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Research paper

Inter-temporal decisions, optimal taxation and non-compliant behaviors in groundwater management

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A R T I C L E I N F O

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A B S T R A C T

A sizable part of the water extracted is unauthorized. This phenomenon may contribute to exacerbate the problem of groundwater over-exploitation. To consider both issues, we study the interaction between the water agency and farmers through a leader–follower differential game in which both agents are perfect foresight. Since the farmers have to pay a tax on individual withdrawals imposed by the water agency to manage the groundwater, illegal behaviors may arise to save this cost. However, if discovered, the farmers are punished with an administrative sanction. The game is solved using feedback Stackelberg solution. Moreover, to enrich the model's policy suggestions, we also consider an alternative context in which farmers can adopt a trigger strategy. Finally, we perform numerical simulations based on the western La Mancha (Spain) aquifer data to better understand both the analytical results and the effects of the sanction mechanism on non-compliant behaviors.

1. Introduction

In recent decades, the water demand, mostly to satisfy a growing food demand, has experienced a dramatic increase around the world. Indeed, as in the case of groundwater, when the resource has a strategic value and the feasibility of its exploitation is threatened, it is generally necessary to establish rules for accessing and managing this resource and to ensure the balance between supply and demand. In most arid and semiarid countries, water resources management is an issue as important as controversial. Many water resources experts admit that water conflicts are not caused by the physical water scarcity, but they are mainly due to poor water management [1]. Water authorities are placing increasingly attention to regulation or, at least, to tracking the use of groundwater since they must face both over-exploitation and unauthorized withdrawals. Indeed, there is evidence of a significant volume of unauthorized or informal water extraction in many areas of the world, which considerably affects its over-exploitation [2].

The illegal water pumping may impact both aquifer ecosystem and legal users who could see their profits decrease while illegal users could gain by breaking the law. It might be the society as a whole that suffers the consequences of this illegal use, since the uncontrolled exploitation of water resources also results in the ecosystem's deterioration. In some areas of the world, such as Southern Europe and Chile, the phenomenon of illegality is more widespread and several studies estimate that it accounts for around 30% and 60% of the total water extraction [3–5]. The water agencies are

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inability to stop this activity, but using some instruments for water management and law enforcement, such as fines or penalties and water taxes, it can try to reduce the illegal pumping phenomenon. The theory of depletable resources such as groundwater is an important field in economic theory, encompassing a large range of analytical results with major contributions to the sustainability of resource exploitation.

The seminal work of Gisser and Sanchez [6] is the starting point of a large literature investigating welfare gains from public intervention. Most of these contributions show a social planner's solution (see, among others, [7,8]), while others compare the strategic behaviors of farmers under perfect competition (see, among others, [9–12]). Some of these studies advocate that the benefits from policy intervention with respect to competition are insignificant and depend on hydrological and economic parameters. The debate has been enriched including environmental externalities into the analytical framework [13,14], firms' heterogeneity [15] or intragenerational and intergenerational competition through overlapping generations approach [16].

The papers cited above do not deal with the problem of the illegal exploitation of the water resource, a phenomenon studied only in a few works. Biancardi et al. [17] studies a hydro-economic model through a differential game between legal and illegal firms (the water agency activity is exogenous). The contribution of taxes and penalties imposed by public authorities are shown in a non-cooperative and cooperative context. Conversely, Biancardi et al. [18] deals with the strategic interaction between compliant and non-compliant firms in an evolutionary context. This approach emphasizes the farmers' myopic behaviors towards the ecosystem since they consider the impact of their decisions on the environment as negligible. Finally, in Biancardi et al. [19], authors introduces the water agency as an active player through a leader–follower differential game. The water agency is the leader while the follower is a population of identical myopic farmers that can decide to not declare all water pumped. Differently from Biancardi et al. [19], in this paper farmers are perfect foresight, emphasizing the strategic interaction between players as well as the effects of policy instruments, taxation, and sanction, on illegal behaviors. Moreover, differently from previous works, we also consider a framework in which some farmers can defect from the agreed strategy. The water agency reacts to a trigger strategy imposing a threat (for an introduction to trigger strategies, see [20]). Numerical simulations, using the widely adopted data of the Western La Mancha region (Spain), complete the study.

The paper is organized as follows. Section 2 presents the model and Section 3 solves the inter-temporal maximization problems of both farmers and water agency. Section 4 proposes numerical simulations to derive policy implications to counter non-compliant behaviors. The trigger strategy framework is developed in Section 5, and, finally, Section 6 concludes.

