



On the dynamics of coordinated hierarchies: Economic growth versus poverty traps

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ABSTRACT

We study development traps generated by payoff-mediated coordination between an institutional layer and a private-sector layer. The leading interpretation is the transition from informality to formal-sector participation: institutional effort raises the return to formalisation, while formal participation raises the fiscal, compliance, and legitimacy returns to institutional effort. The paper develops this mechanism through nested continuous-time replicator models. The one-population benchmark gives the familiar critical-mass and hysteresis result. The two-role follower model shows how a scalar threshold becomes a stable saddle branch. The main contribution is a leader–follower phase-plane analysis: we derive local stability conditions for joint poverty, joint prosperity, and decoupled regimes; give sufficient global conditions under which the stable manifold of an interior saddle is the basin boundary; and prove a comparison result showing that payoff-improving reforms weakly expand the basin of joint prosperity. A multi-role extension records local rare-deviation stability and monotone nullcline shifts, without claiming a complete global classification in four dimensions. The analysis implies that unilateral policy interventions do not fail by definition; they fail when the induced state remains in the basin of the low-activity equilibrium.

1. Introduction

A persistent question in development economics is why apparently similar economies can follow sharply different long-run trajectories. Some economies accumulate state capacity, formal-sector participation, and productive investment, whereas others remain in low-productivity configurations sustained by informality, weak enforcement, and low fiscal capacity. The modern literature attributes such divergence partly to institutions, state capacity, and political incentives (Acemoglu et al., 2001, 2005; Acemoglu and Robinson, 2010, 2012; Aoki, 2001; Bosi et al., 2024; North, 1990; Rodrik, 1997; Stiglitz, 1989, 2003). The poverty-trap and big-push traditions offer a complementary explanation: strategic complementarities can make history and expectations matter because individually costly actions become profitable only when sufficiently many other agents also undertake them (Rosenstein-Rodan, 1943; Murphy et al., 1989; Azariadis and Drazen, 1990; Azariadis

and Stachurski, 2005; Matsuyama, 2008; Bowles et al., 2006; Easterly, 2006).

This paper studies a layered evolutionary version of that mechanism. The leading interpretation is economic formalisation. Leaders represent policymakers, regulators, public officials, or other actors who affect state capacity, enforcement, fiscal capacity, and the reliability of public goods. Followers represent firms, workers, entrepreneurs, or citizens who decide whether to operate in the formal, investment-intensive part of the economy. Each layer has two behavioural profiles. The high profile, denoted by H , means institutional effort for leaders and formal, compliant, investment-oriented behaviour for followers. The low profile, denoted by L , means weak enforcement, rent seeking, or corruption for leaders and informality, low investment, evasion, or subsistence activity for followers. The word “hierarchy” is used in this payoff-mediated sense: there is a public layer and a private layer linked by vertical externalities.¹

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¹ The model is not a Stackelberg game, does not assume that leaders move first, and does not impose an authority relation beyond the fact that institutional choices enter follower payoffs while follower choices enter institutional returns.

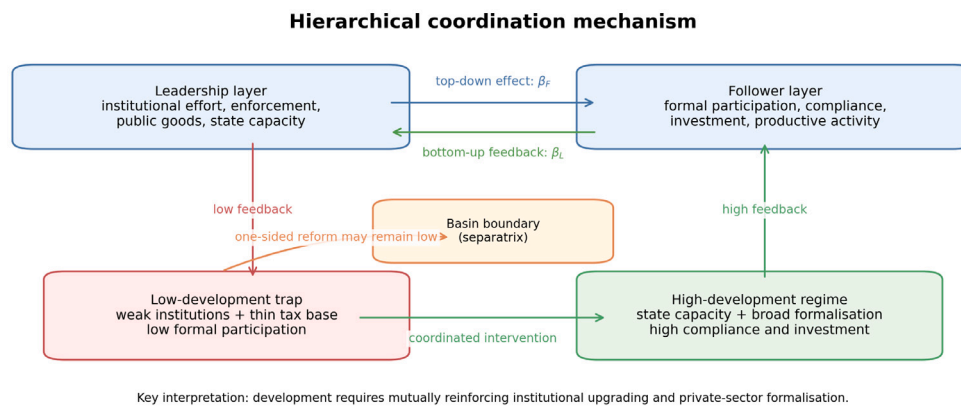


Fig. 1. Hierarchical coordination mechanism in the formalisation/state-capacity interpretation.

The economic logic is vertical strategic complementarity (Vives, 1990, 2005; Vives and Vravosinos, 2024). If leaders expand enforcement and public-good provision while most followers remain informal, the tax base is thin and the return to institutional investment is limited. If followers are willing to formalise but leaders provide unreliable enforcement, predatory regulation, or weak public goods, the private return to formalisation may be too low. Hence neither side can generally sustain the high-profile outcome alone. Broad formalisation requires coordination between institutional quality from above and private formal participation from below.² The interpretation is deliberately stylised, but it is consistent with work on institutions and enforcement (North, 1990; Acemoglu et al., 2001, 2005), state capacity (Besley and Persson, 2009), entry regulation and formal-sector costs (Djankov et al., 2002), fiscal and regulatory incentives (Besley and Case, 1995; Oates, 1972; Varsano, 2002; de Mello, 2008; Coelho, 2021; Tabellini, 2008), and informality (La Porta and Shleifer, 2014).

Table 1 summarises how the example is used throughout the model. The table is not an additional assumption; it is a guide that keeps the economic interpretation fixed while the mathematical environment changes from the one-population benchmark to the leader–follower coevolution model.³

The hierarchy therefore identifies a two-way payoff constraint on development. The public layer shapes the payoff to formalisation from above, but the private layer also shapes the fiscal, compliance and legitimacy returns to institutional effort from below. Fig. 1 summarises this mechanism. The top-down parameter β_F captures the effect of enforcement, public goods, and regulatory predictability on the follower payoff from formalisation. The bottom-up parameter β_L captures the effect of the tax base, compliance, and legitimacy on the leader payoff from institutional effort. The figure also indicates why the relevant policy object is the position of the state relative to the basin boundary, not whether the intervention is labelled unilateral or coordinated.

This paper is also related to a large literature on coordination failures, strategic complementarities, and institutional persistence. Standard coordination games under replicator dynamics already imply stable coordinated corners, unstable thresholds, and path dependence

² The example is economically relevant because informality is pervasive: the International Labour Organization estimates that roughly two billion workers, or 61.2 percent of global employment, are in informal employment (International Labour Organization, 2018).

³ In each case, development is viewed as an issue of coordination involving institutional improvement and the formalisation of the private sector. Weak institutions discourage formal participation, and widespread informality reduces fiscal capacity and incentives for institutional investment. The hierarchical framework is useful because it captures these mutually reinforcing feedback loops, explaining why economies may remain in a state of low development even when reforms only improve incentives on one side of the hierarchy.

Table 1
Running formalisation/state-capacity example.

Model object	Formalisation interpretation	Payoff channel
Follower state x	Share of firms, workers, or citizens choosing formal, compliant, investment-oriented behaviour	Formal activity is costly when rare, but more valuable when many agents participate
Institutional parameter λ	Enforcement, bureaucratic reliability, public-good provision, and regulatory predictability	Better institutions raise the relative payoff of the high-profile action
Two follower roles (x, y)	Complementary private roles, such as firms and workers or upstream and downstream sectors	One role's formal action is more profitable when the complementary role also coordinates
Leader state x_L and follower state x_F	Institutional effort and private formal participation coevolve	Top-down enforcement and bottom-up tax-base/legitimacy feedback jointly determine high-profile viability
Multi-role state z	Two leadership roles and two follower roles, such as enforcement, public-good provision, firms, and workers	Local stability is determined by rare-deviation payoffs; cross-layer links make one-layer-only coordination fragile

(Akao et al., 2025; Accinelli and Sanchez Carrera, 2012; Hofbauer and Sigmund, 1998; Sandholm, 2010; Sanchez Carrera, 2012, 2019; Weibull, 1995). The development literature gives these mechanisms substantive content through big-push models, poverty traps, and threshold externalities (Azariadis and Drazen, 1990; Azariadis and Stachurski, 2005; Bowles et al., 2006; Kikuchi and Vachadze, 2015; Murphy et al., 1989; Rosenstein-Rodan, 1943; Sanchez Carrera et al., 2021). Social-interaction and supermodular models show how complementarities can create multiple aggregate outcomes even when individual incentives are simple (Cooper and John, 1988; Milgrom and Roberts, 1990; Brock and Durlauf, 2001; Vives, 1990; Vives and Vravosinos, 2024; Vives, 2005). Our contribution is therefore not the one-population threshold result itself. It is threefold. First, we embed that benchmark in a layered evolutionary model and keep separate the one-dimensional, two-dimensional, and higher-dimensional claims. Second, for the leader–follower system, we give a sufficient global phase-plane theorem identifying the stable manifold of an interior saddle as the basin boundary. Third, we prove a comparison result for payoff-improving reforms: under the maintained phase-plane restrictions, increasing the payoff advantage of high-profile behaviour weakly expands the basin of joint prosperity. This last result is the formal counterpart of the policy claim that reform success depends on crossing, or moving, the relevant separatrix

Methodologically, we use continuous-time replicator dynamics as a parsimonious representation of payoff-monotonic adaptation (Weibull, 1995; Hofbauer and Sigmund, 1998; Sandholm, 2010; Schlag, 1998). The hierarchical interpretation connects this evolutionary apparatus to models of organisations, agency, leadership, and social hierarchy (Radner, 1992; Williamson, 1995; Ross, 1973; Grossman and Hart, 1983; Acemoglu and Jackson, 2015; Stiglitz, 2015). The global phase-plane discussion in the leader–follower model uses standard tools from planar dynamical systems, including invariant faces, the stable-manifold theorem, and the Bendixson–Dulac criterion; the bifurcation language follows the qualitative theory of dynamical systems (Kuznetsov, 1998).

