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## University admission: Is achievement a sufficient criterion?

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## Research Article

# Alessandro Tampieri* <br> University Admission: Is Achievement a Sufficient Criterion? 

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#### Abstract

We analyse university admissions using a statistical discrimination model where students differ by ability and social group. In this university system, candidates are evaluated on the basis of their expected human capital, which includes both their innate abilities and the knowledge acquired during their schooling. Consequently, students determine their study effort based on the behaviour of universities. Interestingly, we find that students from a less advantaged group need a lower grade to gain admission to the best universities. If a university cannot discriminate between social groups, all students with the same grade will attend universities of the same quality, but with different levels of human capital.


Keywords: discrimination; affirmative action; study effort
JEL Classification: I21; I23; J71

## 1 Introduction

Should universities base their admissions decisions solely on objective measures of achievement such as exam scores and entrance tests, or should other irrelevant factors such as race, gender, the secondary school attended or family social background also play a role in the selection process? While most observers tend to view

[^0][^1]university policies that favour certain applicants on the basis of gender, age, race or social class as clearly discriminatory, many leading universities still engage in such practices. In the United States, the debate revolves around the issue of affirmative action for ethnic minorities. ${ }^{1}$ In the United Kingdom, the focus, however, shifts to the social background and the type of school, whether private or state-funded, attended by the applicants. ${ }^{2}$

The adoption of affirmative action policies in university admission is founded on principles of social justice, aiming to support minorities or disadvantaged groups. However, our paper presents an alternative perspective by showing that a university, guided solely by the objective of admitting the most qualified candidates, would opt for different admission standards for various groups. This finding arises from the well-established dual role of education, which not only enhances human capital but also serves as a means to signal an individual's ability. ${ }^{3}$

We develop a model of statistical discrimination in which students attend school and subsequently undergo a final school test that shapes their university prospects. Students possess varying levels of ability, directly influencing their test outcomes, and are categorised into different social groups. Within each group, the distribution of ability differs: in line with empirical findings, students from higher socio-economic backgrounds exhibit lower variance compared to those from less privileged households. ${ }^{4}$ In addition, we assume that the result of the test is affected by an idiosyncratic "luck of the day". ${ }^{5}$

The university system accommodates students according to their expected human capital, which comprises a combination of ability and the effort they put into their studies whilst in school. Specifically, the most prestigious universities aim at admitting students with the highest human capital. These institutions have access to information about a student's social group and their test results. Using this data, universities form their beliefs about the student's human capital. On the other hand, students understand how universities operate and, based on this understanding, adjust their study efforts accordingly.

[^2]The main finding of our study is that students from groups with a more variable or uncertain signal require a lower grade to gain admission to the top universities. Interestingly, this result contradicts the established model of statistical discrimination in the labour market, as proposed by Phelps (1972) and Lundberg and Startz (1983). In our model, these students from less advantaged groups seem to experience favourable discrimination.

Our result is a consequence of the nature of the information structure and the process of human capital acquisition we postulate. We assume that both human capital and test scores are affected by studying at school and by ability. However, studying has a "comparative advantage" in affecting the school test: an increase in school learning that compensates in the school test exactly for lower ability would not be sufficient to compensate for the reduction in human capital. In other words, given that universities do not observe ability directly but must infer it from the exam results, Spence signalling operates. If a student from a social group characterised by low ability variance achieves a high score on the exam, they are more likely to be perceived as fortunate rather than possessing exceptional ability. On the other hand, a student from a group with higher ability variance who performs well on the exam is more likely to be perceived as genuinely talented. Consequently, when ability variance is lower, a test score becomes less informative and should be more heavily discounted in the admission process.

Next, we examine the impact of an anti-discrimination policy that legally restricts universities from selecting students based on their social group. In equilibrium, we discover that students benefiting from the anti-discrimination policy demonstrate higher expected human capital compared to their peers in highly selective universities. As a result, if policymakers slightly lower the admission threshold for disadvantaged students compared to advantaged ones, disadvantaged students maintain a higher expected human capital. However, if the threshold is lowered too much for disadvantaged students, the outcome is reversed.

In line with these findings, the empirical evidence is mixed. For instance, Melguizo $(2008,2010)$ finds that minorities enrolled in highly selective institutions have a higher probability of attaining a bachelor's degree compared to those in less selective institutions. Wainer and Melguizo (2018), analysing the results of the Brazilian national exam ENADE, discover that the performance of students admitted to university through affirmative action is equivalent to that of other students who did not benefit from it. In contrast, Arcidiacono, Aucejo, and Hotz (2016), focusing on California colleges and STEM majors, find that beneficiaries of affirmative action programmes perform worse than their counterparts. Our framework interprets this ambiguity as stemming from varying intensities of affirmative action programmes. Specifically, setting the admission threshold too low for students from low social groups, in comparison to their socially advantaged counterparts, can effectively reverse their expected human capital outcomes.

The paper is organised as follows. Section 2 briefly surveys some of the related literature, and Section 3 contains the model. Section 4 presents the baseline results, whilst Section 5 analyses the introduction of the anti-discrimination policy. Section 6 extends the analysis by assuming that students are aware of their abilities and introducing competition among universities with different objectives. Section 7 concludes the paper.

## 2 Related Literature

First, the paper contributes to the debate on affirmative action. ${ }^{6}$ Affirmative action policies involve the modification of admission standards based on observable characteristics, such as ethnicity or socioeconomic status, with the aim of enhancing educational and job opportunities for minority or disadvantaged groups. This practice is widespread in many U.S. universities (Bowen and Bok 1998) and has also been adopted outside the United States in recent years. For instance, the British government has implemented policies promoting access to universities for applicants from disadvantaged backgrounds over the past decade, while Brazil introduced a U.S.-style, race-based affirmative action law in the summer of 2012 (BBC, 8th August 2012).

Theoretical models of affirmative action typically fall within the framework of statistical discrimination. In this line of research, seminal papers by Phelps (1972) and Arrow (1973) have contributed significantly to understanding the subject. ${ }^{7}$ In their analysis of labour market discrimination, Coate and Loury (1993) assume that both ability and human capital are unobservable. However, they assert that ability is equally distributed within each social group, while the employer holds bias and perceives one group to be more productive than the other. This prejudice is then reinforced in equilibrium as workers from the perceived less productive group invest less in human capital.

The implementation of an affirmative action policy exacerbates the situation. Due to the belief that one group performs worse than the other, the employer sets lower standards for workers from this group, which, in turn, leads to reduced incentives for them to acquire human capital. ${ }^{8}$

[^3]On the contrary, De Fraja (2005) supports affirmative action programmes and evaluates the intervention of a utilitarian government in the analysis of the second best, considering the presence of asymmetric information on students' ability. He finds that the optimal education policy should allow individuals from disadvantaged groups to pay lower tuition fees and be enrolled in higher education levels compared to otherwise identical individuals from advantaged groups.

The present paper is closely related to De Fraja (2005), but it makes two different assumptions. First, unlike De Fraja (2005), we assume that students have no information about their level of ability. This assumption is a simplification, for the sake of tractability, of the more realistic scenario in which the university and the student both observe a separate, imperfectly correlated signal of the student's ability. ${ }^{9}$ Yet, it is convenient to catch the relevance of students' "self-awareness" and "metacognition", well known by educational psychologists, in their study effort choice. ${ }^{10}$

Another important distinction is that De Fraja (2005) assumes that the distribution of ability among students from higher social groups first-order stochastically dominates that of students from lower ones. In practical terms, this implies that the most socially disadvantaged group tends to have the lowest proportion of individuals with high ability, while the most advantaged group exhibits the highest proportion. ${ }^{11}$ In contrast, we make the weaker assumption that the distribution of ability within the advantaged social group has lower variance than that within the disadvantaged group. This assumption suggests a disparity in the signal precision of ability associated with belonging to a particular social group, a key element in our analysis. The assumption is also consistent with substantial empirical evidence (see footnote 12 for details). Additionally, our focus is not on designing optimal school tuition but on university admission decisions.

In the context of university admission analysis, Epple, Romano, and Sieg (2008) and Chan and Eyster (2004) find that when universities prioritise diversity, the elimination of affirmative action significantly reduces the number of minorities in toptier colleges. On the other hand, Krishna and Tarasov (2016) examine a model of contests among agents differing in ability and social advantage, who choose their effort level to pass a test. They find that affirmative action creates a trade-off between wasteful effort and selection. The effort effect implies that preferences in favour

[^4]of the disadvantaged group increase their average effort level. However, since this effort is exerted solely to improve a student's placement, it becomes inefficient as it comes at the expense of other students. The selection effect, however, results from admitted students from the disadvantaged group having, on average, greater natural ability. If a society prioritises natural ability over the placement system, this effect favours disadvantaged students. Krishna and Tarasov (2016) demonstrate that the selection effect outweighs the effort effect when tuition fees are high, and vice versa. As a policy implication, affirmative action should be implemented only in such cases, similar to the U.S. college system. Compared to Krishna and Tarasov (2016), our analysis includes the role of learning effort, while we do not model wasteful effort as students are not affected by the academic performance of others. Following the Greek university system, De Fraja, Eleftheriou, and Ioakimidis (2021) assess the incentives that drive students to exert effort both at school and during preparation for the "Panhellenic examination" - the test required for admission to Greek universities. To conduct their evaluation, they employ a multi-unit, all-pay model of auction. The study reveals that students with higher abilities tend to invest more effort in their studies.

The paper is also related to the theoretical literature on admission standards. Pioneering works of Costrell (1994) and Betts (1998) theoretically analyse optimal standards using political economy models. In particular, Betts (1998) assumes that students exhibit different levels of ability, and employers cannot fully assess the productivity of job applicants. Unlike these models, we assume that belonging to different social groups has an impact on the beliefs about a student's ability. In Epple et al. (2006), college admissions are modelled as a bargaining game between the college and potential students under asymmetric information, with a particular focus on the evidence of student "profiling" practices in college admission processes. Information about student abilities is revealed during the negotiation process. Gary-Bobo and Trannoy (2008) provide a normative analysis of the admission process and emphasise the double-sided asymmetric information problem of a student's ability. They demonstrate that this issue can be addressed through a mixed policy involving tuition fees and examinations. Hence, the different information held by students and universities explains the need for a policy mix. In contrast, we assume the same lack of information regarding students' abilities for both students and universities. Additionally, we consider students who are members of various social groups.