2. The model

Let us consider a population of N farmers that pump water ($w \geq 0$) to irrigate their crops. According to the seminal work of Gisser and Sanchez [6], the aquifer dynamics is a function of natural recharge ($R > 0$) and of farmers' total withdrawals ($W \geq 0$):

$$\dot{H} = \frac{1}{S_a}\left[R - (1-\gamma)W\right] \tag{1}$$

where $\gamma \in (0, 1)$ is the return flow coefficient and $S_a > 0$ is the area of the aquifer times storativity.

Following Rubio and Casino [11], the demand for irrigation is an affine function of the water price p:

$$W = \alpha - \beta p$$

where $\alpha > 0$ and $\beta > 0$ represent the intercept and the slope, respectively. Assuming symmetry between farmers ($W = wN$) and agricultural competitive markets, by integrating the water price, we get the farmers' revenues:

$$\int p(w)\,\mathrm{d}w = \frac{\alpha}{\beta}w - \frac{N}{2\beta}w^2$$

Differently, pumping costs are a function of both water pumped and aquifer [6]:

$$C(w, H) = (c_0 - c_1 H)w$$

where $c_0 > 0$ is the fixed cost (with respect to the water table) due to the hydrologic cone and $c_1 > 0$ is the marginal cost (with respect to the water table). The ratio $\overline{H} = \frac{c_0}{c_1}$ is the maximum level of the aquifer.

The right of pumping is given by the water agency upon payment of a tax $\tau > 0$ for each Mm^3 [19,21,22]. However, to save the tax cost, farmers can decide to not declare a share $\theta \in [0, 1]$ and face the risk of being sanctioned. We denote $(1-\theta)w$ as the amount of water pumped legally and θw as the amount of water pumped illegally. Profit of the representative farmer is composed of a compliant part, given by revenues, pumping, and taxation costs, and a non-compliant one, given by the overall sanction, namely, the taxation not payed and an administrative sanction, subject to the probability of being discovered:

$$\pi = \frac{\alpha}{\beta}w - \frac{N}{2\beta}w^2 - (c_0 - c_1 H)w - (1-\theta)\tau w - \phi\theta\tau\sigma w$$

where $\phi \in [0, 1]$ is the probability of being discovered and $\sigma > 1$ is the administrative sanction. For the sake of analytically simplicity, the probability of being discovered is uniformed distributed and is a function of the evasion share:

$$\phi = \eta\theta \tag{2}$$

where $\eta \in (0, 1)$ represents the monitoring effort of the water agency. Therefore, the inter-temporal optimization problem, subject to dynamics (1), of the representative farmer is:

$$\max_{\theta,\, w} \pi = \int_0^{+\infty} \left[\frac{\alpha}{\beta} w - \frac{N}{2\beta} w^2 - (c_0 - c_1 H)w - (1-\theta)\tau w - \eta\theta^2 \tau\sigma w\right] e^{-rt}\, \mathrm{d}t \tag{3}$$

where $r > 0$ is the discount rate.

Conversely, the objective of the water agency is to maximize the Social Welfare, composed of legal Net Benefit and Ecosystem Damage. According to Biancardi et al. [19], we define the Net Benefit as the only legal part of farmers' profit:

$$NB = \left[\frac{\alpha}{\beta} w - \frac{N}{2\beta} w^2 - (c_0 - c_1 H)w\right](1-\theta)N \tag{4}$$

Following Esteban and Albiac [13], we define the environmental cost as:

$$ED = [(1-\gamma)wN - R]\delta \tag{5}$$

namely, the volume depleted from the aquifer in each period multiplied by parameter $\delta > 0$, which represents the cost of damage to ecosystem from each cubic meter of aquifer depletion. Therefore, the water agency's problem is:

$$\max_{\tau} SW = \int_0^{+\infty} (NB - ED) e^{-st}\, \mathrm{d}t \tag{6}$$

where $s > 0$ is the water agency discount rate.

3. Analysis of the game

The interaction between the water agency and the farmers is given by a leader-follower differential game, in which the water agency is the leader and the farmers play the role of the follower. The sequence of the game is the following:

(1) the water agency announces the water tax τ;
(2) the representative farmer chooses the optimal share of evasion θ and the optimal pumping level w solving the problem (3) under the dynamics (1);
(3) the water agency chooses the optimal water tax τ solving the problem (6) under the dynamics (1);
(4) adopting a feedback strategy, the water agency derives the steady state value of the water table H.