The model is developed in four steps, but the analytical centre of the paper is the leader–follower phase plane. Setting A studies one follower population under an exogenous institutional parameter. It delivers the benchmark critical-mass and hysteresis logic: the threshold is a point, and the global dynamics on the unit interval are fully characterised by the sign of a single affine payoff difference. Setting B introduces two payoff-relevant follower roles, such as firms and workers or upstream and downstream sectors. The threshold is no longer a scalar; under the stated cross-role coordination assumptions, the relevant separatrix is generated by the interior saddle. Setting C endogenises institutions by allowing leaders and followers to coevolve. It provides explicit local-stability conditions for joint poverty, joint prosperity, and decoupled outcomes, gives a sufficient global separatrix theorem, and proves basin monotonicity for payoff-improving reforms. Setting D is an ancillary extension with two leadership roles and two follower roles; it provides local pure-profile stability, a no-stable-one-layer-only corollary, and monotone nullcline shifts, while avoiding unsupported global four-dimensional basin claims.

The main message is intentionally qualified. In the one-dimensional benchmark, an institutional improvement moves the critical mass and can eliminate the low-activity equilibrium only after a boundary threshold is crossed. In higher-dimensional settings, the same intuition survives, but the geometry of the basin boundary is essential. A policy intervention need not fail merely because it is unilateral; it fails when the state remains in the basin of the low-activity equilibrium. Conversely, institutional reform, subsidies, expectations management, or coordinated sectoral interventions can succeed if they move the system across the relevant basin boundary. This distinction between local stability, one-dimensional global statements, and full basin computations is central for using evolutionary models of poverty traps in a way that is mathematically precise and economically interpretable.

To reach these goals, the remainder of the paper proceeds as follows. Section 2 introduces the evolutionary-game framework and studies the benchmark follower coordination problem under exogenous institutions, first for a single population and then for two complementary follower roles. Section 3 endogenises the institutional layer by allowing leaders and followers to coevolve, deriving the local stability conditions for joint poverty, joint prosperity, and decoupled outcomes, and clarifying the role of separatrices in basin selection. Section 4 provides a numerical illustration of the main dynamical regimes and their economic interpretation. Section 5 concludes by summarising the mechanism, discussing its policy implications, and outlining directions for future research.

2. The model

There are one or more large, well-mixed populations. In each population, agents choose a strategy $s \in \{H, L\}$, and the state variable is the share of agents choosing H . We use continuous-time replicator dynamics as a compact representation of payoff-monotonic adaptation. In the present context, replication should be read broadly: behaviours with higher realised payoffs spread through imitation, social learning, entry and exit, organisational replacement, or differential growth of firms and activities. This modelling choice is deliberately minimal. It allows

Table 2

Assumptions and scope of the main results.

Result	Main restrictions	Scope of conclusion
Setting A	- Coordination inequalities (4); for hysteresis, existence and uniqueness of λ_L, λ_H and $\Delta_s, \Delta_\lambda > 0$	- Global phase-line result on $[0, 1]$
Setting B	- Symmetric cross-role game and Assumption 1	- Complete rest-point classification; stable saddle branch is the basin boundary
Setting C, local	- $\alpha_i > 0, \beta_i \geq 0, \gamma_i > 0$; additional inequalities stated in Proposition 4	- Local stability of corners and decoupled outcomes
Setting C, global	- Mutual dependence, an interior saddle, and the additional restrictions in Proposition 6	- Sufficient phase-plane theorem; otherwise basin geometry is numerical or case-specific
Setting C, reforms	- Two parameterisations satisfying Proposition 6; new payoff differentials	- Basin of joint prosperity weakly expands, Proposition 7
Setting D	- Linear complementarity payoffs (16), $a_i > 0, b_{ij} \geq 0, \gamma_i > 0$; for Corollary 1, same-layer support condition (19)	- Local pure-profile stability; no stable one-layer-only high-profile state under (19); monotone policy shifts

the analysis to focus on the geometry generated by complementarities rather than on a specific microfoundation for revision opportunities.

Table 2 records the restrictions used in the main results. Its purpose is to prevent the one-dimensional global statements from being read as applying automatically to the higher-dimensional systems.

2.1. Setting A: Single-population followers under exogenous institutions

Let $x \in [0, 1]$ be the share of followers choosing H . Institutional quality is an exogenous parameter λ . The follower payoff matrix is

$$A(\lambda) = \begin{pmatrix} a_{HH}(\lambda) & a_{HL}(\lambda) \\ a_{LH}(\lambda) & a_{LL}(\lambda) \end{pmatrix}. \quad (1)$$

The expected payoff difference between H and L is

$$\begin{aligned} \Delta(x; \lambda) &= \pi_H(x; \lambda) - \pi_L(x; \lambda) \\ &= x[a_{HH}(\lambda) - a_{LH}(\lambda)] + (1 - x)[a_{HL}(\lambda) - a_{LL}(\lambda)]. \end{aligned} \quad (2)$$

The follower dynamic is

$$\dot{x} = x(1 - x)\Delta(x; \lambda). \quad (3)$$

We impose the coordination inequalities

$$a_{HH}(\lambda) > a_{LH}(\lambda), \quad a_{LL}(\lambda) > a_{HL}(\lambda). \quad (4)$$

They say that if the opponent plays H , H is the better reply, while if the opponent plays L , L is the better reply.

Proposition 1 (Critical Mass in Setting A). *Suppose (4) holds. Then (3) has the boundary rest points 0 and 1 and a unique interior rest point*

$$x^*(\lambda) = \frac{a_{LL}(\lambda) - a_{HL}(\lambda)}{[a_{HH}(\lambda) - a_{LH}(\lambda)] + [a_{LL}(\lambda) - a_{HL}(\lambda)]} \in (0, 1). \quad (5)$$

The boundary rest points are locally asymptotically stable and $x^(\lambda)$ is unstable. Moreover, $x(0) < x^*(\lambda)$ implies $x(t) \rightarrow 0$, while $x(0) > x^*(\lambda)$ implies $x(t) \rightarrow 1$.*

Proof. Under (4), $\Delta(0; \lambda) = a_{HL} - a_{LL} < 0$ and $\Delta(1; \lambda) = a_{HH} - a_{LH} > 0$. Since Δ is affine in x , it has exactly one zero in $(0, 1)$, given by (5). Also

$$\frac{\partial \Delta}{\partial x} = [a_{HH} - a_{LH}] + [a_{LL} - a_{HL}] > 0.$$

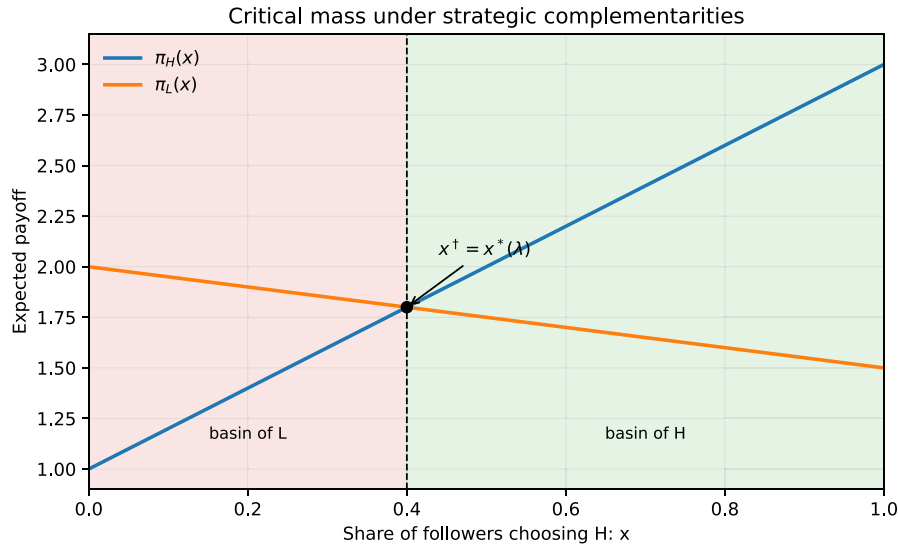


Fig. 2. Strategic complementarities in Setting A. The payoff of H crosses the payoff of L once. The crossing point $x^* = x^*(\lambda)$ is unstable and separates the basin of the low-profile outcome from the basin of broad formalisation.

