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Roadmap on multimode photonics

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Roadmap

Roadmap on multimode photonics

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Abstract

Multimode devices and components have attracted considerable attention in the last years, and different research topics and themes have emerged very recently. The multimodality can be seen as an additional degree of freedom in designing devices, thus allowing for the development of more complex and sophisticated components. The propagation of different modes can be used to increase the fiber optic capacity, but also to introduce novel intermodal interactions, as well as allowing for complex manipulation of optical modes for a variety of applications. In this roadmap we would like to give to the readers a comprehensive overview of the most recent developments in the field, presenting contributions coming from different research topics, including optical fiber technologies, integrated optics, basic physics and telecommunications.

(Some figures may appear in colour only in the online journal)

Contents

1. Introduction

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In the last 30 years optical fibers communications and technologies have experienced a tremendous growth driven by the relentless increase in demand of bandwidth. For this reason, since the early 90s, enormous efforts have been devoted to enhance the bandwidth capacity of the optical transmission links. High bit rate channels have been progressively packed in single transmission fibers, exploiting the systems and components based on different multiplexing schemes. Nevertheless, a fundamental limit in single mode transmission links spectral efficiency is set by the well-known Shannon relation [1, 2] that lead to an upper limit value of around $10\,\mathrm{b\,s^{-1}\,Hz^{-1}}$ that has been approached by recent experimental demonstration using single mode fibers [3]. However, the digital revolution we are witnessing, requires a novel turning point in communication technology. This fact stimulated researchers to evolve from the paradigm of single mode transmission and to overcome the Shannon limit, investigating 'space' as an additional dimension to increase the fiber data capacity. The use of few mode fiber (FMF) to parallelize the optical transmission represents a unique opportunity to overcome the so-called 'capacity-crunch', however this comes with unprecedented technical challenges related to the necessity of precisely handle and manipulate different modes and the linear and nonlinear interactions between them. Space-based multiplexing can be also implemented using the so-called multicore fibers. In this case each transmission channel is transmitted on a separate core of the same fiber, reducing signals crosstalk and interactions when compared with FMFs, but at the same time, increasing the system complexity.

In addition, the propagation of a plurality of modes in the same fiber is set to enable a plethora of novel functions and techniques that go beyond signal processing and communications. Indeed, distinct modes in a fiber carries a specific energy with its own spatial distribution and exhibit propagation constants with peculiar dispersion behaviours, that can be finely tuned by properly designing the fiber cross-sections. On the one hand, the interplay between different modes can add several degrees of freedom in designing nonlinear elements, thus allowing the realization of efficient processes over unprecedented large bandwidth values. Intermodal nonlinear interaction represents a powerful solution to efficiently generate, e.g. optical sources in MIR based on parametric interactions or ultrabroadband supercontinuum radiation. At the same time the capability of control multimode (MM) propagation can be used on short distances for high resolution imaging transmission or can open the path to new opportunities in optical manipulation of multiple particles through spatially controlled and reconfigurable beam patterns.

Short distance communications based on silicon photonics have followed a similar path outlined by long distance optical fiber communication systems in previous years [4]: in this case the need for large transmission capacity has been addresses by using well known multiplexing techniques used in fiber systems and the exploitation of mode-multiplexing techniques is starting to being considered.

MM silicon photonics is currently undergoing a fast development and most of the key building blocks enabling intermodal conversion and MM propagation have been demonstrated in the recent years, including multiplexer and demultiplexer, waveguide crossing and bending.

In addition, the development of switching and active reconfigurable devices in a silicon MM platform is a key towards the practical exploitation of this technology. Dispersion engineering of the interacting modes combined with unprecedented fabrication capability offered by the complementary metal oxide semiconductor (CMOS)-foundries allow the realization of both linear and nonlinear optical devices for switching and signal processing.

This roadmap is aimed at discussing different aspects and trends of MM photonics, targeting to enhance the cross-contamination of ideas between scientists working in different fields of integrated and fiber photonics. The ultimate intent of this paper is to provide guidance for young scientists as well as providing research-funding institutions and stake holders of a comprehensive overview of perspectives and opportunities offered by this, rapidly evolving, research field.

2. High-capacity transmission with multi-mode fibers

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Status

Optical fiber communications systems are the backbone of the global communications infrastructure and support the increased digitization of our society. Data rates in optical fiber networks have grown exponentially over the past decades and are expected to continue growing exponentially for the foreseeable future [5]. Today, high-capacity optical transmission systems rely on single-mode fibers (SMFs). SMF can support data rates of around 100 Tb s^{-1} when considering the traditional low-loss spectral window between 1530 nm and 1610 nm wavelength (C- and L-band). Increasing data rates significantly above 100 Tb s^{-1} in SMF is very challenging, as the Kerr-nonlinearity of the optical fiber poses a limitation to the maximum optical power. This limits the achievable signal to noise ratio and hence the amount of data that can be transmitted in the given low-loss spectral window.

Novel optical fibers have been investigated over the past decade to add a spatial dimension for multiplexing (space-division multiplexing, SDM) and thus a multiplication of the achievable data rates [6]. While different fiber types have been proposed for SDM transmission, multi-mode fibers (MMFs) enable this spatial dimension to transmit different data streams over orthogonal spatial fiber modes. MMFs have the unique feature that the spatial channels share a common fiber core. Therefore, fibers with a very large number of spatial channels can be realized with a cross-section similar to standard SMFs. This provides high spatial efficiency whilst supporting similar cabling technologies as for conventional fibers.

Current and future challenges

Figure 1 shows a schematic of a high data rate wavelength-division multiplexed (WDM)/SDM transmission system based on MMFs. Standard components and fibers, based on SMFs are shown in blue with novel devices required to exploit the spatial domain colored in green. Up to several hundred WDM signals are generated for each fiber mode. Each group of WDM signals is then transmitted at the same time over a different mode of the MMF.