We assume that the water agency does not react strategically to the evasion share θ. This is because farmers seek to maintain hidden from the agency their non-compliant behaviors. Therefore, the value of the evasion share is not known ex-ante by the water agency, but only ex-post. We denote $V_f(H, t)$ as the value function of the representative farmer. To obtain the farmer's reaction function to the water agency's tax announcement, we need to solve the farmer's optimization problem and its Hamilton–Jacobi–Bellman (HJB) equation:

$$\begin{aligned} rV_f(H,t) = \max_{\theta,\, w} \Bigg\{ & \frac{\alpha}{\beta} w(t) - \frac{N}{2\beta} w^2(t) - (c_0 - c_1 H(t))w(t) - (1-\theta(t))\tau(t)w(t) \\ & - \eta\theta^2(t)\tau(t)\sigma w(t) + \frac{V_f'(H,t)}{S_a}\left[R - (1-\gamma)Nw(t)\right] \Bigg\} \end{aligned} \tag{7}$$

Assuming an interior solution and differentiating the right side of (7) with respect to θ and w, we obtain:

$$\theta^* = \frac{1}{2\eta\sigma} \tag{8}$$

and

$$w^* = \frac{\alpha S_a - \beta\left\{S_a[c_0 - c_1 H + \tau(1-\theta+\theta^2\eta\sigma)] + V_f' N(1-\gamma)\right\}}{NS_a} \tag{9}$$

Notice that $\theta^* > 0$ always, while $\theta^* \leq 1$ if

$$\eta \geq \frac{1}{2\sigma}$$

Since $\sigma > 1$, then $\eta \in [\frac{1}{2}, 1]$ is a sufficient condition to ensure $\theta^* \in (0, 1)$.

To solve the water agency's optimization problem, we substitute (9) in its objective (6) and in the dynamics (1). Denoting $V_a(H,t)$ as the value function of the water agency, we write its HJB equation as follows:

$$sV_a(H,t) = \max_{\tau}\left\{\left[\frac{\alpha}{\beta}w^*(t) - \frac{N(1-\theta)}{2\beta}w^{*2}(t) - (c_0 - c_1H(t))w^*(t)\right](1-\theta)N \right. \\ \left. - \delta[(1-\gamma)Nw^*(t) - R] + \frac{V_a'(H,t)}{\Delta S}\left[R - (1-\gamma)Nw^*(t)\right]\right\} \tag{10}$$

Assuming an interior solution and differentiating the right side of (10) with respect to τ, we obtain:

$$\tau^* = \frac{\beta(1-\gamma)V_a' - N\beta(1-\gamma)(1-\theta)^2V_f' + S_a\beta[\theta(c_0-c_1H)(1-\theta) + (1-\gamma)\delta] - \alpha\theta(1-\theta)S_a}{S_a(1-\theta)^2(1-\theta+\eta\sigma\theta^2)\beta} \tag{11}$$

Substituting τ^* in (9) and subsequently θ^* both in the water pumping and in tax, we obtain:

$$w^* = -\frac{4\eta^2\sigma^2\beta(1-\gamma)V_a'}{NS_a(2\eta\sigma-1)^2} + \frac{2\eta\sigma[\alpha - \beta(c_0-c_1H)]}{N(2\eta\sigma-1)} - \frac{4\eta^2\sigma^2\delta\beta(1-\gamma)}{N(2\eta\sigma-1)^2} \tag{12}$$

and

$$\tau^* = \frac{4\eta\sigma}{(4\eta\sigma-1)}\left\{-\frac{N(1-\gamma)V_f'}{S_a} + \frac{4\sigma^2\eta^2(1-\gamma)V_a'}{S_a(2\eta\sigma-1)^2} - \frac{[\alpha-\beta(c_0-c_1H)](2\eta\sigma-1) - 4\eta^2\sigma^2\beta\delta(1-\gamma)}{\beta(2\eta\sigma-1)^2}\right\} \tag{13}$$

Substituting (8), (12), and (13) in the HJB equations (7) and (10) and rearranging the terms, we get the following equations:

$$rV_f = \frac{R}{S_a}V_f' + \frac{2\eta^2\sigma^2\{(2\eta\sigma-1)S_a[\alpha - \beta(c_0-c_1H) - 2\eta(1-\gamma)\beta\sigma(\delta S_a + V_a')]\}^2}{N\beta S_a^2(2\eta\sigma-1)^4} \tag{14}$$

$$sV_a = \frac{2\sigma^2\eta^2\beta(1-\gamma)^2(V_a')^2}{S_a^2(2\eta\sigma-1)^2} + \left\{-\frac{2\eta\sigma(1-\gamma)[\alpha-\beta(c_0-c_1H)]}{(2\eta\sigma-1)} + R + \frac{4\eta^2\sigma^2\delta(1-\gamma)^2}{(2\eta\sigma-1)^2}\right\}\frac{V_a'}{S_a} \\ + \frac{[\alpha-\beta(c_0-c_1H)]^2}{2\beta} + \delta R + \frac{2(1-\gamma)\delta\eta\sigma\{\alpha - \eta\sigma[2\alpha - \delta\beta(1-\gamma)]\}}{(2\eta\sigma-1)^2} \tag{15}$$