Thus $\Delta(x; \lambda) < 0$ for $x < x^*(\lambda)$ and $\Delta(x; \lambda) > 0$ for $x > x^*(\lambda)$. The sign of (3) gives the two basins. Local stability follows from $f_x(0; \lambda) = \Delta(0; \lambda) < 0$, $f_x(1; \lambda) = -\Delta(1; \lambda) < 0$, and $f_x(x^*; \lambda) = x^*(1 - x^*)\Delta_x(x^*; \lambda) > 0$. $\square$

Fig. 2 illustrates the critical-mass threshold. In the running example, the left basin corresponds to an informal economy in which individual formalisation is not profitable; the right basin corresponds to broad formalisation sustained by complementarities.

We now let institutions move the relative payoff of H . Define

$$\lambda_L : \Delta(1; \lambda_L) = 0, \quad \lambda_H : \Delta(0; \lambda_H) = 0.$$

The subscript L means that L ceases to be viable when rare at the high-activity boundary; the subscript H means that H becomes viable when rare at the low-activity boundary.

Proposition 2 (Hysteresis in the One-Dimensional Reduction). Suppose $\Delta_x(x; \lambda) > 0$, $\Delta_\lambda(x; \lambda) > 0$, and the two threshold values λ_L and λ_H exist and are unique. Then $\lambda_L < \lambda_H$ and the following regimes hold.

- (i) If $\lambda < \lambda_L$, then $\Delta(x; \lambda) < 0$ for all $x \in [0, 1]$ and 0 is globally asymptotically stable on $[0, 1]$.
- (ii) If $\lambda \in (\lambda_L, \lambda_H)$, both 0 and 1 are locally asymptotically stable and the unstable threshold is $x^*(\lambda)$.
- (iii) If $\lambda > \lambda_H$, then $\Delta(x; \lambda) > 0$ for all $x \in [0, 1]$ and 1 is globally asymptotically stable on $(0, 1]$.

If, in addition, $\Delta_\lambda(0; \lambda_H) \neq 0$ and $\Delta_x(0; \lambda_H) \neq 0$, the branch $x^*(\lambda)$ exchanges stability with the boundary branch $x = 0$ at λ_H through the standard transcritical mechanism. The analogous statement holds at $x = 1$ after the change of variable $u = 1 - x$.

Proof. Because $\Delta_x > 0$, $\Delta(1; \lambda) > \Delta(0; \lambda)$ for every λ . Since $\Delta_\lambda > 0$, the zero of $\Delta(1; \lambda)$ is reached at a lower value of λ than the zero of $\Delta(0; \lambda)$, hence $\lambda_L < \lambda_H$. The three regimes follow directly from the sign of the affine function $\Delta(\cdot; \lambda)$. At λ_H , write $\mu = \lambda - \lambda_H$. Since $\Delta(0; \lambda_H) = 0$,

$$\dot{x} = x(1 - x)[\Delta_\lambda(0; \lambda_H)\mu + \Delta_x(0; \lambda_H)x + o(|\mu| + |x|)].$$

The boundary branch is $x = 0$, whereas the interior branch satisfies

$$x = -\frac{\Delta_\lambda(0; \lambda_H)}{\Delta_x(0; \lambda_H)}\mu + o(\mu).$$

Their stability changes when the branches cross. In the extended one-dimensional system this is the transcritical normal form up to smooth

non-zero rescaling. Restricted to the feasible interval, the same calculation says that the interior threshold collides with the boundary and then exits or enters the simplex. The proof at $x = 1$ is identical after setting $u = 1 - x$. $\square$

The economic implication is deliberately stated in conditional terms. Inside the bistable interval, marginal institutional improvements move the threshold but do not by themselves eliminate the low-activity basin. Escape requires either an expectations or coordination shock that moves x across $x^*(\lambda)$, or an institutional change large enough to pass λ_H . Fig. 3 shows this finite hysteresis loop. Economically, the bifurcation represents a regime shift: incremental institutional improvements shrink the low-activity basin, whereas crossing λ_H eliminates it in the one-dimensional benchmark.⁴

2.2. Setting B: Two follower roles

Many transactions are cross-role rather than within-role: firms match with workers, upstream suppliers with downstream producers, borrowers with lenders, or exporters with customs officials. Let x be the share of role 1 followers choosing H and y the share of role 2 followers choosing H . For a symmetric cross-role game, role 1 payoffs are given by (1), while role 2 payoffs are given by the transpose. The payoff differences are

$$\Delta_1(y; \lambda) = y(a_{HH} - a_{LH}) + (1 - y)(a_{HL} - a_{LL}), \quad (6)$$

$$\Delta_2(x; \lambda) = x(a_{HH} - a_{HL}) + (1 - x)(a_{LH} - a_{LL}), \quad (7)$$

where the dependence on λ is suppressed on the right-hand side. The two-population replicator dynamic is

$$\dot{x} = x(1 - x)\Delta_1(y; \lambda), \quad \dot{y} = y(1 - y)\Delta_2(x; \lambda). \quad (8)$$

Assumption 1 (Cross-Role Coordination). For the parameter value under consideration,

$$a_{HH} > \max\{a_{LH}, a_{HL}\}, \quad a_{LL} > \max\{a_{LH}, a_{HL}\}.$$

⁴ In the example of formalisation, such a shock could take the form of comprehensive judicial reform, the large-scale digitalisation of public administration, temporary tax incentives for formal firms, or a coordinated programme that simultaneously encourages business registration and institutional upgrading.

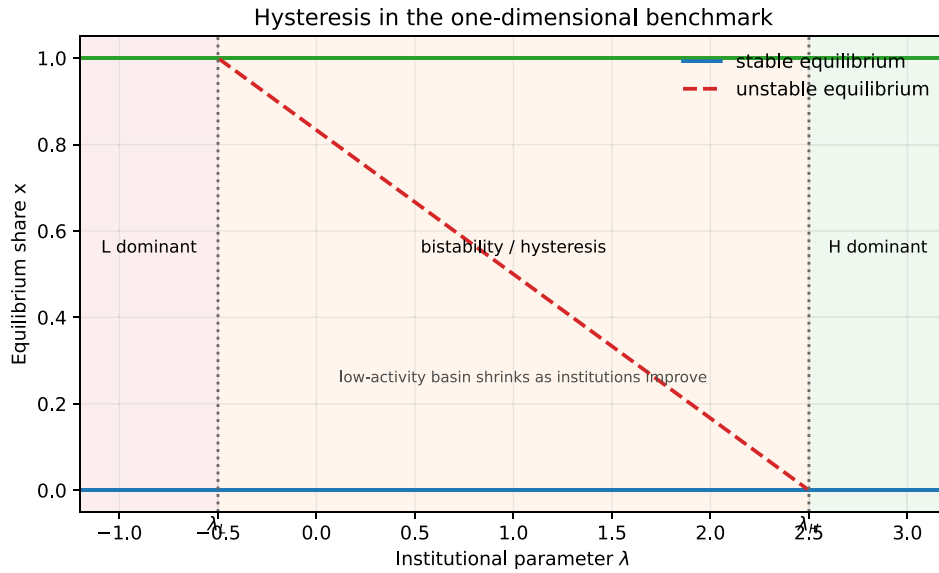


Fig. 3. Hysteresis in the one-dimensional follower dynamic. Solid branches are locally stable and dashed branches are unstable. Between λ_L and λ_H , both boundary outcomes are locally stable; institutional improvements shrink the low-activity basin but do not eliminate it until the upper threshold is crossed.

Proposition 3 (Equilibria and Local Stability in Setting B). *Under Assumption 1, system (8) has no edge equilibria other than the four corners and has one interior equilibrium*

$$x^* = \frac{a_{LL} - a_{LH}}{a_{HH} - a_{HL} - a_{LH} + a_{LL}}, \quad y^* = \frac{a_{LL} - a_{HL}}{a_{HH} - a_{HL} - a_{LH} + a_{LL}}. \quad (9)$$

The corners (0, 0) and (1, 1) are locally asymptotically stable, the corners (1, 0) and (0, 1) are unstable nodes, and (x^*, y^*) is a saddle point.