Several contributions have also explored the effects of introducing admission tests in a political economy setting. De Fraja (2001) considers a framework where students differ in ability and income, pay university fees, and face uncertainty regarding their future income. Admission tests are implemented alongside a uniform subsidy for university attendance, financed by a proportional tax on parents'
income. He shows the emergence of a participation gap, where wealthier parents are more likely to enrol their children at university, thereby taking on the financial risk associated with uncertain returns to education. In a recent paper, De Donder and Martinez-Mora (2017) extend De Fraja's work to a general equilibrium setting, where parents vote on the admission levels of universities and decide whether to invest in private tutoring to help their children pass the test. They find that a university participation gap emerges between rich and poor students, as wealthier parents invest more in tutoring.

A recent related paper is Kaganovich and Su (2019), which focuses on heterogeneity in the gains of human capital and competition among colleges. In this paper, the key concept is a college's "curricular standard". This concept differs from the admission threshold as it influences the gains in human capital, based on a student's natural aptitude.

The current framework bears similarities to MacLeod and Urquiola (2015). The information structure is identical, but there are slight differences in their education technology, mainly because we exclude schools' teaching efforts from our analysis. Additionally, unlike MacLeod and Urquiola (2015), we do not consider variations in social groups among students. Instead, in our study, students enter the job market after completing their schooling, rather than applying for university admission. Here, employers, not universities, are responsible for their selection.

Furthermore, while MacLeod and Urquiola (2015) primarily focus on the political economy aspects of school funding and the interactions between different school types (e.g. state and private, for-profit schools), our research hones in on the choices made within a single educational system and examines the differences in the distribution of abilities among students from different social groups.

## 3 The Model

Our study examines the interaction between students and a university system. After completing their education at school, students undergo an examination and subsequently gain admission into the university system. As per the admission process, the student may have access to one or more universities which belong to the university system. The primary focus of our analysis is on the university admission process.

### 3.1 Students

There exists a large population of students. Before receiving any education, each student is characterised by a pair $(s, \theta) \in\{d, a\} \times \mathbb{R}$. The variable $s$ is common
knowledge and captures the social background of the individual, where $d$ indicates a disadvantaged background, and $a$ indicates an advantaged background. The variable $\theta$ measures the innate ability of the person, which remains unknown. It is the realisation of a random variable drawn from the normal distribution $\mathcal{N}\left(\bar{\theta}_{s}, \frac{1}{\rho_{s}}\right)$, where $\bar{\theta}_{s}$ is the mean ability and $\rho_{s}=\frac{1}{\sigma_{s}^{2}}$ represents the precision of a normal distribution with variance $\sigma_{s}^{2}$.

Assumption 1. The distribution of the social group $a$ and $d$ satisfies $\rho_{a}>\rho_{d}$.

In practice, social groups exhibit systematic differences, which can form the basis of policy. In particular, the advantaged social group has less dispersion in ability, as suggested by empirical evidence. ${ }^{12}$

Students attend school to be admitted to university. We denote by $e$ the individual schooling effort which generates an unobservable level of human capital

$$
\begin{equation*}
h=\theta+\eta e \tag{1}
\end{equation*}
$$

where $\eta \in[0,1]$ is a commonly known parameter. ${ }^{13}$ We assume that the level of study effort is known only to students and, by the additive form of (1), it is independent of her ability. This functional form is based on the assumption that students are unaware of their ability level.

At the end of their school career, students take a test, administered by acentral agency. ${ }^{14}$ A student's result in this test is given by

$$
\begin{equation*}
t=\theta+e+\varepsilon \tag{2}
\end{equation*}
$$

[^5]The variable $\varepsilon$ represents the "luck" on the day of the exam. It is drawn from the normal distribution $\mathcal{N}\left(0, \frac{1}{\rho_{\varepsilon}}\right)$.

### 3.1.1 Student's Utility

A student who enters a university $u$ of the university system, obtains a benefit given by $h_{u}$, which is the level of human capital necessary to be admitted to $u$ (see the next subsection for details on the university admission). However, a student does not yet know which university she will attend ex-ante. Moreover, the university's decision rule will depend on test results and therefore by (2) on the student's effort. To capture this relationship, we denote by $\widehat{h}_{i s}\left(e_{i s}\right)$ as the expected benefit of a student $i$ from social group $s$ as a function of her effort and by

$$
\begin{equation*}
S_{i s}(e)=\widehat{h}_{i s}\left(e_{i s}\right)-\frac{\psi e_{i s}^{2}}{2}, \tag{3}
\end{equation*}
$$

her expected utility, where $\frac{\psi e_{i s}^{2}}{2}$ measures the utility cost of effort with positive parameter $\psi>0$. The exact form of $\widehat{h}_{i s}\left(e_{i s}\right)$ depends on the universities' admission criteria in equilibrium and will be discussed below.

### 3.2 Universities

At the end of the school, students apply to enter the university system. The university system consists of a continuum of universities with measure $U \in \mathbb{R}^{+}$indexed by $u$. We abstract away from the capacity of universities: each university can accommodate any number of students. Universities belonging to the university system are perfectly ordered according to their prestige: they are perfectly ranked in terms of tradition and reputation.

Students are aware of the ranking system and adapt their preferences accordingly, as does the labour market: attending a higher-ranked university yields a higher return in terms of job opportunities (not modelled here). Admission requirements thus depend on the position in the ranking. In this way, we abstract away from the competition between universities, which is tangential to the question at hand, and allow us to focus on the analysis of admission criteria. It also allows us to focus on the behaviour of a single university, since each university will behave in the same way in its ranking position, without strategic interaction with other universities.

The admission requirement is in terms of expected human capital, which is inferred by the universities on the basis of the observable variable "social group" $s$ and the result of the final examinations $t$. In particular, the goal of university $u$
is to admit students who have an expected level of human capital of at least $\hat{h}_{u}$, which is determined by its ranking position in the university system. Therefore, if university $u$ has lower prestige than university $u^{\prime}$, then the expected human capital of a student will be $\widehat{h}_{u}<\widehat{h}_{u^{\prime}}{ }^{15}$

Admission is designed by selecting a vector of admission thresholds. Thus university $u$ 's objective function can be represented as

$$
\pi_{u}=\widehat{h}_{u}\left(\tau_{u}\right),
$$

where $\tau_{u}=\left[\tau_{u d}, \tau_{u a}\right]$ is the vector of admission standards for each social group. Formally, if university $u$ sets an admission standard $\tau_{u s}, s \in\{d, a\}$, a student from group $s$ may attend university $u$ if and only if she achieves a test score $t \geq \tau_{u s}$.

An example of a similar university system and admission mechanism can be found in the public universities of Italy. In Italy, students take a test known as TOLC (Test OnLine CISIA). Public universities in Italy set admission thresholds that allow students who score a certain number of points to choose to enrol in any university with requirements equal to or lower than their score.

### 3.3 The Game and Equilibrium Concept

The equilibrium concept is subgame perfect equilibrium. Figure 1 illustrates the timing of events. Each academic year, each institution in the university system sets its admissions criteria, before students decide on their study effort based on this


Figure 1: The game between universities and students.

15 The decision to focus on the fully stratified case of a university system implies that a student can find a perfect fit for her effort in terms of admission. If there were gaps among universities, a student might work very hard to get into the "next" higher-ranked university if she is close to the admission threshold, but not as hard if she falls somewhere in the middle. The fully stratified case we are considering allows us to abstract away from these effects. However, it is worth noting that this is not too distant from reality, considering the globalised world with thousands of academic institutions. Moreover, the assumption of university rank is supported by empirical evidence (Arcidiacono and Lovenheim 2016; Long 2004, among others).
information. Specifically, in the first stage, each university simultaneously chooses its vector of admission standards, followed by students choosing their effort in the second stage.

In addition, students know how their peers make their choices, which allows them to form accurate beliefs about the effort of others. Information about these variables becomes crucial for individual students, as universities take them into account when evaluating the effort that a student is perceived to have put into his or her education.

## 4 Results

### 4.1 University Ranking

Figure 2 illustrates an example with two universities, $u$ and $u^{\prime}$, ranked within the university system $U$. Suppose $f_{a}(\theta)$ and $f_{d}(\theta)$ are the densities of the universities' beliefs about the human capital of advantaged and disadvantaged students, respectively. Universities have accurate beliefs about the ability, study effort, and "day of exam" error of these student groups.

Students from both groups, whose believed human capital exceeds $\widehat{h}_{u^{\prime}}$, receive offers from both universities. However, based on their preferences, they choose to enrol in university $u^{\prime}$. The interpretation is straightforward: at the end of the school period, a student expects that graduating from a top university will corresponds to a higher chance of getting a higher salary or a better job in the labour market. Subsequently, students whose expected human capital $\widehat{h}$ lies between $\widehat{h}_{u}$ and $\widehat{h}_{u^{\prime}}$ will receive offers from $u$ but not $u^{\prime}$, leading them to attend university $u$. Since we


Figure 2: Distribution of expected human capital in the two social groups.
are considering a continuum of perfectly ranked universities, the expected human capital intervals in Figure 2 are infinitesimally small.

In a fully stratified university system, the most prestigious university admits students with the highest expected human capital. The second most prestigious university admits students who fall within the next interval of the distribution of expected human capital, and this stratification continues accordingly down to those students whose human capital is believed to be the lowest in the distribution, who receive an offer only from the university ranked the lowest in the university system. ${ }^{16}$

### 4.2 Students' Effort

Since universities are perfectly ranked, not only by students but also by employers (although not explicitly modelled), attending a higher ranked university offers greater benefits to the student. The university system serves as a signal of the student's expected human capital. Understanding the relationship between this signal and their effort, students base their decisions on Equations (1) and (2).

Consequently, if a student receives admission offers from several universities, they will simply choose the one with the highest ranking among them. This establishes a functional relationship between the test score and the best university to which the student is admitted.

An is student's problem is represented by:

$$
\begin{equation*}
\max _{e_{i s}} S\left(e_{i s}\right)=\widehat{h}_{i s}\left(e_{i s}\right)-\frac{\psi e_{i s}^{2}}{2} . \tag{4}
\end{equation*}
$$

In (4) the expected human capital $\widehat{h}_{i s}\left(e_{i s}\right)$, for a student $i$ with ability $\theta_{i}$ from social group $s$, who obtained $t_{i s}$ in the test, is given by:

$$
\begin{equation*}
\widehat{h}_{i s}\left(e_{i s}\right)=E\left(\theta+\eta e_{i s} \mid t_{i s}, s\right) \tag{5}
\end{equation*}
$$

where $e_{i s}$ is the level of effort exerted by student $i$ in social group $s$. From the Bayes rule applied to normal distributions (see Appendix for details, also applied in Macleod and Urquiola 2015), the expected university (5) is the weighted average of the two signals. We can extend this to derive the following expression for a student's expected human capital and hence the university she can attend:

$$
\begin{equation*}
\hat{h}_{i s}\left(t_{i}\right)=\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}}\left(\bar{\theta}_{s}+\eta \bar{e}_{s}\right)+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} t_{i s}, \tag{6}
\end{equation*}
$$

[^6]where $\bar{e}_{s}$ is the average effort of a student from social group $s$.
If that student exerts effort $e_{i s}$, we can use (2) to derive her expected test score, and so obtain her expected human capital as a function of her chosen effort level:
\[

$$
\begin{equation*}
\hat{h}_{i s}\left(e_{i s}\right)=\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}}\left(\bar{\theta}_{s}+\eta \bar{e}_{s}\right)+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}}\left(\bar{\theta}_{s}+e_{i s}\right) . \tag{7}
\end{equation*}
$$

\]

In Equation (7), note that $\bar{e}_{s}$ is the expected effort's from students coming from social group $s$, while $e_{i s}$ is student is's effort. In equilibrium, these two effort values are the same. It is now straightforward to determine the student's optimal choice of effort.