As the game is of the linear-quadratic variety, we postulate quadratic functions of the form:

$$V_f(H,t) = AH^2(t) + BH(t) + C; \quad V_a = EH^2(t) + FH(t) + G$$

with the first derivative:

$$V_f'(H,t) = 2AH(t) + B; \quad V_a'(H,t) = 2EH(t) + F$$

where A, B, C, E, F, and G are constant parameters of the unknown value function which are to be determined. Substituting the equations $V_f(H,t)$, $V_a(H,t)$, $V_f'(H,t)$, and $V_a'(H,t)$ in the HJB given in (14) and (15), we obtain a system of six Riccati equations for the coefficients of the value functions:

$$rA = \frac{2\eta^2\sigma^2\beta[c_1S_a(2\eta\sigma-1) - 4\eta\sigma(1-\gamma)E]^2}{NS_a^2(2\eta\sigma-1)^4} \tag{16}$$

$$rB = \frac{2RA}{S_a} + \frac{4c_1\eta^2\sigma^2(\alpha-\beta c_0)}{N(2\eta\sigma-1)^2} + \frac{32\beta(1-\gamma)^2(S_a+\delta F)\eta^4\sigma^4E}{N(2\eta\sigma-1)^4S_a^2} \tag{17}$$

$$rC = \frac{RB}{S_a} + \frac{2\eta^2\sigma^2[(\alpha-\beta c_0)(2\eta\sigma-1)S_a - 2\eta\sigma\beta(1-\gamma)(\delta S_a+F)]^2}{N\beta(2\eta\sigma-1)^4S_a^2} \tag{18}$$

$$sE = \frac{\beta[(2\eta\sigma-1)S_ac_1 - 4\eta\sigma E(1-\gamma)]^2}{2S_a^2(2\eta\sigma-1)^2} \tag{19}$$

$$sF = \frac{-2(1-\gamma)\beta\eta\sigma[c_1S_a(2\eta\sigma-1) + 4\eta\sigma(1-\gamma)E]}{(2\eta\sigma-1)^2S_a^2}F + \\ + \frac{2RE + c_1S_a(\alpha-\beta c_0)}{S_a} + \frac{8\eta^2\sigma^2\delta\beta(1-\gamma)^2E}{S_a(2\eta\sigma-1)^2} \tag{20}$$

$$sG = \frac{R(S_a\delta+F)}{S_a} + \frac{[(\alpha-\beta c_0)(2\eta\sigma-1)S_a - 2\eta\sigma(1-\gamma)(F+\delta S_a)]^2}{2S_a^2(2\eta\sigma-1)^2\beta} \tag{21}$$

Eq. (19) admits two real and distinct solutions:

$$E = \frac{S_a(2\eta\sigma-1)[4\eta\sigma c_1\beta(1-\gamma) + sS_a(2\eta\sigma-1) \pm \sqrt{\Delta}]}{16\beta\sigma^2\eta^2(1-\gamma)^2}$$

in which

$$\Delta = sS_a(2\eta\sigma - 1)[sS_a(2\eta\sigma - 1) + 8\beta\eta\sigma c_1(1-\gamma)]$$

is always positive. The Eqs. (16), (17), and (20) give us the following solution:

$$A = \frac{2\beta\sigma^2\eta^2[c_1S_a(2\eta\sigma - 1) - 4\eta\sigma(1-\gamma)E]^2}{NrS_a^2(2\sigma\eta - 1)^4}$$

$$B = \frac{2RA}{rS_a} - \frac{16\sigma^3\eta^3\alpha(1-\gamma)E}{NrS_a(2\eta\sigma - 1)^3} + \frac{4\eta^2\sigma^2c_1[\alpha(2\eta\sigma - 1) + \beta(c_0 - 2\eta\sigma(c_0 + (1-\gamma)\delta))]}{Nr(2\eta\sigma - 1)^3} - \frac{8\eta^3\sigma^3\beta(1-\gamma)\{c_1S_a(2\eta\sigma - 1)F - E[2S_ac_0(2\eta\sigma - 1) + 4(1-\gamma)\eta\sigma(\delta S_a + F)]\}}{NrS_a^2(2\eta\sigma - 1)^4}$$

$$F = -\frac{2S_aE\left\{-2(1-g)\eta\sigma(2\eta\sigma - 1)(\alpha - \beta c_0) + R(2\eta\sigma - 1)^2 + 4E(1-\gamma)^2\delta\beta\eta^2\sigma^2\right\}}{8\eta^2\sigma^2\beta(1-\gamma)^2E - S_a^2(2\eta\sigma - 1)^2s - 2S_a(2\eta\sigma - 1)\sigma\eta(1-\gamma)c_1\beta} - \frac{S_a^2c_1(2\eta\sigma - 1)[(2\eta\sigma - 1)(\alpha - \beta c_0) - 2\delta\beta\eta\sigma(1-\gamma)]}{8\eta^2\sigma^2\beta(1-\gamma)^2E - S_a^2(2\eta\sigma - 1)^2s - 2S_a(2\eta\sigma - 1)\sigma\eta(1-\gamma)c_1\beta}$$