Proof. At an equilibrium, each factor in (8) must vanish. If $x \in \{0, 1\}$ and $y \in (0, 1)$, then $\dot{y} = 0$ would require $\Delta_2(0) = 0$ or $\Delta_2(1) = 0$. Assumption 1 gives $\Delta_2(0) = a_{LH} - a_{LL} < 0$ and $\Delta_2(1) = a_{HH} - a_{HL} > 0$, so this cannot occur. The same argument excludes edge equilibria with $y \in \{0, 1\}$ and $x \in (0, 1)$. The only edge equilibria are therefore the four corners. The interior equilibrium is obtained from $\Delta_1(y^*) = 0$ and $\Delta_2(x^*) = 0$, yielding (9); Assumption 1 puts both coordinates in (0, 1).

The Jacobian is

$$J(x, y) = \begin{pmatrix} (1-2x)\Delta_1(y) & x(1-x)\Delta_1'(y) \\ y(1-y)\Delta_2'(x) & (1-2y)\Delta_2(x) \end{pmatrix}.$$

At (x^*, y^*) , the diagonal entries vanish, while the off-diagonal entries are strictly positive. Hence $\det(J(x^*, y^*)) < 0$, so the interior rest point is a saddle. At (0, 0) the eigenvalues are $a_{HL} - a_{LL} < 0$ and $a_{LH} - a_{LL} < 0$; at (1, 1) they are $-(a_{HH} - a_{LH}) < 0$ and $-(a_{HH} - a_{HL}) < 0$. At (1, 0) the eigenvalues are $-\Delta_1(0) = -(a_{HL} - a_{LL}) > 0$ and $\Delta_2(1) = a_{HH} - a_{HL} > 0$, and at (0, 1) they are $\Delta_1(1) = a_{HH} - a_{LH} > 0$ and $-\Delta_2(0) = -(a_{LH} - a_{LL}) > 0$. $\square$

The separatrix statement can be made explicit for this separable two-population system, but one must distinguish the stable and unstable saddle branches. Let us then state this formality.

Lemma 1 (First Integral and Stable Branch in Setting B). *On $(0, 1)^2$, define*

$$\Psi(x, y) = \int_{x^*}^x \frac{\Delta_2(s; \lambda)}{s(1-s)} ds - \int_{y^*}^y \frac{\Delta_1(r; \lambda)}{r(1-r)} dr.$$

Then Ψ is constant along every interior trajectory. The critical level set $\Psi(x, y) = \Psi(x^, y^*)$ contains the local stable and unstable branches of the saddle. Only the two branches tangent to the negative-eigenvalue eigenspace of $J(x^*, y^*)$ form the local basin boundary between (0, 0) and (1, 1); the branches tangent to the positive-eigenvalue eigenspace are unstable saddle branches and flow towards the coordinated corners.*

Proof. Along any interior trajectory,

$$\frac{d}{dt} \Psi(x(t), y(t)) = \frac{\Delta_2(x)}{x(1-x)} \dot{x} - \frac{\Delta_1(y)}{y(1-y)} \dot{y} = \Delta_2(x)\Delta_1(y) - \Delta_1(y)\Delta_2(x) = 0.$$

Thus interior trajectories lie on level curves of Ψ . At (x^*, y^*) , both first derivatives of Ψ vanish and the Hessian is diagonal with entries

$$\frac{\Delta_2'(x^*)}{x^*(1-x^*)} > 0, \quad -\frac{\Delta_1'(y^*)}{y^*(1-y^*)} < 0.$$

Hence the critical level set has a saddle crossing. The stable-manifold theorem identifies the two branches tangent to the negative-eigenvalue eigenspace of the vector field as the stable manifold. The remaining branches are the unstable manifold. In the symmetric benchmark, this distinction is visible: the diagonal branch is invariant and unstable, while the transverse branch is the basin boundary. $\square$

This observation is also why the coordinates (x^*, y^*) alone are not a complete description of basin size.

The signs of (8) are organised by the two indifference lines $y = y^*$ and $x = x^*$. The basin boundary between low-low and high-high outcomes is the stable manifold of the saddle, not merely the point (x^*, y^*) . When λ shifts both payoff differences upward, $x^*(\lambda)$ and $y^*(\lambda)$ move down under the corresponding monotonicity conditions; however, the size and shape of a basin depend on the entire stable manifold. Boundary bifurcations occur when one of the indifference lines reaches the boundary of the unit square. In contrast to Setting A, these events need not be generic scalar transcritical bifurcations unless additional non-degeneracy and simultaneity conditions are imposed.⁵

Fig. 4 illustrates this point. The indifference lines locate where each role's payoff difference changes sign, but the economically relevant threshold is the stable branch of the saddle. Hence policy in the two-role follower economy is not characterised by a single participation rate; it depends on the joint location of complementary private roles relative to the basin boundary.

⁵ In the formalisation example, Setting B captures the fact that firms cannot sustain formal investment without skilled and compliant workers, while workers cannot profit from skill acquisition without formal firms. The “critical mass” is therefore a curve in (x, y) -space.

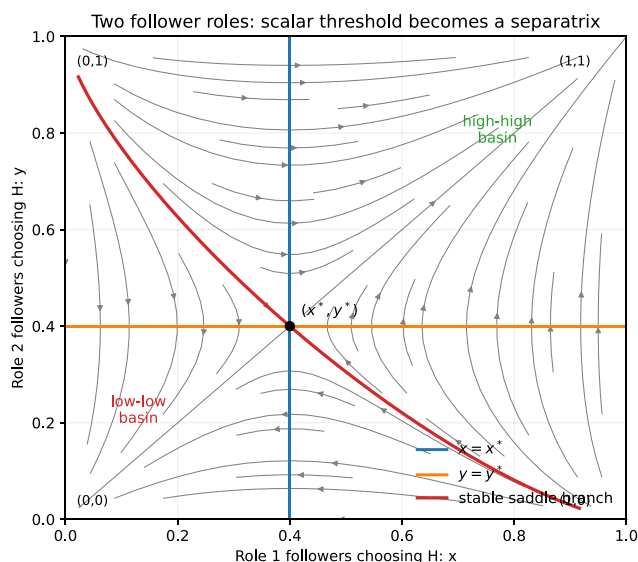


Fig. 4. Two follower roles in Setting B. The scalar threshold of the one-population benchmark becomes a basin boundary in (x, y) -space. The stable saddle branch, not only the coordinate (x^*, y^*) , separates convergence to the low-low and high-high outcomes.

3. Endogenous institutions and coevolution

The institutional parameter is exogenous in Settings A and B. We now let leaders and followers coevolve. Let x_L be the share of leaders choosing high institutional effort and x_F the share of followers choosing the high-profile private action. The leader state should be read as an aggregate adoption variable rather than as a literal count of a single decision maker. High institutional effort can spread through political turnover, bureaucratic promotion, imitation across agencies, diffusion of administrative practices across jurisdictions, or replacement of low-capacity organisations by high-capacity ones. This interpretation keeps the model evolutionary while recognising that the leadership layer is not a mass of price-taking firms in the usual sense. We study

$$\dot{x}_I = x_I(1 - x_I)(\alpha_I x_I + \beta_I x_F - \gamma_I), \quad (10)$$

$$\dot{x}_F = x_F(1 - x_F)(\beta_F x_L + \alpha_F x_F - \gamma_F), \quad (11)$$

where $\alpha_i > 0$ is within-layer complementarity, $\beta_i \geq 0$ is cross-layer feedback, and $\gamma_i > 0$ is the effective cost of adopting H for $i \in \{L, F\}$. The parameter β_F is top-down institutional effectiveness. The parameter β_L is bottom-up fiscal, legitimacy, or compliance feedback.

A minimal enforcement/tax-base interpretation gives these reduced-form terms directly. Let the leader's payoff gain from high institutional effort rather than low effort be

$$\underbrace{b_L x_L}_{\text{bureaucratic complementarity}} + \underbrace{\tau x_F}_{\text{tax base, legitimacy, compliance}} - c_L,$$

bureaucratic complementarity tax base, legitimacy, compliance

and let the follower's payoff gain from formal high-profile behaviour rather than informality be

$$\underbrace{b_F x_F}_{\text{formal-sector complementarity}} + \underbrace{e x_L}_{\text{enforcement and public goods}} - c_F.$$

formal-sector complementarity enforcement and public goods

Eqs. (10)–(11) are obtained by setting $\alpha_L = b_L$, $\beta_L = \tau$, $\gamma_L = c_L$, $\alpha_F = b_F$, $\beta_F = e$, and $\gamma_F = c_F$. Thus the coefficients are not behavioural primitives: they summarise institutional technology, fiscal returns, formal-sector network effects, and the cost of high effort.