Proposition 1. A student from social group s exerts effort given by:

$$
\begin{equation*}
e_{s}^{*}=\frac{\rho_{\varepsilon}}{\psi\left(\rho_{s}+\rho_{\varepsilon}\right)} . \tag{8}
\end{equation*}
$$

Proof. Given universities' equilibrium beliefs about the effort exerted bya student in her social group, a student will choose her own effort $e_{i s}$ to maximise the difference between (7) and the cost of effort $\frac{\psi e_{i s}^{2}}{2}$ :

$$
\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}}=\psi e_{i s} .
$$

In equilibrium, $e_{i s}=e_{s}$, which gives (8). The second order condition, is satisfied as $\frac{\partial^{2} S\left(e_{i s}\right)}{\partial e_{i s}^{2}}=-\psi<0$.

A quick look at Proposition 1 shows that, in general, the effort exerted by a student depends on the relative precision of the random error in the test and the distribution of ability within their social group. This difference between groups leads to different levels of effort exerted by students from different social backgrounds. This variation is understandable given the dual role of effort: it directly increases human capital and, as pointed out by Spence (1973), it also affects the signal that influences a university's belief in the student's ability, which is an essential component of human capital.

However, the benefit of attempting to change the signal depends on the accuracy of the signal itself. In the extreme case where all students in a particular social group $s$ have similar abilities ( $\rho_{s}$ is very high), universities perceiving a student with a very high test score will attribute it to luck, assuming that the student has only put in the "average" effort for group $s$. Consequently, universities will not be significantly influenced by the test in assessing the student's ability. As a result, students in group $s$ receive little benefit from their effort and therefore have little incentive to exert themselves.

Conversely, if the test is a highly accurate measure of ability (high $\rho_{\varepsilon}$ ), the influence of the test becomes crucial. With high $\rho_{\varepsilon}$, effort becomes a highly effective means of improving one's post-school prospects, and the student thus has a strong incentive to exert significant effort.

A university evaluating a student with an unexpectedly good test score, given her social group s, will attribute the positive result to a combination of high ability and good luck. If the probability that the student has very high ability is extremely low (due to the low variance in her social group, resulting in very small tails), then her good test score will be attributed primarily to luck, with minimal impact on the university's assessment of her overall ability. As a result, this student will have little incentive to work hard, since trying to "outperform" her peers will have little impact on the university's perception of her ability. This situation is similar to the intuition underlying Dewatripont, Jewitt, and Tirole (1999a, 1999b)'s explanation of the lower effort levels of managers with multiple tasks: a good performance is more likely to be attributed to luck rather than to ability.

It is interesting to compare the level of effort derived in (8) with the efficient level of effort. This is the level that would be exerted without the bias caused by the unobservability of ability, and is easy to calculate. A student whose ability is perfectly observable to universities, would choose $e_{i}$ to maximise $\theta_{i}+\eta e_{i}-\frac{\psi}{2} e_{i}^{2}$ and so choose $e_{i}^{*}=e_{o}^{*}$, which satisfies:

$$
\begin{equation*}
e_{o}^{*}=\frac{\eta}{\psi}, \tag{9}
\end{equation*}
$$

where subscript $o$ stands for "observable ability". Unlike the value derived in (8) when ability is unobserved, $e_{o}^{*}$ is independent of the social groups and of the parameters of the distribution of test errors. Note also that effort can be inefficiently high: this happens when

$$
\frac{\eta}{\psi}<\frac{\rho_{\varepsilon}}{\psi\left(\rho_{s}+\rho_{\varepsilon}\right)},
$$

as in Spence's (1973) signalling model, where $\eta=0$.

### 4.3 Admission and Social Group

Each university $u$ sets its admission threshold $\tau_{u s}=\left[\tau_{u d}, \tau_{u a}\right]$ based on the level of expected human capital established by its position in the ranking of the university system, $\widehat{h}_{u}$. The university system possesses knowledge about the distribution of human capital in each social group, enabling it to determine the functional relationship between a student's test score within a particular social group and their expected human capital. Additionally, universities are aware of the students' ranking preferences, which allows them to predict which university each student will choose from those that have extended offers to them. Thus, a student who receives
offers from university $u$ but also from any other university positioned higher than $u$ in the ranking of the university system, it will simply enrol in the latter. This implies the following.

Remark 1. All students enrolled on a certain university exhibit the same level of expected human capital, regardless of their social group. This level of expected human capital corresponds to that below which a university is not willing to admit the student.

We are now in a position to compare the admission criteria for students from different social groups, for given test score. Indeed, Equation (6) represents the relationship between the admission threshold, in terms of expected human capital $\hat{h}_{s}(t)$ of a student from social group $s$ and the test score $t$. In particular, the relationship depicted by (6) is linear, with slope $\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}}$ and intercept, $\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}}\left(\bar{\theta}_{s}+\eta e_{s}^{*}\right)$. Comparing the slopes of (6), we obtain

$$
\frac{\rho_{\varepsilon}}{\rho_{a}+\rho_{\varepsilon}}<\frac{\rho_{\varepsilon}}{\rho_{d}+\rho_{\varepsilon}}
$$

while the difference between the intercepts is ambiguous:

$$
\begin{equation*}
\frac{\rho_{a}}{\rho_{a}+\rho_{\varepsilon}}\left(\bar{\theta}_{a}+\eta e_{a}^{*}\right) \gtrless \frac{\rho_{d}}{\rho_{d}+\rho_{\varepsilon}}\left(\bar{\theta}_{d}+\eta e_{d}^{*}\right) . \tag{10}
\end{equation*}
$$

Substituting the equilibrium efforts from Proposition 1 and rearranging, one gets:

$$
\bar{\theta}_{a}-g \bar{\theta}_{d} \gtrless \frac{\eta}{\psi} \frac{\left(\rho_{a} \rho_{d}-\rho_{\varepsilon}^{2}\right)\left(\rho_{a}-\rho_{d}\right) \rho_{\varepsilon}}{\rho_{a}\left(\rho_{d}+\rho_{\varepsilon}\right)^{2}\left(\rho_{a}+\rho_{\varepsilon}\right)},
$$

where $g \equiv \frac{\rho_{d}\left(\rho_{a}+\rho_{\varepsilon}\right)}{\rho_{a}\left(\rho_{d}+\rho_{\varepsilon}\right)}<1$. For the sake of exposition, we denote

$$
\Delta \bar{\theta} \equiv \bar{\theta}_{a}-g \bar{\theta}_{d} \in \mathbb{R}, \Phi \equiv \frac{\eta}{\psi} \frac{\left(\rho_{a} \rho_{d}-\rho_{\varepsilon}^{2}\right)\left(\rho_{a}-\rho_{d}\right) \rho_{\varepsilon}}{\rho_{a}\left(\rho_{d}+\rho_{\varepsilon}\right)^{2}\left(\rho_{a}+\rho_{\varepsilon}\right)} \in \mathbb{R} .
$$

Suppose first $\Delta \bar{\theta}<\Phi$, implying that the average ability of students coming from social group $d$ is quite higher than that of students from $a\left(\bar{\theta}_{d}>\frac{1}{g} \bar{\theta}_{a}\right)$, and or the precision of the exam outcome is quite lower than that of the distribution of abilities ( $\rho_{\varepsilon}<\rho_{d}$ ). This case appears less relevant empirically, but shows that our results do not require any restrictions on the average ability of students coming from different social groups. When $\Delta \bar{\theta}<\Phi$, the expected human capital of $d$
students is systematically higher than that of $a$ students, $\widehat{h}_{d}(t)>\widehat{h}_{a}(t)$, because $\widehat{h}_{a}(t)$ exhibits a lower slope and a lower intercept than $\widehat{h}_{d}(t)$.

Proposition 2. If $\Delta \bar{\theta}<\Phi$ then, for every university $u$, students from social group a are admitted with a higher test score than students from social group d: $\tau_{u a}>\tau_{u d}$.

Figure 3 shows the relationship between all possible test outcomes, on the horizontal line, and the expected human capital required by the highest-ranked university that extends an offer to a student from a given social group with a test score of $t$. Thus, the dashed line $\widehat{h}_{a}(t)$ corresponds to the expected level of human capital for a student from social group $a$ with a test score $t$, while the solid increasing line $\widehat{h}_{d}(t)$ represents the corresponding relationship for students from social group $d$. The fact that $\hat{h}_{d}(t)>\widehat{h}_{a}(t)$ implies that each university requires a higher test score to $a$ than $d$ students. For example, university $u$ requires $\tau_{u d}=t^{\prime}$ and $\tau_{u a}=t^{\prime \prime}$, which both signal the same level of human capital $\hat{h}_{u}\left(\tau_{u}\right)=\widehat{h}_{d}\left(t^{\prime}\right)=\widehat{h}_{a}\left(t^{\prime \prime}\right)$.

Figure 3 also provides a clear illustration of why students in the group with higher variance exert more effort: a given improvement in the exam result translates into a greater improvement in the most preferred university that makes them


Figure 3: Relationship between the expected university and the test results for students coming from social group dor $a: \Delta \bar{\theta}<\Phi$.
an offer than for students from the group with lower variance, and so their incentive to exert effort is stronger.

Next, suppose $\Delta \bar{\theta}>\Phi$, implying that the average ability of $a$ students is sufficiently higher than that of $d$ students. This case is likely more relevant, given the consistent empirical regularity across the world that shows a positive correlation between ability and social background. ${ }^{17}$ From $\Delta \bar{\theta}>\Phi$, it follows that $\widehat{h}_{a}(t)$ exhibits higher intercept than $\widehat{h}_{d}(t)$. This in turn implies the existence of a single crossing condition at a certain test result, which we denote as $t^{*}$.

