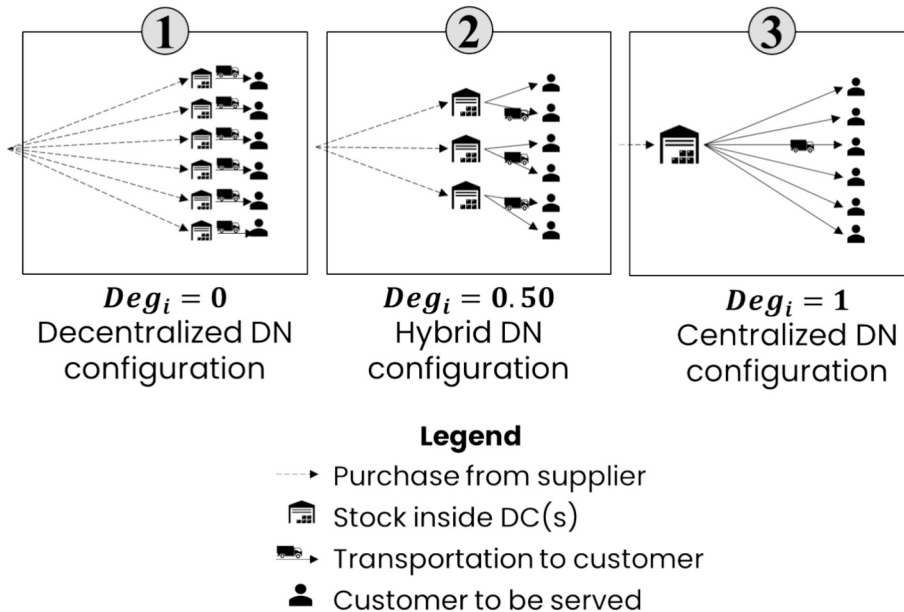


Fig. 2. Three stock deployment policies evaluated in an example company with six customers.

$$Deg_i = \text{degree of centralisation} = \begin{cases} 1, & \text{if } i = 3 (\text{centralization}) \\ 1 - \frac{DC_i}{N}, & \text{else} \end{cases} \quad (3)$$

To assist retailers in choosing between centralized, decentralized, or hybrid DNs, the literature offers various mathematical formulation. A recent example is proposed by Cantini et al. (2024b), whose formulation is summarized in Appendix A and used in this paper to illustrate how BT can be applied to stock deployment decisions. This formulation is chosen for its simplicity, allowing this paper to remain focused on the application of BT rather than on complex mathematical details. Nonetheless, as shown in Appendix B, BT can be applied to any mathematical formulation. While the mathematical formulation by Cantini et al. (2024b) is reported in Appendix A, to enhance the readability of this paper, we only mention here that it relies on the influential variables listed in Table 1. Since Cantini et al. (2024b) provide little insight into how variables' mutual interactions affect stock deployment performance, this area seems fertile ground for the application of the BT. In fact, the BT enables a more thorough examination of influential variables' behavior, and how they influence the total cost of different stock deployment policies, in turn facilitating informed decisions regarding the optimal alternative to choose (Girard, 2024).

3.3. Buckingham theorem in stock deployment problems

To apply the BT to stock deployment problems and identify dimensionless groups, the three-step procedure explained by Langhaar (1951) was adopted. In step 1, the phenomenon under investigation was described through a mathematical relation. Yang and Lin (2021) provide guidance on selecting this mathematical relation, recommending to first define the output or performance metric to be understood via BT (such as the total cost of stock deployment policies as in Equation A(2), or other metrics like service level, lead times, etc.), and then to identify an equation that captures the underlying relationships between this metric (C_{TOT_i}) and its influential variables. The influential variables can be identified using domain knowledge, literature review, expert consultation, or empirical observation, as long as they form a comprehensive set and satisfy the conditions for BT validity outlined in Section 3.1. Also causal

Table 1
Notation adopted to formulate the stock deployment problem.

Indexes	Description	Unit measure
i	Considered stock deployment policy. i assumes integer values between 1 and 3	—
Influential variables (input data)	Description	Unit measure
Deg_i	Degree of centralization associated with the stock deployment policy i . It can assume values 0, 0.50, and 1, according to Fig. 2.	—
SL	Pre-established service level for the considered SKU. It is associated with the service factor (Z) in a standard normal distribution.	—
$\bar{D}$	Average annual demand emitted by 1 customer for the SKU. It depends on the average customer targeted by the DN (not on the DC where the SKU is stored).	units/time
σ	Standard deviation of the demand emitted by 1 customer.	units/time
N	Total number of customers served by the DN.	—
b	Unitary backorder cost of the considered SKU.	€/backorder
l	Average procurement lead time for receiving the SKU from the supplier.	time
c	Unitary cost of purchasing the SKU from the supplier.	€/unit
o	Cost of issuing one stock replenishment order.	€/order
h	Annual holding cost rate for keeping inventory of the SKU in a DC.	time ⁻¹
d_c	Average distance from the central DC to customers in the centralized DN configuration ($i = 3$).	km
t	Unitary transportation cost for distributing stocks from DCs to customers.	€/km*vehicle
q	Average quantity of products ordered by a customer each time it demands it.	units/demand
C	Capacity of the vehicles used to transport products from DCs to customers.	units
Decision variables	Description	Unit measure
Q_i	Optimal order quantity of the SKU in each DC.	units
RP_i	Reorder point associated with the SKU in each DC.	units
SS_i	Safety stocks of the SKU in each DC.	units
DC_i	Total number of DCs in the DN.	—
D_i	Annual demand received by an individual DC for the specific SKU under consideration. It depends on the total number of customers served by the DC.	units/time
N_v	Average number of vehicles required to transport the SKU each time it is distributed from a DC to customers.	vehicles/ transportation
N_b	Average number of annual transportations performed.	transportations/time
d_i	Average distance from a DC to customers.	km
N_b	Average number of annual backorders for the SKU in a DC.	orders/time
N_o	Average number of annual orders issued for the SKU in a DC.	orders/time
C_{TOT_i}	Total cost of the considered stock deployment policy.	€/time
C_{P_i}	Annual purchase cost.	€/time
C_{H_i}	Annual holding cost.	€/time
C_{O_i}	Annual ordering cost.	€/time
C_{B_i}	Annual backorder cost.	€/time
C_{T_i}	Annual transportation cost.	€/time

diagrams can be used, which are graphical tools used to represent cause-effect relationships among variables, helping to clarify which factors influence the metric of interest (Rodriguez et al., 2021). In this paper, following Yang and Lin (2021) and focusing on stock deployment policy decisions, step 1 was carried out leveraging, as an example, the mathematical formulation in Appendix A, where Equation A(2) (described in turn by Equations A3-A17) plays the role of Equation (1).

In step 2, the influential variables representing the phenomenon were listed, resulting in a set of $n = 14$ variables (first column of Table 2). These variables were extracted from Table 1, specifically from the subsection titled ‘influential variables (input data)’. Consequently, the selection of these variables (and the resulting value of $n = 14$) was determined by the mathematical formulation used to describe stock deployment problems (Section 3.2), with Table 1 providing detailed definitions for each variable. While acknowledging that the chosen variables depend on the specific problem formulation, the application of the BT and its findings (herein presented) are easily extendable to other mathematical models. In fact, previous studies have modeled stock deployment decisions using a similar number and type of influential variables, as demonstrated in the following examples (Cantini et al., 2025; Daskin et al., 2002; Tapia-Ubeda et al., 2020) and in Appendix B.

To complete step 2, each of the 14 influential variables was associated with the respective physical dimensions, obtaining the so-called ‘dimensional matrix’ (Table 2). No matter which mathematical formulation is adopted for stock deployment problems, the literature claims that the basic physical dimensions in supply chain and operations management problems are time, quantity (e.g., of stocks), and money (Naddor, 1966; Vignaux, 2001), which are conventionally expressed by capital letters enclosed in square brackets ([T], [Q], and [\$], respectively). To these dimensions, according to Suvanjumrat et al. (2024), the length must be added, which characterizes Equation A14 and is denoted by [L]. Therefore, $k = 4$ physical dimensions were achieved. Accordingly, cells in Table 2 were filled with the value of exponents (powers) by which each influential variable was related to the respective physical dimension.

Taking $n = 14$ as the number of influential variables and $k = 4$ as the number of physical dimensions, following Section 3.1, the BT allowed us to identify a set of $(n - k) = 10$ independent dimensionless groups through which the phenomenon of stock deployment could be reformulated and explained. In this regard, Lira (2013) stressed that dimensionless groups are not unique and the method for obtaining them is completely irrelevant. In simple cases, they can be reached by inspection; otherwise, they can be derived by applying the matrix-method described by Deb and Deb (1986) (which constitutes step 3 and is detailed below). A subset of $k = 4$ influential variables was selected whose dimensional sub-matrix has a non-zero determinant. The sub-matrix was obtained by removing from Table 2 all but the rows and columns in correspondence of the four chosen variables. Based on Rouché-Capelli theorem (Horbiychuk et al., 2021), the selected influential variables constituted a subset of linearly independent vectors. We selected the following subset of independent variables: $\bar{D}$, l , c , and t . Therefore, an underdetermined system of linear algebraic equations was set up to find the exponents (conventionally denoted by the letter α) to which to raise the physical dimension of the $(n - k) = 10$ remaining variables so that their ratio was dimensionless. The resulting ratios (one per each remaining variable) generated a set of 10 dimensionless groups (herein denoted by the letter π). For example, focusing on the variable b , exponents (α) were identified that make dimensionless the ratio in Equation (4). This translated into determining the exponents that verified Equation (5), namely $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 1$, and $\alpha_4 = 0$. The resulting dimensionless group (π_1) was that in Equation (6).

$$\pi_1 = \frac{b}{\bar{D}^{\alpha_1} \bullet l^{\alpha_2} \bullet c^{\alpha_3} \bullet t^{\alpha_4}} \quad (4)$$

$$[\$] = [Q \bullet T^{-1}]^{\alpha_1} \bullet [T]^{\alpha_2} \bullet [\$ \bullet Q^{-1}]^{\alpha_3} \bullet [\$ \bullet L^{-1}]^{\alpha_4} \quad (5)$$

$$\pi_1 = \frac{b}{\bar{D} \bullet l \bullet c} \quad (6)$$

By repeating this approach for the other influential variables, the dimensionless groups in Equations 7–15 were obtained. Similarly, in Equation (16), the total cost C_{TOT_i} was made dimensionless.

Table 2

Dimensional matrix related to the stock deployment problem. [Q], [T], [\$], and [L] mean quantity, time, money, and length, respectively.

Influential variables	Physical dimensions			
	[Q]	[T]	[\$]	[L]
Deg_i	0	0	0	0
SL	0	0	0	0
$\bar{D}$	1	−1	0	0
σ	1	−1	0	0
N	0	0	0	0
b	0	0	1	0
l	0	1	0	0
c	−1	0	1	0
o	0	0	1	0
h	0	−1	0	0
d_c	0	0	0	1
t	0	0	1	−1
q	1	0	0	0
C	1	0	0	0

$$\pi_2 = N \quad (7)$$

$$\pi_3 = Deg_i \quad (8)$$

$$\pi_4 = SL \quad (9)$$

$$\pi_5 = \frac{\sigma}{D} \quad (10)$$

$$\pi_6 = \frac{o}{D \bullet l \bullet c} \quad (11)$$

$$\pi_7 = h \bullet l \quad (12)$$

$$\pi_8 = \frac{d_c \bullet t}{D \bullet l \bullet c} \quad (13)$$

$$\pi_9 = \frac{q}{D \bullet l} \quad (14)$$

$$\pi_{10} = \frac{C}{D \bullet l} \quad (15)$$

$$\pi^* = \frac{C_{TOT_i}}{D \bullet c} \quad (16)$$

To conclude, Equation A(2) was rewritten as a function of ten dimensionless groups, leading to Equation (17). The next Section discusses the practical relevance of these dimensionless groups.

$$\frac{C_{TOT_i}}{D \bullet c} = \Phi \left(\frac{b}{D \bullet l \bullet c}, N, Deg_i, SL, \frac{\sigma}{D}, \frac{o}{D \bullet l \bullet c}, h \bullet l, \frac{d_c \bullet t}{D \bullet l \bullet c}, \frac{q}{D \bullet l}, \frac{C}{D \bullet l} \right) \quad (17)$$

Before proceeding with this discussion, we would like to remind the reader that, although applying Equations 4–17 may appear complex and time-consuming, the process of deriving dimensionless groups from a set of influential variables can be simplified thanks to various automated tools available to support the application of the BT across different contexts. Examples include the Python libraries ‘SymPy’ and ‘DimPy’, MATLAB’s ‘Dimensional Analysis Tool’, the ‘Engineering Equation Solver (EES)’, and the ‘Modelica’ modeling language.

4. Results and discussion

As an immediate consequence of applying the BT, the number of influential variables to be considered to optimize stock deployment policies has been reduced from 14 to 10. This reduction may seem insufficient for streamlining stock deployment decisions and gaining insights into this problem. However, the value of the BT results can be discussed from another perspective. By definition, dimensionless groups enable similarity and scaling considerations to be made. Therefore, by plotting the variation of dimensionless groups and their consequent impact on the cost performance of different stock deployment policies, performance maps can be obtained that offer two outcomes. First, they provide deeper insights into the decision-making problem by visually illustrating the relationships between influential variables and their impact on centralized, decentralized, and hybrid DN choices. Secondly, they enable direct comparisons of these different DNs, guiding retailers toward similar stock deployment decisions in all contexts where the same dimensionless group values apply (even if the underlying influential variables differ). Thus, performance maps expand and complement the results of the mathematical formulas presented in [Appendix A](#) as follows. Rather than analyzing individual DNs separately, an intuitive graphical representation is offered that reveals that similar stock deployment decisions can be pooled across DNs governed by the same dimensionless group values, albeit with different initial characteristics.

Although creating performance maps out of ten dimensionless groups may seem challenging, [Sonin \(2004\)](#) suggests how to do so. The number of dimensionless groups representing a problem can be reduced by fixing the values of influential variables that are beyond the control of decision-makers. Building on this idea, three distinct applications of the BT in stock deployment decisions have been herein identified and discussed, providing the corresponding performance maps and showing their benefits.

Notably, the validity of these performance maps was assessed through two approaches: conceptual validation and testing on a case study. The latter is presented in [Section 5](#), while the conceptual validation was conducted as follows. A critical analysis of the performance maps was carried out in a two-step process. First, the trends and relationships among variables were examined (cf. [Sections 4.1–4.3](#)), confirming their full consistency with the theoretical foundations and qualitative discussions scattered in previous literature studies like ([Gregersen and Hansen, 2018](#); [Holmström et al., 2010](#); [Milewski, 2020](#)). Next, the performance maps were reviewed by a panel of six experts, thereby completing the conceptual validation, as detailed in [Appendix C](#).

4.1. First application: Performance maps for inventory classification

Let us consider the typical case – shown by Cantini et al. (2024a) for two international companies – of a retailer that has partnership relationships with a supplier (l fixed), owns DCs with known geographical locations (d_c fixed), operates a fleet of distribution vehicles (t and C fixed), has already installed management software to communicate with the supplier and issue replenishment orders (o fixed), and serves a set of customers ensuring a pre-established service level (N and SL fixed). In this context, the BT allows for creating performance maps that suggest the optimal stock deployment policy (Deg_i) as the dimensionless groups vary, with all influential variables fixed except those related to the considered SKU (b , c , and h) and its demand ($\bar{D}$, σ , and q).

By fixing influential variables that retailers have no power to change, the degrees of freedom in the stock deployment problem are reduced, and Equation (17) translates into Equation (18). Notably, the groups in Equation (18) are no longer dimensionless. However, according to Sonin (2004), they represent the most economical scaling parameters for comparing similar DNs in this application context.

$$C_{TOTi} = \Phi\left(\frac{b}{\bar{D} \cdot c}, Deg_i, \frac{\sigma}{\bar{D}}, h, \frac{q}{\bar{D}}\right) \quad (18)$$

By plotting the value of C_{TOTi} when the groups $\frac{b}{\bar{D} \cdot c}$, Deg_i , $\frac{\sigma}{\bar{D}}$, h , and $\frac{q}{\bar{D}}$ vary, the resulting performance maps are achieved. These maps indicate the optimal choice among centralized, decentralized, and hybrid DNs that retailers should associate with each SKU. For instance, Fig. 3 shows the performance maps obtained for an example company with the input characteristics in Table 3. By fixing a point in the graph representing a specific DN (given by a combination of $\frac{b}{\bar{D} \cdot c}$, $\frac{\sigma}{\bar{D}}$, h , and $\frac{q}{\bar{D}}$ variables), comparing the colors assumed by the maps at the fixed point as Deg_i changes, it is possible to determine the most cost-effective stock deployment policy (blue colors). The direct comparison of stock deployment policies shows how their cost-effectiveness change as the groups on the x, y, and z axes change, pinpointing their reciprocal interactions.