When using fiber modes as a spatial dimension for multiplexing, devices to address individual fiber modes with independent data, called mode-multiplexers (MUXs), become crucial. When designing MUXs, the most important parameters to optimize are the number of addressable fiber modes, insertion loss, loss difference between modes (mode-dependent loss (MDL)) and the mode-selectivity.

Signals propagating in different modes of MMFs exhibit some level of coupling. Additionally, signals in different fiber modes generally propagate with unequal group delay, often described by the differential mode delay (DMD). The combination of DMD and coupling leads to a spatio–temporal spread that signals experience during propagation. Minimizing the modal delay spread in MMFs by optimizing the refractive index profile is a common approach for traditional short-reach transmission systems. However, for MMF-based SDM transmission, optimum profiles need to be designed for the spectral region of lowest loss around 1550 nm wavelength. Additional strategies to minimize the delay spread include DMD management, where fibers with inverse relative group velocity of lower- and higher-order modes (HOMs) are combined, requiring a very careful control over the group delay of individual modes and being especially challenging for fibers supporting many modes. To maximize transmission capacity using wide-band optical signals, low delay spread also needs to be ensured over a wide optical bandwidth, e.g. more than 80 nm covering the C- and L-bands. Additionally, it is important that MMFs for SDM transmission have similar loss in all spatial modes, as unequal loss leads to an effect called MDL that fundamentally limits the transmission capacity.

Optical amplification in SMF transmission is often achieved by erbium-doped fiber amplifiers (EDFAs). Although MMF systems using single-mode EDFAs are possible, for fully integrated MMF-based SDM transmission systems, multi-mode (MM-) EDFAs are required. Key optimization parameters for those are high gain and total output power and low gain differences between spatial modes (mode-dependent gain, MDG) as well as broadband operation.

Novel equalization techniques are required to recover signals that suffered from the spatio–temporal spread during propagation in MMFs. Those can be implemented as digital signal processing (DSP), incorporating multiple-input/multiple-output (MIMO) equalization, previously used in wireless communications. MIMO equalization is often implemented with finite-impulse response filters that need to have a sufficient number of filter taps to include the temporal spread of signals due to DMD and the spatial spread of signals due to modal coupling.

Advances in science and technology to meet challenges

Tremendous advancements have been reported to meet the challenges described in the previous section in the past decade, starting with the first demonstration using all modes of a three-mode MMF simultaneously at the same wavelength in 2011 [7]. MUXs, based on multi-plane light conversion (MPLC) have been demonstrated with up to 1035 modes [8]. A 45 mode MUX was developed to interface with a graded-index (GRIN) MMF with standard core- and cladding-diameters, optimized for minimal DMD at 1550 nm wavelength. The combination of those and a coherent 90×90 MIMO receiver facilitated the transmission over 45 spatial modes with a data rate exceeding 100 Tb s^{-1}, using only 20 WDM channels [9]. A different experiment has shown an optimization of MUXs and a 15-mode MMF for wideband transmission covering more
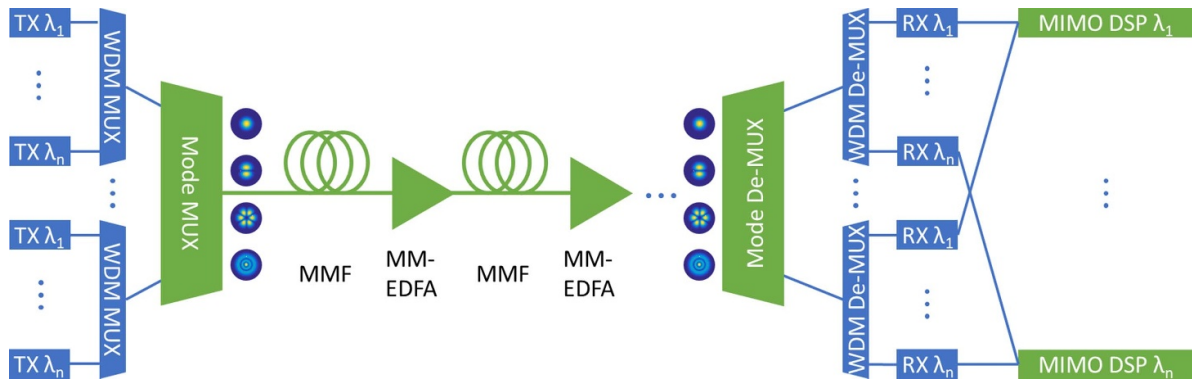
Figure 1. Schematic of a wavelength- and space-division multiplexed (WDM and SDM) transmission link using multi-mode fibers (MMFs). Blue building blocks refer to traditional single-mode fiber technology, while green blocks symbolize novel technologies required for SDM transmission in MMFs.

than 80 nm optical bandwidth. In combination with highly spectral efficient 64-QAM modulated signals, this demonstration exceeded 1 Pb s^{-1} in a fiber with standard cladding diameter [10]. A transmission demonstration with reduced complexity MIMO equalization used a MMF structure with lower modal coupling between groups of modes and allowed for a data rate of 402 Tb s^{-1} in a weakly coupled ten mode MMF [11]. Another transmission demonstration focused on maximizing the data rate in each fiber mode reached 93 Tb^{-1}s^{-1} mode in a three-mode MMF, covering both C- and L-bands, using highly spectral efficient modulation [12]. High data rate and long-distance transmission over 1045 km was reported in a three-mode FMF, applying DMD management techniques effectively minimizing the total delay spread over more than 75 nm optical bandwidth [13]. 3000 km transmission was demonstrated at 40 Tb s^{-1} and more than 6000 km at low data rate employing cyclic mode permutation to lower the total delay spread [14].