Lemma 1. There exists a test outcome $t^{*}$, such that university $u^{*}$ admits students coming from social groups $a$ and $d$ who got the same test score: $\tau_{u^{*}}=\left[\tau_{u a}^{*}, \tau_{u d}^{*}\right]$ where $\tau_{u a}^{*}=\tau_{u d}^{*}$.

The following proposition is a direct consequence of Lemma 1.

Proposition 3. Each university such that $\tau \geq t^{*}$ admits students from social group a with a higher test score than students from social group d: $\tau_{u a}>\tau_{u d}$. The opposite applies for each university such that $\tau<t^{*}$.

Figure 4 depicts the relationship between test scores and admitting university, for different social groups, when $\Delta \bar{\theta}>\Phi$. If a student obtained a test score higher than $t^{*}$, she will enrol in a higher-ranked university if she comes from social group d. In this case, the intuition is the same as in Proposition 2.

Nonetheless, now the admission criterion of Proposition 2 is confirmed only for universities that expect to admit students with a relatively high human capital. Below a certain threshold $t^{*}, a$ students are expected to have a higher human capital than $d$ students given the same test score. The intuition is simple: when the test score is low (below $t^{*}$ ) its weight at explaining the expected human capital is relatively lower than the average ability of the social group. For $\Delta \bar{\theta}>\Phi$, this effect favours $a$ students.

We conclude the section by considering the case with complete information. Now universities observe each student's ability directly, so that a student's (known)

17 The association between social class and children's cognitive abilities is empirically well documented. Students with a disadvantaged social background enter primary school behind their more advantaged peers (Lee and Burkam 2002; Marton, Hounsell, and Entwistle 1997; Mayer 1997), and these initial cognitive differences increase as children progress through school (Downey et al. 2004; Phillips et al. 1998; Reardon 2003). What is postulated is that an advantaged household and social environment can help develop cognitive skills (Carneiro and Heckman 2003; Cunha et al. 2006; Joshi and McCulloch 2001).


Figure 4: Relationship between the expected university and the test results for students coming from social group $d$ or $a$ : $\Delta \bar{\theta}>\Phi$.
human capital is:

$$
\begin{equation*}
h_{i}\left(t_{i}\right)=\theta_{i}-(1-\eta) e_{o}^{*}+t_{i} . \tag{11}
\end{equation*}
$$

In this case, the relationship between test score and university admission strictly depends on the student's ability level.

## 5 Anti-Discrimination Policy

In this section, we assume that universities are not permitted to set different admission standards for different social groups. An example of this policy is the so-called "Top 10 \% Rule", implemented in Texas since 1998, which mandates public universities in Texas to ensure automatic admission to all high school seniors who graduate in the top tenth of their class (Niu, Tienda, and Cortes 2006, among others). Our setup can easily be adapted to this case by adding the additional constraint that $\tau_{u s}$ be constant in $s$.

The first consequence of this restriction is that the test score becomes the sole signal of ability that universities can take into account. From a student's viewpoint, the expected test score will reflect the level of university where a student $i$, coming from social group $s$, expects to be admitted and, as a result, her benefit from
attending university, i.e.

$$
S_{i s}(e)=\hat{t}_{i s}\left(e_{i s}\right)-\frac{\psi e_{i s}^{2}}{2},
$$

where

$$
\begin{equation*}
\hat{t}_{i s}=\bar{\theta}_{s}+e_{i} . \tag{12}
\end{equation*}
$$

From a university point of view instead, because a university cannot discriminate based on social groups, the admission results must be the same for each admitted student. Therefore, all students with the same grade will attend the same university level. It follows that the ranking of the university system cannot be based on students' expected human capital anymore (which would account also for the information of social group) but must rely only on the ranking of test results. Thus, in this case, a monotonic transformation turns test score $t$ into university $u$.

The second, related consequence is that a student's expected human capital (6) does not determine which university she will attend anymore:

$$
\begin{equation*}
\hat{h}_{i s}=\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}}\left(\bar{\theta}_{s}+\eta \bar{e}_{s}\right)+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} t_{i} \tag{13}
\end{equation*}
$$

It follows that students coming from different social groups who obtained the same grade will attend the same university, but their expected human capital differs. In contrast, in the baseline model all students attending the same university had the same expected human capital but different test results.

By substituting (12) into the students' optimisation problem, we can obtain the result for the case with no discrimination across social groups. Begin by considering students. A student $i$ 's problem becomes

$$
\max _{e_{i}} \bar{\theta}_{s}+e_{i}-\frac{\psi e_{i}^{2}}{2},
$$

and her optimal choice of effort is given by

$$
\begin{equation*}
e^{A D}=\frac{1}{\psi}, \tag{14}
\end{equation*}
$$

where superscript $A D$ stands for "Anti-Discrimination". Therefore,
Proposition 4. Suppose universities cannot differentiate admission standards across social groups. Then study effort would be the same for all students.

Now all students exert the same level of effort. Also, all of them exert higher effort than in the baseline case in which universities can discriminate across students, since the marginal benefit of education is now higher:

$$
e^{A D}=\frac{1}{\psi}>\frac{\rho_{\varepsilon}}{\psi\left(\rho_{s}+\rho_{\varepsilon}\right)}=e_{s}^{*}
$$

for every $s$.
Next, we turn to the university system. Having already clarified that university admission will follow the level of test scores, we are left to determine the differences in terms of human capital of students by different social groups admitted in the same university.

Students from $s$ admitted in university $u$ have expected human capital

$$
\hat{h}_{s}=\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}}\left(\bar{\theta}_{s}+\eta e^{A D}\right)+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} u .
$$

Like in the baseline case, the slope of $\widehat{h}_{a}$ is always lower than that of $\widehat{h}_{d}$. Comparing the intercept of $\widehat{h}_{a}$ and $\widehat{h}_{d}$, we get

$$
\frac{\rho_{a}}{\rho_{a}+\rho_{\varepsilon}}\left(\bar{\theta}_{a}+\eta e^{A D}\right) \gtrless \frac{\rho_{d}}{\rho_{d}+\rho_{\varepsilon}}\left(\bar{\theta}_{d}+\eta e^{A D}\right) .
$$

Substituting $e^{A D}$ from (14) and rearranging, one obtains $\Delta \bar{\theta} \gtrless \Phi^{A D}$, where

$$
\Phi^{A D} \equiv-\frac{\eta}{\psi} \frac{\left(\rho_{a}-\rho_{d}\right) \rho_{\varepsilon}}{\rho_{a}\left(\rho_{d}+\rho_{\varepsilon}\right)}<0
$$

The case $\Delta \bar{\theta}<\Phi^{A D}$ requires that the average ability of $d$ students is much higher than that of $a$ students, i.e.

$$
\bar{\theta}_{d}>\Phi^{A D}+\frac{\rho_{a}\left(\rho_{d}+\rho_{\varepsilon}\right)}{\rho_{d}\left(\rho_{a}+\rho_{\varepsilon}\right)} \bar{\theta}_{a}
$$

Even if theoretically admissible, this case is not supported by empirical evidence, and will be left aside to ease the exposition.

Thus, $\Delta \bar{\theta}>\Phi^{A D}$ implies the existence of a single-crossing condition between $\widehat{h}_{a}(u)$ and $\widehat{h}_{d}(u)$ : we denote as $u^{* *}$ the university whose students from social groups $a$ and $d$ yield the same level of expected human capital, $\widehat{h}_{a}^{A D}\left(u^{* *}\right)=\widehat{h}_{d}^{A D}\left(u^{* *}\right)$. It follows that

Corollary 1. Within any university u positioned above $u^{* *}$ in the university system, a students have a lower expected human capital than d students. The opposite applies for every university set below $u^{*}$.

Figure 5 illustrates the relationship between universities and human capital. Unlike the baseline case depicted in Figures 3 and 4, where students were accepted for admission from the same university even with different test results, in this scenario each university of a certain quality requires the same test score for admission. Consequently, students with differing levels of human capital attend the same university. In the example shown in the figure, university $u$ is being attended by students who achieved $\tau_{u}$ : those from social group $d$ have expected human capital $\widehat{h}_{d}(u)$ and students from social group $a$ with expected human capital $\widehat{h}_{a}(u)$, where $\widehat{h}_{d}(u)>\widehat{h}_{a}(u)$. As the quality of the university increases, so does the difference in human capital between students from different social groups.

In terms of policy implications, the direct consequence of Corollary 1 is that, when social groups have different distribution of unobservable ability, the introduction of affirmative action (i.e. lowering the admission threshold for disadvantaged students) would be optimal in terms of admission decisions. Firstly, it allows for the more precise identification of students' expected human capital, aligning with the university's objectives. In turn, it narrows down the differences in expected human capital among students from different groups.


Figure 5: Relationship between human capital and universities when an anti-discrimination policy is implemented.

However, lowering the admission threshold for students from low social groups below a certain point would effectively reverse the expectation of their human capital. The mixed empirical evidence discussed in the Introduction (Arcidiacono, Aucejo, and Hotz 2016; Melguizo 2008, 2010; Wainer and Melguizo 2018) suggests that our model may indeed provide an interpretation for the potentially different effects of adopting affirmative action policies.

## 6 Extensions

In this section, we extend our analysis to verify if our results are robust to different assumptions. Firstly, we will assume that the students observe their ability level perfectly. Secondly, we will incorporate university competition, also considering universities with different objectives.