To satisfy the stability condition $\frac{d\dot{H}}{dH} < 0$, we substitute (12) in the dynamics of the water table (1), and considering that $V_a'(H,t) = 2EH(t) + F$, we obtain:

$$\frac{d\dot{H}}{dH} = \frac{-2\eta\sigma\beta(1-\gamma)[(2\eta\sigma - 1)c_1S_a - 4\eta\sigma(1-\gamma)E]}{S_a^2(2\eta\sigma - 1)^2} < 0 \tag{22}$$

that is satisfied by the solution

$$E_2 = \frac{S_a(2\eta\sigma - 1)[4\eta\sigma c_1\beta(1-\gamma) + sS_a(2\eta\sigma - 1) - \sqrt{\Delta}]}{16\beta\sigma^2\eta^2(1-\gamma)^2}$$

A unique steady state for the feedback Stackelberg equilibrium of the game exists:

$$H^* = -\frac{Y}{\widetilde{Y}} \tag{23}$$

where

$$\widetilde{Y} = -\frac{2(1-\gamma)\beta\eta\sigma[c_1S_a(2\eta\sigma - 1) - 4E_2\eta\sigma\,(1-\gamma)]}{{S_a}^2(2\sigma\eta - 1)^2} \tag{24}$$

and

$$Y = \frac{1}{S_a}\left\{R - \frac{2(1-\gamma)\eta\sigma[S_a(\alpha - \beta c_0)(2\sigma\eta - 1) - 2\eta\sigma\beta(1-\gamma)(F_2 + \delta S_a)]}{S_a\,(2\sigma\eta - 1)^2}\right\} \tag{25}$$

$\widetilde{Y}$ is a constant that depends on the parameters of the model. Moreover, the feedback Stackelberg equilibrium water table trajectory is given by:

$$H(t) = H^* + (H_0 - H^*)e^{\widetilde{Y}t} \tag{26}$$

where H_0 is the initial value of the water table. Finally, the optimal values of water pumped and tax are:

$$w^* = \frac{2\beta\eta\sigma}{(2\eta\sigma - 1)N}\left[c_1 - \frac{4\eta\sigma(1-\gamma)E_2}{S_a(2\eta\sigma - 1)}\right]H(t) - \frac{4\eta^2\sigma^2\beta(1-\gamma)F_2}{S_aN(2\eta\sigma - 1)^2} + \frac{2\eta\sigma(\alpha - \beta c_0)}{(2\eta\sigma - 1)N} - \frac{4\eta^2\sigma^2\delta\beta(1-\gamma)}{N(2\eta\sigma - 1)^2} \tag{27}$$