Transmission using MM-EDFAs has been demonstrated with up to ten fiber modes [15], while a 45 mode MM-EDFA has been reported [16], however not yet been used for optical fiber transmission.

Advanced MIMO algorithms schemes using interference cancellation techniques have been demonstrated to be effective in scenarios where significant levels of MDL lower the performance. While most MMF-based SDM transmission experiments have made use of offline DSP, advances have been made in the recent past to perform MIMO equalization in real-time. While real-time MIMO equalization for weakly coupled MMF transmission has been shown at low symbol rates [17], the extension of this to large number of modes and high symbol rates is yet to be demonstrated. However, we note that real-time MIMO demonstrations reported so far were based on field-programmable gate arrays and it is expected that high symbol rates could be achieved using application-specific integrated circuits.

Concluding remarks

Research on SDM transmission in MMFs has seen a tremendous progress over the past decade. Yet, several challenges are still to be tackled. Those include demonstrating long distance transmission using MMFs with a large number of modes (e.g. more than 30), combined with large optical bandwidths (e.g. C- and L-bands). Another remaining challenge for an industrial roll-out of MMF-based SDM transmission systems remains the development of transceivers that can perform MIMO equalization in real-time for high symbol-rate signals. However, numerous highly recognized publications have clearly highlighted the strong potential of MMF-based SDM transmission.

3. Signal propagation in space division fiber transmission systems

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Status

Since the prediction of the capacity crunch of the global fiber optic network in 2009, SDM came into the spot-light of optical communications research. The idea of SDM is to increase the information throughput of a fiber communications system by integrating multiple light-paths into a single optical link. A straightforward implementation of SDM can be pursued by simply bundling multiple single-mode-fibers together, but it is believed that in order to reduce the cost per bit, a higher degree of integration is beneficial. In particular, the integration of multiple light-paths into a single optical waveguide, in the form of a MM or a multi-core fiber, has the benefit of increasing spatial efficiency and, as we discuss in what follows, it increases the tolerance to nonlinear-propagation-induced signal distortions. Interestingly, the earliest generations of optical communication systems were implemented over MMFs simply because no reliable process for producing SMFs existed. However, since in those days there were no means of addressing the individual modes, or even controlling their number, the multiplicity of modes could not be exploited for communications, and instead it was responsible for deleterious multi-path interference.

Currently, reliable methods for manufacturing MM and multi-core fibers (to which we will refer jointly as SDM fibers in what follows) are available. In addition, multiple efforts conducted over the past decade have produced promising solutions for signal multiplexing/de-multiplexing in SDM fibers, as well as for integrated MM amplification. Progress in the realm of transmitter/receiver integration and multiple-input-multiple-output (MIMO) DSP is also anticipated. Our focus in this section is on reviewing the fundamental aspects of signal propagation in SDM fibers, and their implications to system performance.

From the standpoint of communications, propagation in SDM fibers requires the consideration of mode coupling, modal dispersion (MD), MDL, and optical nonlinearity. Mode coupling refers to mixing of signals propagating in different modes as a result of unavoidable perturbations to the ideal fiber structure, whereas MD refers to the frequency dependence of the mode coupling [18]. These phenomena are unitary in nature so that in principle their effect can be undone by the receiver. MDL is a nonunitary phenomenon that leads to a fundamental reduction of the information capacity of the channel [19]. In particular, a fundamental capacity loss per scalar mode of the order of 0.25 bits was reported for 10 dB of average MDL in [20]. The MD and MDL phenomena have been studied intensively over the past decade and their effect on

system performance is relatively well understood [20]. Nonlinearity, on the other hand, poses a more difficult challenge and its modelling in combination with the linear propagation effects is based on the coupled nonlinear Schrödinger equations [21]. However, the complexity of these equations prevents one from extracting relevant physical insights that are necessary for effective system design. It turns out that a major simplification follows from dividing the modes supported by an SDM fiber into groups, where modes belonging to the same group are quasi-degenerate in the sense that they have very similar propagation constants. Such modes are strongly coupled, meaning that the signals propagating in them mix randomly on a very short length-scale (shorter than most other relevant length-scales characterizing the propagation). This is contrary to the case of nondegenerate modes (i.e. modes that do not belong to the same group), which are weakly coupled. Within this framework, the description of nonlinear propagation in SDM fibers reduces to the much simpler form of the coupled multicomponent Manakov equations [22, 23], which generalizes the famous Manakov equation describing nonlinear propagation in SMFs with random polarization-mode coupling. Finally, the case of intermediate coupling has been investigated experimentally, but theoretical insights into its dynamics are still lacking [24].

Current and future challenges

Improving the modelling of propagation in SDM fibers is an important challenge as the insights that it is likely to provide are necessary for advancing impairment-mitigation strategies in SDM system design.

Furthermore, focusing on the context of propagation, the most important promise of SDM transmission from the fundamental standpoint, lies in the fact that the nonlinear interference between information-carrying signals reduces in the regime of strong coupling [21]. The reason for this has to do with the fact that in the presence of mode mixing, the optical power that is responsible for nonlinear phase modulation is the sum of independent contributions from the individual modes, and therefore the nonlinear interference noise accumulates incoherently during propagation. Taking advantage of this property requires designing fibers with large groups of strongly-coupled modes. At the same time it requires maintaining MD sufficiently low, in order to keep the receiver complexity at acceptable levels. This imposes a challenging constraint on the fiber design.

While the modelling of nonlinearity is well understood in the regime of strong mode coupling, its understanding in the regime of partial coupling between modes requires further investigation. In particular, the phenomenon of enhanced nonlinearity in the presence of partial coupling between modes reported in [24] is yet to be understood. It should also be noted that the extent of coupling between modes has important implications to the effectiveness of DSP techniques that can be used for the compensation of nonlinear distortions [25].