The performance maps in Fig. 3 offer several contributions. Firstly, they depict the interactions between parameters related to SKUs and customer demand and their impact on the cost-effectiveness of stock deployment policies. This visualization offers valuable insights into the decision-making problem. For instance, considering the example company in Table 3 and assuming a combination of SKU-customer demand represented by the dot in Fig. 3 (where dashed lines show the dot's projections on the x, y, and z axes and the color reflects the DN total cost according to the color legend), the performance maps convey the following knowledge. As the degree of centralization (Deg_i) increases – i.e., moving to the right in Fig. 3 –, the total cost of the stock deployment policy (C_{TOTi}) rises (i.e., the

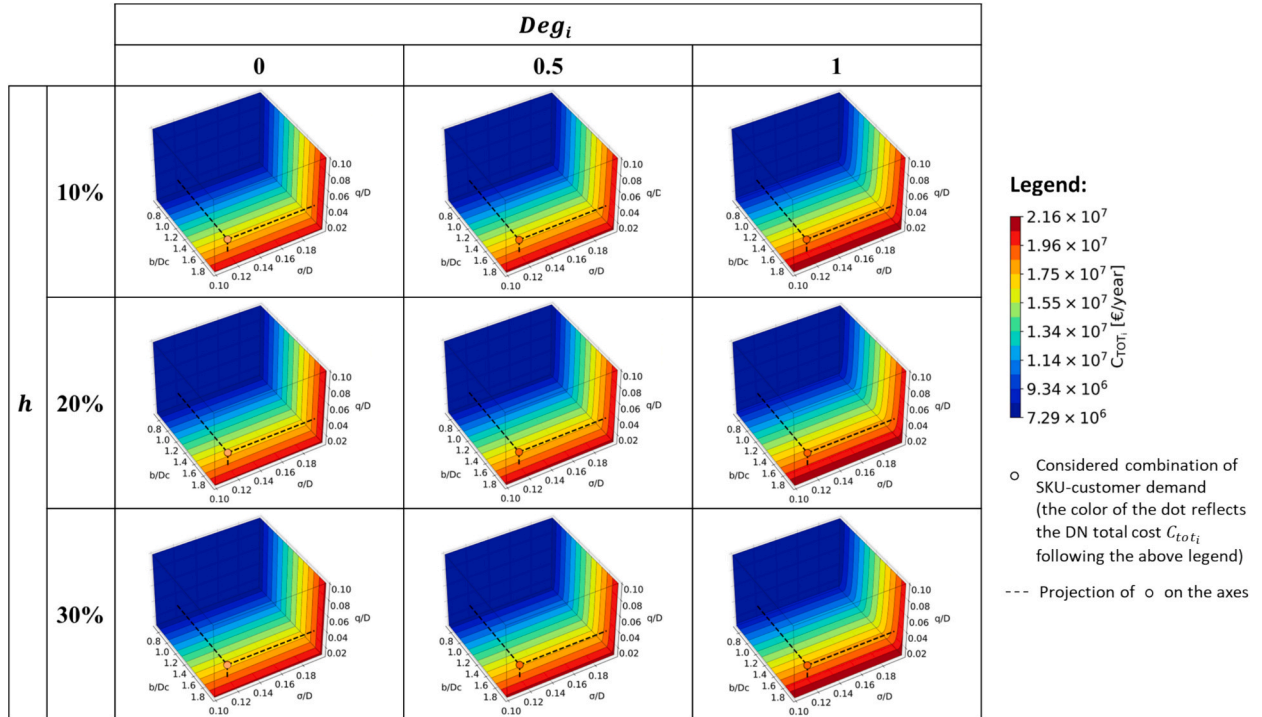


Fig. 3. Performance maps for the company described in Table 3. For each combination of SKU-customer demand (i.e., fixing specific values of b , c , $\bar{D}$, σ , and q , which determine the position of the dot), the most cost-effective stock deployment can be identified by comparing the maps' colors at the dot as Deg_i (columns) and h (rows) vary, according to the color legend. The x, y, and z axes of the performance maps reflect the variables' reciprocal interactions when impacting cost-effectiveness.

Table 3
Characteristics of an example company in the first application context.

Known variable	Value	Unit measure
l	0.01	[years]
d_c	100	[km]
t	0.7	[€/km*vehicle]
C	10,000	[units]
o	5	[€/order]
N	200	—
SL	0.95	—

color of the curves shifts from light orange to dark orange, resulting in a cost increase of around 10 %). Accordingly, for the considered combination of SKU and customer demand, decentralization ($Deg_i = 0$) is the optimal (most cost-effective) stock deployment policy. However, for the same dot, C_{TOT_i} slightly increases while increasing the annual holding cost rate (h) (i.e., sliding down in Fig. 3). This is visible comparing the three left graphs of Fig. 3 (with $Deg_i = 0$ and $h = 10\%$, 20%, and 30%, respectively), thus noticing that a dark red-shaded area appears. As the impact of h on C_{TOT_i} is more pronounced in the left-hand graphs, the performance maps reveal that convenience in decentralization decreases as much as h rises. Indeed, centralization (through risk-pooling) enables reducing inventory levels and the consequent holding costs (C_{H_i} , Equation A(4)). All the aforementioned considerations are also shown in Fig. 4, which breaks down the 3D performance maps into two dimensions, showing, as an example, the cross-sections of the top- graphs (with $h = 10\%$) at the fixed dot. A comprehensive analysis of 2D plots for different dots (i.e., not just the one in Fig. 3) is provided in Appendix D.

Additionally, within each performance map in Fig. 3 (e.g., focusing on the bottom-right one, with $Deg_i = 1$ and $h = 30\%$), the total cost of the considered stock deployment policy changes as follows. Along the x-axis, C_{TOT_i} increases as the ratio $\frac{b}{D \cdot c}$ increases (i.e., colors

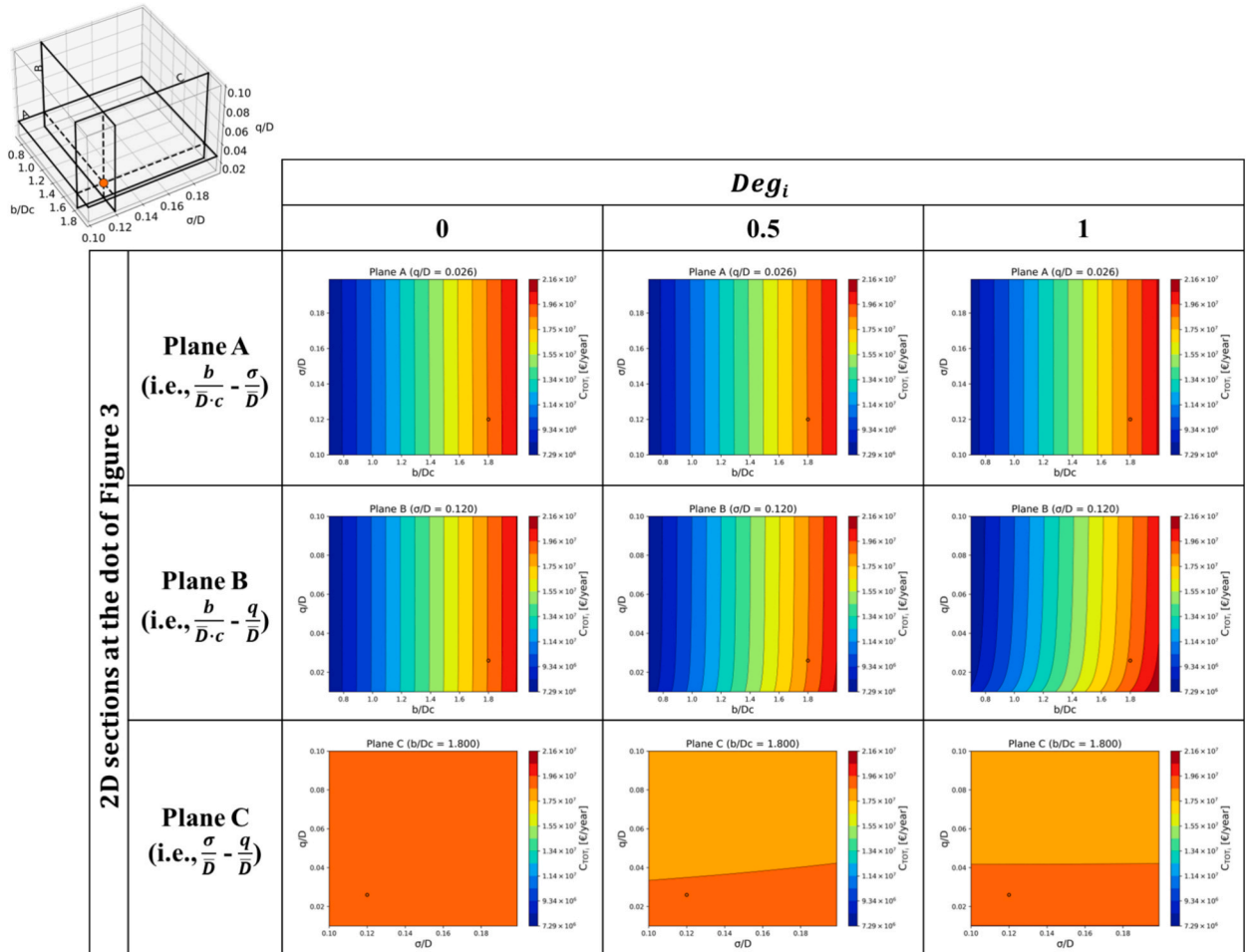


Fig. 4. Two-dimensional breakdown (rows) of the performance map with $h = 10\%$ and variable Deg_i (columns), corresponding to the dot considered in Fig. 3 (shown here in the top-left 3D graph).

shift from blue to red). This increase is higher in centralized DNs because a unitary backorder cost that is higher than the average demand valued in money (i.e., $b > \bar{D} \bullet c$) requires flexible and responsive deliveries to contain backorder costs ($C_{B,i}$, Equation A(6)). However, centralized stock deployment policies (where the central DC has a higher distance from customers compared to decentralized DCs) hinder the DN responsiveness, justifying the aforementioned increase in C_{TOTi} . Along the y-axis, C_{TOTi} seems to remain constant as $\frac{q}{\bar{D}}$ increases, meaning that the variation in customer demand has less impact on the decision-making problem than the other parameters of Fig. 3. Finally, along the z-axis, C_{TOTi} slightly increases as the value of $\frac{q}{\bar{D}}$ increases (i.e., the curves are not perpendicular, but slightly twisted). This occurs because, given a fixed capacity of transportation vehicles (C), if clients emit sporadic orders for large quantities of stocks, this leads to a greater number of deliveries, resulting in an increase in transport costs ($C_{T,i}$, Equation A(7)). Again, the increase is much greater the more centralized the stock deployment policy is because of the greater distance between DCs and customers.

As the second (and most important) contribution, the performance maps in Fig. 3 constitute user-friendly tools for inventory classification. By leveraging similarity and scaling considerations, all SKUs characterized by identical values for the scaling parameters in Equation (18) can be linked to the same optimal stock deployment policy. This approach reduces the evaluation efforts for retailers managing thousands of SKUs, comparing the cost performance of different stock deployment policies into a graphical decision support tool, which can be consulted for each SKU and any time its demand fluctuates. For example, consider the two SKUs in Table 4. Despite showing different characteristics in terms of demand ($\bar{D}$, σ , and q) and costs (c , b , and h), they share identical values for $\frac{b}{\bar{D} \bullet c}$, h , $\frac{q}{\bar{D}}$, and $\frac{q}{\bar{D}}$. Therefore, they both correspond to the orange dot in the top-left graph in Fig. 3, being associated with the same optimal stock deployment policy (decentralization).

Similarly, if a retailer adds a new SKU to its portfolio (e.g., by expanding into a new market) or if the demand for an existing SKU changes over time, the optimal stock deployment policy can be identified by referencing a similar SKU (already managed by the retailer), easily reviewing the DN configuration.

4.2. Second application: Performance maps for supplier and transportation selection

Let us consider an application – similar to that described by PrasannaVenkatesan and Goh (2016) – in which a retailer, among all SKUs managed, serves one of them to a well-consolidated market (i.e., N , $\bar{D}$, σ , q , b , c , and h fixed). For the considered SKU, the retailer follows an established procedure for ordering replenishments from suppliers (o fixed). In this context, the BT allows for creating performance maps that help to evaluate the optimal stock deployment policy (Deg_i) to be associated with the considered SKU as the dimensionless groups vary, with all influential variables fixed except SL , l , d_c , t and C . The latter variables may take arbitrary values since the retailer can make decisions on the provided service level, the supplier who procures SKUs (upstream of DCs), and the vehicle used to deliver stocks (downstream of DCs). By fixing influential variables that retailers have no power to change, the degrees of freedom in the stock deployment problem are reduced, and Equation (17) translates into Equation (19).

$$C_{TOTi} = \Phi \left(Deg_i, SL, \frac{d_c \bullet t \bullet C}{l} \right) \quad (19)$$

The resulting performance maps are provided in Fig. 5, considering an example company with the characteristics in Table 5. The performance maps in this application context pinpoint how the current choices of service level, supplier, and transportation mode impact the DN cost-effectiveness.

The performance maps in Fig. 5 provide the following contributions. First, for the considered SKU, they show the interplay between parameters characterizing service level, supplier, and transportation mode, and their impact on the total cost of different stock deployment policies. Therefore, these maps provide the retailer with a visual guide on how to adjust supply and distribution decisions to minimize C_{TOTi} . For example, considering a combination of service level, supplier, and transportation mode represented by the dot in Fig. 5, the retailer can make the following observations. As the degree of centralization (Deg_i) increases – moving to the right in Fig. 5 –, C_{TOTi} decreases (i.e., the color of the curves shifts from water green to blue, resulting in a cost reduction of 0.99 %). This finding are also shown in 2D in Fig. 6, and demonstrate that, for the considered SKU and the specific supplier-distribution combination, centralization ($Deg_i = 1$) is the optimal stock deployment policy although the cost-effectiveness is minimal given the small percentage decrease in costs.

Additionally, within each performance map in Fig. 5 (e.g., focusing on the right one, which has $Deg_i = 1$), the total cost of the considered stock deployment policy changes as follows. Along the x-axis, C_{TOTi} increases as the service level (SL) decreases, as

Table 4
Characteristics of two SKUs.

Influential variable [unit measure]	SKU 1	SKU 2
b [€/backorder]	1,790	4,475
$\bar{D}$ [units/time]	1,000	5,000
σ [units/time]	120	600
c [€/unit]	1.00	0.50
h [time ⁻¹]	0.10	0.10
q [units/demand]	25	125

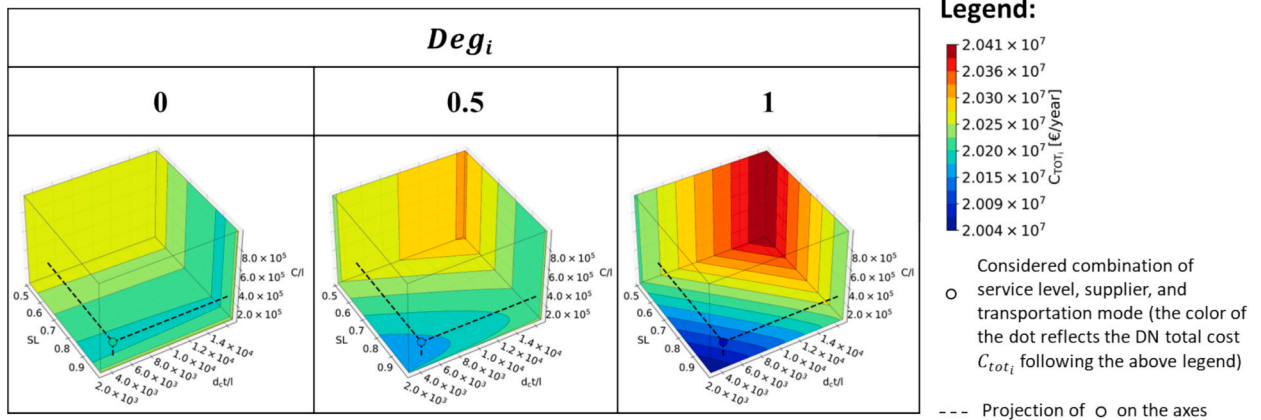


Fig. 5. Performance maps for the company described in Table 5. For each combination of service level, supplier, and transportation mode (i.e., fixing specific values of SL, C, l, t, and d_c , which determine the position of the dot), the most cost-effective stock deployment can be identified by comparing the maps' colors at the dot as Deg_i (columns) varies, according to the color legend. The cost-effectiveness changes as the groups on the x, y, and z axes change, pinpointing their reciprocal interactions.

Table 5
Characteristics of an example company in the second application context.