and

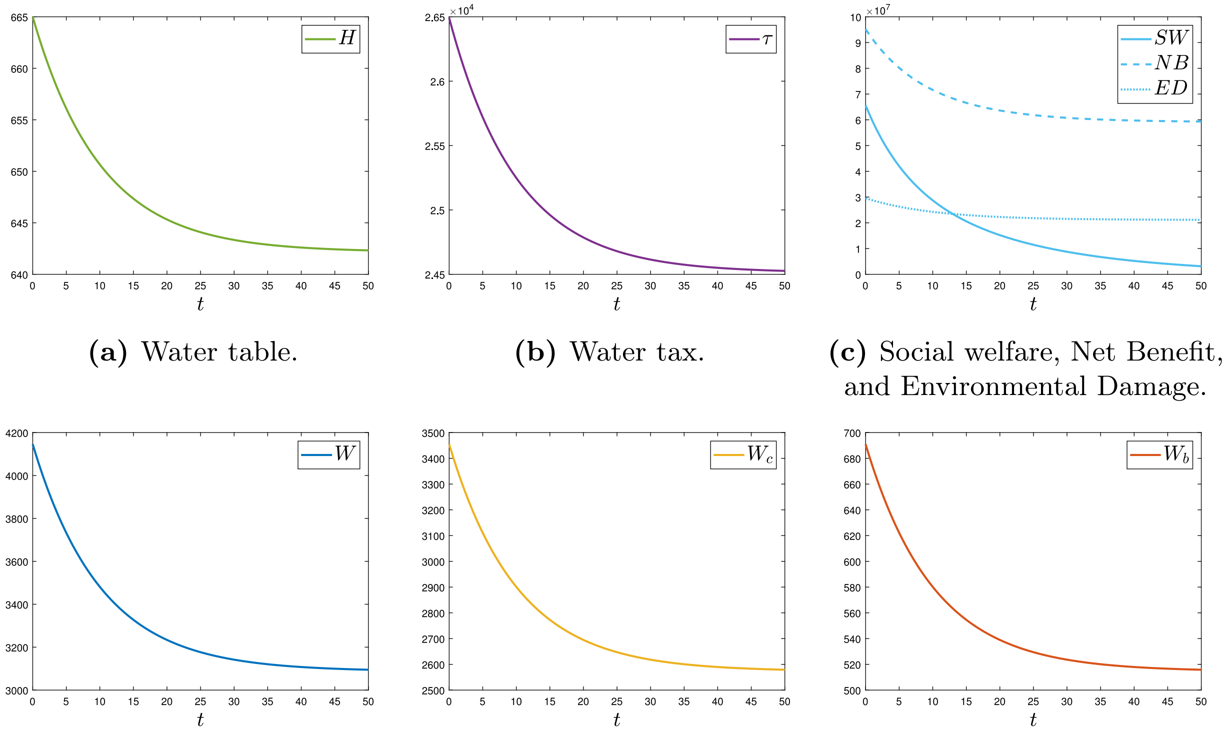
$$\tau^* = \frac{4\eta\sigma}{(4\eta\sigma - 1)}\left[-\frac{2NA_2(1-\gamma)}{S_a} + \frac{8\eta^2\sigma^2(1-\gamma)E_2}{S_a(2\eta\sigma - 1)^2} - \frac{c_1}{(2\eta\sigma - 1)}\right]H(t) - \frac{4\eta\sigma}{(4\eta\sigma - 1)}\left[\frac{(\alpha - \beta c_0)}{\beta(2\eta\sigma - 1)} + \frac{NB_2(1-\gamma)}{S_a} + \frac{4\eta^2\sigma^2(1-\gamma)(\delta S_a + F_2)}{S_a(2\eta\sigma - 1)^2}\right] \tag{28}$$

4. Numerical simulations

In this section, we perform numerical simulations to derive some policy implications. The data used are from Western La Mancha aquifer, widely adopted in literature (see, among others, [19,21,23,24]). Table 1 contains the parameter values.

Table 1
Parameter values.

Parameters	Description	Units	Value
α	Intercept of the water demand	€/Mm^3	4400.73
β	Slop of the water demand	€/Mm^3	0.097
c_0	Intercept of the pumping cost	€/Mm^3	266000
c_1	Slope of the pumping cost	€/Mm^{3}m	400
γ	Return flow coefficient	–	0.2
S_a	Aquifer area	Mm^2	126.5
R	Natural recharge	Mm^3	360
δ	Ecosystem Damage cost	€	10000
$\overline{H}$	Maximum water level and initial condition	m	665
r	Farmers discount rate	–	0.1
s	Water agency discount rate	–	0.05
η	Monitoring effort	–	0.75
σ	Administrative sanction	–	4
N	Number of farmers	–	100