### 6.1 Ability Observable by Students

Since students can now observe their ability, they will set their study effort accordingly. A way to model this is to assume a multiplicative relationship between effort and ability in the human capital and test function, i.e.

$$
\begin{equation*}
h=\eta e \theta \tag{15}
\end{equation*}
$$

and

$$
\begin{equation*}
t=e \theta+\varepsilon \tag{16}
\end{equation*}
$$

As a consequence, following the same procedure as the baseline model, a student is's expected human capital is given by

$$
\begin{align*}
\hat{h}_{i s}\left(e_{i s}\right) & =E\left(\eta e_{i s} \theta \mid t_{i s}, s\right)  \tag{17}\\
& =\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta e_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} e_{i s} \theta_{i} .
\end{align*}
$$

In turn, that student's problem is:

$$
\begin{equation*}
\max _{e_{i s}} S\left(u_{i s}\left(e_{i s}\right)\right)=\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta e_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} e_{i s} \theta_{i}-\frac{\psi}{2} e_{i s}^{2} \tag{18}
\end{equation*}
$$

whose first order condition is

$$
\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} \theta_{i}-\psi e_{i s}=0
$$

Remember that ability is distributed over the real numbers, and thus may also be negative. In this case, given the student's knowledge of her ability, her best choice is not to exert any effort at all:

$$
e_{i s}^{*}=\max \left\{\frac{\theta_{i}}{\psi} \frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}}, 0\right\}
$$

Effort now depends on both ability and social group, so that it increases with ability, since it yields a higher return for given effort when $\theta$ is high. Also, it decreases with the precision of the social group:

$$
\begin{equation*}
\frac{\partial e_{i s}^{*}}{\partial \theta_{i}}=\frac{1}{\psi} \frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}}>0, \frac{\partial e_{i s}^{*}}{\partial \rho_{s}}=-\frac{\theta_{i}}{\psi} \frac{\rho_{\varepsilon}}{\left(\rho_{s}+\rho_{\varepsilon}\right)^{2}}<0 \tag{19}
\end{equation*}
$$

By considering the population of different social groups, these contrasting effects may turn into an aggregate non-monotonic relationship between effort and ability: looking at the cross derivative of equilibrium effort with respect to $\theta$ and $\rho_{s}$, one gets

$$
\begin{equation*}
\frac{\partial^{2} e_{i s}^{*}}{\partial \theta_{i} \partial \rho_{s}}=-\frac{1}{\psi} \frac{\rho_{\varepsilon}}{\left(\rho_{s}+\rho_{\varepsilon}\right)^{2}}<0 \tag{20}
\end{equation*}
$$

Equation (20) states that a high-ability student from social group a may study less than a less able student from social group $d$. Moreover, as universities also consider the average effort within social groups when evaluating expected human capital, the signal of social group becomes non-monotonic, especially if $\bar{\theta}_{a}>\bar{\theta}_{d}$. Indeed, denoting the average equilibrium effort as

$$
e_{s}^{*}=\frac{\bar{\theta}_{s}}{\psi} \frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}}
$$

this may be higher or lower for a certain social group depending on whether the effect of average ability $\bar{\theta}_{a}>\bar{\theta}_{d}$ prevails $\left(e_{a}^{*}>e_{d}^{*}\right)$ or not $\left(e_{d}^{*}>e_{a}^{*}\right)$ over the effect of average precision $\rho_{a}>\rho_{d}$.

This outcome could offer an explanation for certain recent empirical findings indicating a potential negative correlation between students' ability and effort (Chadi, de Pinto, and Schultze 2019). According to our framework, this nonmonotonic relationship between ability and effort might be explained by the influence of social groups in university admissions criteria.

### 6.2 Universities of Different Types

A potential limitation of this framework is related to the focus on a university system that accommodates any possible level of utility-maximising expected human
capital. In this section we relax this assumption by considering a small number of universities. Moreover, there may be interest in exploring a scenario in which universities are endowed with distinct objective functions, and have limited student capacity.

To fix ideas, consider an economy with three universities, while students are still aware of their admission criteria. Each university has a different objective: one university, denoted by $E$ ("elitist") maximises the average level of expected human capital of its student body, denoted by $\widehat{h}_{E}$, subject to filling its student capacity, $\bar{c}_{E}$. Another university, denoted by $T$ ("teaching") has lower academic prestige than university $E$. Its objective is to maintain a "good" average level of human capital, $0<$ $\widehat{h}_{T}<\widehat{h}_{E}$, subject to its capacity, $\bar{c}_{T}$. Finally a third university, denoted by $P$ ("public"), simply sets a lower admission standard to be more inclusive, $0 \leq \widehat{h}_{P}<\widehat{h}_{T}$. Since it is public (such as a university system) we assume that it has unlimited capacity. Given that students are aware of the different nature of universities, each of them ranks the three universities as $E>T>P$. The consequence of having only three universities and not a university system that accommodates all possible human capital levels implies that all students whose test score is below the lowest admission threshold are simply not admitted to any university.

To keep things tractable, we assume that a university capacity constraint is binding. This assumption avoids the unrealistic situation in which a university admission strategy is solely focused on the quality and not the quantity of the body of students: for university $E$, this strategy would narrow down the admission only to the students with the highest expected human capital.

In line with the different objectives, we assume that the elitist university sets its threshold $\tau_{E s}$ to maximise the expected human capital of its class and subject to $c_{E}=$ $\bar{c}_{E}$. As the maximisation of human capital without limit would result in university $E$ admitting only students with the highest expected human capital, the presence of a binding capacity constraint implies

$$
\begin{aligned}
\tau_{E s} & =\arg \max \hat{h}_{E}\left(\tau_{E s}\right) \\
\text { s.t. } c_{E} & =\bar{c}_{E}
\end{aligned}
$$

In words, university $E$ lowers its objective of expected human capital (and in turn its admission threshold) until its capacity is fulfilled.

University $T$ also maximises the expected human capital subject to capacity constraint $c_{T}$ : University $T$ is also aware that it cannot compete with University $E$ in terms of academic prestige, i.e. any student admitted to both University $E$ and $T$ will choose $E$. It follows that, in order to fill all the available student placements, it must set its admission threshold lower than that of University $E$, which in turn is
a function of University $E$ 's capacity. Thus the admission threshold of University $T$ depends not only on its own capacity, but also on University E's, i.e. ${ }^{18}$

$$
\begin{aligned}
\tau_{T s} & =\arg \max \hat{h}_{T}\left(\tau_{T s}\right), \\
\text { s.t. } \hat{h}_{P} & <\widehat{h}_{T}<\widehat{h}_{E}, \\
c_{E} & =\bar{c}_{E} \\
c_{T} & =\bar{c}_{T}
\end{aligned}
$$

Finally, University $P$ provides access to education also for weaker students, aiming to reduce disparities in educational attainment and to promote social equity. Consistent with this i.e. we assume

$$
\tau_{P s}=\arg \left(\hat{h}_{P} \mid \hat{h}_{P}=0\right)
$$

Given unlimited capacity, all students whose expected human capital is larger or equal to zero are admitted to University $P$. Note that, as with anti-discrimination policy, the test score is the same for students coming from different social groups, while their expected human capital differs. In what follows, we will consider the two scenarios where ability is unobservable for students and the alternative case in which it is observable.

### 6.2.1 Unobserved Ability

Since students are not aware of their ability, they are not sure about how much they must study to obtain admission. Hence, they solve the usual maximisation problem. ${ }^{19}$ Considering the students' preferences $E>T>P$, university competition is contingent on their capacity, where $E$ holds an advantage in terms of expected human capital over both $T$ and $P$, while $T$ has an advantage over $P$.

[^7]Consequently, $P$ accommodates the residual students with non-negative expected human capital, based on $E$ and $T$ capacity levels. The baseline results regarding admission selection and study effort across different social groups remain unchanged.

### 6.2.2 Observed Ability

In this case, like in Section 6.1, students may target the level of effort necessary to be admitted to a certain university. Note that, since each of the three universities requires a specific level of expected human capital, the effort exerted by students will generally be suboptimal: ${ }^{20}$ they will compare their utility obtained by exerting effort to be admitted to different universities and then choose that which yields the higher utility. We may expect that more able students have an advantage at choosing to exert effort to be admitted to university $E$, average-ability students to exert effort to be admitted to University $T$ and all the rest the effort necessary to be admitted at University $P$.

First, we define as $e_{i s}\left(\theta_{i}, \tau_{k s}\right)$ the effort necessary for student from $s$ with ability $\theta_{i}$ to be admitted to university $k, k \in\{P, T, E\}$. Then, we define $\theta_{E}$ or $\theta_{T}$ as the ability level of a student who is admitted to universities $E$ or $T$, respectively, as a result of utility maximisation. Since $\tau_{E s}>\tau_{T s}$, it follows that $\theta_{E}>\theta_{T}$ and, according to (20), $e_{i s}\left(\theta_{i} \tau_{E s}\right)>e_{i s}\left(\theta_{i}, \tau_{T s}\right)$ : this implies that any student $i$ will exert more effort to join $E$ than $T$.

The fact that University $P$ admits all students with human capital at least zero implies that students admitted at $P$ have no incentives to exert effort: $e_{i s}\left(\theta_{i}, \tau_{P s}\right)=0$ and, since human capital is multiplicative over effort, even their human capital will be zero. University $P$ anticipates this and expects human capital equal to zero: both the test and the social group lose their role of signals.

We are now able to sort all the students in the population. To ease the exposition, below we will use $e_{i s}\left(\tau_{k s}\right)$ referring to $e_{i s}\left(\theta_{i}, \tau_{k}\right)$ without loss of generality. A generic student who is admitted at university $k$ will get utility given by $S\left(\theta_{i}, e_{i s}\left(\tau_{k}\right)\right)$. Clearly, any student with ability $\theta>\theta_{E}$ will choose university $E$ since that is the closest to their optimum effort. By contrast, any student with ability $\theta_{i} \in\left(\theta_{T}, \theta_{E}\right)$ must compare $S\left(\theta_{i}, e_{i s}\left(\theta_{i}, \tau_{E s}\right)\right)$ with $S\left(\theta_{i}, e_{i s}\left(\theta_{i}, \tau_{T s}\right)\right)$. This difference is positive if

[^8]\[

$$
\begin{aligned}
& \frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta \bar{e}_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} e_{i s}\left(\tau_{E s}\right) \theta_{i}-\frac{\psi e_{i s}^{2}\left(\tau_{E s}\right)}{2} \\
&> \frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta \bar{e}_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} e_{i s}\left(\tau_{T s}\right) \theta_{i}-\frac{\psi e_{i s}^{2}\left(\tau_{T s}\right)}{2} \\
& \Longleftrightarrow \\
& \frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}}\left(e_{i s}\left(\tau_{E s}\right)-e_{i s}\left(\tau_{T s}\right)\right) \theta_{i} \\
&> \frac{\psi}{2}\left(e_{i s}\left(\tau_{E s}\right)-e_{i s}\left(\tau_{T s}\right)\right)\left(e_{i s}\left(\tau_{E s}\right)+e_{i s}\left(\tau_{T s}\right)\right)
\end{aligned}
$$
\]

which holds for $\theta_{i}>\widetilde{\theta}_{E s}$, where ${ }^{21}$

$$
\theta_{i}>\widetilde{\theta}_{E s} \equiv \frac{\rho_{s}+\rho_{\varepsilon}}{\rho_{\varepsilon}} \frac{\psi}{2}\left(e_{i s}\left(\tau_{E s}\right)+e_{i s}\left(\tau_{T s}\right)\right)
$$