Known variable	Value	Unit measure
N	100	—
$\bar{D}$	1,000	units/time
σ	150	units/time
q	100	units/demand
b	4	€/backorder
c	200	€/unit
h	0.20	time ⁻¹
o	5	€/order

backorder costs impact the stock deployment performance (C_{Bi} , Equation A(6)). Along the y-axis, C_{TOTi} rises when $\frac{d_c \cdot t}{l}$ increases (namely, when the procurement lead time is less than the product between the unitary transportation cost and the distance from the central DC to customers), and this is particularly visible when $Deg_i = 1$ (right graph). In contrast, along the z-axis, C_{TOTi} remains constant as the value of $\frac{c}{l}$ increases. This suggests that the ratio between the transportation vehicle capacity and the procurement lead time has a lesser impact on C_{TOTi} compared to the other dimensionless groups. Given that the performance maps in Fig. 5 are strongly three-dimensional, dimensionless groups on the x and y axes (SL and $\frac{d_c \cdot t}{l}$) not only show a marked effect on C_{TOTi} but also a strong mutual interaction when influencing the cost-effectiveness of stock deployment policies.

Another contribution of the performance maps is their role as intuitive visual tools that enable the retailer to make similarity and scaling considerations. By grouping all suppliers and transportation modes with identical dimensionless group values from Equation (19), the retailer can assign them the same optimal stock deployment policy. This approach translates the results of the mathematical model from Section 3.2 into a graphical tool, allowing the retailer to review the stock deployment policies whenever vendor ratings or transportation decisions are updated.

4.3. Third application: Performance maps for following demand fluctuations

Finally, let us consider an application – similar to that described by Mohammed et al. (2023) – in which management efforts are focused on a specific SKU (b , c , and h fixed). Let the location of DCs, the upstream supplier, and the downstream distribution vehicles be known (i.e., l , d_c , t and C fixed). Finally, let the management system for issuing replenishment orders be known (o fixed). Suppose a retailer is interested in evaluating how changes in market demand and customer requirements impact the cost-effectiveness of a stock deployment policy. Under these conditions, the stock deployment problem can be reformulated by translating Equation (17) into Equation (20).

$$C_{TOTi} = \Phi\left(N, Deg_i, SL, \frac{\sigma}{\bar{D}}, \frac{q}{\bar{D}}\right) \quad (20)$$

The consequent performance maps achieved through the BT are depicted in Fig. 7, considering an example company with the characteristics in Table 6.

The first contribution of the performance maps in Fig. 7 is their ability to illustrate the interplay between customer market

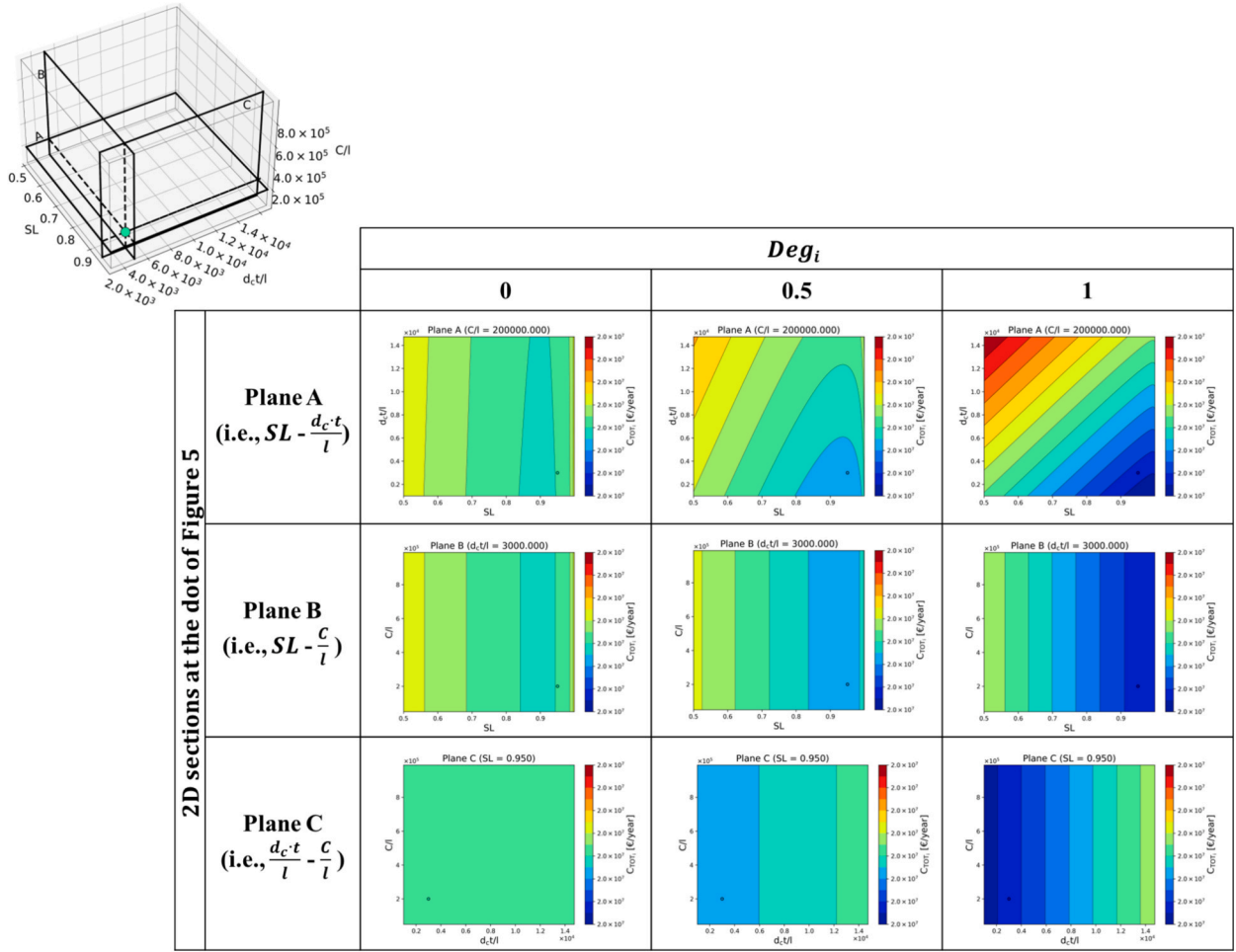


Fig. 6. Two-dimensional breakdown (rows) of the performance map with variable Deg_i (columns), corresponding to the dot considered in Fig. 5 (shown here in the top-left 3D graph).

parameters and their impact on the total cost of different stock deployment policies for the considered SKU (Equation (20)). Through these visual maps, retailers can adjust stock deployment choices in response to market changes by comparing the impact of different stock deployment decisions on C_{TOTi} . These insights allow retailers to identify good management strategies while deepening the understanding of the decision-making process. For example, assuming a combination of service level, supplier, and transportation mode represented by the dot in Fig. 7, retailers can observe the following. While maintaining the service level (e.g., fixing $SL = 90\%$), increasing the degree of centralization (Deg_i) – moving to the right in Fig. 7 – leads to a rise in C_{TOTi} (indicated by the color shift from blue to light blue, corresponding to a cost reduction from $1.07 \cdot 10^5 \frac{\text{€}}{\text{year}}$ to $5.85 \cdot 10^3 \frac{\text{€}}{\text{year}}$). Specifically, at the dot, while fixing $SL = 90\%$ both the decentralized ($Deg_i = 0$, left graph) and hybrid DNs ($Deg_i = 0.5$, central graph) are shaded blue. Conversely, centralization ($Deg_i = 1$, right graph) is colored light blue. This finding demonstrates that, for the considered SKU and supplier-distribution combination, both decentralization and the hybrid policy are cost-effective compared with centralization. The performance maps further reveal no significant difference between decentralized and hybrid policies under the cost items composing C_{TOTi} (see Equation A(2)). This means that, in this case, the retailer can opt for a hybrid policy to reduce the number of DCs and their management efforts, or decentralization to minimize delivery times due to shorter distances between DCs and customers. All the aforementioned considerations are also shown in Fig. 8, which breaks down the 3D performance maps into two dimensions, showing, as an example, the cross-sections of the top-right graph (with $SL = 90\%$ and $deg_i = 0$) at the fixed dot.

As SL increases (i.e., sliding down in Fig. 7), the bending of the colored areas becomes more pronounced in decentralized DNs (left-hand graphs). This reveals that decentralization becomes advantageous for more combinations of N , $\frac{a}{D}$, and $\frac{q}{D}$ as SL rises. The reason is that, when customers expect a high service level, decentralization reduces transport costs (C_{Ti}) and enhances DN responsiveness while ensuring the same number of backorders as other stock deployment policies. For instance, fixing $SL = 99\%$, the colored curves at the dot in the case of the hybrid DN are not blue anymore (rather shift to light blue) while decentralization remains blue. Additionally, as SL rises, the colors in centralized DNs (right-hand graphs) change from red to orange. Although this change is not very marked, the performance maps reveal that while increasing SL , backorder costs (C_{Bi}) decrease, thus lowering C_{TOTi} especially in centralized DNs.

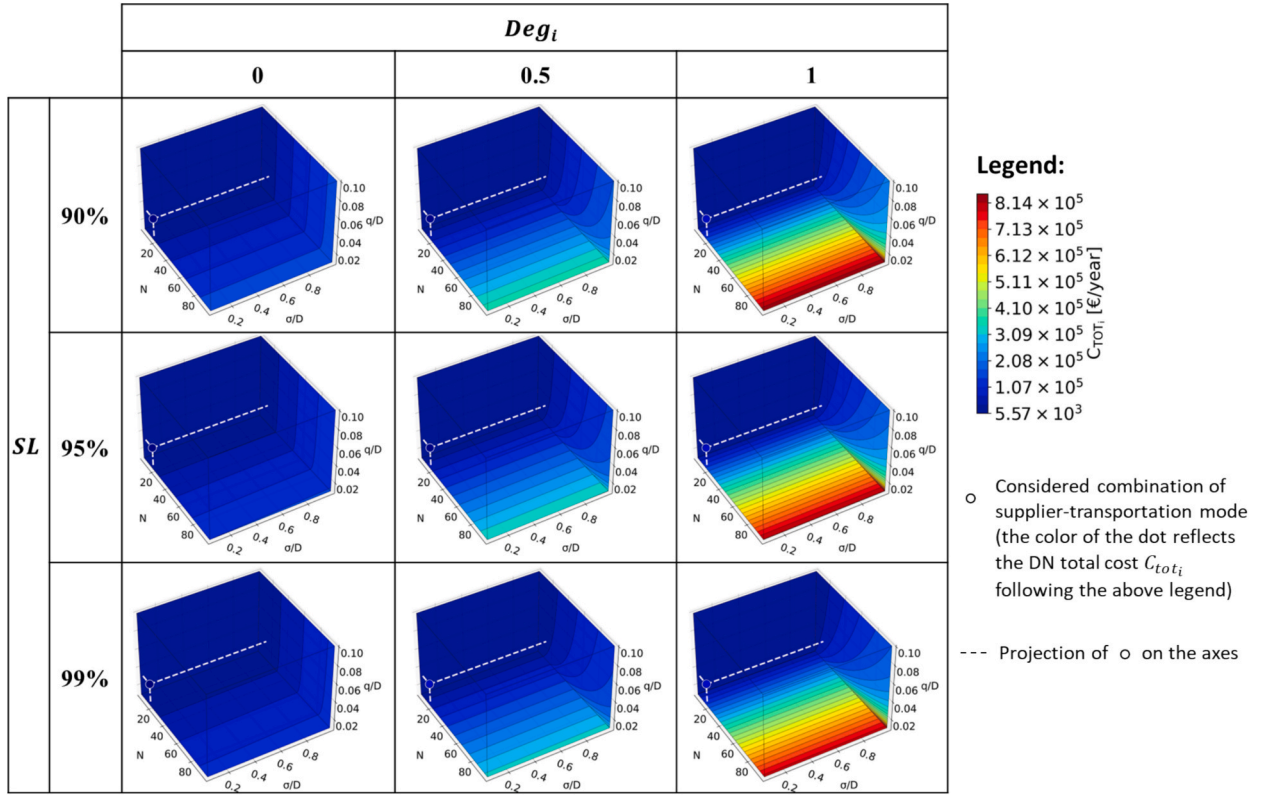


Fig. 7. Performance maps for the company described in Table 6. For each combination of supplier, and transportation mode (i.e., fixing specific values of N , $\bar{D}$, σ , and q , which determine the position of the dot), the most cost-effective stock deployment can be identified by comparing the maps' colors at the dot as Deg_i (columns) and SL (rows) vary, according to the color legend. The cost-effectiveness changes as the groups on the x, y, and z axes change, pinpointing their reciprocal interactions.

Table 6
Characteristics of an example company in the third application context.

Known variable	Value	Unit measure
b	5	€/backorder
c	1	€/unit
h	0.20	time ⁻¹
l	0.01	[years]
d_c	100	[km]
t	0.7	[€/km*vehicle]
C	10,000	[units]
o	5	[€/order]

Moreover, while decentralization remains more cost-effective than other policies, it requires higher inventory levels to meet demand and avoid backorders. This inventory increase is more significant in decentralized DNs because they do not benefit from risk-pooling. Consequently, as SL increases, the differences in holding costs (C_{Hi}) across stock deployment policies diminish, leading to smaller differences in C_{TOTi} as i changes. Therefore, for higher SL values, an incorrect stock deployment decision by retailers has less severe economic consequences compared to when SL is lower.

Additionally, within each performance map in Fig. 7 (e.g., focusing on the top-left one with $Deg_i = 0$ and $SL = 90\%$), the total cost of the selected stock deployment policy changes as follows. Along the x-axis, C_{TOTi} increases as the number of customers (N) grows. This is because the overall demand to be met increases, with consequences on inventory levels, number of transports, etc. Along the y-axis, C_{TOTi} rises when the value of $\frac{q}{D}$ increases, and this is particularly visible when $Deg_i = 0$ (left-hand graphs). This is because decentralized DNs do not benefit from risk pooling and are more sensitive to demand fluctuations. Conversely, along the z-axis, C_{TOTi} decreases as the value of $\frac{q}{D}$ increases, and this is particularly evident when $Deg_i = 1$ (right graphs). This occurs because a lower value of $\frac{q}{D}$ implies fewer transports (N_{ti} , Equation A13), resulting in a lower transportation cost (C_{Ti}), which heavily impacts centralized stock deployment policies due to the distance between the central DC and customers.

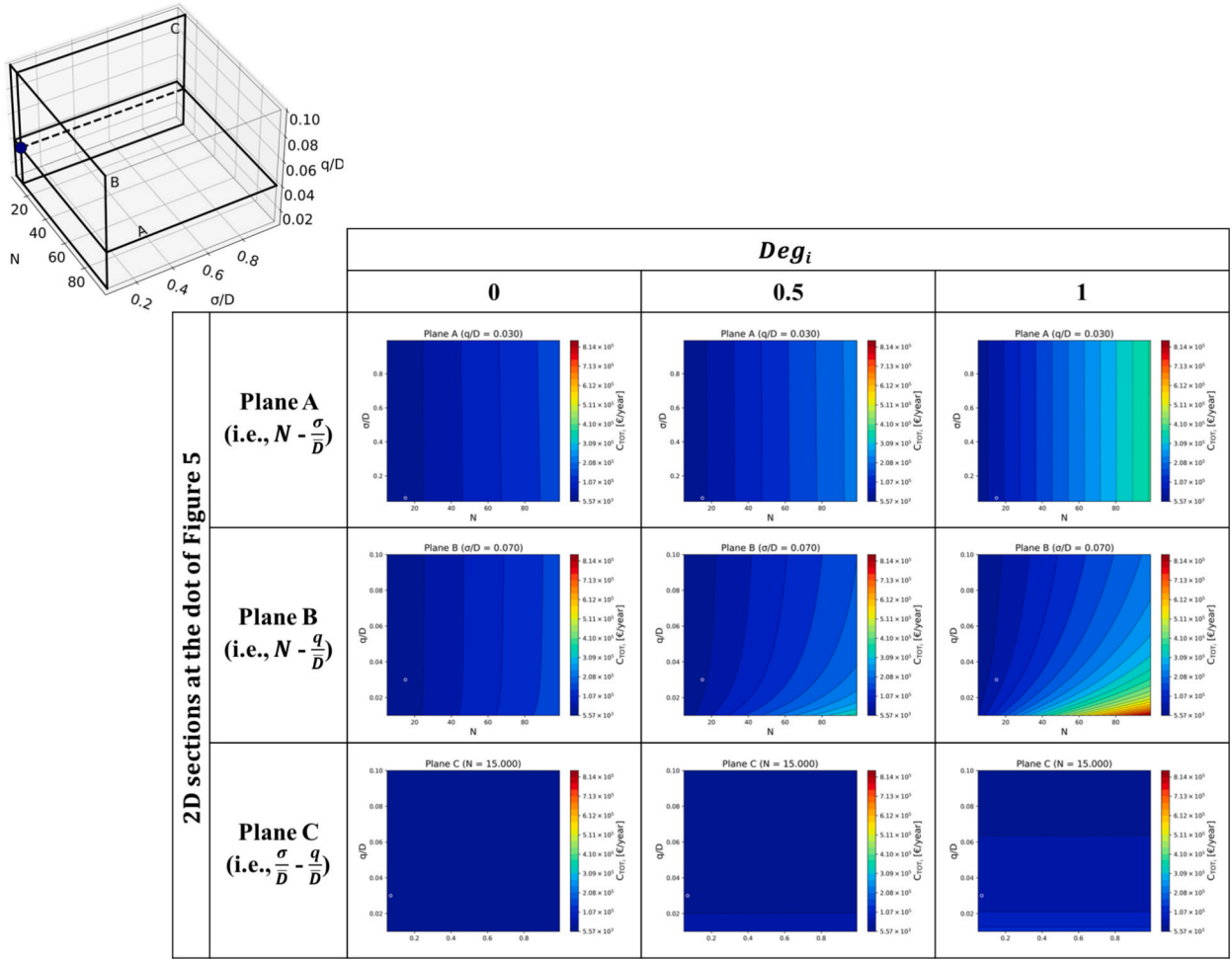


Fig. 8. Two-dimensional breakdown (rows) of the performance map with $SL = 90\%$ and variable Deg_i (columns), corresponding to the dot considered in Fig. 7 (shown here in the top-left 3D graph).