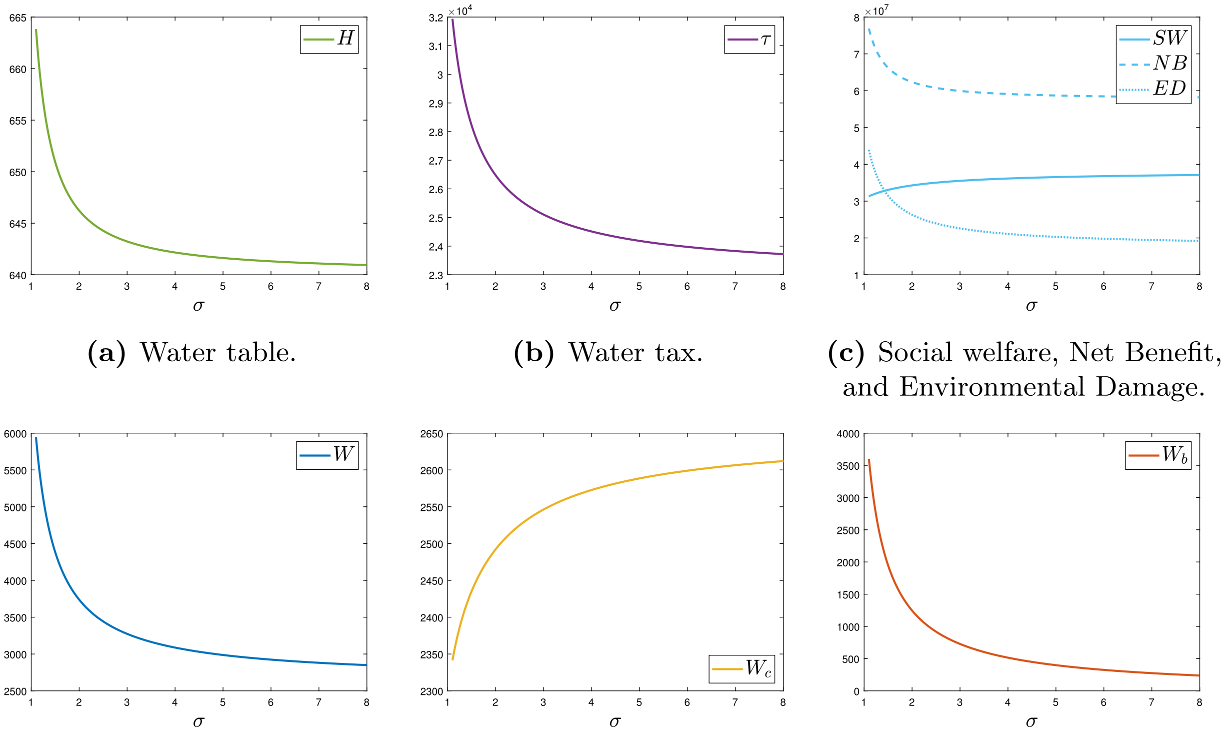
(a) Water table. **(b)** Water tax. **(c)** Social welfare, Net Benefit, and Environmental Damage.

(d) Total water pumped. **e)** Compliant water pumped. **(f)** Non-compliant water pumped.

Fig. 1. Optimal trajectories of H, τ, SW, W, W_c, and W_b.

Fig. 1 shows the optimal trajectories of aquifer level H, water tax τ, Social Welfare SW (composed of Net Benefit NB and Environmental Damage ED), and total water pumped W, composed of compliant (W_c) and non-compliant (W_b). The value of the total withdrawals is $W = w^*N$, of the legal withdrawals is $W_c = (1 - \theta^*)w^*N$, while of the illegal ones is $W_b = \theta^* w^* N$, with θ^* and w^* given by (8) and (27), respectively. Since $H_0 = \overline{H}$, the water table trajectory decreases with time increases (see Fig. 1(a)). A lower water table implies higher pumping cost which decreases the total water pumped (Fig. 1(d)) as well as compliant and non-compliant withdrawals (see Figs. 1(e) and 1(f), respectively). Another effect of lower water pumped is a decrease of water tax (Fig. 1(b)). In terms of Social Welfare, a fall of water pumped leads to a decrease of both Net Benefit and Environmental Damage (see Fig. 1(c)).

To understand the effects of sanction to counter non-compliant behaviors, we perform numerical comparative static analysis in the steady state (Fig. 2). Indeed, an increase of sanction σ causes a (low) decrease of non-compliant water (see Fig. 2(f)) and, on the other hand, an (high) increase of compliant water pumped (see Fig. 2(e)). The effect is a fall of total water pumped (Fig. 2(d)), which it is followed by a decreasing taxation (Fig. 2(b)). Interestingly, although the decrease of



(a) Water table. **(b)** Water tax. **(c)** Social welfare, Net Benefit, and Environmental Damage. **(d)** Total water pumped. **(e)** Compliant water pumped. **(f)** Non-compliant water pumped.

Fig. 2. Comparative statics of H, τ, SW, W, W_c, and W_b at increasing values of the sanction σ in the steady state. In the previous figure $\sigma = 4$.

W, an increase of σ causes a fall of the water table (Fig. 2(a)). Moreover, diminishing W and H imply both lower NB and ED. However, since the Net Benefit decreases slower than the Environmental Damage, then the Social Welfare increases. We can conclude that sanction is a key policy instrument since it counters non-compliant behaviors without reducing the Social Welfare.