This also separates different policy mechanisms. Institutional reform that makes formal activity more profitable raises β_F or lowers

γ_F ; administrative investment or anti-corruption reform lowers γ_L or raises α_L ; tax-base improvements and compliance feedback raise β_I ; subsidies to firms or workers lower γ_F ; and expectation-management or direct coordination policies move the state variables x_L and x_F rather than only changing a parameter. These mechanisms can have different effects on the basin boundary.

Proposition 4 (Corner Stability and Decoupling). *For system (10)–(11), the eigenvalues of the Jacobian matrix at each of the four corners are*

<i>Corner</i>	<i>Eigenvalues of the Jacobian matrix</i>
(0, 0)	$-\gamma_L, -\gamma_F$
(1, 1)	$-(\alpha_L + \beta_L - \gamma_L), -(\alpha_F + \beta_F - \gamma_F)$
(1, 0)	$-(\alpha_L - \gamma_L), \beta_F - \gamma_F$
(0, 1)	$\beta_L - \gamma_L, -(\alpha_F - \gamma_F)$

Hence $(0, 0)$ is always locally asymptotically stable. If

$$\alpha_i < \gamma_i < \alpha_i + \beta_i, \quad i \in \{L, F\}. \quad (12)$$

then $(1, 1)$ is also locally asymptotically stable and the decoupled corners are unstable. Moreover, $(1, 0)$ is locally asymptotically stable if and only if $\alpha_L > \gamma_L$ and $\beta_F < \gamma_F$; symmetrically, $(0, 1)$ is locally asymptotically stable if and only if $\alpha_F > \gamma_F$ and $\beta_L < \gamma_L$.

Proof. Let $P_L(x_L, x_F) = \alpha_L x_L + \beta_L x_F - \gamma_L$ and $P_F(x_L, x_F) = \beta_F x_L + \alpha_F x_F - \gamma_F$. The Jacobian is

$$\begin{pmatrix} (1-2x_L)P_L + \alpha_L x_L(1-x_L) & \beta_L x_L(1-x_L) \\ \beta_F x_F(1-x_F) & (1-2x_F)P_F + \alpha_F x_F(1-x_F) \end{pmatrix}.$$

At every corner, the off-diagonal entries vanish, so the eigenvalues are the diagonal entries listed in the table. The stated stability conditions follow immediately. $\square$

Condition (12) formalises mutual dependence. Neither layer can sustain the high action alone, but joint adoption makes high effort worthwhile. This rules out stable decoupling locally. Outside this mutual-dependence region, stable decoupled outcomes are economically meaningful rather than anomalous. A stable $(1, 0)$ can represent persistent institutional effort without private formalisation, for example when the state has strong internal incentives to enforce rules but followers still face high formal-sector costs. A stable $(0, 1)$ can represent private high-profile activity under weak institutions, for example enclaves of formal investment that are sustained by within-sector complementarities rather than by public capacity. These cases are useful precisely because they show why the hierarchy matters. Condition (12) does not, by itself, prove a global statement about all interior trajectories; global claims require the basin boundary.

Proposition 5 (*Interior Rest Point and Separatrix Qualification*). Let $D = \alpha_I \alpha_E - \beta_I \beta_E$. If $D \neq 0$, any interior rest point must be

$$x_L^* = \frac{\alpha_F \gamma_L - \beta_L \gamma_F}{D}, \quad x_F^* = \frac{\alpha_L \gamma_F - \beta_F \gamma_L}{D}. \quad (13)$$

If this point lies in $(0, 1)^2$, then its Jacobian has trace

$$T = \alpha_I x_I^* (1 - x_I^*) + \alpha_F x_F^* (1 - x_F^*) > 0$$

and determinant

$$\det(J(x_I^*, x_E^*)) = x_I^*(1 - x_I^*)x_E^*(1 - x_E^*)D.$$

Consequently, if $D < 0$, the interior rest point is a saddle and its stable manifold is a local separatrix. If $D > 0$, the interior rest point is unstable with positive real-part eigenvalues, and basin conclusions cannot be inferred from a stable manifold of that point. If $D = 0$, the two nullclines are parallel or coincident; hence there is either no interior rest point or a continuum of interior rest points on the common nullcline, and the preceding saddle/source classification does not apply.

Proof. At an interior rest point the two payoff differentials vanish, giving the linear system

$$\alpha_L x_L + \beta_L x_F = \gamma_L, \quad \beta_F x_L + \alpha_F x_F = \gamma_F.$$

If $D \neq 0$, Cramer's rule yields (13). If $D = 0$, the two affine nullclines are either parallel or coincident, which gives the degenerate alternatives stated in the proposition. At the interior point, the terms multiplying $(1 - 2x_i)$ vanish in the Jacobian. The remaining matrix is

$$J(x_L^*, x_F^*) = \begin{pmatrix} \alpha_L r_L & \beta_L r_L \\ \alpha_F r_F & \alpha_F r_F \end{pmatrix}, \quad r_i = x_i^*(1 - x_i^*) > 0.$$

The stated trace and determinant follow. If $D < 0$, the determinant is negative, hence the eigenvalues have opposite signs. If $D > 0$, the determinant and trace are positive, so both eigenvalues have positive real parts. $\square$

Proposition 5 is deliberately local. It identifies the interior rest point, classifies it by the sign of $D = \alpha_L \alpha_F - \beta_L \beta_F$, and shows when this point can generate a local separatrix. This is not yet a global basin result. To use the stable manifold as the boundary between joint poverty and joint prosperity, one must also rule out attracting decoupled corners, additional attracting boundary rest points, and recurrent interior dynamics. The next proposition therefore adds explicit mutual-dependence and boundary-saddle restrictions and uses a Bendixson–Dulac argument to obtain a sufficient global phase-plane statement.

Proposition 6 (A Sufficient Global Phase-Plane Result). Assume (12), $\beta_L < \gamma_L$, $\beta_F < \gamma_F$, $D < 0$, and that the interior rest point $E = (x_L^*, x_F^*)$ lies in $(0, 1)^2$. Then the only locally attracting rest points are $(0, 0)$ and $(1, 1)$, the interior rest point is a saddle, and there are no periodic orbits in the interior of the square. Every interior trajectory that is not on the stable manifold $W^s(E)$ converges to either $(0, 0)$ or $(1, 1)$. The stable manifold $W^s(E)$ is therefore the basin boundary. In the boundary-saddle case covered by $\beta_L < \gamma_L$ and $\beta_F < \gamma_F$, its two branches have backward limits at the off-diagonal corners $(0, 1)$ and $(1, 0)$.

Proof. The unit square is positively invariant because each equation contains the factor $x_i(1 - x_i)$. The corner classification follows from Proposition 4: under (12), $(0, 0)$ and $(1, 1)$ are locally asymptotically stable, while the additional inequalities $\beta_L < \gamma_L$ and $\beta_F < \gamma_F$ make the off-diagonal corners boundary saddles rather than sinks. Proposition 5 gives a unique interior saddle because $D < 0$ and $E \in (0, 1)^2$.

There may also be boundary rest points on the top and right edges. On the bottom edge, $x_F = 0$, the leader payoff differential is $\alpha_L x_L - \gamma_L < 0$ for all $x_L \in [0, 1]$ because $\gamma_L > \alpha_L$. Hence there is no non-corner rest point on the bottom edge. Similarly, there is no non-corner rest point on the left edge. On the top edge, a boundary rest point is

$$T = \left(\frac{\gamma_L - \beta_L}{\alpha_L}, 1 \right),$$

and on the right edge a boundary rest point is

$$R = \left(1, \frac{\gamma_F - \beta_F}{\alpha_F} \right).$$

When these points lie on their respective edges, they are not attracting from the interior. Indeed, the tangential eigenvalue at T is positive, and the transverse eigenvalue is

$$-\left[\beta_F \frac{\gamma_L - \beta_L}{\alpha_L} + \alpha_F - \gamma_F \right] > 0,$$

where the inequality follows from $D < 0$ and $x_F^* \in (0, 1)$. The argument for R is symmetric. Thus no interior trajectory converges to these boundary points.