Given the three-dimensionality of the performance maps, Fig. 5 demonstrates that the groups on the x , y , and z axes not only have a marked effect on C_{TOT_i} but also strong mutual interactions when influencing the cost-effectiveness of stock deployment policies. Finally, another key contribution is the role of performance maps as intuitive visual tools for retailers to make similarity and scaling considerations. Retailers can group customer markets that share identical values of dimensionless groups (i.e., scaling parameters in Equation (20) and assign them the same optimal stock deployment policies.

5. Case study

The applicability of the BT was tested in a real company. Company A is an Italian retailer, which owns 5 DCs. Among the various

Table 7
Characteristics of Company A.

Known variable	Value	Unit measure
N	200	—
$\bar{D}$	1,000	units/year
σ	300	units/year
q	200	units/demand
b	5	€/backorder
c	1	€/unit
SL	0.95	—
h	0.20	year ⁻¹
o	5	€/order
d_c	100	km

SKUs managed by Company A, one particular is distributed to a well-consolidated market with the characteristics outlined in Table 7 (i.e., N , $\bar{D}$, σ , q , b , c , SL , and h fixed). For this SKU, Company A follows a standardized procedure for emitting stock replenishment orders from a supplier (o fixed). The current stock deployment policy for this SKU is centralization ($Deg_i = 1$), with inventory stored in one of the five DCs (d_c fixed).

Company A seeks to understand how the total cost of its current DN (C_{TOT_i}) is influenced by the choice of the supplier (upstream DCs) and delivery transportation mode (downstream DCs). Additionally, Company A wants to determine whether the current stock deployment policy (centralization) is optimal or should be replaced with a decentralized or hybrid one to reduce C_{TOT_i} . Finally, Company A wishes to compare the current combination of supplier and transportation mode with a new alternative outlined in Table 8, aiming to evaluate if the new distribution option could be more cost-effective and whether adjustments in the supplier and transportation mode would necessitate changes in stock deployment.

In this scenario, the BT helps obtain the performance maps shown in Fig. 9, identifying the most cost-effective stock deployment policy (Deg_i) for the given SKU as the characteristics of the supplier (I) and transportation mode (SL , d_c , t and C) vary, while fixing all other variables (Table 7). This approach leads to Equation (19) and the application context of Section 4.2.

Concerning how C_{TOT_i} is affected by the choice of supplier and transportation mode, the performance maps in Fig. 9 uncovered similar findings to those already discussed in Section 4.2 (c.f. description of performance trends as the x , y , z axes vary). Although Fig. 9 shares similarities with Fig. 5, there are evident differences, especially in the performance maps for hybrid and decentralized DNs (central and left graphs). In Fig. 9, blue, straight lines replace the green and azure, curved ones of Fig. 5. These differences reveal that despite considering the same application context (Section 4.2), companies can be advised to adopt different stock deployment policies based on their input characteristics (Table 5 and Table 8, respectively). For instance, unlike the example company in Table 5, Company A serves twice the number of customers ($N = 200$ instead of 100) but meets a similar average demand ($\bar{D} = 1000 \frac{\text{units}}{\text{year}}$) with greater variability ($\sigma = 300 \frac{\text{units}}{\text{year}}$ vs. $150 \frac{\text{units}}{\text{year}}$). Additionally, Company A faces a slightly higher unit backorder cost ($b = 5 \frac{\text{€}}{\text{backorder}}$ vs. $4 \frac{\text{€}}{\text{backorder}}$), but has a much lower purchase cost ($c = 1 \frac{\text{€}}{\text{unit}}$ vs. $100 \frac{\text{€}}{\text{unit}}$). As a result, Company A's performance maps (Fig. 9) show C_{TOT_i} values that are two orders of magnitude lower than those in Fig. 5 (with a maximum of $9.0 \cdot 10^5 \frac{\text{€}}{\text{year}}$ compared to $2.04 \cdot 10^7 \frac{\text{€}}{\text{year}}$) and exhibit a less curvilinear pattern in the colored curves.

Concerning the optimal stock deployment policy for the considered SKU and the current supplier-transportation mode (see the dot in Fig. 9), through the performance maps Company A understood that centralization ($Deg_i = 1$) was not cost-effective. Rather, a shift to decentralization ($Deg_i = 0$) was recommended, keeping stocks in all DCs. Indeed, the projections of the dot along the x , y , and z -axes fall within an azure-shaded area in decentralized DNs maps, while being water green and green for hybrid and centralized DNs, respectively. Consulting the legend in Fig. 9, this color difference indicates that reducing Deg_i results in a cost savings of 47.8 % when moving from centralization to decentralization.

Finally, when evaluating the optimal supplier-transportation mode combination, Company A established that, based on Table 8, the current and new combinations of supplier-transportation mode show identical values of $of_{T_i}^{\text{€}} = 7'000$ and $c_i^{\text{€}} = 500'000$. This means they are both represented by the same dot in Fig. 9. Under similarity conditions, both combinations are associated with the same optimal stock deployment policy (decentralization), resulting in the same economic impact. Given no difference in C_{TOT_i} , Company A decided to retain the current supplier and transportation mode.

6. Conclusions

This paper introduces the first application of the BT to assist retailers in identifying the most cost-effective stock deployment policy (i.e., centralized, decentralized, or hybrid DN configurations). After mathematically modeling the stock deployment problem, the BT checks the meaningfulness of its governing equations, pinpoints influential variables, and gains knowledge on how they interact when impacting the costs of stock deployment policies. For the first time, the BT enables the creation of performance maps that graphically compare the performance of different stock deployment policies, suggesting similar decisions for scaled DNs, thus facilitating retailers' informed decisions about the most cost-effective option to select.

This paper proposed three applications of the BT in stock deployment decisions, each supported by performance maps to assist retailers in their decision-making processes. In the first application, by fixing the stocks' supplier and the transportation mode, the BT provides performance maps to associate different SKUs with the optimal stock deployment policy, serving as a visual tool for inventory classification. In the second application, considering a specific SKU, the BT provides performance maps to guide decisions on the optimal supplier and transportation mode for delivering stocks. Finally, in the third application, by fixing a specific SKU, its supplier, and transportation mode, the BT provides performance maps to optimize stock deployment decisions and DN resilience in response to customer demand changes (driven by emerging trends, geopolitical shifts, or catastrophic events).

Table 8

Characteristics of the current and new combination of supplier and transportation mode.

Influential variable [unit measure]	Current combination supplier-transport mode	New combination supplier-transport mode
$t[\text{€}/\text{km} \cdot \text{vehicle}]$	0.70	1.05
$C[\text{units}]$	5,000	7,500
$I[\text{years}]$	0.010	0.015

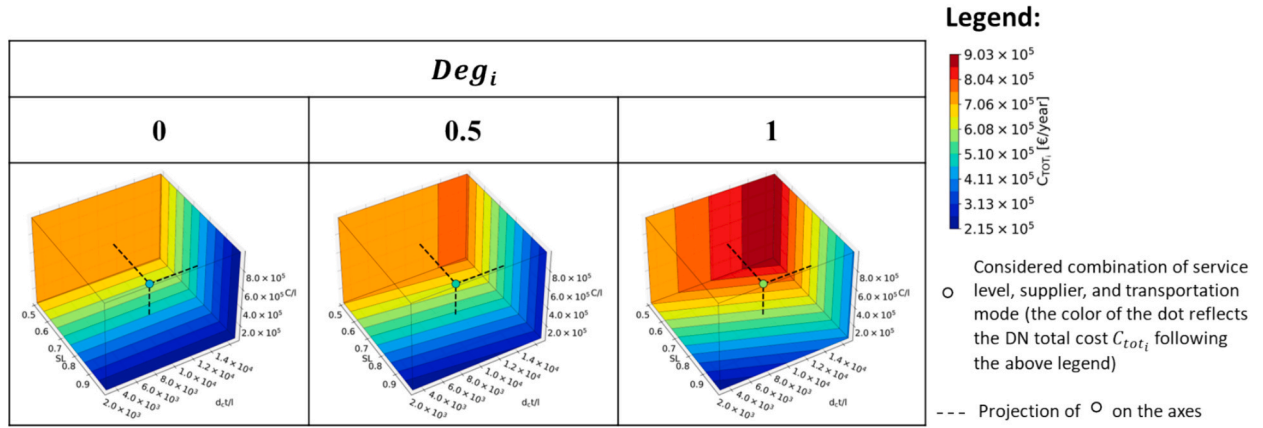


Fig. 9. Performance maps derived for Company A. The dot represents both supplier and transportation mode combinations in Table 8 (where SL, C, l, t, and d_c are fixed). By analyzing color variations at the dot across different Deg_i values (columns), the most cost-effective stock deployment policy is identified (blue colors indicate lower C_{TOTi}). Additionally, changes along the groups on the x, y, and z axes highlight the mutual interactions among these factors, influencing overall cost-effectiveness. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The applicability of the BT was tested on the case study of Company A, focusing on the second application context. The derived performance maps allowed Company A to assess: (i) the impact of different variables on the total DN cost (C_{TOTi}); (ii) the cost-effectiveness of modifying the current stock deployment policy; and (iii) the possibility of changing the current supplier and transportation mode, thus reviewing the supply chain actors. Moving from the current stock deployment policy (centralization) to the optimal one recommended by BT (decentralization) resulted in a 47.8 % cost reduction. Moreover, Company A was encouraged to maintain the current supplier and transportation mode, as similitude considerations indicated that the new alternatives were not more cost-effective.

Concerning the main contributions of this study, at a theoretical level, this paper is the first to propose the application of BT (and, more broadly, DA) in the context of stock deployment decisions, while also being one of the few studies to apply BT within the field of supply chain and operations management. As such, it lays the groundwork for exploring the potential of a well-established tool (commonly used in other fields such as turbomachinery) in a new and underexplored context. Moreover, the performance maps presented in this study are the first visual tools that enable a direct comparison of the cost-effectiveness of centralized, decentralized, and hybrid DNs.

At a practical level, the provided performance maps serve as a graphical tool to support retailers in stock deployment decisions. They provide insights into how various suppliers, transportation modes, SKUs, and customer demand influence DN costs. Moreover, they help reduce DN costs by identifying cost-effective stock deployment policies. Finally, they enable revisions of stock deployment decisions over time, thus ensuring DN resilience without requiring significant time or computational efforts (simple graphical tools must be consulted). This contribution is particularly beneficial for companies that operate in turbulent environments, characterized by variable demand or unstable geopolitical situations.

The main limitations of this study are six. Firstly, the BT can only be applied under specific conditions, including the assumption of independent influential variables, the requirement for dimensional homogeneity, and the capability of describing the investigated phenomenon through physical (quantifiable) variables, thereby neglecting any non-physical ones. The latter assumption may be considered quite strong, as real-world stock deployment decisions may also be influenced by behavioral and policy-related (non-physical) factors like trust between DN partners, collaboration intensity, or the probability of disruption risks. Since such variables cannot be captured through dimensional analysis alone, future research could address this limitation by exploring hybrid modeling approaches that combine BT-based performance maps with insights derived from qualitative methods (e.g., expert judgment, Delphi studies), behavioral models, or simulation techniques. For example, one possible integration could involve using BT to navigate the analysis of the physical aspects of a DN, while complementing the decision-making process with behavioral weights or policy constraints derived from surveys or agent-based modeling. This multi-layered approach would enable more comprehensive decision support by accounting for both the quantitative and qualitative dimensions of stock deployment decisions.

As the second limitation, the BT involves time-consuming calculations to identify the dimensionless groups, which may be prone to error. While acknowledging that no future research developments could solve this limitation, we do not believe that it undermines the validity and usefulness of this study.

As the third limitation, the application of the BT proposed in this paper relies on the mathematical model proposed by Cantini et al. (2024b), which is valid under specific assumptions. For example, it adopts an (RP, Q) inventory control policy, assumes that SKUs follow a normal demand distribution, etc. To address this limitation, future research could reapply the BT while relaxing some of these assumptions. For instance, studies could consider SKUs with different demand distributions (e.g., Poisson), explore alternative inventory control policies, and enhance the mathematical model presented in Appendix A by incorporating additional cost items (e.g., facility rental costs, lead time delay penalties, etc.). Alternatively, the BT could be applied to other mathematical models available in the literature that address stock deployment decisions, as exemplified in Appendix B. Notably, the BT is highly adaptable to different initial formulations, as the specific mathematical equations may vary, but the three-step method for implementing the BT (Section 3.3) remains valid.

As the fourth limitation, this study derives performance maps by targeting exclusively economic goals (Equation A(1)). Future research could incorporate other sustainability goals, making DNs resilient not only in economic terms but also in environmental and social ones.

As the fifth limitation, only three applications of the BT have been explored in Section 4, all of which focus exclusively on stock deployment decisions. However, future research could investigate a much broader range of applications where performance maps may prove beneficial. For instance, remaining in the stock deployment domain, BT's scaling capabilities could be leveraged to translate full-scale, real-world supply chains into small-scale physical models, enabling universities to set up laboratory environments and test benches for experimental and educational purposes. Beyond stock deployment, future studies could explore the use of BT in other areas of supply chain and operations management, leveraging performance maps to support a wider variety of decisions. Potential applications include: facility location and DN configuration (where BT could help derive generalizable rules for optimal facility placement under varying cost, demand, and geographic constraints); inventory control and warehouse stock management (where BT could aid in optimizing stock positioning on the shelves or storage and retrieval procedures); technology and investment decisions (where BT could support the development of performance maps to compare alternatives such as production machines, warehouse systems, or other assets in terms of economic, environment, social, or operational performance); etc. Future studies may develop new mathematical models tailored to these contexts and apply BT to derive novel dimensionless groups and corresponding performance maps. These maps could further democratize access to strategic insights and assist decision-makers in rapidly adapting supply chains to dynamic and uncertain environments.

Finally, while the applicability of the proposed method was tested on a real case study, to enhance the generalizability of the findings and strengthen the validity of BT-based performance maps, future research could replicate the application of BT across other companies and industries. In particular, DNs operating in different sectors could be explored, such as DNs of spare parts, perishable goods, fashion and luxury items, etc. Comparing BT outcomes across these diverse contexts would help assess the robustness of the method and confirm whether it offers value beyond a niche application.

CRedit authorship contribution statement

Alessandra Cantini: Writing – original draft, Visualization, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Leonardo Leoni:** Writing – review & editing, Validation, Supervision, Methodology, Formal analysis. **Saverio Ferraro:** Writing – review & editing, Validation, Supervision, Formal analysis, Data curation. **Filippo De Carlo:** Writing – review & editing, Visualization, Validation, Supervision, Project administration, Methodology, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

This section offers a concise overview of the mathematical model recently proposed by Cantini et al. (2024b) to address stock deployment decisions. A brief description is provided here, as the model serves to illustrate the application of BT in this paper. For a more detailed explanation of the model's Equations and their interpretations, readers are referred to Cantini et al. (2024b).