Finally, any student from $s$ with ability $\theta_{i} \in\left(0, \theta_{T}\right)$ chooses her effort to be admitted to university $T$ if $S\left(\theta_{i}, e_{i s}\left(\tau_{T s}\right)\right)>S\left(\theta_{i}, e_{i s}\left(\tau_{P s}\right)\right)$, where $S\left(\theta_{i}, \tau_{P s}\right)=0$, given that the student does not exert any effort if she plans to be admitted at $P$. Comparing the two utilities, one obtains

$$
\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta \bar{e}_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} e_{i s}\left(\tau_{T s}\right) \theta_{i}>\frac{\psi e_{i s}^{2}\left(\tau_{T s}\right)}{2}
$$

which holds for $\theta_{i}>\widetilde{\theta}_{T s}$, where

$$
\tilde{\theta}_{T s} \equiv \frac{\rho_{s}+\rho_{\varepsilon}}{\rho_{\varepsilon}} \frac{\psi}{2} e_{i s}\left(\tau_{T s}\right)-\frac{\rho_{s}}{\rho_{\varepsilon}} \eta \bar{e}_{s} \bar{\theta}_{s} .
$$

We may thus summarise the following.
Corollary 2. For $\theta \in\left[0, \widetilde{\theta}_{T s}\right]$, $e_{i s}\left(\tau_{P s}\right)=0$ is such that $\widehat{h}_{P i s}\left(\tau_{P s}\right)=0$. For $\theta \in$ $\left(\widetilde{\theta}_{T s}, \widetilde{\theta}_{E s}\right), e_{i s}\left(\tau_{T s}, \theta_{i}\right)$ is such that

$$
\begin{equation*}
\hat{h}_{i s}\left(\tau_{T s}\right)=\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta e_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} e_{i s}\left(\tau_{T s}\right) \theta_{i} . \tag{21}
\end{equation*}
$$

[^9]For $\theta \in\left(\widetilde{\theta}_{T s}, \widetilde{\theta}_{E s}\right), e_{i s}\left(\tau_{E s}, \theta_{i}\right)$ is such that

$$
\begin{equation*}
\hat{h}_{i s}\left(\tau_{E s}\right)=\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta e_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} e_{i s}\left(\tau_{E s}\right) \theta_{i} . \tag{22}
\end{equation*}
$$

Corollary 2 sorts the students in each university according to their ability. What remains unanswered is how, for a given ability, a university's admission policy changes according to social group. To ease the exposition and avoid cumbersome comparison between discrete levels of social groups, in what follows we will assume a continuum of social groups $s \in R^{+}$, where, in line with the previous assumptions, $\frac{\partial \rho_{s}}{\partial s}=\rho_{s}^{\prime}>0$. We also focus on the case in which average ability is the same across social groups, $\frac{\partial \bar{\theta}_{s}}{\partial s}=0$ and $\theta>0$. Considering a generic university $k$ whose expected human capital is represented by

$$
\begin{equation*}
\hat{h}_{k}\left(\tau_{k s}\right)=\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta \bar{e}_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} \tau_{k s}(\theta), \tag{23}
\end{equation*}
$$

we may differentiate (23) with respect to $s$ to evaluate differences in the admission test score. We obtain:

$$
\begin{gather*}
0=\frac{\rho_{\varepsilon} \rho_{s}^{\prime}}{\left(\rho_{s}+\rho_{\varepsilon}\right)^{2}} \eta \bar{e}_{s} \bar{\theta}_{s}+\frac{\rho_{s}}{\rho_{s}+\rho_{\varepsilon}} \eta \bar{\theta}_{s} \frac{\mathrm{~d} \bar{e}_{s}\left(\tau_{k s}\right)}{\mathrm{d} \tau_{k s}} \frac{\mathrm{~d} \tau_{k s}}{\mathrm{~d} s} \\
-\frac{\rho_{\varepsilon} \rho_{s}^{\prime}}{\left(\rho_{s}+\rho_{\varepsilon}\right)^{2}} \tau_{k s}+\frac{\rho_{\varepsilon}}{\rho_{s}+\rho_{\varepsilon}} \frac{\mathrm{d} \tau_{k s}}{\mathrm{~d} s}, \\
\left(\rho_{\varepsilon}+\rho_{s} \eta \bar{\theta}_{s} \frac{\mathrm{~d} \bar{e}_{s}\left(\tau_{k s}\right)}{\mathrm{d} \tau_{k s}}\right) \frac{\mathrm{d} \tau_{k s}}{\mathrm{~d} s}=-\frac{\rho_{\varepsilon} \rho_{s}^{\prime}}{\left(\rho_{s}+\rho_{\varepsilon}\right)} \eta \bar{e}_{s} \bar{\theta}_{s}+\frac{\rho_{\varepsilon} \rho_{s}^{\prime}}{\left(\rho_{s}+\rho_{\varepsilon}\right)} \tau_{k s}, \tag{24}
\end{gather*}
$$

In (24), $\frac{\mathrm{d} \bar{e}_{s}\left(\tau_{k s}\right)}{\mathrm{d} \tau_{k s}}>0$ since study effort increases, for given ability, with the admission threshold. Calling $\bar{t}_{s}=\bar{e}_{s} \overline{\bar{\theta}}_{s}$ as the expected test score of the student with average group ability $\bar{\theta}_{s}$, and solving with respect to $\frac{\mathrm{d} \tau_{k s}}{\mathrm{~d} s}$, one obtains

$$
\begin{equation*}
\frac{\mathrm{d} \tau_{k s}}{\mathrm{~d} s}=\frac{1}{\rho_{\varepsilon}+\rho_{s} \eta \bar{\theta}_{s} \frac{\mathrm{~d} \bar{e}_{s}\left(\tau_{k}\right)}{\mathrm{d} \tau_{k}}} \frac{\rho_{\varepsilon} \rho_{s}^{\prime}}{\left(\rho_{s}+\rho_{\varepsilon}\right)}\left(\tau_{k s}-\bar{t}_{s}\right) \tag{25}
\end{equation*}
$$

from which we may state

Corollary 3. If the admission grade is higher than the expected grade of the student with an average group ability, $\tau_{k s}>\bar{t}_{s}$, then the admission threshold of students from a higher social group is also higher. Vice versa when $\tau_{k s}<\bar{t}_{s}$.

The findings in Corollary 3 results seem consistent with the baseline results: for instance, university $E$ likely sets its admission threshold above the test score
obtained by a student with average group ability, and should admit students with a lower $s$ with a lower grade. The opposite applies to University $P$. This situation mirrors the situation described in Figure 4 and Proposition 3.

## 7 Concluding Remarks

In this paper, we have investigated how potentially discriminatory characteristics, such as sex, race, or social background, could influence university admissions. To achieve this, we examined the interaction between students and universities during the admission process. The findings show that higher-ranked universities tend to set lower grade requirements in the admission test for students coming from disadvantaged groups. Conversely, lower-ranked universities follow the opposite pattern. With the implementation of an anti-discrimination policy, students with the same grades will attend universities at the same level. However, they will still exhibit varying levels of human capital based on their social group affiliation.

The results of this study critically rely on the assumption of higher precision of ability in the advantaged social group. While footnote 12 presents a substantial body of empirical evidence, the paper does not directly verify the validity of this assumption through an empirical exercise, which somewhat limits the relevance of the current analysis. Nevertheless, it is reasonable to anticipate a distinct shape in the distribution of ability between students from different social groups. In simpler terms, the slopes of Figure 4 are expected to differ, a priori. A comprehensive empirical investigation to confirm whether disadvantaged students indeed exhibit a more volatile distribution of ability is left for future research.

Finally, our analysis abstracts away from discrimination stemming from bias against members of a specific targeted group (see Becker 1957), which could potentially influence the university selection process in the real world.

The analysis conducted can be adapted to study job recruitment instead of university admission, especially when explaining the role of observable characteristics that may not be relevant to the job. Evidence from field and experimental studies, since Goldin and Rouse's (2000) seminal work, strongly suggests that if employers are prevented from considering characteristics deemed irrelevant to job performance, their hiring decisions are often significantly altered. ${ }^{22}$ Recruitment services are now proliferating, offering employers protection against discrimination by filtering out all "non-relevant" information provided by job applicants before passing the CV to employers. ${ }^{23}$

22 See also Edin and Lagerström (2006) and Aslund and Skans (2012), among others.
23 For example, www.2apply.co.uk.

## Appendix

## Derivation of the Expected University

In this appendix we derive the expected university from Equations (5) to (6). For further details, a more formal derivation may be found in De Groot (1970), page 167, Theorem 1.

As stressed in Remark 1, the expected university corresponds to a student's expected human capital, $\widehat{u}(e)=\widehat{h}$. However, when they formulate their expectations of human capital, universities treat student $i$ 's effort $e_{i s}$ and expected students from social group s's effort $e_{s}$ as constants, which just shifts the expectation. This is due to the fact that admission strategies are established before students choose their effort. Hence in what follows, to simplify the exposition, we abstract away from effort by focusing on the expectation of ability (rather than human capital) given the two signals "social group" and "test score". In addition, below we employ the relationship between precision and variance $\rho=\frac{1}{\sigma^{2}}$ and use variance and standard deviation. We will then switch back to precision at the end to match the formula in the main text.

Denote the expectation of ability conditioned to the two signals, social group $s$ and test score $t$, as $\bar{\theta}_{p}$. In terms of Bayesian updating, $\bar{\theta}_{p}$ corresponds to the mean of the posterior function.

The prior function of ability coming from social group $s$ is represented by the distribution of ability $\mathcal{N}\left(\bar{\theta}_{s}, \sigma_{s}^{2}\right)$ in the social group considered. Thus, the prior follows a normal distribution with exponential quadratic probability density function:

$$
f_{s}(\theta)=\frac{1}{\sigma_{s} \sqrt{2 \pi}} \exp \left\{-\frac{1}{2 \sigma_{s}^{2}}\left(\theta-\bar{\theta}_{s}\right)^{2}\right\}
$$

Setting aside effort, the test score of student with ability $\theta$ is $t=\theta+\varepsilon$. It is distributed according to the likelihood distribution with random error $\varepsilon \sim \mathcal{N}\left(0, \sigma_{\varepsilon}^{2}\right)$,

$$
f_{\varepsilon}(\theta)=\frac{1}{\sigma_{\varepsilon} \sqrt{2 \pi}} \exp \left\{-\frac{1}{2 \sigma_{\varepsilon}^{2}}\left(\theta-\bar{\theta}_{s}\right)^{2}\right\}
$$

This is also a normal distribution with exponential quadratic probability density function. We assume that the variance of the likelihood distribution $\sigma_{\varepsilon}^{2}$ is known, so that we have to estimate only the expected value of the posterior distribution, $\bar{\theta}_{p}$. In particular, the relationship between distribution is

$$
f_{p}(\theta) \propto f_{s}(\theta) \times f_{\varepsilon}(\theta)
$$

Note that both priors are exponential quadratic density functions. Such priors are also called coniugate priors, given their similar form. It follows that also the posterior distribution will be exponential quadratic.