It remains to rule out other recurrent behaviour. On $(0, 1)^2$, use the Dulac function

$$B(x_L, x_F) = \frac{1}{x_L(1 - x_L)x_F(1 - x_F)}.$$

For the vector field in (10)–(11),

$$\frac{\partial}{\partial x_L}(B \dot{x}_L) + \frac{\partial}{\partial x_F}(B \dot{x}_F) = \frac{\alpha_L}{x_F(1 - x_F)} + \frac{\alpha_F}{x_L(1 - x_L)} > 0.$$

The Bendixson–Dulac criterion excludes periodic orbits and other compact recurrent curves in the interior. By the Poincaré–Bendixson theorem for planar systems, an interior omega-limit set must therefore be an equilibrium. The stable sets of the off-diagonal boundary saddles lie on the boundary, so an interior trajectory can converge only to $(0, 0)$, $(1, 1)$, or E . The set of initial states converging to E is exactly $W^s(E)$. Since the two attracting basins are open and nonempty and the flow is continuous, $W^s(E)$ is their common boundary. The branch endpoint statement follows from the boundary flow and the absence of any additional invariant set on which a branch could accumulate. $\square$

The global result above is a statement about a given vector field. Policy analysis, however, requires comparing two vector fields: the pre-reform system and the post-reform system. Such a comparison is meaningful only under common phase-plane restrictions ensuring that the relevant basin boundary is the stable manifold of an interior saddle in both systems. Proposition 7 therefore studies payoff-improving reforms under these maintained restrictions and establishes the corresponding monotonicity of the high-development basin.

Proposition 7 (Basin Monotonicity Under Payoff-Improving Reforms). Consider two parameter vectors, indexed by $k \in \{0, 1\}$, for system Eq. (10)–Eq. (11). Let

$$P_L^k(x_L, x_F) = \alpha_L^k x_L + \beta_L^k x_F - \gamma_L^k, \quad P_F^k(x_L, x_F) = \beta_F^k x_L + \alpha_F^k x_F - \gamma_F^k.$$

Assume that both systems satisfy the hypotheses of Proposition 6. If

$$P_L^1(x_L, x_F) \geq P_L^0(x_L, x_F), \quad P_F^1(x_L, x_F) \geq P_F^0(x_L, x_F)$$

for all $(x_L, x_F) \in [0, 1]^2$,

then the basin of attraction of $(1, 1)$ for system 0 is contained in the basin of attraction of $(1, 1)$ for system 1. Equivalently, payoff-improving reforms weakly expand the set of initial conditions that converge to joint prosperity and weakly shrink the basin of joint poverty.

Proof. Let F^k denote the vector field of system k . Condition Eq. (14) implies $F_i^1(x) \geq F_i^0(x)$ for each component $i \in \{L, F\}$ and every $x \in [0, 1]^2$, because the factors $x_i(1 - x_i)$ are non-negative. Each system is cooperative in the interior because the cross-partial derivatives $\beta_L^k x_L(1 - x_L)$ and $\beta_F^k x_F(1 - x_F)$, which are non-negative. The Kamke comparison argument for cooperative differential equations therefore gives

$$\phi_t^1(x_0) \geq \phi_t^0(x_0) \quad \text{componentwise for all } t \geq 0,$$

where ϕ_t^k is the flow of system k from the same initial condition x_0 . If x_0 is in the basin of $(1, 1)$ under system 0, then $\phi_t^0(x_0) \rightarrow (1, 1)$. For every $\varepsilon > 0$, there is T such that $\phi_t^0(x_0) \geq (1 - \varepsilon, 1 - \varepsilon)$ for all $t \geq T$. The comparison inequality then implies $\phi_t^1(x_0) \geq (1 - \varepsilon, 1 - \varepsilon)$ for all $t \geq T$. Because $(1, 1)$ is locally asymptotically stable for system 1, choose $\varepsilon > 0$ small enough that the rectangle $[1 - \varepsilon, 1]^2$ is contained in its local basin of attraction. The comparison inequality implies $\phi_t^1(x_0) \in [1 - \varepsilon, 1]^2$ for all sufficiently large t . Hence the trajectory of system 1 converges to $(1, 1)$. Therefore every initial state in the old prosperity basin remains in the prosperity basin after the payoff-improving reform. $\square$

Proposition 5 also corrects an important possible overstatement. The location of (x_L^*, x_F^*) is informative, but the basin of attraction of the poverty trap depends on the entire separatrix. In the saddle case, the stable manifold must be computed analytically or numerically. Fig. 5 does this for the symmetric mutual-dependence parameterisation $\alpha_L = \alpha_F = 3$, $\beta_L = \beta_F = 4$, and $\gamma_L = \gamma_F = 4.5$. The interior saddle is at $(4.5/7, 4.5/7)$, and the stable manifold connects the off-diagonal corners. A unilateral reform that jumps to $x_L = 1$ succeeds if the new point lies in the basin of $(1, 1)$ and fails otherwise; the model therefore recommends crossing the basin boundary, not unilateral action per se.

The diagonal in Fig. 5 is shown because the illustrative parameters are symmetric. The next lemma states exactly what symmetry buys.

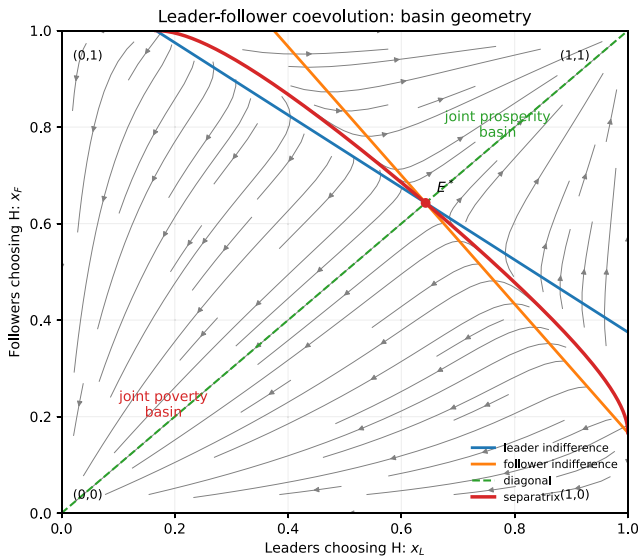


Fig. 5. Coevolutionary dynamics in Setting C for $\alpha_L = \alpha_F = 3$, $\beta_L = \beta_F = 4$, and $\gamma_L = \gamma_F = 4.5$. The shaded regions indicate the basins of joint poverty and joint prosperity. The interior rest point is a saddle and the plotted stable manifold is the separatrix between the two basins.

Lemma 2 (Diagonal Invariance Under Symmetry). Assume $\alpha_L = \alpha_F = \alpha$, $\beta_L = \beta_F = \beta$, and $\gamma_L = \gamma_F = \gamma$. Then the diagonal $x_L = x_F$ is invariant. For a point (t, t) on the diagonal, deviations transverse to the diagonal contract locally if

$$q(t) = (3\alpha + \beta)t^2 - 2(\alpha + \gamma)t + \gamma > 0. \quad (15)$$

For the numerical parameters in Fig. 5, $q(t) = 13t^2 - 15t + 4.5 > 0$ for all $t \in [0, 1]$.

Proof. Under symmetry, interchanging x_L and x_F leaves the vector field unchanged, so $x_L = x_F$ is invariant. Let $d = x_L - x_F$. Direct subtraction gives

$$\dot{d} = -d[\alpha(x_L^2 + x_L x_F + x_F^2 - x_L - x_F) + \beta x_L x_F - \gamma(x_L + x_F) + \gamma].$$

At $x_L = x_F = t$, the expression in braces equals $q(t)$. Thus $\dot{d} = -q(t)d + o(d)$, which proves local transverse contraction when $q(t) > 0$. For $\alpha = 3$, $\beta = 4$, and $\gamma = 4.5$, the discriminant of $13t^2 - 15t + 4.5$ is negative and the leading coefficient is positive; hence it is positive on $[0, 1]$. $\square$

Moreover, Fig. 6 translates the same phase-plane geometry into policy language. A reform that mainly raises institutional effort can still fail when private formalisation remains too low and the new state stays below the separatrix. By contrast, a coordinated programme that moves both the institutional and private-sector components can cross the basin boundary and trigger convergence to joint prosperity. Proposition 7 adds a comparative-dynamic qualification: when a reform raises the payoff differential of the high-profile action, the old prosperity basin is not lost, and the poverty basin can only shrink under the maintained phase-plane assumptions. This is stronger than the statement that a particular shock crosses a particular boundary, but it still does not provide a closed-form formula for the separatrix.