In the study by Cantini et al. (2024b), stock deployment is formulated as an optimization problem based on Equations A1-A2, where Table 1 in Section 3.2 lists the notation used. The mathematical model follows a single-product approach, which, according to Cohen et al. (2006), means that the optimal choice among centralized, decentralized, and hybrid policies is made for each individual SKU by considering its specific characteristics and balancing its requirements for network responsiveness, high service level, and minimization of costs. In other words, a specific SKU is analyzed at a time, solving the following Equations. Equation A(1) is the objective function, which aims to identify the minimum total cost (C_{TOT_i}) among those associated with the three stock deployment policies (i) in Fig. 2. Once the minimum C_{TOT_i} is identified, the model can return the corresponding value of i , thus recommending the most cost-effective stock deployment policy for the considered SKU. Equation A(2) clarifies that C_{TOT_i} is the sum of several cost items, comprising the cost for purchasing stocks from suppliers (C_{P_i} , Equation A(3)), the inventory holding cost (C_{H_i} , Equation A(4)), the ordering cost for replenishing DCs (C_{O_i} , Equation A(5)), the backorder cost (C_{B_i} , Equation A(6)), and the transportation cost for delivering products to customers (C_{T_i} , Equation A(7)).

$$\min [C_{TOT_i}] \text{ for } i = 1, 2, 3 \quad (\text{A1})$$

$$C_{TOT_i} = C_{P_i} + C_{H_i} + C_{O_i} + C_{B_i} + C_{T_i} \quad (\text{A2})$$

Where:

$$C_{P_i} = c \bullet D_i \bullet DC_i \quad (\text{A3})$$

$$C_{H_i} = h \bullet c \bullet \left(\frac{Q_i}{2} + SS_i \right) \bullet DC_i \quad (A4)$$

$$C_{O_i} = o \bullet N_{o_i} \bullet DC_i \quad (A5)$$

$$C_{B_i} = b \bullet N_{b_i} \bullet DC_i \quad (A6)$$

$$C_{T_i} = t \bullet N_v \bullet N_{t_i} \bullet d_i \bullet DC_i \quad (A7)$$

The cost items in Equations A3-A7 can be computed by applying Equations A8-A17, which are valid under these assumptions: (i) a two-echelon DN is considered. No generality is lost by considering a two-echelon DN since any multi-echelon DN can be decomposed into a series of interconnected two-echelon DNs, where the supplier of one echelon is the customer of the preceding one (Ding and Kaminsky, 2020; Rizqi and Chou, 2024); (ii) suppliers have a deterministic procurement lead time, which depends only on the SKU (not on the geographical location of DCs) (Nasr et al., 2021); (iii) no capacity constraints are considered for suppliers' warehouses and retailer's DCs (Tapia-Ubeda et al., 2020); (iv) customer demand is stochastic and normally distributed (Suryawanshi and Dutta, 2021). It is worth mentioning that Cantini et al. (2024b) considered a Poisson demand since they focused on spare parts DNs. Instead, this study investigates a normal demand to make the mathematical formulation agnostic for the industrial sectors and not only focused on spare parts DNs. However, the results of this study can be extended to other demand distributions; (v) given a fixed stock deployment policy, the average annual demand of all customers in the DN is evenly distributed among the existing DCs (Cantini et al., 2025; Sherbrooke, 1968); (vi) a continuous (RP, Q) inventory policy is selected to control inventories (Ivanov, 2021), where RP is the reorder point and Q is the optimal order quantity; (vii) no lateral transshipments are admitted (Firoozi et al., 2013); (viii) the type of vehicle used to transport products is the same in all stock deployment alternatives. It has a capacity constraint on the maximum number of products transportable in each trip. Moreover, it has a unit transportation cost that is constant per kilometer (Ivanov, 2021); (ix) the period considered to develop the analysis is one year (Daskin et al., 2002); (x) a single-product approach is adopted, which allows choosing for each SKU the optimal stock deployment policy (Stoll et al., 2015); (xi) no fixed costs for purchasing or renting DCs are considered since retailers already own DCs and they only want to choose in which of them to stock SKUs.

$$D_i = \frac{\bar{D} \bullet N}{DC_i} \quad (A8)$$

$$DC_i = \begin{cases} 1, & \text{if } i = 3 \\ [(1 - Deg_i) \bullet N]^+, & \text{else} \end{cases} \quad (A9)$$

$$N_{o_i} = \frac{D_i}{Q_i} \quad (A10)$$

$$N_{b_i} = [(1 - SL) \bullet D_i]^+ \quad (A11)$$

$$N_v = \left\lceil \frac{q}{C} \right\rceil^+ \quad (A12)$$

$$N_{t_i} = \frac{D_i}{q} \quad (A13)$$

$$d_i = \begin{cases} d_c, & \text{if } i = 3 \\ (0.7644 \bullet Deg_i^2 + 0.2009 \bullet Deg_i + 0.0161) \bullet d_c, & \text{else} \end{cases} \quad (A14)$$

$$Q_i = \sqrt{\frac{2 \bullet D_i \bullet o}{h \bullet c}} \quad (A15)$$

$$RP_i = D_i \bullet l + SS_i \quad (A16)$$

$$SS_i = Z \bullet \sigma \bullet \sqrt{l \bullet \frac{N}{DC_i}} \quad (A17)$$

Appendix B

To demonstrate the effectiveness of the proposed method when applied to mathematical models beyond that of Cantini et al. (2024b), this appendix presents the application of the BT to the stock deployment problem formulated by Shen et al. (2003). This problem formulation was selected for three reasons: (i) it is a seminal contribution to the literature on stock deployment decisions; (ii) it introduces a more complex mathematical model compared to Cantini et al. (2024b), offering a nonlinear integer programming formulation for optimizing joint location-inventory decisions; and (iii) it relaxes several assumptions made in Cantini et al. (2024b).

For example, it allows for heterogeneous customer demand across the DN and does not require the average annual demand to be evenly distributed among DCs. Given a two-echelon DN with a set I of retailers (each facing independent and uncertain demand), the model by Shen et al. (2003) determines how many DCs to establish, where to locate them, which retailer to assign to each DC, how frequently to reorder at each DC, and the appropriate level of safety stock to meet the demand. The objective is to minimize the total DN cost (C_{TOTij}), including location, inbound and outbound transportation, working inventory, and safety stock costs. For a detailed explanation of the joint location-inventory model and its underlying assumptions, readers are referred to Shen et al. (2003). Here, for the purpose of applying the BT, we only provide a concise summary of the problem formulation and a description of its variables.

The model is formulated as an objective function that minimizes C_{TOTij} (Equations B1-B2) while respecting constraints on the set of retailers that may be selected as DCs (Equations B3-B6). The model relies on the notation listed in Table B1, where the meaning of each variable is explained.

$$\min [C_{TOTij}] \quad (B1)$$

$$C_{TOTij} = \sum_{j \in I} \left\{ f_j X_j + \beta \left(\sum_{i \in I} \mu_i d_{ij} Y_{ij} \right) + \sqrt{2\theta h' (F_j + \beta g_j)} \sqrt{\sum_{i \in I} \mu_i Y_{ij}} + \beta a_j \left(\sum_{i \in I} \mu_i Y_{ij} \right) + \theta h' z_\alpha \sqrt{\sum_{i \in I} \sigma_i^2 Y_{ij}} \right\} \quad (B2)$$

$$\sum_{j=1} Y_{ij} = 1, \text{foreach } i \in I \quad (B3)$$

$$Y_{ij} - X_j \leq 0 \text{foreach } i, j \in I \quad (B4)$$

$$Y_{ij} \in \{0, 1\} \text{foreach } i, j \in I \quad (B5)$$

$$X_j \in \{0, 1\} \text{foreach } j \in I \quad (B6)$$

Table B1

Notation adopted to formulate the joint location-inventory problem.

Indexes	Description	Unit measure
i	Retailer (from a set of I retailers) served by the DCs in the DN, and thus representing a customer.	—
j	Retailer (from a set of I retailers) selected to serve as a DC in the DN.	—
Influential variables (input data)	Description	Unit measure
f_j	Fixed (annual) cost of locating a regional DC at retailer j .	€/time
β	Weight factor associated with the transportation cost.	—
θ	Weight factor associated with the inventory cost.	—
z_α	Service factor associated with the desired service level in a standard normal distribution, such that $P(z \leq z_\alpha) = \alpha$.	—
h'	Inventory holding cost per unit of product per year.	€/units*time
F_j	Fixed cost of placing a replenishment order at DC j	€
μ_i	Mean (yearly) demand at retailer i	units/time
σ_i	Standard deviation of demand during the lead time	units/time
d_{ij}	Cost per unit to ship from retailer j to retailer i	€/unit
g_j	Fixed inbound shipping cost from the upstream supplier to the DC located at j	€
a_j	Per-unit inbound shipping cost from the upstream supplier to the DC located at j	€/unit
Decision variables	Description	Unit measure
X_i	Binary variable, which is 1 if retailer j is selected as DC, and 0 otherwise for each $j \in I$.	—
Y_{ij}	Binary variable, which is 1 if retailer i is served by a DC based at retailer j , and 0 otherwise for each $i, j \in I$.	—
C_{TOTij}	Total (annual) cost of the DN	€/time
C_{fixed}	Annual fixed cost for locating DCs in the DN	€/time
$C_{outbound}$	Annual outbound transportation cost for distributing stocks from DCs (j) to customers (i)	€/time
$C_{workinginv.}$	Annual working inventory cost in DCs	€/time
$C_{inbound}$	Annual inbound transportation cost for receiving stocks from suppliers to DCs	€/time
$Q_{safetystock}$	Total quantity of safety stock in the DN	units

Following the notation by Shen et al. (2003) and grouping the variables according to their meaning, Equation B(2) can be rewritten as in Equation B(7), where the description of each term is reported in Equations B8-B12 and in Table B1.

$$C_{TOTij} = C_{fixed} + \beta C_{outbound} + C_{workinginv.} + \beta C_{inbound} + \theta h' z_\alpha Q_{safetystock} \quad (B7)$$

$$C_{fixed} = \sum_{j \in I} f_j X_j \quad (B8)$$

$$C_{outbound} = \sum_{j \in I} \sum_{i \in I} (\mu_i d_{ij} Y_{ij}) \quad (B9)$$

$$C_{workinginv.} = \sum_{j \in I} \left(\sqrt{2\theta h' (F_j + \beta g_j)} \cdot \sqrt{\sum_{i \in I} \mu_i Y_{ij}} \right) \quad (B10)$$

$$C_{inbound} = \sum_{j \in I} a_j \cdot \sum_{i \in I} \mu_i Y_{ij} \quad (B11)$$

$$Q_{safetystock} = \sum_{j \in I} \sqrt{\sum_{i \in I} \hat{\sigma}_i^2 Y_{ij}} \quad (B12)$$

Under these conditions, the problem appears to be function of $n = 9$ influential variables characterized by $k = 3$ physical dimensions, as shown in Equation B13 and Table B2.

$$C_{TOTij} = F(C_{fixed}, C_{outbound}, C_{workinginv.}, C_{inbound}, Q_{safetystock}, \beta, \theta, h', z_\alpha) \quad (B13)$$

Table B2

Dimensional matrix related to the joint location-inventory problem. [Q], [T], and [\$] mean quantity, time, and money, respectively.

Influential variables	Physical dimensions		
	[Q]	[T]	[\$]
C_{fixed}	0	−1	1
$C_{outbound}$	0	−1	1
$C_{workinginv.}$	0	−1	1
$C_{inbound}$	0	−1	1
$Q_{safetystock}$	1	0	0
β	0	0	0
θ	0	0	0
h'	−1	−1	1
z_α	0	0	0

Applying BT, and selecting a subset of $k = 3$ linearly independent variables (e.g., C_{fixed} , h' , and $Q_{safetystock}$), the problem can be expressed in terms of $n - k = 6$ dimensionless groups (Equations B14–B21).

$$\pi^* = \Phi(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6) \quad (B14)$$

$$\pi_1 = \frac{C_{inbound}}{C_{fixed}} \quad (B15)$$

$$\pi_2 = \frac{C_{outbound}}{C_{fixed}} \quad (B16)$$

$$\pi_3 = \frac{C_{workinginv.}}{C_{fixed}} \quad (B17)$$

$$\pi_4 = \theta \quad (B18)$$

$$\pi_5 = \beta \quad (B19)$$

$$\pi_6 = z_\alpha \quad (B20)$$

$$\pi^* = \frac{C_{TOTij}}{C_{fixed}} \quad (B21)$$

This reformulation enables several key considerations. While the original problem posed by [Shen et al. \(2003\)](#) identifies the most cost-effective DN by specifying the exact DCs to open – represented by the values of X_j and Y_{ij} – based on case-specific input variables (Table B1), the application of the BT offers complementary insights. At the core of the BT application lies Equation B14, which serves as the fundamental expression of the joint location-inventory problem, expressing the total cost of a DN in terms of six dimensionless groups: π_1 , π_2 , and π_3 (representing operational efficiency drivers), π_4 and π_5 (strategic weights characterizing the DN), and π_6 (which captures the desired service level). This decomposition enables a structured approach: DN managers can first make strategic choices by selecting π_4 , π_5 , and π_6 , thereby defining the DN operating regime. Then, they can optimize π_1 , π_2 , and π_3 to reduce the total costs of the DN. Finally, they can benchmark their DN's economic performance against others characterized by different values of the dimensionless groups. To support this benchmarking process, BT enables the construction of performance maps: visual tools that illustrate how each dimensionless group affects DN total cost. For instance, Fig. B1 shows the performance maps for an example company whose characteristics align with the case study analyzed by [Shen et al. \(2003\)](#), namely: with the variables listed in Table B3, and with β and θ ranging from 0.001 to 0.005 and from 0.1 to 20, respectively.

Table B3
Characteristics of an example company.

Known variable	Value	Unit measure
h'	10	€/units*year
z_a	1.960	—
C_{fixed}	600,000	€/year
$Q_{safetystock}$	200	units

For this company, the performance maps in Fig. B1 enable direct comparison between its economic performance and the performance of different DNs, allowing for similitudes and comparative analyses that were not immediately apparent through Shen et al. (2003). While Shen et al. determine the optimal stock deployment policy, they do not show how the corresponding DN total cost (C_{TOTij}) differs from other stock deployment policies. In contrast, BT-based performance maps reveal this cost information and visually highlight that all DNs with the same π^* value exhibit equivalent economic performance and stock deployment optimality. In fact, although Shen et al. (2003) suggests a stock deployment policy with a high degree of centralization (i.e., 7 out of 88 DCs when $\theta = 20$ and $\beta = 0.005$) to optimize DN costs, it does not highlight the following aspect, which instead clearly emerges through the use of BT. Since θ is three orders of magnitude greater than β , the total cost of the DN is primarily influenced by the dimensionless group π_4 . As a result, variations in β (i.e., π_5) do not lead to significant changes in C_{TOTij} , whereas shifts in θ cause noticeable changes in C_{TOTij} only when the increase in θ is substantial (specifically, when it rises from 1 to 20). In this case, the performance maps show light blue regions where they were previously blue, meaning that the absolute values of C_{TOTij} double, as indicated in the color legend.

Moreover, when θ and β are fixed – for example, assuming $\theta = 0.1$ and $\beta = 0.001$ (top-left graph in Fig. B1) – the performance maps confirm Shen et al.'s (2003) recommendation to adopt stock deployment policies with a high degree of centralization. Also, the maps reveal the following insight: because θ is three orders of magnitude greater than β , inventory costs ($C_{working}$) have a much greater impact than transportation costs ($C_{inbound}$ and $C_{outbound}$). Consequently, the company's economic performance varies almost exclusively along the z-axis of the performance maps, while showing negligible changes along the x- and y-axes. Therefore, regardless of where the DCs are located in the DN (whether closer to suppliers or to customers), it is always preferable to maintain a low number of DCs in order to benefit from risk pooling and reduce inventory levels.

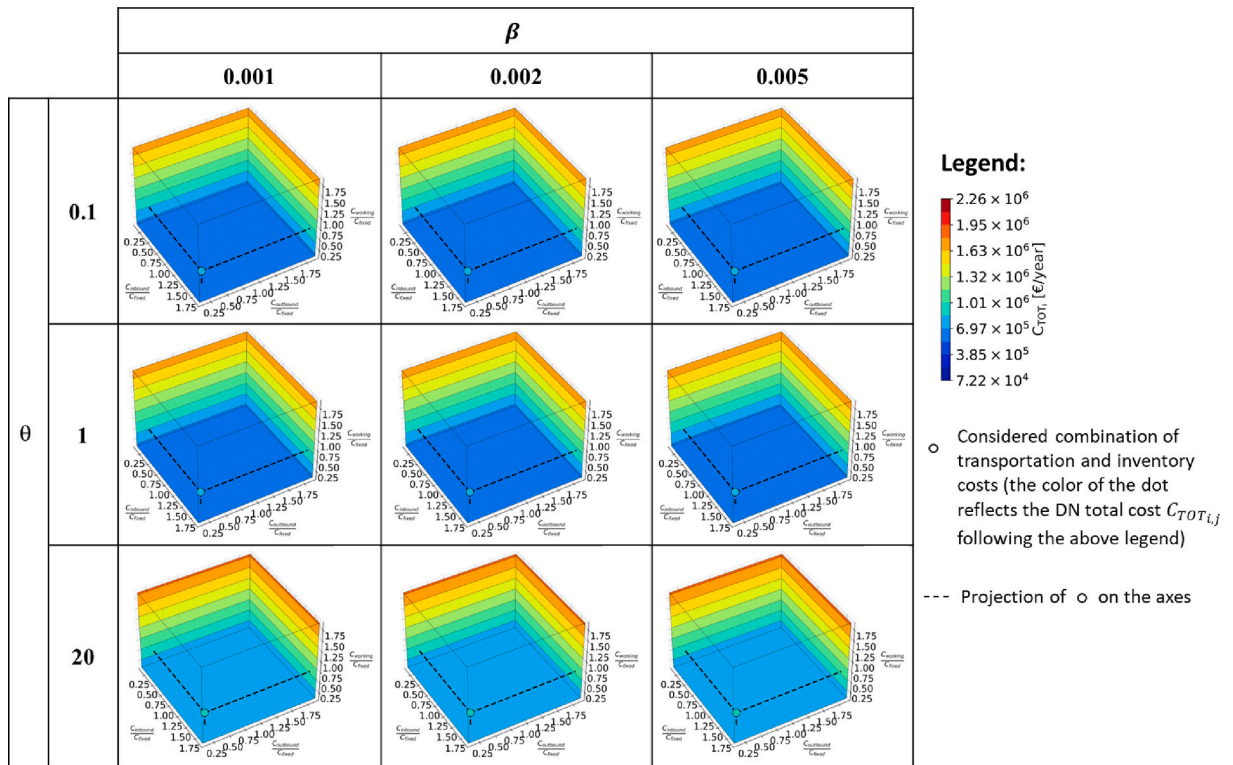


Fig. B1. Performance maps for the company described in . For each combination of transportation-inventory costs (i.e., fixing specific values of C_{fixed} , $C_{inbound}$, $C_{outbound}$, and $C_{working}$, which determine the position of the dot), the most cost-effective stock deployment policy can be identified by comparing the maps' colors at the dot when as β (columns) and θ (rows) vary, according to the color legend. The x, y, and z axes of the performance maps reflect the variables' reciprocal interactions when impacting cost-effectiveness

Let us now consider a scenario where the same variables of Table B3 are assumed but β and θ are allowed to assume similar values, ranging from 0.5 to 1, as shown in Fig. B2. Let us also consider that the company has already established its DN based on the model by Shen et al. (2003) but market conditions (e.g., customer demand) change. In this context, the BT-based performance maps allow for rapid reassessment. For example, assuming that the current economic performance is identified by the dot in Fig. B2, based on the color of the dot and the color legend in Fig. B2, if the current DN aligns with a minimum-cost area on the performance maps (i.e., any dot located in a blue area), managers can leave the current stock deployment policy unchanged. Conversely, if lower-cost regions are identified, then it is advisable to adjust the stock deployment policy accordingly. The maps, therefore, serve as diagnostic and prescriptive tools, providing general guidelines to update the stock deployment policies by showing which cost items (fixed costs, inbound and outbound transportation costs, working inventory costs, and safety stock costs) mainly contribute to the total cost of the DN, thus suggesting (through their definitions in Equations B8-B12) how to act on the decision variables (X_j and Y_{ij} of Table B1) to reduce them.