We can then calculate the posterior distribution, which is generally given by

$$
\begin{equation*}
f_{p}(\theta)=K f_{s}(\theta) \times f_{\varepsilon}(\theta) \tag{26}
\end{equation*}
$$

where $K$ is a constant. The probability density function (26) may thus be rewritten as

$$
f_{p}(\theta)=K \times \frac{1}{\sigma_{s} \sqrt{2 \pi}} \exp \left\{-\frac{1}{2 \sigma_{s}^{2}}\left(\theta-\bar{\theta}_{s}\right)^{2}\right\} \times \frac{1}{\sigma_{\varepsilon} \sqrt{2 \pi}} \exp \left\{-\frac{1}{2 \sigma_{\varepsilon}^{2}}\left(\theta-\bar{\theta}_{s}\right)^{2}\right\}
$$

We define $K^{\prime} \equiv K \frac{1}{\sigma_{s} \sqrt{2 \pi}} \frac{1}{\sigma_{\varepsilon} \sqrt{2 \pi}}$ and apply the properties of the product of exponentials to get

$$
\begin{equation*}
f_{p}(\theta)=K^{\prime} \exp \left\{-\frac{1}{2}\left[\frac{1}{\sigma_{s}^{2}}\left(\theta-\bar{\theta}_{s}\right)^{2}+\frac{1}{\sigma_{\varepsilon}^{2}}\left(\theta-\bar{\theta}_{s}\right)^{2}\right]\right\} \tag{27}
\end{equation*}
$$

For convenience, we denote the term in the square brackets in (27) as follows:

$$
\begin{equation*}
A \equiv \frac{1}{\sigma_{s}^{2}}\left(\theta-\bar{\theta}_{s}\right)^{2}+\frac{1}{\sigma_{\varepsilon}^{2}}\left(\theta-\bar{\theta}_{s}\right)^{2} . \tag{28}
\end{equation*}
$$

Hence the posterior distribution of ability $\theta$ is distributed following $\theta \sim$ $\mathcal{N}\left(\bar{\theta}_{p}, \sigma_{p}^{2}\right)$, whose probability density function is

$$
f_{p}(\theta)=\frac{1}{\sigma_{p} \sqrt{2 \pi}} \exp \left\{-\frac{1}{2 \sigma_{p}^{2}}\left(\theta-\bar{\theta}_{p}\right)^{2}\right\} .
$$

For convenience, we define

$$
\begin{equation*}
B \equiv \frac{1}{\sigma_{p}^{2}}\left(\theta-\bar{\theta}_{p}\right)^{2} . \tag{29}
\end{equation*}
$$

We are now in a position to determine the values of $\bar{\theta}_{p}$ and $\sigma_{p}^{2}$ in terms of $\bar{\theta}_{s}$ and $\sigma_{s}^{2}$. To do so, we note that Equations (28) and (29) are identical. We can expand $A$ to get

$$
\begin{equation*}
A \equiv \theta^{2}\left(\frac{1}{\sigma_{s}^{2}}+\frac{1}{\sigma_{\varepsilon}^{2}}\right)-2 \theta\left(\frac{\bar{\theta}_{s}}{\sigma_{s}^{2}}-\frac{\bar{\theta}_{s}}{\sigma_{\varepsilon}^{2}}\right)+\bar{\theta}_{s}^{2}\left(\frac{1}{\sigma_{s}^{2}}+\frac{1}{\sigma_{\varepsilon}^{2}}\right) . \tag{30}
\end{equation*}
$$

We can expand $B$ to get

$$
\begin{equation*}
B=\frac{1}{\sigma_{p}^{2}}\left(\theta-\bar{\theta}_{p}\right)^{2}=\frac{\theta^{2}}{\sigma_{p}^{2}}+\frac{\bar{\theta}_{p}^{2}}{\sigma_{p}^{2}}-\frac{2 \theta \bar{\theta}_{p}}{\sigma_{p}^{2}} . \tag{31}
\end{equation*}
$$

From (31), the coefficient of $\theta^{2}$ can be rewritten as

$$
\frac{1}{\sigma_{p}^{2}}=\frac{1}{\sigma_{s}^{2}}+\frac{1}{\sigma_{\varepsilon}^{2}}
$$

which, rearranged, yields

$$
\begin{equation*}
\sigma_{p}^{2}=\frac{\sigma_{s}^{2} \sigma_{\varepsilon}^{2}}{\sigma_{\varepsilon}^{2}+\sigma_{s}^{2}} \tag{32}
\end{equation*}
$$

This amounts to the standard deviation for the posterior distribution in terms of the known standard deviations of the prior distribution of social group and the likelihood distribution of the test score.

Equating the coefficients of $-2 \theta$ in Equations (30) and (31) yields

$$
\frac{\bar{\theta}_{p}}{\sigma_{p}^{2}}=\frac{\bar{\theta}_{s}}{\sigma_{s}^{2}}+\frac{\bar{\theta}_{s}}{\sigma_{\varepsilon}^{2}}
$$

which may be rewritten as

$$
\begin{equation*}
\bar{\theta}_{p}=\sigma_{p}^{2}\left[\frac{\bar{\theta}_{s} \sigma_{\varepsilon}^{2}+\bar{\theta}_{s} \sigma_{s}^{2}}{\sigma_{s}^{2} \sigma_{\varepsilon}^{2}}\right] . \tag{33}
\end{equation*}
$$

Next, we may plug (32) into (33) to get

$$
\begin{equation*}
\bar{\theta}_{p}=\frac{\bar{\theta}_{s} \sigma_{\varepsilon}^{2}+\bar{\theta}_{s} \sigma_{s}^{2}}{\sigma_{\varepsilon}^{2}+\sigma_{s}^{2}} \tag{34}
\end{equation*}
$$

Remembering that $\sigma^{2}=\frac{1}{\rho}$, (34) may be rewritten as

$$
\bar{\theta}_{p}=\frac{\bar{\theta}_{s} \frac{1}{\rho_{\varepsilon}}+\bar{\theta}_{s} \frac{1}{\rho_{s}}}{\frac{1}{\rho_{\varepsilon}}+\frac{1}{\rho_{s}}}
$$

which, rearranged, yields

$$
\bar{\theta}_{p}=\frac{\rho_{s} \bar{\theta}_{s}+\rho_{\varepsilon} \bar{\theta}_{s}}{\rho_{s}+\rho_{\varepsilon}}
$$

Finally, we may add to the formula the expected effort components, namely, $\eta e_{s}$ for the prior distribution and $e_{i s}$ for the likelihood distributions of test score, which are treated as constants in the evalutation of the posterior distribution of ability:

$$
u_{i s}\left(e_{i s}\right)=\widehat{h}\left(e_{i s}\right)=\frac{\rho_{s}\left(\bar{\theta}_{s}+\eta e_{s}\right)+\rho_{\varepsilon}\left(\bar{\theta}_{s}+e_{i s}\right)}{\rho_{s}+\rho_{\varepsilon}}
$$

In this way, we obtain the formula of the expected human capital in the main text.

## References

Anger, S., and D. D. Schnitzlein. 2017. "Cognitive Skills, Non-Cognitive Skills, and Family Background: Evidence from Sibling Correlations." Journal of Population Economics 30: 591-620.
Arcidiacono, P., and M. Lovenheim. 2016. "Affirmative Action and the Quality-Fit Trade-Off." Journal of Economic Literature 54: 3-51.
Arcidiacono, P., E. M. Aucejo, and V. J. Hotz. 2016. "University Differences in the Graduation of Minorities in STEM Fields: Evidence from California." The American Economic Review 106: 525-62.
Arrow, K. J. 1973. "The Theory of Statistical Discrimination." In Discrimination in Labor Markets, edited by Ashenfelter, and Rees. Princeton: Princeton University Press.
Aslund, O., and O. N. Skans. 2012. "Do Anonymous Job Application Procedures Level the Playing Field?" Industrial and Labor Relations Review 65: 82-107.
BBC News. 2012. "Brazil Approves Affirmative Action Law for Universities." http://www.bbc.co.uk/ news/world-latin-america-19188610.
Becker, G. S. 1957. The Economics of Discrimination. Chicago: University of Chicago Press.
Betts, J. R. 1998. "The Impact of Educational Standards on the Level and Distribution of Earnings." The American Economic Review 88: 266-75.
Bowen, W. G., and D. Bok. 1998. The Shape of the River: Long-Term Consequences of Considering Race in College and University Admissions. Princeton: Princeton University Press.
Carneiro, P., and J. J. Heckman. 2003. "Human Capital Policy." In Inequality in America: What Role for Human Capital Policies? edited by J. J. Heckman, and A. B. Krueger. Cambridge: The MIT Press.
Chadi, A., M. de Pinto, and G. Schultze. 2019. "Young, Gifted and Lazy? The Role of Ability and Labor Market Prospects in Student Effort Decisions." Economics of Education Review 72: 66-79.
Chan, J., and E. Eyster. 2004. "Does Banning Affirmative Action Harm College Quality?" The American Economic Review 94: 858-72.
Coate, S., and G. C. Loury. 1993. "Will Affirmative-Action Policies Eliminate Negative Stereotypes?" The American Economic Review 83: 1220-40.
Costrell, R. M. 1994. "A Simple Model of Educational Standards." The American Economic Review 84: 956-71.
Cunha, F., J. J. Heckman, L. Lochner, and D. V. Masterov. 2006. "Interpreting the Evidence on Life Cycle Skill Formation." Handbook of the Economics of Education 1: 697-812.
De Donder, P., and F. Martinez-Mora. 2017. "The Political Economy of Higher Education Admission Standards and Participation Gap." Journal of Public Economics 154: 1-9.
De Fraja, G. 2001. "Policies: Equity, Efficiency and Voting Equilibrium." Economic Journal 111: 104-19.
De Fraja, G. 2005. "Reverse Discrimination and Efficiency in Education." International Economic Review 46: 1009-31.
De Fraja, G., T. Oliveira, and L. Zanchi. 2010. "Must Try Harder. Evaluating the Role of Effort on Examination Results." The Review of Economics and Statistics 92: 577-97.
De Fraja, G., K. Eleftheriou, and M. Ioakimidis. 2021. "A Note on University Admission Tests: Simple Theory and Empirical Analysis." The B.E. Journal of Economic Analysis \& Policy 22: 623-32.
De Groot, M. H. 1970. Optimal Statistical Decisions. New York: McGraw-Hill.
Dewatripont, M., I. Jewitt, and J. Tirole. 1999a. "The Economics of Career Concerns, Part I: Comparing Information Structures." The Review of Economic Studies 66: 183-98.
Dewatripont, M., I. Jewitt, and J. Tirole. 1999b. "The Economics of Career Concerns, Part II: Application to Missions and Accountability of Government Agencies." The Review of Economic Studies 66: 199-217.