Another important problem that calls for a solution is the modelling of the interplay between MD and nonlinearity,

which is missing even in the relatively well-studied regime of strong mode coupling. In this case, propagation is described by the multi-component Manakov equation supplemented by the random MD term, which makes the characterization of the nonlinear interference noise considerably more difficult [21]. A similar gap exists in the context of MDL analysis in the presence of nonlinear propagation.

Another important open problem in the context of uncoupled-core multi-core fibers is the modelling of the time-skew dynamics between cores [26], which impairs low-latency applications as well as applications where synchronization between cores is required.

Finally, the proper validation of models requires comparing analysis and experimental data. In this context it is worth emphasizing the importance of conducting studies also on deployed SDM fibers, whose properties may differ significantly from those tested in laboratory conditions [27].

Advances in science and technology to meet challenges

Topics that need to be addressed in SDM-fiber transmission include:

(a) In order to exploit the potential of SDM transmission in the nonlinear propagation regime, fibers with large groups of strongly coupled modes should be developed. In this context it is interesting to consider schemes where intentional perturbations are introduced along the fiber link in order to produce strong mode coupling on the length scale of a few kilometers.

(b) High-mode count SDM systems will necessitate devising links with low MD, a goal that requires investing further research efforts into fiber design, and possibly also into optical schemes for MD mitigation. In addition, large MIMO processing capabilities at the SDM receiver are necessary, which requires further advances in high-speed electronics and DSP.

(c) Faster electronics needs to be supplemented by more computationally efficient DSP techniques. These are necessary for compensating linear propagation impairments, including MD and MDL, and may enable the implementation of practical schemes for the mitigation of nonlinear impairments.

(d) Understanding the important regime of partial mode coupling and its consequences in terms of nonlinear signal distortions requires further effort in the context of modelling nonlinear propagation in SDM fibers, with the goal of extracting insights that allow more efficient system design.

(e) Relying on recent studies [28] where it was shown that MD values can be optimized to reduce nonlinear transmission penalties, it is desirable to develop methods for designing fibers with controllable MD levels.

(f) As propagation effects in practice may differ considerably from those observed in a laboratory setting, it is important to experiment with deployed SDM fiber testbeds. This is also key to characterizing the time dynamics of the most important fiber properties, including, as an outstanding example, time skew in uncoupled-core multi-core fibers. Experimental results may facilitate the devising of useful models for the characterization of time changes in the input-output fiber-channel mapping.

Concluding remarks

SDM transmission remains the leading candidate for scaling the capacity of future optical communication networks. Its implementation through MM and multi-core fibers introduces numerous challenges, many of which relate to fundamental issues of light propagation. Addressing these challenges is key to the successful deployment of SDM systems in the future.

Acknowledgments

4. Quantum communications and quantum key distribution based on multimode fibers

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Status

Quantum communication between distant locations has been investigated exploiting free-space, satellites, and underwater channels, however, the transmission of quantum states through optical fibers constitutes the most reasonable and promising way to build future quantum networks as, nowadays, more than 4.5 billion kilometers of optical fibers are deployed worldwide, connecting people, cities, and countries [29]. Nonetheless, a full deployment of quantum technologies based on the transmission of quantum information is still prevented by non-trivial issues, e.g. the low information rates compared to classical communication, and limited transmission distance. Two techniques can be used to increase the rate, that is: multiplexing quantum channels [30, 31] or using highdimensional quantum states (also called *qudits*) [32]. Lately, MM fibers have been proven to play an important role in quantum communication, benefiting both the aforementioned approaches. They can be divided into two main classes, namely, *high-order mode fibers* (HOMFs) and *multicore fibers* (MCFs), shown in figure 2 [30]. As suggested by their name, HOMFs allows for the propagation of modes that are different from the fundamental TEM_{00}, hence the space within which it is possible to encode and transmit the information is larger compared to standard SMFs. As such, HOMFs have been considered as channels to transmit qudits, i.e. quantum bits encoded in a Hilbert space with dimension larger than 2 [32]. In 2019, the first quantum communication over a 1.2 km long HOMF using high-dimensional quantum states has been reported [33]. The authors developed a communication system where qudits encoded in the orbital angular momentum (OAM) of light were transmitted and detected. As an example of concrete application, they also performed two- and four-dimensional quantum key distribution (QKD) protocols. The demonstration showed a clear enhancement of the secret key rate-figure of merit for QKD systems-in the four-dimensional case compared to the two-dimensional one. Later the same year, more works appeared leveraging on the OAM to transmit highdimensional states. In particular, entanglement and hybridentanglement distribution have been achieved [34, 35], see figure 3(a). Recently, a huge effort has been made to extend the overall link distance of these fibers, and the transmission over a 25 km long HOMF of OAM encoded qudits has been reported [36]. Besides high-dimensional quantum communication, HOMF can also be used for quantum channel multiplexing. Indeed, during the propagation through a HOMF, modes have different group velocities and hence move away from each other, lowering their spatial and temporal overlap, so that the low crosstalk between them make possible the
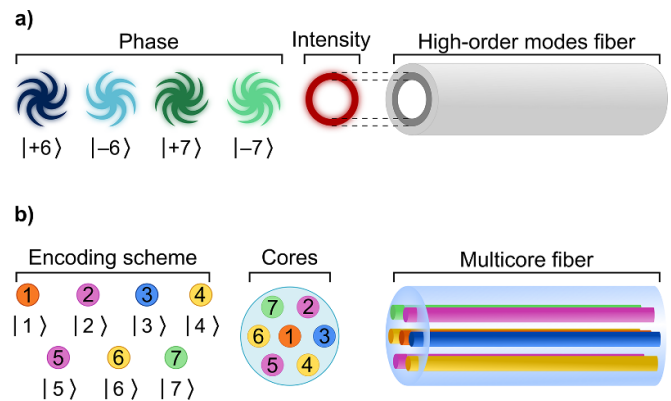