The performance maps enable the identification of economic indifference curves (i.e., isocolor contours in the three-dimensional space defined by (π_1, π_2, π_3) , for any fixed value of π^* , π_4 , and π_5 (i.e., θ and β respectively). Along each indifference curve, all DNs exhibit the same economic performance, despite variations in the underlying input variables. These contours reveal performance frontiers and make visible the trade-offs between cost components (such as fixed costs, inbound and outbound transportation costs, working inventory costs, and safety stock costs) that remained implicit in the original formulation by Shen et al. (2003). For example, Fig. B3 presents a two-dimensional breakdown of the 3D performance maps from Fig. B2, showing the cross-sections of the top graphs (with $\theta = 0.5$) at the fixed dot. Plan A (i.e., the $\pi_1 - \pi_2$ plane) shows that, at the dot, inbound transportation costs (x-axis) are significantly higher than outbound costs (y-axis). All DNs whose stock deployment policies shift this point into areas of the performance maps with the same color will maintain identical economic performance. Therefore, such configurations are not worth pursuing, as they involve reorganizing the DN without achieving any cost benefits. Conversely, stock deployment policies that increase the number of DCs and position them closer to customers can reduce inbound transportation costs. If this reduction is sufficient to offset the increase in outbound costs, the total DN cost will decrease. The performance maps visually depict the critical points where the partial derivatives of π^* in Equation B22 with respect to π_1 and π_2 are equal, thus representing the boundary between the cost-effectiveness of centralized and decentralized deployment policies as a function of inbound and outbound transportation costs.

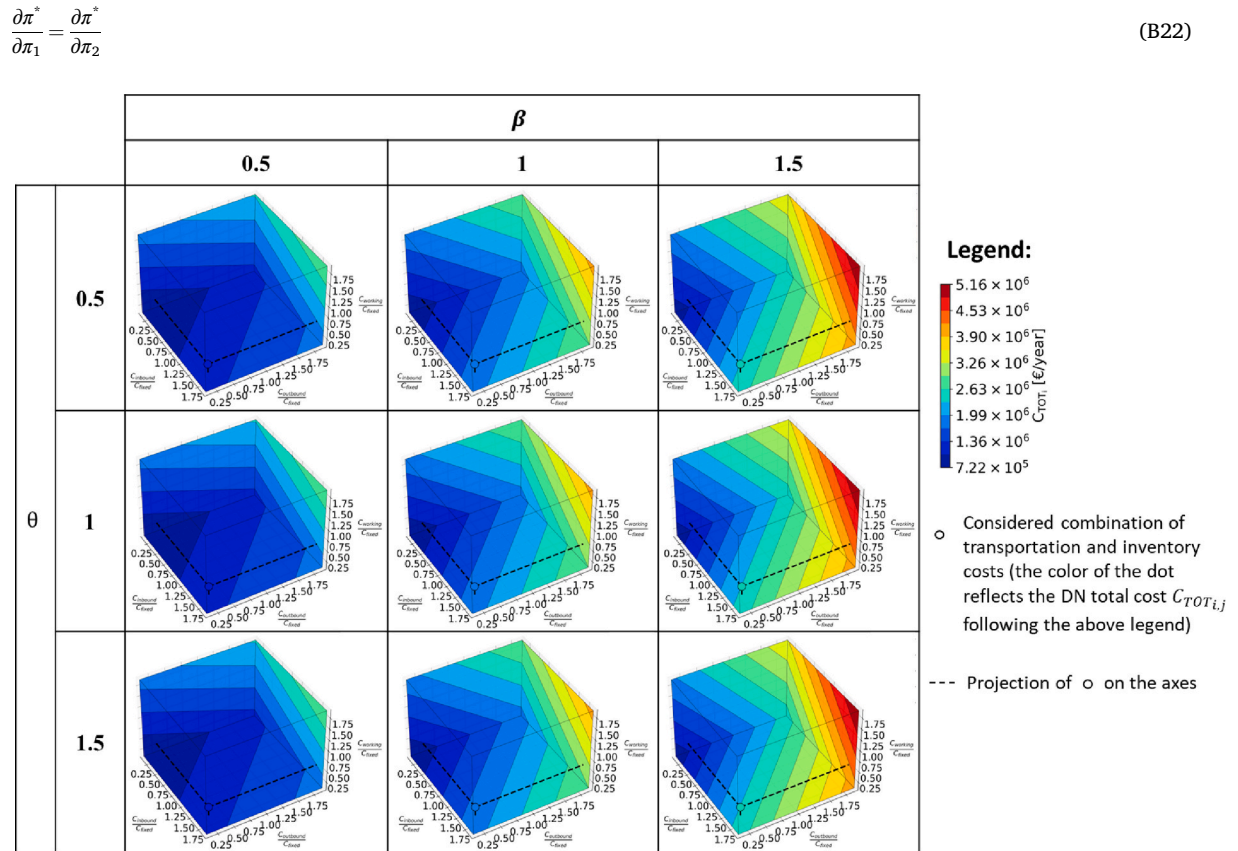


Fig. B2. Performance maps for the same company considered in but allowing β and θ to assume values of 0.5; 1; 1.5 (instead of $\beta = 0.001$; 0.002; 0.005 and $\theta = 0.01$; 1; 20, respectively)

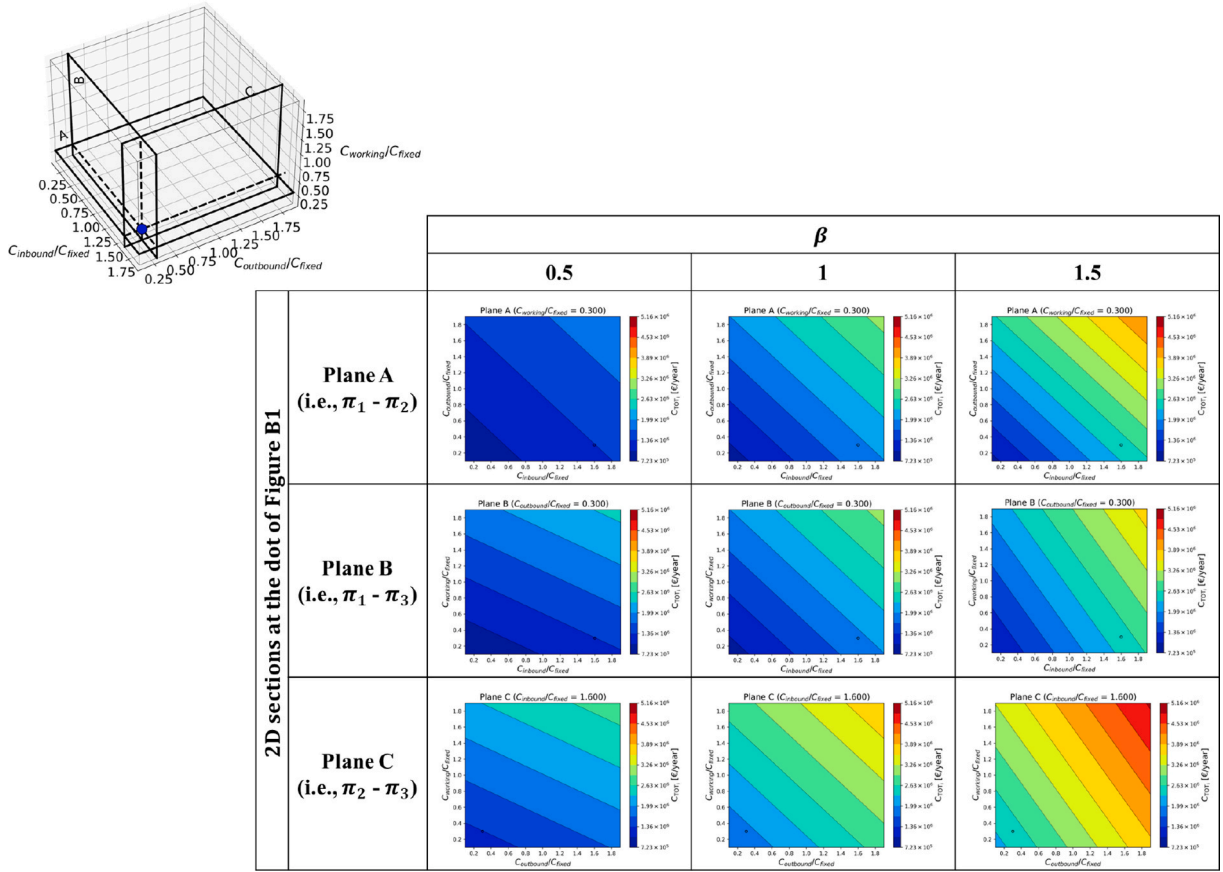


Fig. B3. Cross-sections of the performance maps in , assuming $\theta = 0.5$

By generalizing the above considerations and analyzing different combinations of π_1 , π_2 , and π_3 , three dominant regions can be identified. When $\pi_1 \gg \pi_2$ and π_3 , high and distant suppliers dominate the stock deployment decisions in the DN. In this case, the optimal stock deployment policy is to minimize the number of DCs and locate them close to suppliers to reduce inbound transportation costs. This case is typical of import-heavy DNs or DNs of global commodities. When $\pi_2 \gg \pi_1$ and π_3 , last-mile delivery costs dominate the stock deployment decisions. Thus, the optimal stock deployment policy shifts toward having many small DCs located near customers, a situation common in e-commerce and fresh food DNs. Finally, when $\pi_3 \gg \pi_1$ and π_2 , inventory costs dominate the stock deployment decisions. Here, a fully centralized policy becomes optimal, minimizing the number of DCs to achieve economies of scale through risk pooling. This case typically applies to DNs managing high-value, slow-moving inventory. All the aforementioned insights enables cross-sectoral and cross-dimensional comparisons, which are not directly available through Shen et al.'s model (2003).

Focusing on the three-dimensional structure of the performance maps (e.g., fixing the values of θ and β and considering the top-left graph in Fig. B2), it becomes clear that π_1 , π_2 , and π_3 are strongly interrelated. On the two-dimensional ($\pi_1 - \pi_2$) plane, as said, a geographic trade-off in DC placement becomes needed to balance inbound and outbound transportation costs (highlighted by 45° sloped isocolor curves in Fig. B3). Locating DCs closer to suppliers, in fact, reduces π_1 (due to Equation B11), but increases π_2 (due to Equation B(9)); the reverse is true when DCs are placed near customers. Hence, managers can visually understand how the number of DCs impact the total transportation cost of their DN. Whereas, the dimensionless group π_3 captures the impact of economies of scale in the DN. A centralized stock deployment policy, with fewer DCs serving larger demand shares, yield lower π_3 values due to risk-pooling. The opposite effect occurs in more decentralized DNs.

Looking further at how θ and β (i.e., π_4 and π_5) shape the DN total cost and stock deployment optimality, the performance maps in Fig. B2 show that, for the company described in Table B3 – and allowing θ (rows) and β (columns) to assume the values 0.5, 1, and 1.5 –, π_5 has a stronger impact on the DN total cost than π_4 due to the double presence of β in Equation B(7). In contrast, the opposite effect is observed in Fig. B1, where θ takes values that are three orders of magnitude higher than β . Therefore, since θ and β can assume a wide range of values, the performance maps help identify two extreme operating regimes: a “transport-heavy” regime and an “inventory-

heavy” regime. The transport-heavy regime is characterized by $\pi_5 \gg \pi_4$ (i.e., $\beta \gg \theta$). In this regime, π^* becomes mainly sensitive to π_1 and π_2 , while π_3 becomes secondary. As a result, stock deployment policies with a lower degree of centralization tend to be optimal in accordance with Equation B(7), recommending several DCs (located closer to suppliers or customers depending on whether $\pi_1 > \pi_2$ or vice versa). Conversely, the inventory-heavy regime, marked by $\pi_5 \ll \pi_4$, makes π^* primarily sensitive to π_3 , reducing the influence of π_1 and π_2 . Here, stock deployment policies with higher degree of centralization become preferable, with fewer DCs used to minimize inventory costs via risk pooling.

Beyond analytical depth, BT also offers computational advantages compared to Shen et al. (2003) by offering intuitive visual tools rather than requiring the resolution of complex nonlinear programming problems. To prove the computational simplification, it is worth mentioning that looking at the BT performance maps requires a few seconds to extract insights on the cost-effectiveness of different stock deployment policies. Conversely, Shen et al. (2003) demonstrate that their nonlinear integer programming model needs to be reformulated as set-covering problem to be computationally solved. Despite this reformulation, Shen et al.’s model may still require more than 800 CPU seconds to be solved using CPLEX 5.0 on a Sun SPARCstation running the SunOS 4.1.3u5 operating system, where the computation times exclude the time lost for inputting the values of the starting influential variables characterizing the DN in the model.

In conclusion, BT does not replace the detailed optimization model by Shen et al. (2003), which remains essential for capturing heterogeneity among DCs and customers, and for specifying the DN configuration via indices i and j (see Table B1). Rather, BT enriches the perspective on stock deployment by decomposing the total DN cost (C_{TOTij}) into dimensionless groups, offering graphical performance maps that enable rapid decision-making, strategic benchmarking (i.e., similitudes), and a clearer understanding of trade-offs between centralized, decentralized, and hybrid DNs.

Appendix C

This Appendix provides details on the sample of experts, working in DNs, who were consulted during an online workshop to conceptually validate the study’s findings (particularly the content of the performance maps in Figs. 3, 5, and 7). As detailed in Table C, six experts were consulted: three university professors specializing in supply chain management and three industry managers working in retail companies. These experts were intentionally selected that work across different geographic areas and industrial settings, and with a minimum of five years of experience in DN and inventory management/research. The experts remarked on the appropriateness of the content of performance maps, highlighting their usefulness for visualizing and directly comparing the economic impact of different stock deployment choices. The industrials also reported that these maps could serve as democratic decision-support tools for the following reason. Their visual explanatory nature makes the underlying stock deployment dynamics accessible even to operators with limited mathematical skills, thereby enabling any retailer to enhance DN performance.

Table C. Sample of experts consulted to validate the findings of this study.

ID	Position	Experience (Years)	Working Sector	Country
1	Full professor of Industrial Logistics	11	University	Norway
2	Associate Professor of Operations Management and Industrial Plants	13	University	Italy
3	Assistant Professor of Supply Chain Management	5	University	France
4	Supply Chain Manager	6	Furniture and houseware retail company	Sweden
5	Head of Logistics and Supply Chain Department	9	Agri-food retail company	France
6	Maintenance Business Unit Director	10	Spare parts distribution company	Germany

Appendix D

This appendix presents, for each performance map in Figs. 3, 5, and 7, a series of two-dimensional cross-sections obtained by slicing the 3D plots at various positions along the axes. This comprehensive breakdown (shown in Figs. D1-D7) improves the readability of the isocost curves discussed in Sections 4.1-4.3, offering deeper insight into their behavior not only at the specific example dots illustrated in Figs. 3, 5, and 7, but also across other regions of the maps. In Figs. D1-D7, the first column indicates the plane under consideration, the second column displays the corresponding 2D cross-section, and the remaining columns present the performance map slices at

different values of Deg_i . Figs. D1-D3 correspond to the first application context (Section 4.1), assuming values of h equal to 10 %, 20 % and 30 %, respectively. Fig. D4 refers to the second application context (Section 4.2), while Figs. D5-D7 relate to the third application context (Section 4.3), with SL set at 90 %, 95 %, and 99 %, respectively.