DfES. 2003. The Future of Higher Education, 5735. Richmond: Department for Education and Skills, HMSO.
Downey, D. B., P. T. von Hippel, and A. B. Beckett. 2004. "Are Schools the Great Equalizer? Cognitive Inequality during the Summer Months and the School Year." American Sociological Review 69: 613-35.
Edin, P. A., and J. Lagerström. 2006. Blind Dates: Quasi Experimental Evidence in Discrimination, Vol. 4. Uppsala: Institute for Evaluation of Labour Market and Education Policy.
Epple, D., R. Romano, S. Sarpca, and H. Sieg. 2006. "Profiling in Bargaining Over College Tuition." Economic Journal 11: 6459-479.
Epple, D., R. Romano, and H. Sieg. 2008. "Diversity and Affirmative Action in Higher Education." Journal of Public Economic Theory 10: 475-501.
Fang, H., and A. Moro. 2011. "Theories of Statistical Discrimination and Affirmative Action: A Survey." In Handbook of Social Economics, 133-200. The Netherlands: Handbook of Social Economics.
Flavell, J. H. 1976. "Metacognitive Aspects of Problem Solving." In The Nature of Intelligence, edited by L. B. Resnick, 231-6. Hillsdale: Erlbaum.

Flavell, J. H. 1979. "Metacognition and Cognitive Monitoring: A New Area of Cognitive - Developmental Inquiry." American Psychologist 34 (10): 906-11.
Galindo-Rueda, F., and A. Vignoles. 2003. Class Ridden or Meritocratic? An Economic Analysis of Recent Changes in Britain, Vol. 32. London: Centre for the Economics of Education, London School of Economics.
Gary-Bobo, R., and A. Trannoy. 2008. "Efficient Tuition Fees and Examinations." Journal of the European Economic Association 6: 1211-43.
Goldin, C., and C. Rouse. 2000. "Orchestrating Impartiality: The Impact of "Blind" Auditions on Female Musicians." The American Economic Review 90: 715-41.
Gottschling, J., E. Hahn, C. R. Beam, F. M. Spinath, S. Carroll, and E. Turkheimer. 2019. "Socioeconomic Status Amplifies Genetic Effects in Middle Childhood in a Large German Twin Sample." Intelligence 72: 20-7.
Hanscombe, K. B., M. Trzaskowski, C. M. A. Haworth, O. S. P. Davis, P. S. Dale, and R. Plomin. 2012. "Socioeconomic Status (SES) and Children’s Intelligence (IQ): In a UK-Representative Sample SES Moderates the Environmental, Not Genetic, Effect on IQ." PLoS One 7: 1-7.
Hauser, R. M. 2002. Meritocracy, Cognitive Ability, and the Sources of Occupational Success, Vol. 32. Madison: Center for Demography and Ecology, University of Wisconsin-Madison.
Joshi, H. E., and A. McCulloch. 2001. "Neighbourhood and Family Influences on the Cognitive Ability of Children in the British National Child Development Study." Social Science \& Medicine 53: 579-91.
Kaganovich, M., and X. Su. 2019. "College Curriculum, Diverging Selectivity, and Enrollment Expansion." Economic Theory 67: 1019-50.
Krishna, K., and A. Tarasov. 2016. "Affirmative Action: One Size Does Not Fit All." American Economic Journal: Microeconomics 8: 215-52.
Lee, V. E., and D. T. Burkam. 2002. "Inequality at the Starting Gate: Social Background Differences." In Achievement as Children Begin School. Washington: Economic Policy Institute
Long, M. C. 2004. "College Applications and the Effect of Affirmative Action." Journal of Econometrics 121: 319342.
Lundberg, S. J., and R. Startz. 1983. "Private Discrimination and Social Intervention in Competitive Labor Market." The American Economic Review 73: 340-7.

MacLeod, W. B., and M. Urquiola. 2015. "Reputation and School Competition." The American Economic Review 105: 3471-88.

Marton, F., D. Hounsell, and N. Entwistle. 1997. The Experience of Learning. Edinburgh: Scottish Academic Press.
Mayer, S. E. 1997. What Money Can't Buy: Family Income and Children's Life Chances. Cambridge: Harvard University Press.
Melguizo, T. 2008. "Quality Matters: Assessing the Impact of Attending More Selective Institutions on College Completion Rates of Minorities." Research in Higher Education 49: 214-36.
Melguizo, T. 2010. "Are Students of Color More Likely to Graduate from College if they Attend More Selective Institutions? Evidence from a Cohort of Recipients and Nonrecipients of the Gates Millennium Scholarship Program." Educational Evaluation and Policy Analysis 20: 1-19.
Metcalfe, J., and A. P. Shimamura. 1994. Metacognition: Knowing about Knowing. Cambridge: MIT Press.
New York Times. 2012a. "Rethinking Affirmative Action." http://www.nytimes.com/2012/10/14/ sunday-review/rethinking-affirmative-action.html?
New York Times. 2012b. "Affirmative Action - A Complicated Issue for Asian-Americans." http://www .nytimes.com/2012/11/04/education/edlife/affirmative-action-a-complicated-issue-for-asianamericans.html?
New York Times. 2016. "Affirmative Action Isn’t Just a Legal Issue. It’s Also a Historical One." https:// www.nytimes.com/2016/06/25/opinion/affirmative-action-isnt-just-a-legal-issue-its-also-a-historical-one.html.
Niu, S. X., M. Tienda, and K. Cortes. 2006. "College Selectivity and the Texas Top 10\% Law." Economics of Education Review 25: 259-72.
Pacelli, K. A. 2011. "Fisher V. University of Texas at Austin: Navigating the Narrows Between Grutter and Parents Involved." Maine Law Review 63: 569-92.
Phelps, E. 1972. "The Statistical Theory of Racism and Sexism." The American Economic Review 62: 659-61.
Phillips, M., J. Brooks-Gunn, G. J. Duncan, and P. K. Klebanov. 1998. "Family Background, Parenting Practices, and the Black-White Test Score Gap." In The Black-White Test Score Gap, edited by Charles Jencks, and Meredith Phillips. Washington: The Brookings Institute.
Reardon, S. 2003. "The Growth of Racial - Ethnic and Socioeconomic Test Score Gaps in Kindergarten and First Grade." Paper presented at the Annual Conference of the American Sociological Association.
Spence, M. A. 1973. "Job Market Signalling." Quarterly Journal of Economics 873: 55-74.
Sowell, T. 2004. Affirmative Action Around the World: An Empirical Study. New Haven: Yale University Press.
Wainer, J., and T. Melguizo. 2018. "Inclusion Policies in Higher Education: Evaluation of Student Performance Based on the Enade from 2012 to 2014." Educação e Pesquisa 44. https://doi.org/10 .1590/S1517-9702201612162807.


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[^2]:    1 Pacelli (2011) reviews the four US Supreme Court landmark cases - Bakke, Hopwood, Grutter, and Parents Involved - in her analysis of the latest such case to appear before the Supreme Court, Abigail Fisher's claim that she was denied admission by University of Texas Austin, while less qualified Latinos and African-Americans were offered places. The issue received media coverage in the New York Times (2012a), (2012b) and (2016), among others.
    2 See DfES (2003). Several other countries in the world impose some form of affirmative action (Sowell 2004). Most recently, Brazil passed a law reserving half of the university places for students from state schools and increasing the number of students of African descent.
    3 Throughout the paper, we use the term "ability" to indicate a combination of several factors affecting a student’s cognitive skills and potential. In Section 3.1, we give a more specific definition.
    4 See, among others, Galindo-Rueda and Vignoles (2003) and De Fraja, Oliveira, and Zanchi (2010).
    5 See Section 3.1 for further discussion.

[^3]:    6 In a recent contribution, Arcidiacono and Lovenheim (2016) survey the empirical literature on the effects of affirmative action programs.
    7 Fang and Moro (2011) provide an exhaustive survey of this literature.
    8 Compared the present analysis, the distribution of ability differs among social groups and universities are aware of these distributions (i.e. there are no prejudices).

[^4]:    9 See below the discussion of Gary-Bobo and Trannoy (2008).
    10 Seminal works are Flavell $(1976,1979)$, while a more recent discussion can be found in Metcalfe and Shimamura (1994).
    11 To be more precise, De Fraja (2005) denotes as "potential to benefit from education" what we call ability to highlight the fact that it is determined by a combination of natural and environmental features.

[^5]:    12 For example, in early UK longitudinal studies, the measure of ability used by Galindo-Rueda and Vignoles (2003) and De Fraja, Oliveira, and Zanchi (2010) shows that the ability distribution of children from higher socioeconomic groups has higher mean and lower variance than that of children from less favoured households. In a more indirect test, Hauser (2002) regresses the mean against the standard deviation in several occupational groups, and finds a weak negative relationship. More recently, Hanscombe et al. (2012), Anger and Schnitzlein (2017), and Gottschling et al. (2019) provide further evidence in this direction.

    13 Parameter $\eta$ is familiar in the Spence signalling set-up (Spence 1973): $\eta=0$ is the extreme case of education as a pure signal, adding nothing to a person's human capital. The restriction that $\eta$ be at most 1 implies that, relative to innate characteristics, the student's effort at school is more effective in helping students do well in the test than in helping them to be successful in their postuniversity career.
    14 We use the term "test", but extending its meaning to include other relatively objectively measures, such as a written personal statement or extracurricular activities would be straightforward.

[^6]:    16 The case where not all students attend university, but receive a labour market payoff which is increasing with their human capital can be easily incorporated in this framework.

[^7]:    18 Note that some students with an expected human capital of $\widehat{h}_{E}\left(\tau_{E s}\right)$ may not secure admission to University $E$ due to capacity constraints. In other words, the capacity constraint may not be able to accommodate all students with the lowest expected level of human capital, leading them to enrol in university $T$. We may assume that (i) university $E$ makes its selection before university $T$, and (ii) it randomly selects amongst students with an expected human capital of $\widehat{h}_{E}\left(\tau_{E s}\right)$ to fill the remaining positions.
    19 It is true that students may undergo training for the test by taking 'mock exams' to assess their preparation and deduce information about the admission level required. However, even to prepare for the mock exam and gather information, the student must exert effort.

[^8]:    20 More precisely, it is suboptimal unless a student has exactly the level of ability such that the optimal effort associated is right enough to be admitted to a certain university.

[^9]:    21 Given that each student is an atom, we may assume that her choice does not affect the average choice of her social group, $\bar{e}_{s}$, which is taken as given.