Figure 2. A pictorial representation of a HOMF and a MCF is shown. (a) The image represents a particular HOMF allowing for the propagation of optical modes owing a non-zero orbital angular momentum. Reproduced from [43]. CC BY 4.0. In that case, the information is encoded in the orbital angular momentum values $|\pm i\rangle$, where the sign indicates the helicity of the phase. Reproduced from [33]. CC BY 4.0. (b) The image shows a 7-cores MCF, whose cross section is also represented. In this case, the encoding scheme is based on the number of cores, which label a specific optical path. Reproduced from [44]. CC BY 4.0.

transmission of independent channels, each of them labelled by a mode, within the same fiber. These characteristics have been exploited in [32] where two two-dimensional QKD protocols have been carried within the same fiber. The authors also showed how multiplexing channels is more effective than using high-dimensional encoding to increase the final information rate. Furthermore, in [37], three OAM modes excited by an integrated photonic chip were multiplexed and simultaneously transmitted through an 800 m long HOMF.

Within a more general context, multimode fibers (MMFs) have been investigated in a variety of quantum applications, e.g. highdimensional linear optical circuit manipulation [38], QKD using space encoded technique [39], programmable quantum network [40], entanglement unscrambling [41], and quantum walk application useful for quantum simulation process [42].

MCFs consist of *N* SMFs within the same cladding, and they are inherently more stable than *N* independent SMFs [45]. As for HOMFs, also MCFs have been used both for high-dimensional and multiplexed quantum communication. In this case, quantum states are path-encoded, and each path is associated to a core of the MCF. The first two experiments involving qudits and MCFs were published in 2017 [46, 47]. In [46] a QKD protocol was demonstrated over 300 m of MCFs, whereas in [47] the transmitter and the receiver performing QKD were both implemented on silicon chips, and they were linked by 5 m of MCF. In both works, the length of the quantum link was rather short and mostly limited by technological issues. However a new experiment, setup shown in figure 3(b), reported a high-dimensional QKD protocol using MCFs over a 2 km long channel, achieving a secret key rate as high as 6.3 Mbit s^{-1} [44]. Also quantum correlated photons have been distributed using MCFs. Indeed, in [48], four-dimensional path-entangled pairs have been successfully
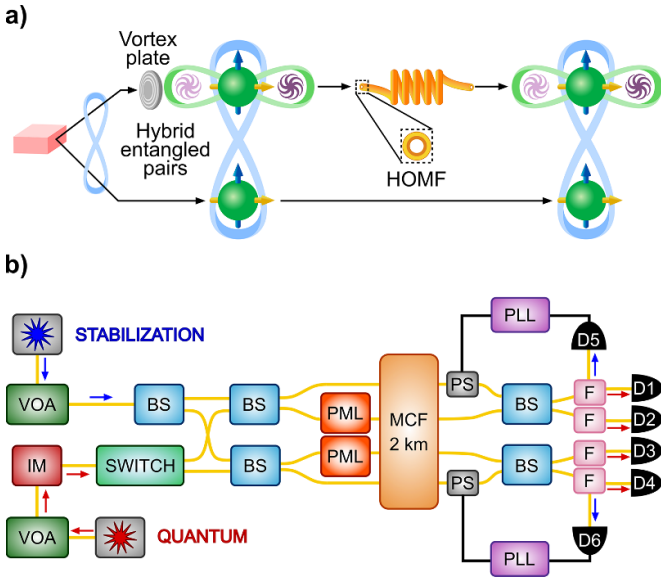
Figure 3. (a) Experimental setup used to distribute hybrid entangled photons exploiting a HOMF. Reproduced from [35]. CC BY 4.0. (b) Experimental setup used to implement a four-dimensional QKD protocol over a 2 km long MCF . Reproduced from [44]. CC BY 4.0. VOA: variable optical attenuator; IM: intensity modulator; BS: beam-splitter; PML: phase modulation loop; PS: phase shifter; PLL: phase-locked loop; F: wavelength division multiplexing filters; D1, D2, D3, and D4: superconducting nanowire single-photon detectors; D5 and D6: InGaAs single-photon detectors.

transmitted over a 11 km long MCF. Channel multiplexing using MCFs has been investigated as an important technique to boost the final secret key rate of a QKD protocol. In [31], 37 time-bin QKD signals have been multiplexed and transmitted through the 37 cores of a MCF, thus achieving the current state of the art secret key rate of 105.7 Mbit s^{-1}. The authors also transmitted a classical communication signal simultaneously with the quantum ones. It is worth mentioning that MCFs have been used also for continuous-variable QKD [49].

Current and future challenges

Despite the notable and impressive results achieved hitherto, diverse challenges affect both HOMFs and MCFs depending on the type of quantum communication performed, whether high-dimensional or multiplexed. The main issue of HOMFs exploited for highdimensional quantum communication is the different mode velocities within the fiber [43]. Indeed, waveguide designs will yield modes with different group velocities through a fiber, and the longer the fiber, the greater temporal separation between modes. As such, the coherence of quantum states, especially those in a quantum superposition, is strongly jeopardized by this *group velocity dispersion* property, in such a way that the propagation distance acts as a threshold value for receiving coherent states at the receiver. Ideally, it is required that all the optical modes arrive at the same time at the receiver side, so a pre-compensation method of the optical path is needed before seeding the qudits into the fiber. At the same time, the different propagation speeds could

open a back door for quantum eavesdropping. Conversely, when performing multiplexed quantum communication, the time separation between the different optical modes could be exploited to guarantee low interference effects between the multiplexed optical modes. In this case then, the more separated the modes the better, at the price of a lower repetition rate. Nonetheless, the number of independent modes, i.e. with very low crosstalk, that can be used for this kind of quantum communication is still limited if compared, for instance, to MCFs.

