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The paper answers affirmatively a question raised in [Brandenburger et al., *Econometrica* 76(2008);MR2388774] (henceforth, BFK), as part of a research in the epistemic foundation of *iterated admissibility* (IA). IA is iterated maximal elimination of weakly dominated strategies of all players in each round of a finite game. BFK shows that promising notions in the characterization of the IA set of strategies are epistemic conditions corresponding to *rationality and common assumption of rationality* (RCAR) and *rationality and  $m$ -th order assumption of rationality* (R $m$ AR). RCAR is the assumption that every player is rational (i.e. chooses the strategy that is optimal given the opponents' possibilities) and that the assumption is mutual, up to every possible level. R $m$ AR is the same, except that it admits mutual assumptions of rationality among players up to the  $m$ -th level. The positive results of BFK are drawn in the context of lexicographic type structures (LTS) from [Blume et al., *Econometrica*, 59(1991);MR1085584] and amounts to showing that (i) for every game there exists some LTS in which the strategies predicted by RCAR are exactly the IA ones, and (ii) for every game and every complete LTS, there exists an  $m$  such that the strategies predicted by R $m$ AR are exactly the IA ones. A negative result BFK proves, however, is an "impossibility theorem" stating that RCAR is impossible in any continuous and complete LTS. BFK raises the question whether, for every finite game, there exists a (discontinuous) complete LTS in which RCAR is possible. This paper answers the question affirmatively (Theorem 3.4) and also shows (Theorem 3.5) that in every LTS such as those that are proved to exist by the previous result,

the strategies predicted by RCAR are exactly the IA ones. Theorem 3.4 is game-dependent and the question whether a complete LTS exists for every game with the same strategy sets is left open.

As a first step in the attempt of making the above result uniform, the authors investigate the relation between LTSs generated by the same strategy sets that only differ topologically. This is used to formalize a notion of difference in the *degree of caution* in making assumptions that the two structures allow, and in making precise when a given structure  $U$  is a *refinement* under this respect of  $T$ . It is shown (Theorem 4.10) that every LTS has a continuous complete refinement generating the same hierarchy of beliefs. If the original LTS is complete and admits RCAR, therefore, its continuous refinement will not, while in both structures the same hierarchy of beliefs is available. This is taken to show that admissibility of RCAR does not depend on how “rich” the structure is in terms of the hierarchy of beliefs it generates, but is also due to “a kind of balance between its topology and its family of Borel sets”. The relation between these results and those coming from some related literature is further investigated in Appendix D of the paper.