This appendix also presents a sensitivity analysis (Figs. D8-D10), conducted as an illustrative example using the performance maps associated with the second application context (i.e., Fig. 5 in Section 4.2). This sensitivity analysis demonstrates the robustness of the insights discussed in this paper with respect to variations in the DN input variables. As summarized in Table D1, alternative values of average demand ($\bar{D}$), standard deviation (σ), and the holding cost rate (h) were tested relative to those fixed in Table 5. Each parameter in Table D1 was varied individually, while all other parameters from Table 5 were held constant.

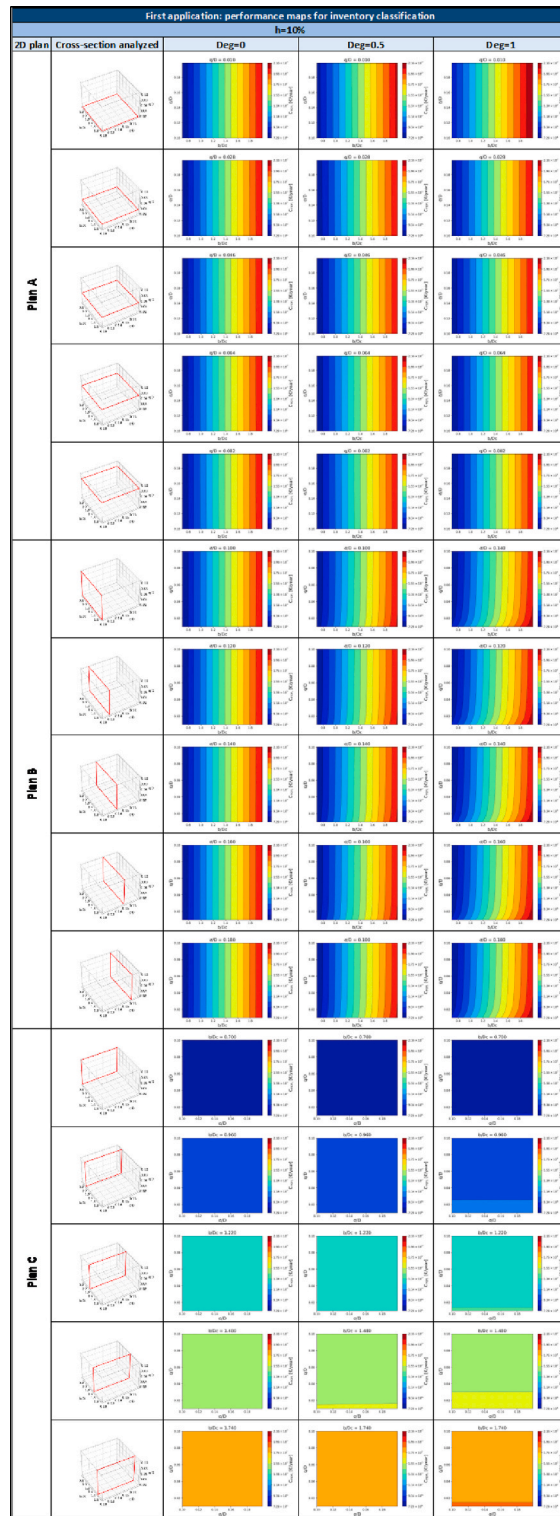
Table D1

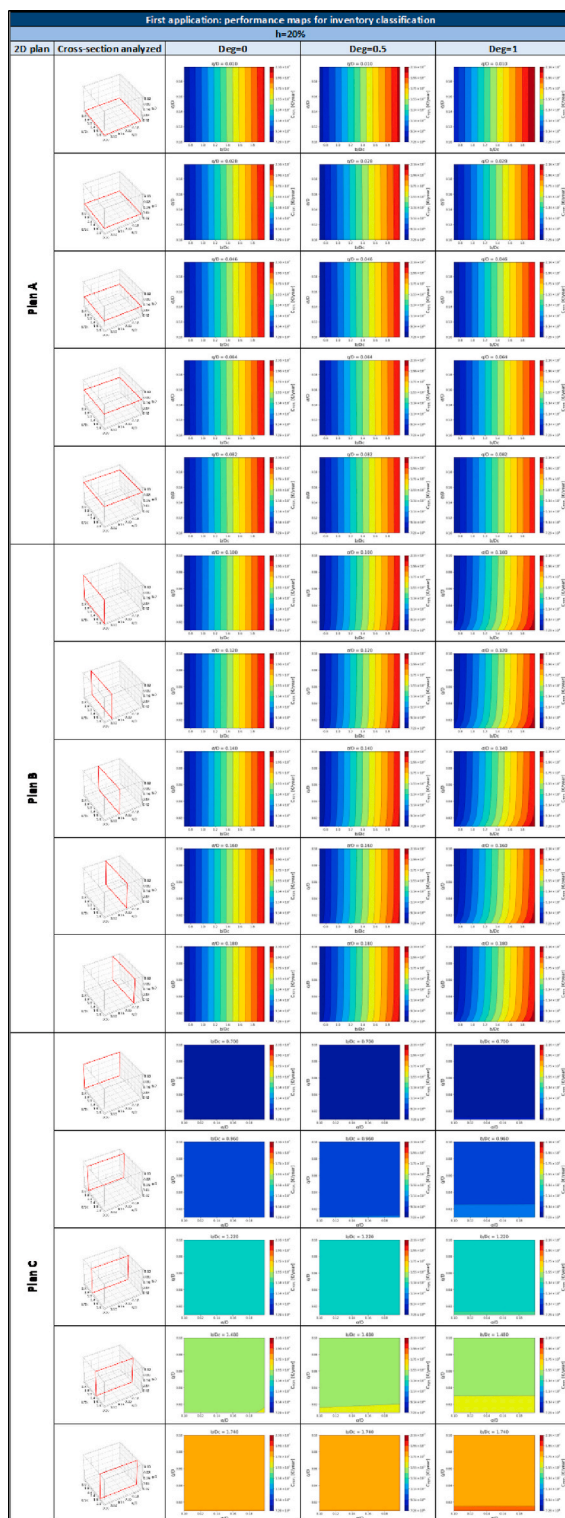
Values of average demand, standard deviation, and holding cost rate tested in the sensitivity analysis. Bold numbers indicate the baseline values used in the performance maps shown in Fig. 5.

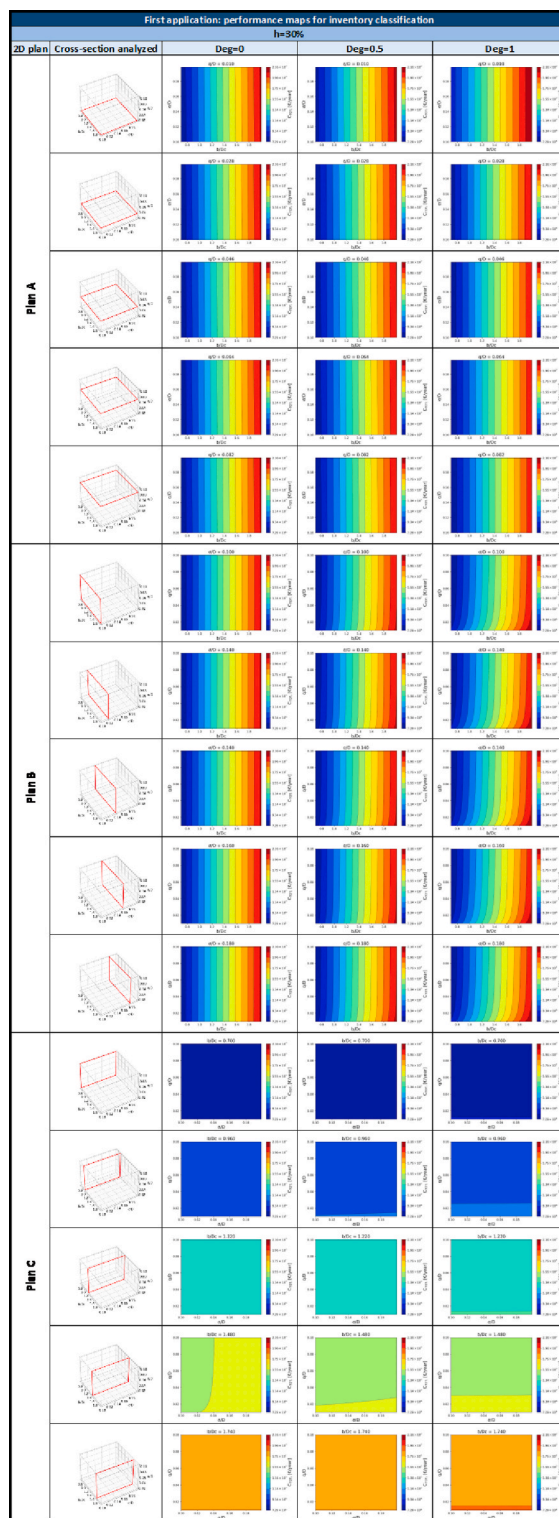
Variable	Tested values	Unit measure
$\bar{D}$	1,000 ; 5,000; 10,000	units/time
σ	150 ; 300; 450	units/time
h	0.10; 0.20 ; 0.30	time ⁻¹

The results in Figs. D8-D10 show how variations in input variables influence the recommended stock deployment policies. Although the absolute values of C_{TOT_i} differ across the graphs (as shown in the last column of Figs. D8-D10), the overall structure of the performance maps and the pattern of the isocolor curves confirm the consistency of the stock deployment recommendations presented in Section 4.2. For instance, in Fig. D10, when the value of h is decreased (10%) or increased (30%) compared relative to the baseline of Fig. 5 ($h = 20\%$), the color patterns in the performance maps remain similar. The main variation lies in the values of C_{TOT_i} , which, while having the same maximum value across all graphs, exhibit different minimum values: $2.003 \cdot 10^7$, $2.004 \cdot 10^7$, and $2.005 \cdot 10^7$ for h equal to 10 %, 20 %, and 30 %, respectively. This variation is easily explained by the increase in holding costs, particularly in decentralized DNs that do not benefit from risk pooling. Despite these differences in C_{TOT_i} values, the recommended stock deployment policy remains unchanged. In all cases, regardless of the value of h , centralization ($Deg_i = 1$) is consistently the most cost-effective stock deployment policy, as indicated by the blue coloring at the corresponding dot.

Similar observations apply to the sensitivity analysis of $\bar{D}$ and σ . For example, in Fig. D8, increasing $\bar{D}$ (to 5,000 or 10,000 units/year) relative to the baseline ($\bar{D} = 1,000$ units/year) does not significantly alter the color pattern in the performance maps. Again, the main difference lies in the values of C_{TOT_i} , which increase by an order of magnitude when the demand increases by the same proportion (i.e., from 1,000 to 10,000 units/year), leading to higher inventory and transportation costs. Nevertheless, for any value of $\bar{D}$, the isocost curves at the reference dot continue to support centralization ($Deg_i = 1$) as the most cost-effective stock deployment policy. Notably, some changes in the curvature and coloring trends of the isocost curves are observed under decentralized and hybrid stock deployment policies, while no significant differences are found under the centralized policy. This is because, while demand uncertainty (σ) remains fixed, increasing the average demand ($\bar{D}$) attenuates the risk-pooling effect, thereby reducing the cost differential between centralized and decentralized DNs. To illustrate this, consider the same reference dot: when $\bar{D} = 1,000$, the dot's color shifts from green to blue when moving from decentralized ($Deg_i = 0$) to centralized ($Deg_i = 1$) stock deployment policies. In contrast, when $\bar{D} = 10,000$, the dot's color changes only from light blue to blue as Deg_i increases, indicating a smaller cost difference between centralized and decentralized stock deployment policies.

Fig. D1. Cross-sections of the performance maps for the first application context (), assuming $h = 10\%$.

Fig. D2. Cross-sections of the performance maps for the first application context (), assuming $h = 20\%$.

Fig. D3. Cross-sections of the performance maps for the first application context (), assuming $h = 30\%$.

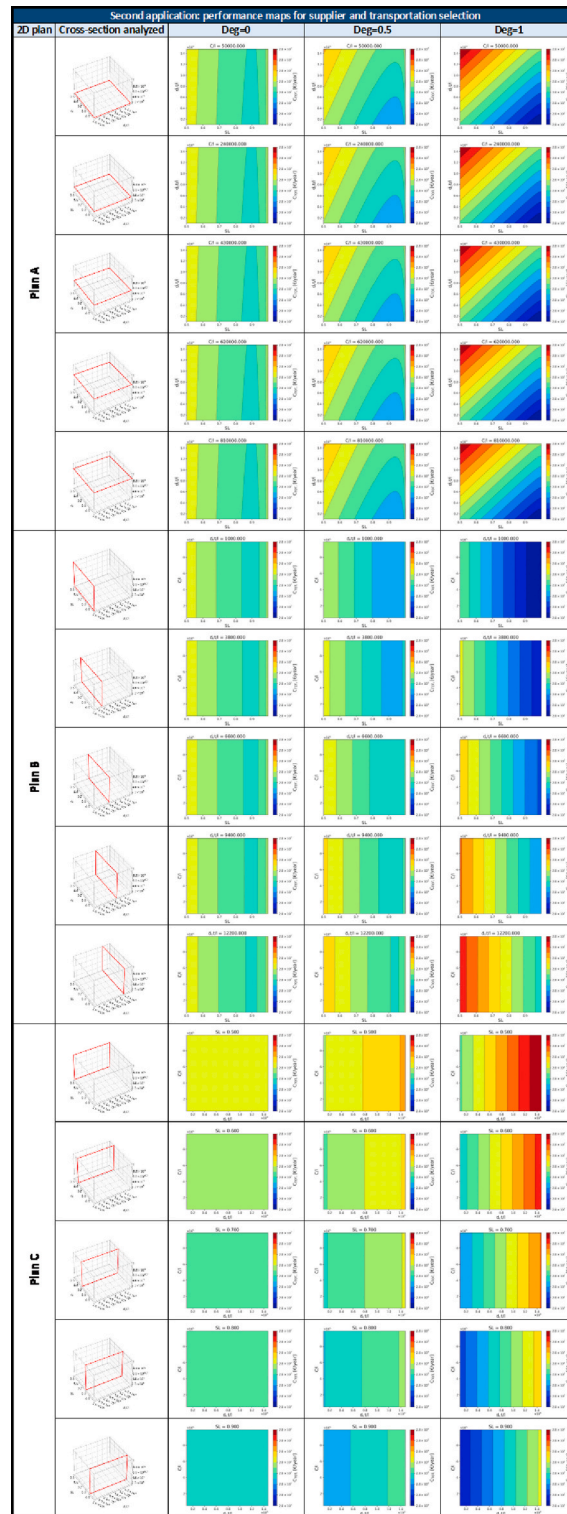


Fig. D4. Cross-sections of the performance maps for the second application context ().

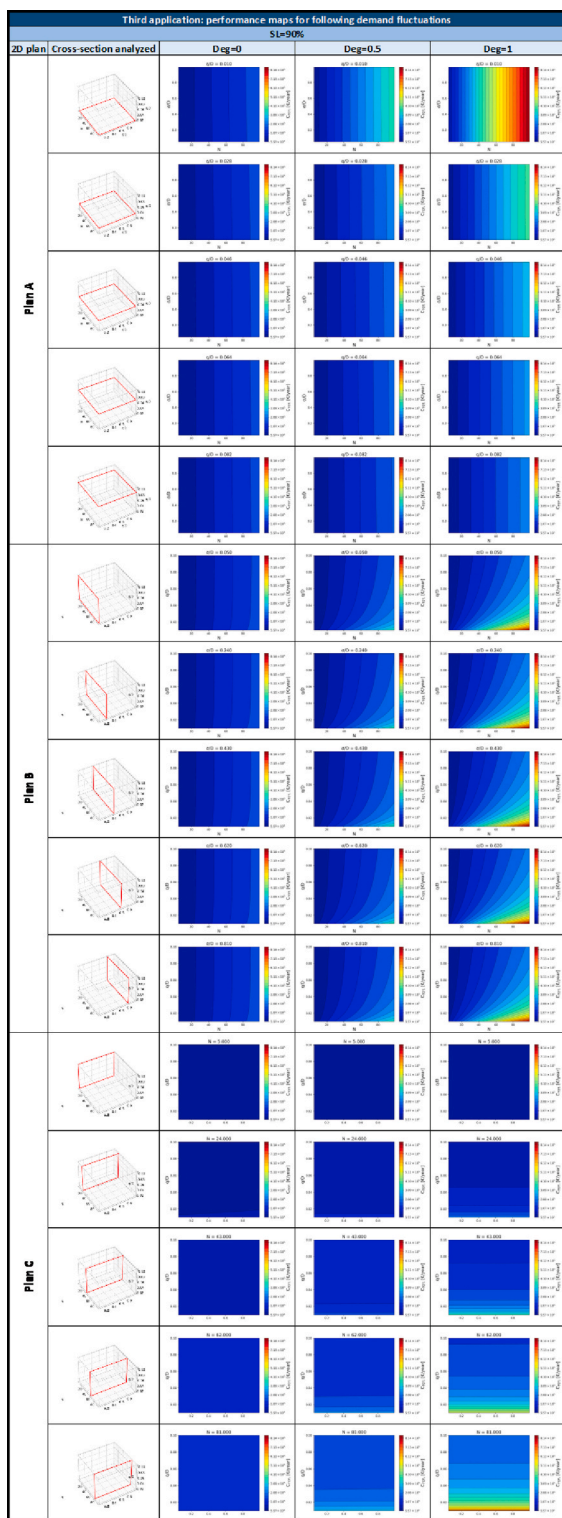


Fig. D5. Cross-sections of the performance maps for the third application context (), assuming SL = 90 %.