3.1. Setting D: A multi-role hierarchical extension

The preceding coevolutionary model has one leadership population and one follower population. We now show how the same logic extends to a richer hierarchy with two leadership roles and two follower roles. The purpose of this extension is not to provide a complete

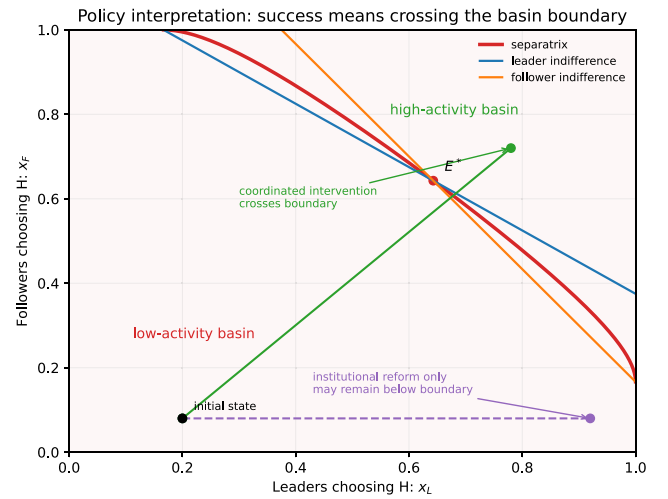


Fig. 6. Policy interpretation in the leader-follower model. A policy shock succeeds when it moves the state across the separatrix into the basin of joint prosperity. A reform that changes only one side of the hierarchy can still fail if the new state remains in the low-activity basin.

global classification in four dimensions, but to record the local stability and comparative-statics implications of a multi-role hierarchical complementarity structure.

Let

$$z = (z_{L_1}, z_{L_2}, z_{F_1}, z_{F_2}) \in [0, 1]^4,$$

where z_i is the share of population i choosing the high-profile action. The leadership roles may represent enforcement and public-good provision, while the follower roles may represent firms and workers, or upstream and downstream private sectors. Let $I = \{L_1, L_2, F_1, F_2\}$. For each population $i \in I$, let the payoff difference between the high and low profile be

$$\Delta_i(z) = a_i z_i + \sum_{j \neq i} b_{ij} z_j - \gamma_i, \quad a_i > 0, \quad b_{ij} \geq 0, \quad \gamma_i > 0. \quad (16)$$

The coefficient a_i captures within-population complementarity, b_{ij} captures the effect of high-profile behaviour in population j on the payoff of high-profile behaviour in population i , and γ_i is the effective cost of adopting the high profile. The four-population replicator system is

$$\dot{z}_i = z_i(1 - z_i)\Delta_i(z), \quad i \in I. \quad (17)$$

This specification nests the one-leader-one-follower model as a two-dimensional special case. It also makes explicit the sense in which the hierarchy is multi-role: cross-layer terms b_{ij} connect leadership and follower roles, while same-layer terms can capture coordination within the public or private side. The next result is the standard boundary linearisation for multipopulation replicator dynamics, stated in the notation of the present hierarchy (Weibull, 1995; Hofbauer and Sigmund, 1998; Sandholm, 2010).

Proposition 8 (Pure-Profile Stability in the Multi-Role Hierarchy). Consider system (17). For any pure profile $z^S \in \{0, 1\}^4$, where $S \subseteq I$ is the set of populations choosing H , the Jacobian at z^S is diagonal. Moreover, z^S is locally asymptotically stable if and only if

$$\Delta_i(z^S) > 0 \quad \text{for all } i \in S, \quad \Delta_i(z^S) < 0 \quad \text{for all } i \notin S. \quad (18)$$

Equivalently, every incumbent high-profile population must prefer H to a rare deviation to L , and every incumbent low-profile population must prefer L to a rare deviation to H .

Proof. For $i \neq j$,

$$\frac{\partial \dot{z}_i}{\partial z_j} = z_i(1 - z_i)b_{ij},$$

which equals zero at any pure profile because $z_i \in \{0, 1\}$. The Jacobian is therefore diagonal at z^S . The own derivative is

$$\frac{\partial \dot{z}_i}{\partial z_i} = (1 - 2z_i)\Delta_i(z) + z_i(1 - z_i)a_i.$$

At $z_i = 0$, this derivative equals $\Delta_i(z^S)$. At $z_i = 1$, it equals $-\Delta_i(z^S)$. Local asymptotic stability requires all diagonal entries to be negative, which is exactly condition (18). $\square$

Proposition 8 is local, but it already delivers a substantive implication for hierarchical coordination. Let $\mathcal{L} = \{L_1, L_2\}$ and $\mathcal{F} = \{F_1, F_2\}$ denote the leadership and follower layers. If same-layer support alone is insufficient to compensate the cost of the high profile, then a pure state in which only one layer coordinates on H cannot be locally stable.

Corollary 1 (No Stable One-Layer-Only High-Profile State). *Suppose that high-profile behaviour cannot be sustained by within-layer support alone:*

$$a_i + \sum_{\substack{j \neq i \\ j \in \mathcal{L}}} b_{ij} < \gamma_i \quad \text{for every } i \in \mathcal{L}, \quad a_i + \sum_{\substack{j \neq i \\ j \in \mathcal{F}}} b_{ij} < \gamma_i \quad \text{for every } i \in \mathcal{F}. \quad (19)$$

Then no pure profile in which all high-profile populations are contained entirely in one layer is locally asymptotically stable.

Proof. Consider a pure profile z^S in which $S \subseteq \mathcal{L}$ or $S \subseteq \mathcal{F}$. If $S = \emptyset$, this is the all-low profile and is not a one-layer-only high-profile state. If S is nonempty and contained in one layer, then for any $i \in S$,

$$\Delta_i(z^S) \leq a_i + \sum_{\substack{j \neq i \\ j \text{ in the same layer as } i}} b_{ij} - \gamma_i < 0.$$

Thus at least one incumbent high-profile population has an incentive to deviate to L , so the pure profile fails the stability condition in **Proposition 8**. $\square$

The corollary formalises the economic idea that institutional effort without private formalisation, or private formalisation without institutional support, may be locally fragile when high-profile activity relies on cross-layer complementarities. This does not rule out stable decoupled outcomes in general; it rules them out under the explicit “same layer is not enough” restriction (19). If that restriction fails, stable partial hierarchies can exist and should be interpreted as genuine regimes rather than anomalies.

Proposition 9 (Monotonicity and Policy Shifts). *If $b_{ij} \geq 0$ for all $i \neq j$, then the interior vector field of (17) is cooperative:*

$$\frac{\partial \dot{z}_i}{\partial z_j} = z_i(1 - z_i)b_{ij} \geq 0, \quad i \neq j. \quad (20)$$

Moreover, lowering γ_i , increasing a_i , or increasing some b_{ij} weakly raises the payoff differential Δ_i of population i . Hence subsidies or reforms that lower the cost of high-profile behaviour, reforms that strengthen within-role spillovers, and reforms that strengthen cross-layer complementarities move the corresponding nullcline in the direction favourable to high-profile adoption.

Proof. Eq. (20) follows by direct differentiation of (17). The comparative statics follow from

$$\frac{\partial \Delta_i}{\partial \gamma_i} = -1, \quad \frac{\partial \Delta_i}{\partial a_i} = z_i \geq 0, \quad \frac{\partial \Delta_i}{\partial b_{ij}} = z_j \geq 0.$$

Thus a lower cost parameter or a higher complementarity coefficient weakly increases the payoff advantage of H for the affected population at every state. $\square$

Setting D is deliberately modest. It records which parts of the argument survive in a multi-role hierarchy without making global four-dimensional basin claims. The rare-deviation stability test, the fragility of one-layer-only high-profile coordination, and the direction of policy-induced nullcline shifts extend directly once several public and private roles interact. A complete description of four-dimensional basin boundaries would require additional restrictions or numerical computation and is left for future work; the core global results of the paper remain the two-dimensional leader–follower results in **Propositions 6** and **7**.

4. Numerical illustration

This section is a numerical illustration, not an empirical calibration. The parameter λ is put on a WGI-like scale only to make the institutional axis interpretable. The Worldwide Governance Indicators report governance dimensions, including Government Effectiveness, in standard-normal units with higher values denoting better governance (World Bank, 2025a,b). The Human Development Index summarises achievements in health, education, and income as a geometric mean of normalised dimension indices (United Nations Development Programme, 2025). These sources motivate the language of low, intermediate, and high institutional environments; they are not used here to estimate the payoff matrix, rank countries, or assign any country to a basin of attraction.⁶

Consider the payoff matrix

$$A(\lambda) = \begin{pmatrix} 12 + 2\lambda & 5 + 2\lambda \\ 11 & 10 \end{pmatrix}. \quad (21)$$

The constants 12, 5, 11, 10 and the slope 2 are deliberately simple. They keep payoffs positive on the WGI-like interval used in the figure and place both thresholds within the WGI-like scale used here. Institutional quality raises the relative return to the high-profile action, including the payoff of investing when others do not. The payoff difference is

$$\Delta(x; \lambda) = x(1 + 2\lambda) + (1 - x)(2\lambda - 5),$$

so the lower and upper thresholds are

$$\lambda_L = -0.5, \quad \lambda_H = 2.5, \quad x^*(\lambda) = \frac{5 - 2\lambda}{6} \quad \text{for } \lambda \in (-0.5, 2.5).$$

Thus, for $\lambda < -0.5$, the low-profile action is strictly dominant; for $-0.5 < \lambda < 2.5$, the system is bistable; and for $\lambda > 2.5$, the high-profile action is strictly dominant.