If we now consider MCFs, the main challenge involves highdimensional quantum communication. In fact, using MCFs for multiplexing quantum communication protocols has no main drawback as the different cores of the MCFs behave as independent fibers. Therefore, degrees of freedom that easily propagate through SMFs can be chosen to encode the information, e.g. time or polarization. On the other hand, when implementing a high-dimensional protocol, each core of the MCFs represents a quantum state, and the superposition states are coherent superpositions of light on two or more cores. This final aspect, in particular, makes the challenge arise. Indeed, to measure the superposition states, interference measurements are needed at the receiver side as the MCF act as a long fiberbased interferometer, whose arms are as long as the channel length. Fiber-based interferometer are inherently unstable and feedback loops (to lock the different relative phase between the cores) are required to successfully perform an interference measurement. Many phase-locked loops have been studied over the years, but recently an innovative and promising method has been investigated, whose results are reported in [44, 45, 50]. Another important effect to be considered when using these fibers is the cross-talk between modes or cores. In particular, the higher the cross-talk the worse quantum states propagate independently and the lower their fidelity. Regarding the MCFs, state of the art fibers have already demonstrated the possibility of achieving a large number of cores with ultralow-crosstalk exploiting trench-assisted techniques [51]. On the other hand, regarding HOMFs, the property of the low cross-talk is inversely proportional to the time difference of the different modes. In other words, to get a low crosstalk, the different modes have to be temporally separated as much as possible.

Advances in science and technology to meet challenges

As for many scientific challenges, technological advances can definitively ease, if not totally resolve, the challenges outlined before both for HOMFs and MCFs. The time delay pre-compensation required to implement high-dimensional quantum communication protocols with HOMFs could be avoided by investigating the design of new HOMFs, where different optical modes face the same effective index throughout their propagation, so that the pre-compensation between them would not be required anymore. Alternatively, implementing the time delay pre-compensation in an automated fashion using programmable phase-delays and integrated dispersion units linked to self-monitored feedback loops, could also

represents an important step for this technology. Both the advances would further boost the range of applicability of this kind of fibers. In particular, the transmission of qudits over very long distances using HOMFs would raise much interest due to the potential scalability of the fiber-based approach. Especially, if such states involve quantum correlations, their robustness to decoherence and preservation after the fiber transmission could be further investigated. On the other hand, a step ahead for quantum communication with HOMFs using multiplexed channel could be the fabrication of fibers whose number of independent optical modes can be as high as state of the art MCFs [52].

Regarding the exploitation of MCFs, as we have seen in the previous section, the main challenge concerns highdimensional quantum communication. Indeed, it derives from the need to stabilize a fiber based interferometer, whose arms comprise the MCF itself, to perform interference measurements, essential to detect superposition states. Smart feedback loops are therefore essential for MCF-based highdimensional quantum communications. In [44, 45, 50], a promising and novel scheme is presented, which can be further boosted by integrating the whole scheme on photonic integrated chips and by exploiting fast electrical components so that faster phase oscillations can be compensated. In addition, further devices can be considered while working with MCFs. For example, integrated multi-port beam splitters based on MCFs [53] open up for a wide range of new applications like the generation of entanglement in multicore fibers [54], and selftesting mutually unbiased bases protocol in higher dimensions [55].

Concluding remarks

Remarkable progress has been achieved over the last decades in the field of quantum communication. In particular, the transmission of quantum states over long distances using optical fibers has seen a fast-growing interest during the last ten years. Despite the promising results obtained so far, there are still several challenges, some practical, some technological and some perceptual, which must be addressed before HOMFs and MCFs can play an active, and potentially important role in quantum communication.