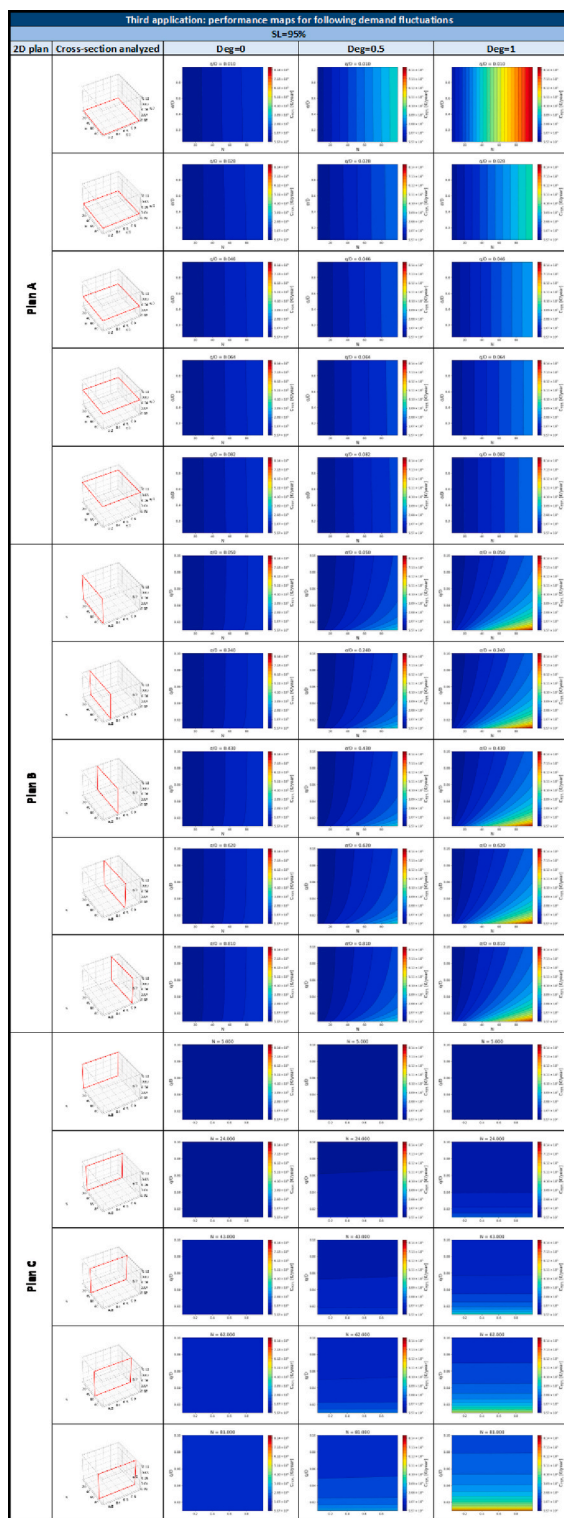


Fig. D6. Cross-sections of the performance maps for the third application context (), assuming SL = 95 %.

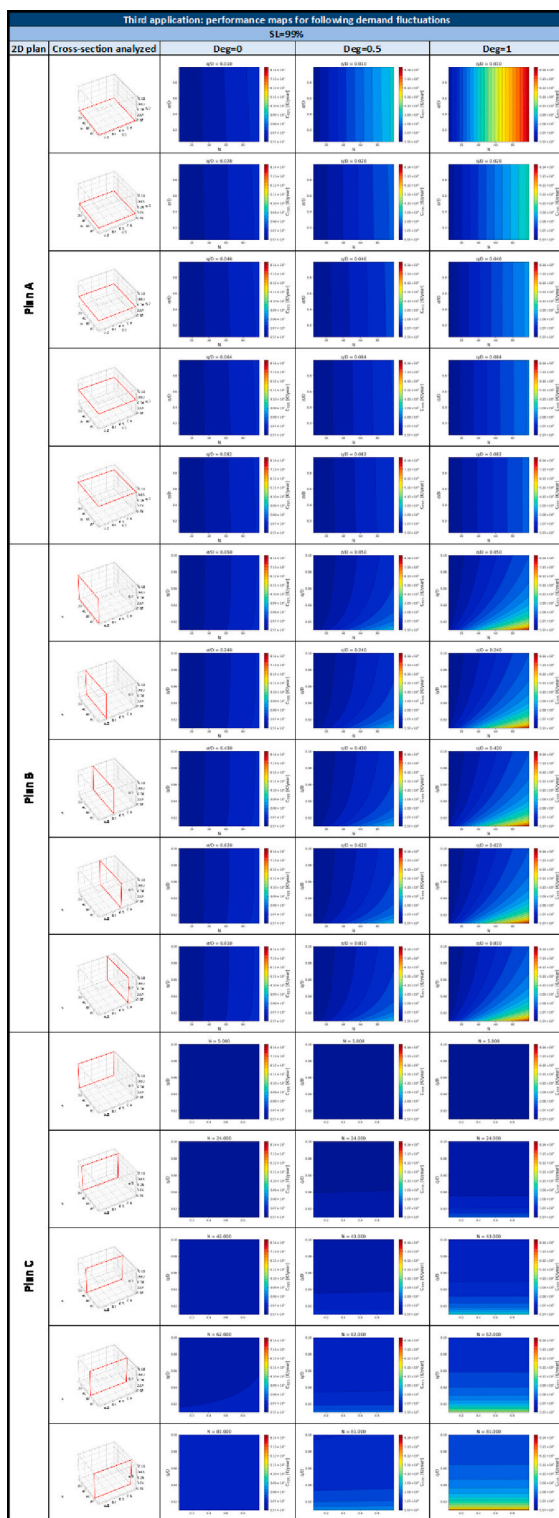


Fig. D7. Cross-sections of the performance maps for the third application context (), assuming SL = 99 %.

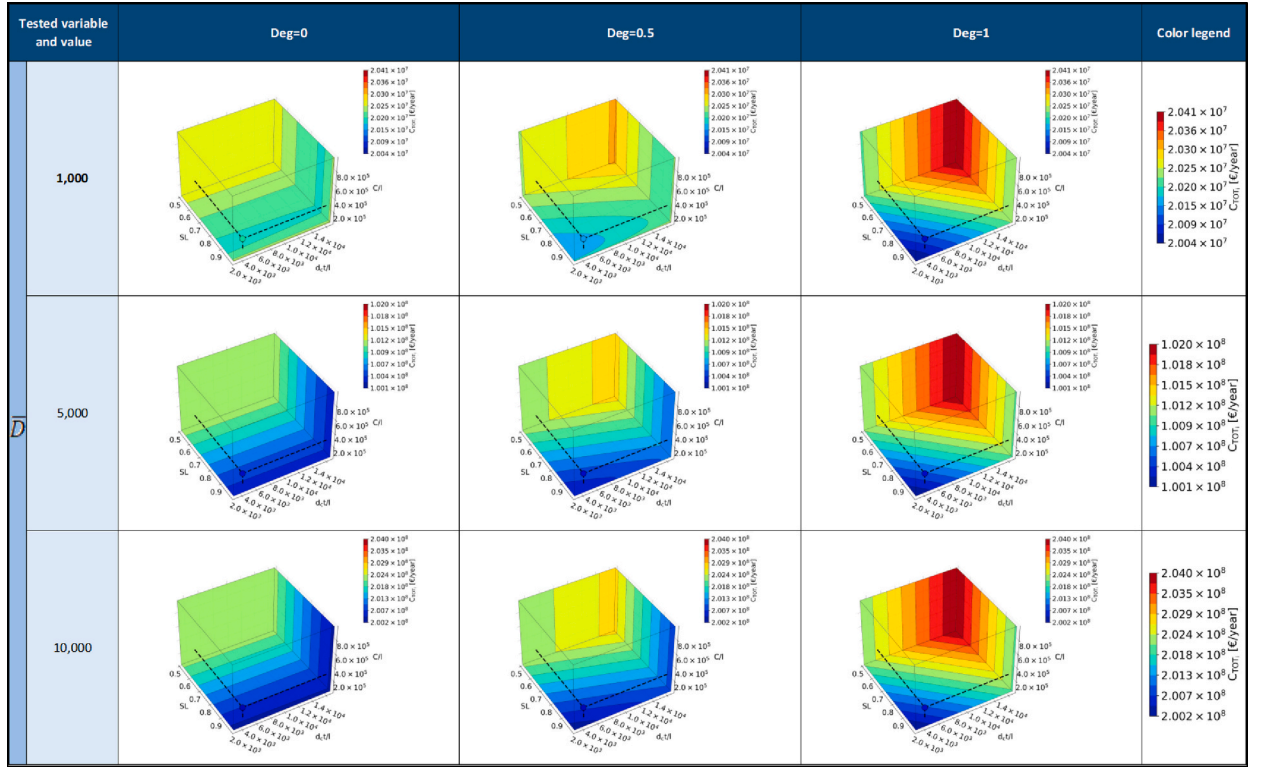


Fig. D8. Sensitivity analysis on the performance maps associated with the second application context (i.e., Fig. 5) and obtained by varying D . The bold number in the 2nd column represent the baseline value of used in to build Fig. 5.

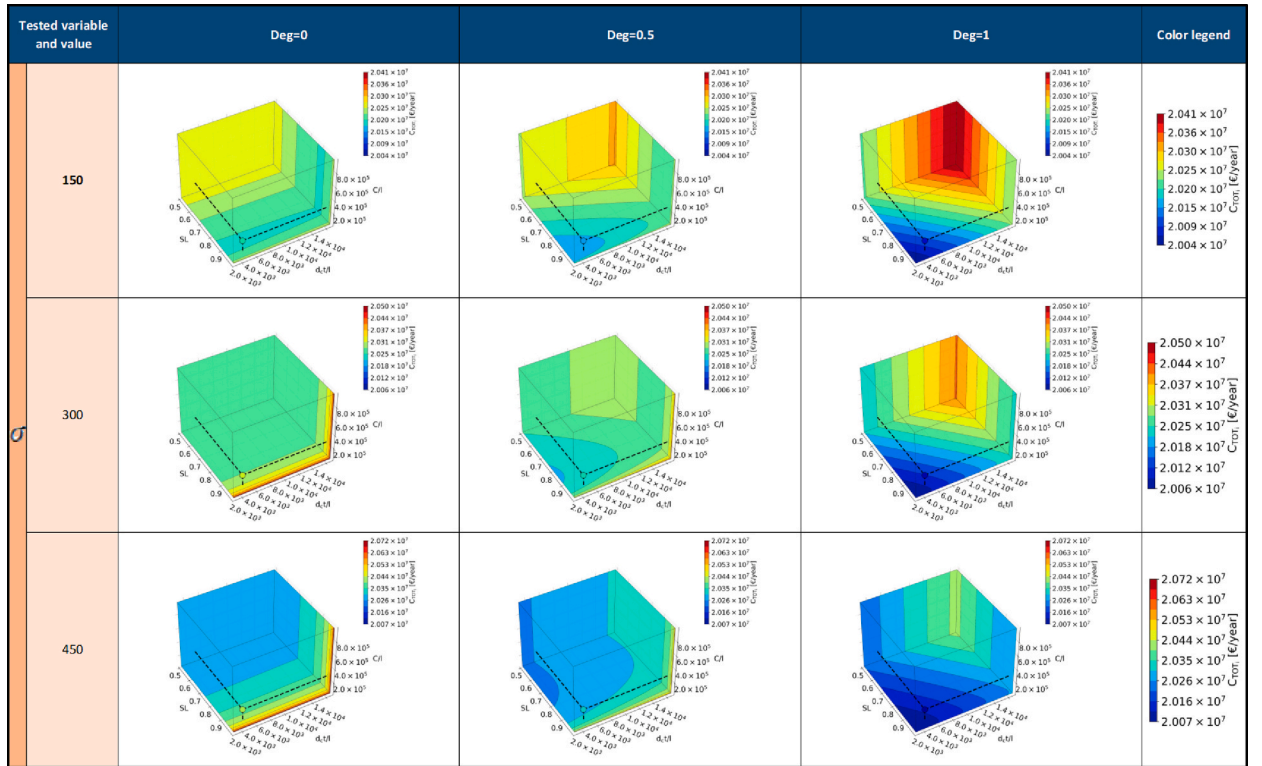


Fig. D9. Sensitivity analysis on the performance maps associated with the second application context (i.e., Fig. 5) and obtained by varying σ . The bold number in the 2nd column represent the baseline value of used in to build Fig. 5.

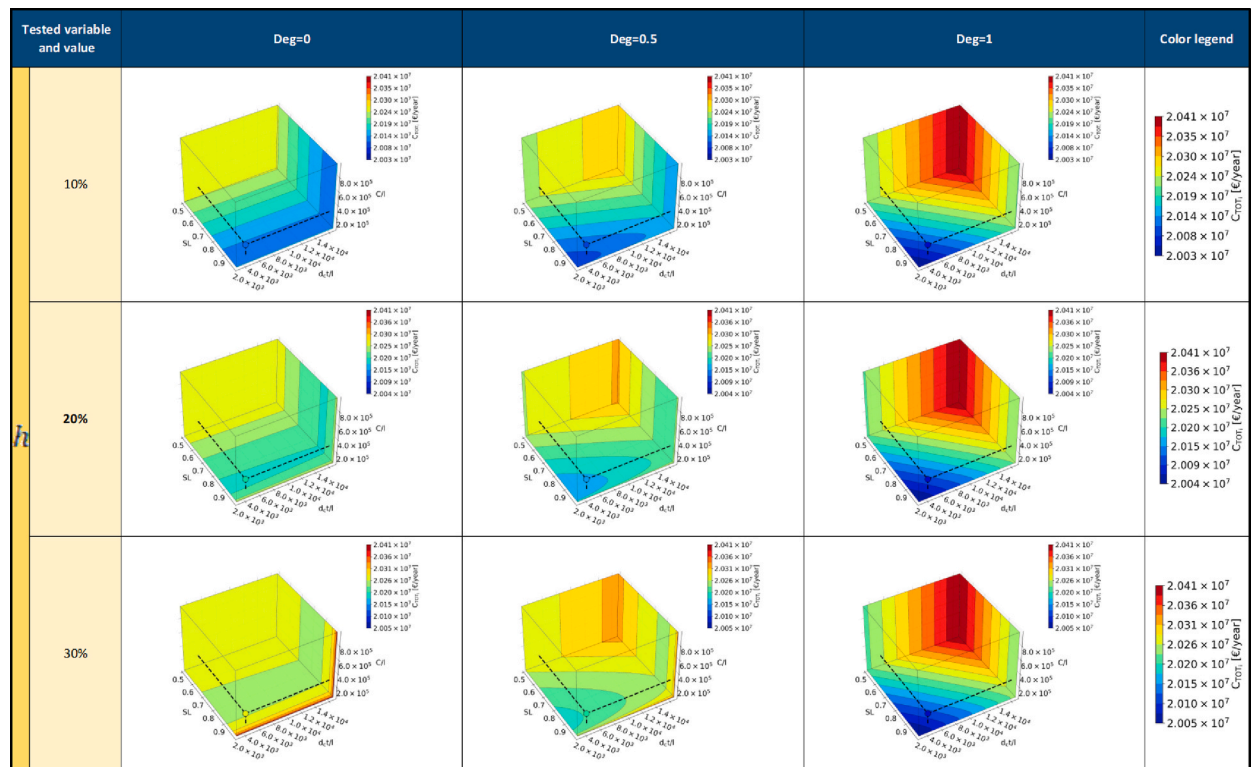


Fig. D10. Sensitivity analysis on the performance maps associated with the second application context (i.e., Fig. 5) and obtained by varying . The bold number in the 2nd column represent the baseline value of used in to build Fig. 5.

Data availability

Data will be made available on request.

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