Table 3 reports exactly which values are used in **Fig. 7** and why. The cases are not economies or country groups. They are three points in parameter space.

Fig. 7 plots the vector field for these three archetypes. At $\lambda = -2.00$, the vector field is negative throughout the interior and the economy converges to the low-profile state. At $\lambda = -0.55$, the economy is close to the lower threshold but remains in the low-dominance region, showing that being near a threshold is not the same as crossing it. At $\lambda = 1.80$, the system is bistable and the critical mass is $x^*(1.80) = 1.4/6 \approx 0.233$. This last case illustrates a strong but not decisive institutional environment: institutions have reduced the size of the poverty-trap basin, but have not eliminated it.

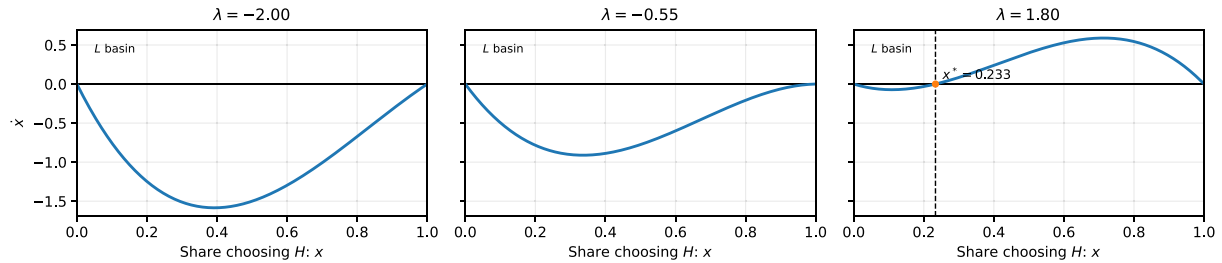
Together, **Figs. 2–7** show how the same economic mechanism becomes geometrically richer as the model moves from a benchmark coordination game to a hierarchy. In Setting A, the threshold is a point; in Setting B and Setting C, it is a basin boundary. The interpretation is therefore not that institutions alone determine development, but that

⁶ A country calibration would require at least three additional steps: a country-year measure of institutional quality, such as Government Effectiveness or Rule of Law; a country-year measure of the adoption state x , such as formal-sector participation, tax compliance, schooling, or investment; and an estimation or normalisation procedure that maps those observables into payoff differences. None of these steps is undertaken in this paper.

Table 3

Parameter values used in the numerical illustration. The entries are archetypes, not country estimates.

Case	λ	Dynamical regime	Reason for inclusion
Low-institution archetype	-2.00	L strictly dominant	Severe institutional environment
Near-threshold archetype	-0.55	L strictly dominant	Just below $\lambda_L = -0.5$
High-institution archetype	1.80	Bistable	Small low-activity basin, $x^* = 0.233$

**Fig. 7.** Numerical vector fields for the payoff matrix (21). Each panel plots $\dot{x} = x(1-x)\Delta(x; \lambda)$ for a different institutional parameter. Negative values imply movement towards the low-profile state, positive values imply movement towards formalisation, and zeros identify rest points. The curves are archetypal parameter cases, not country estimates.

institutional reform and private formalisation must be evaluated jointly through their effect on the position of the state relative to the relevant separatrix.

Another exercise is to consider that the baseline system uses

$$\alpha_L = \alpha_F = 3, \quad \beta_L = \beta_F = 4, \quad \gamma_L = \gamma_F = 4.5.$$

These values satisfy the mutual-dependence inequalities (12), have $D = \alpha_L \alpha_F - \beta_L \beta_F = -7 < 0$, and place the interior saddle at

$$E^0 = \left(\frac{4.5}{7}, \frac{4.5}{7} \right) \approx (0.643, 0.643).$$

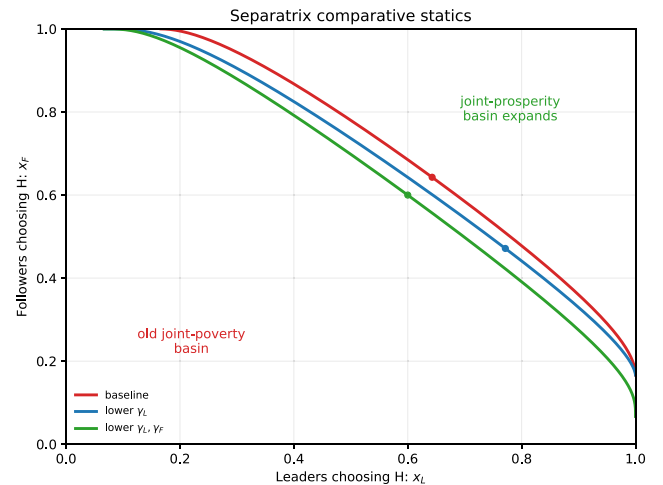
Thus the hypotheses of Proposition 6 hold. The stable manifold of E^0 separates the basin of joint poverty from the basin of joint prosperity.

Fig. 8 compares this baseline with two reforms. The first is a leadership-cost reform, γ_L falling from 4.5 to 4.2, holding the other parameters fixed. The second is a coordinated cost reform, $\gamma_L = \gamma_F = 4.2$. Both reforms raise the payoff differential of high-profile behaviour in at least one equation and preserve the phase-plane assumptions. Proposition 7 therefore implies that the old prosperity basin is contained in the new prosperity basin. The figure illustrates the result geometrically: the separatrix moves so that more initial conditions converge to (1, 1). The unilateral reform changes the boundary unevenly, whereas the coordinated reform shifts it more symmetrically.

The numerical exercise also clarifies the policy language used in the paper. A reform may improve payoffs and still fail from a particular initial state if the state remains below the post-reform separatrix. Conversely, the reform need not be coordinated in order to succeed: a sufficiently large one-sided reform can move either the boundary or the state enough to place the economy in the basin of joint prosperity. What the model rules out is the inference that better institutions alone mechanically imply convergence to prosperity from every initial condition.

5. Concluding remarks

This paper has developed a hierarchical evolutionary-game account of poverty traps and growth transitions. The central mechanism is vertical coordination. A leadership layer changes institutional payoffs, while a follower layer chooses whether to undertake costly high-profile private actions. When payoffs exhibit complementarities, both the low-activity and the high-activity configurations can be locally stable. Institutions then matter not only because they change current payoffs, but also because they move the boundary between basins of attraction.

**Fig. 8.** Separatrix comparative statics in the leader-follower model. The baseline uses $\alpha_L = \alpha_F = 3$, $\beta_L = \beta_F = 4$, and $\gamma_L = \gamma_F = 4.5$. The leadership-cost reform lowers γ_L to 4.2. The coordinated cost reform lowers both γ_L and γ_F to 4.2. The plotted stable manifolds show the basin boundary between joint poverty and joint prosperity. The old prosperity basin is weakly contained in the new one under payoff-improving reforms.

The nested models clarify what can and cannot be concluded from this mechanism. In the one-dimensional follower economy, the critical mass is explicit and the hysteresis result is global on the unit interval. In the two-role follower economy, the threshold becomes a separatrix in the unit square, generated by the stable manifold of the interior saddle under the maintained assumptions. In the leader-follower model, local stability of the corners is governed by simple inequalities, but global basin claims require additional information. When the interior rest point is a saddle, its stable manifold is the natural separatrix; when it is not a saddle, the location of that rest point alone is insufficient for policy or comparative-dynamic conclusions. The multi-role extension shows that the same rare-deviation logic can be applied to several leadership and follower roles, and that one-layer-only high-profile coordination is unstable under explicit cross-layer-dependence restrictions.

The policy implication is therefore more nuanced than the slogan that development requires a “big push”. Marginal reforms may fail inside a low-activity basin, but a unilateral reform can succeed if it

moves the economy across the relevant basin boundary. The relevant object is not the policy instrument by itself, but the induced displacement of the state in the full hierarchical system. Future work could introduce stochastic shocks, heterogeneous revision protocols, network structure, more than two actions, and empirical procedures for mapping observed measures of state capacity, formalisation, and investment into the payoff differences and cross-layer feedback parameters studied here.

CRedit authorship contribution statement

Elena A. Gubar: Writing – review & editing, Writing – original draft, Investigation, Formal analysis, Conceptualization. **Livia Owen:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **Edgar J. Sanchez Carrera:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **Tatyana V. Gerschuk:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Conceptualization.

Declaration of competing interest

The authors that they have NO affiliations with or involvement in any organisation or entity with any financial interest (such as honoraria, educational grants; participation in speakers' bureaus, membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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Data availability

No data was used for the research described in the article.

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