Acknowledgments

5. Optical transmission based on orbital angular momentum mode division multiplexing

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Status

Through the history of optical communications, WDM, time-division multiplexing, polarization-division multiplexing (PDM) and advanced modulation formats play important roles in increasing the transmission capacity, which exploit multiple physical dimensions of lightwaves such as wavelength, time, polarization and complex amplitude. These conventional techniques now are approaching their capacity limit. The space domain of lightwaves that has not yet been fully developed, is of great importance for sustainable capacity scaling through SDM. When tailoring the spatial structure of lightwaves, helically phased light beams having a phase term of $\exp(i\ell\varphi)$ (ℓ: topological charge; φ: azimuthal angle) and carrying OAM have recently received increasing interest (figure 4(a)). In general, there are two methods of using OAM modes for communications. One is to use different OAM modes for representing different data information, i.e. OAM coding/decoding or OAM modulation, which is analogous to advanced modulation formats in the complex amplitude domain (figure 4(b)); while the other is to use different OAM modes as different carriers or independent data channels, i.e. OAM mode division multiplexing (MDM), which is analogous to WDM in the wavelength domain (figure 4(c)). Very recently, the latter one, a subset of SDM, has made considerable progress in free-space, underwater and fiber-based high-capacity optical communications [56, 57]. In free-space optical transmission, using 20 Gbaud 16 ary quadrature amplitude modulation (16-QAM) signals over two groups of concentric rings of polarization multiplexed 8 OAM modes, we demonstrated a 2.56 Tbit s^{-1} transmission capacity together with a 95.7 bit s^{-1} Hz^{-1} spectral efficiency [58]; using 5.8 Gbaud Nyquist 32-QAM signals over polarization multiplexed 52 OAM modes, an ultra-high spectral efficiency up to 435 bit s^{-1} Hz^{-1} was obtained [57]; using 54.139 Gbit s^{-1} orthogonal frequency-division multiplexing 8-QAM signals over 368 WDM polarization multiplexed 26 OAM modes, an ultra-high transmission capacity up to 1.036 Pbit s^{-1} was achieved [57]. In underwater optical transmission, using 10 Gbit s^{-1} on-off keying signals over four green OAM beams, a 40 Gbit s^{-1} communication link was demonstrated [59]. In fiber-based optical transmission, data carrying OAM multiplexing (OAM$_{+3}$, OAM$_{+4}$) communication through 300 km ring-core fiber (RCF) was demonstrated. The obtained results indicate that OAM MDM provides an effective way for capacity scaling in diverse communication scenarios.
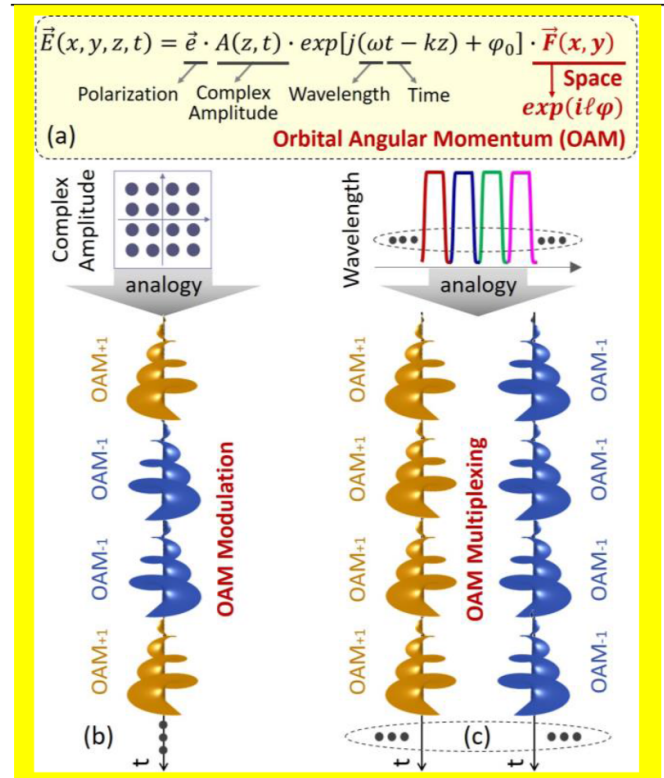

Figure 4. Orbital angular momentum (OAM) and its applications in optical communications. (a) Multiple physical dimensions of lightwaves including OAM with helical phase structure. (b) OAM modulation communications. (c) OAM multiplexing communications.

Current and future challenges

A typical OAM MDM communication system comprises the transmitter, OAM multiplexer, transmission mediums, OAM demultiplexer and receiver (figure 5). Although optical transmission based on OAM MDM has been widely demonstrated, there are still lots of challenges towards current performance improvement and future practical use. The OAM (de)multiplexer is the key element in OAM MDM transmission, which plays the similar role of wavelength (de)multiplexer (e.g. arrayed waveguide grating) in WDM transmission. Many early OAM multiplexing transmission works simply employed beam combiners or diffractive gratings to combine different OAM modes together, which are lossy and unscalable, especially for a large number of OAM modes highly desired in sufficient capacity scaling. Accordingly, efficient and scalable OAM (de)multiplexer is of great importance. It is believed that OAM multiplexing transmission is applicable to multi-scale mediums (μm to hundreds of km), ranging from short-reach optical interconnects (chip, underwater, free space) to long-distance optical communications (fiber) (figure 5). For chip-scale OAM interconnects, the challenge is optimized structures supporting OAM modes and their multiplexing on diverse photonic integration platforms (e.g. silicon, silica). For underwater OAM communications, i.e. propagation through water but not in submarine fiber-optic
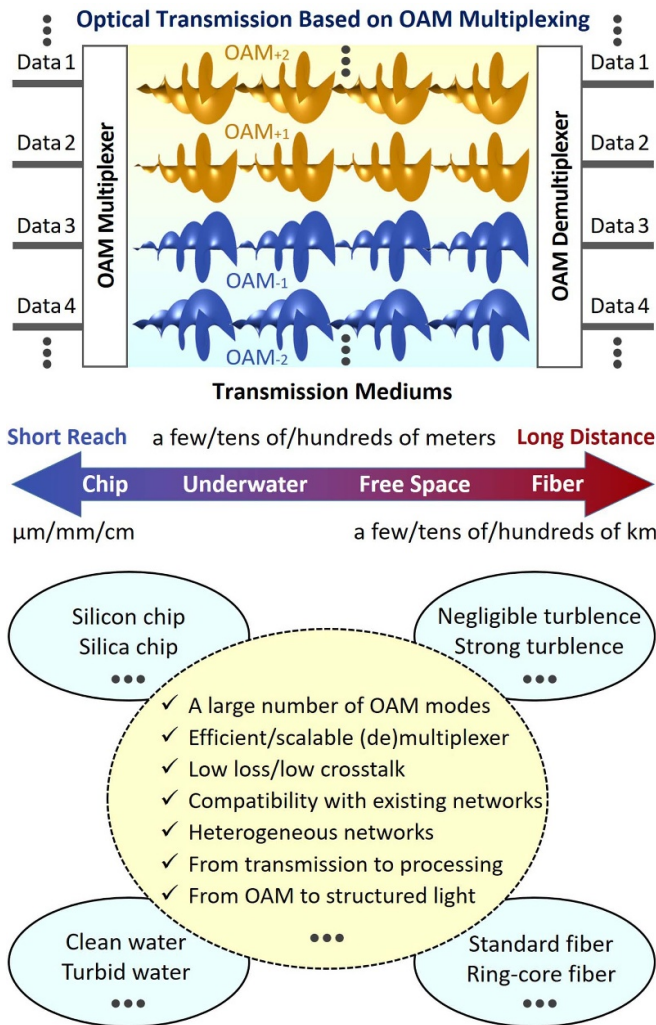
Figure 5. Architecture and challenges of the OAM mode division multiplexing communication system towards multi-scale mediums (chip, underwater, free space, fiber).