5. Trigger strategy

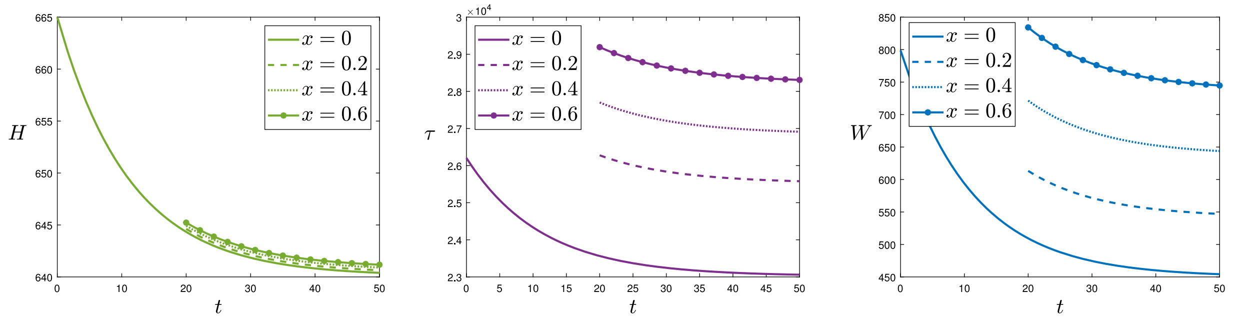
In this section, we change the sanction mechanism. Suppose now that farmers use history-dependent strategy such as trigger strategy [20]. We assume that all farmers are fully compliant, $\theta = 0$, and cooperate pumping the optimal water level. This "agreement" phase lasts as long as some farmers defect. In such a case, the game enters in a "punishment" phase in which the water agency punishes all farmers that play the defect strategy [25–27]. More formally, we assume that at time $t = 0$ all farmers cooperate. However, at time $t = m'$ some farmers defect. The water agency reacts to this defection after a fixed positive time delay d. Therefore, the punishment starts at time $m = m' + d$. A trigger strategy for the representative farmer can be defined as follows:

$$\psi(w,t) = \begin{cases} \widehat{w}^* & \text{if no farmer has defected at or before time } t-d \\ \mu & \text{if a defection has occurred at or before time } t-d \end{cases} \tag{29}$$

where $\widehat{w}^*$ is the optimal pumping level and $\mu = \lambda\widehat{w}^*$ is the threat. According to Mason et al. [28], $0 < \lambda \leq 1$, namely, the punishment consists in a lower pumping level that lasts for ever (if $\lambda = 1$, no punishment occurs).

Using data of Table 1, numerical simulations on the effect of the trigger strategy on aquifer level H and tax are shown in Fig. 3. We denote $x \in [0, 1]$ as the share of farmers that play the defect strategy, therefore, if $x = 0$, then all farmers play the agreed strategy. Moreover, we assume that $m' = 20$, namely, all farmers play the agreed strategy for $t \in [0, 20]$, while for $t \in (20, 50]$ some farmers may defect. As expected, since the trigger strategy provides for a lower pumping level ($\lambda = 0.9$), the water table increases with increasing value of the share of defecting farmers (Fig. 3(a)). A consequence of a lower water table is less pumping cost and, therefore, the total water pumped increases (see Fig. 3(c)). Finally, to preserve the ecosystem from higher pumping, the water tax raises for higher values of x (Fig. 3(b)).

Comparing the two sanction mechanisms, it emerges that both tools are effective. The administrative sanction can reduce the non-compliant behaviors while the trigger strategy can increase the aquifer level.



(a) Water table. (b) Water tax. (c) Total water pumped.

Fig. 3. Trigger strategy.

6. Conclusions

In this paper, we built a model in which the farmers, to save a tax on withdrawals, can evade a share of the water pumped facing the risk of being sanctioned. They choose the pumping level and the illegal share maximizing their intertemporal profits. On the other hand, the water agency knows that non-compliant behaviors may arise, but it does not know the ex-ante value of the evasion share, since it is hidden by the farmers. The game is played through a leader–follower differential approach in which both the farmers and the water agency are perfect foresight. We obtain an analytical solution which describes the optimal water table trajectory as well as the optimal water tax and pumping level.

Numerical simulations, using Western La Mancha data, show the evolution of all key functions of the model, namely, water table, social welfare, and total water pumped (both compliant and non-compliant). They all decrease over time. With regard to the effects of sanctions, numerical comparative static analysis in the steady state shows that an increase of sanction reduces illegal behaviors but increases compliant water pumped. The overall effect is a reduction of the water table. Finally, the water tax decreases when the sanction increases, this may mean that there could be a trade-off between the two tools.

To enrich the policy implications of the model, we introduce an alternative framework, assuming that farmers are fully compliant but may defect from the agreed equilibrium adopting a trigger strategy. The time is so composed of two parts: an "agreement" phase and a "punishment" one. Numerical simulations show that for increasing values of defected farmers, the aquifer, as well as the tax and the water pumped, increases.

CRediT authorship contribution statement

Marta Biancardi: Conceptualization, Methodology, Formal analysis. **Gianluca Iannucci:** Conceptualization, Methodology, Formal analysis. **Giovanni Villani:** Conceptualization, Methodology, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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