cables, the challenge is the relatively large loss and the impairments (bubbles, obstructions, water current and thermal gradient induced turbulence and vibration) in turbid water environment (e.g. sea water). For free-space OAM communications, the challenge is the divergence of light beams and the impairments from atmospheric turbulence (aerosol, dust, fog, haze, smog, rain) and vibration, especially for practical out-door long-distance transmission with strong turbulence. For fiber-optic OAM communications, one challenge is the optimized structure and fabrication technique towards high-performance specialty fiber (e.g. RCF) with ultra-low loss and low crosstalk, and the other challenge is the accompanied devices and techniques required for transmission in standard fibers that are commercially available and widely deployed, such as SMF and MMF (e.g. OM1–OM5). For OAM interconnects/communications in multi-scale mediums, various random defects and fabrication variability play critical roles in scattering OAM modes and causing crosstalks. Moreover, heterogeneous networks incorporating multi-scale mediums are the future trend and the compatibility with

existing networks (e.g. SMF networks) is of great importance. In addition to OAM transmission, OAM processing is also important at network nodes to enhance the OAM management. Additionally, more general structured light multiplexing transmission beyond OAM modes is also the future trend.

Advances in science and technology to meet challenges

To meet challenges of future robust optical transmission based on OAM MDM, advances in science and technology are expected for superior OAM (de)multiplexer and diverse transmission mediums. The early implementation of OAM (de)multiplexer based on beam splitters is not scalable as each beam splitter induced 3 dB loss limits the number of OAM modes. The design strategy is to find scalable methods, in which the (de)multiplexing loss does not significantly increase with the number of OAM modes. Fortunately, log-polar optical transformation method [60] and its improved high-resolution version adding a fan-out element can be used to enable efficient and scalable OAM (de)multiplexing, which are similar to the wavelength (de)multiplexer and its dense version with reduced channel spacing. Moreover, MPLC method and optical diffractive deep neural networks can be also used to facilitate high-performance OAM (de)multiplexer for a large number of OAM modes. All-fiber OAM (de)multiplexing is another possible approach for a small number of OAM modes due to the cascading loss. Note that all OAM (de)multiplexers are preferred to provide SMF interfaces to be compatible with existing SMF networks. For multi-scale transmission mediums, trench structure assisted silicon waveguides can be used to support chip-scale OAM interconnects [61]. More flexible 3D structures in silica directly inscribed by femtosecond laser can also support chip-scale OAM interconnects. In turbid underwater OAM transmission, visible OAM light beams (e.g. blue and green light) are still preferred to reduce the loss. Meanwhile, high-sensitive detectors can be employed to enable weak light detection. In free-space OAM transmission, diffraction-free Bessel beams carrying OAM can reduce the divergence. Adding a proper beam expander at the transmitter is also an effective way to control the divergence of light. To compensate turbulence induced distortions in underwater and free space situations, various adaptive optics and signal processing techniques can be adopted [62]. Machine learning approaches can support effective OAM detection in turbulent transmission via convolutional neural networks. Meanwhile, feedback control with a fast auto-alignment system can be used to ensure stable detection against turbulence. In fiber-optic OAM transmission, multi-parameter structure optimization of the weakly-guiding RCF and precise profile control fabrication technique can be applied to further reduce the loss and crosstalk. Moreover, OAM mode engineering can be developed by neural networks to flexibly control all OAM mode properties. For standard fibers, perfect mode group excitation/selection can benefit favourable MMF-based OAM transmission

[63]. Multidimensional entanglement transport through SMF is achievable based on hybrid spin-orbit entangled states [64]. In future robust and extended OAM communications, various OAM processing techniques such as OAM exchange, OAM conversion, OAM switching, OAM add/drop, OAM filtering, OAM multicasting and OAM routing are expected. Generalized structured light with spatially variant amplitude, phase and polarization distributions (e.g. Hermite–Gaussian beams, OAM beams, vector beams [65]) can be developed to fully access the spatial structure of lightwaves.

Concluding remarks

OAM modes accessing the spatial structure of lightwaves provide an effective approach to sustainable capacity scaling. The demonstrations in the past ten years show their great potential in not only short-reach optical interconnects but also long-distance optical communications. To further facilitate practical applications, efficient/scalable/compatible OAM (de)multiplexer supporting a large number of OAM modes and other key devices and techniques supporting multi-scale transmission mediums (chip, underwater, free space, fiber) are to be developed. OAM amplifiers such as EDFAs and Raman amplifiers are also important for ultra-long haul transmission. Seamlessly connected heterogeneous OAM networks are attractive in the future. One can image superior OAM communication networks, provided that the challenges towards high performance (high capacity, high efficiency, high scalability, high reliability, high intelligence, full compatibility, low loss, low crosstalk) and low cost are appropriately addressed.

Acknowledgments

6. Multimode and multicore fiber amplifiers

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Status

Over the last decade, SDM [2, 66] has been recognized as one of the most promising technical approaches to increase the transmission capacity of optical fiber networks while simultaneously reducing the cost-per-bit. A substantial body of theoretical and experimental work has consequently been carried out both on MMF and/or MCF implementations. To date, up to 45 spatial mode transmission has been demonstrated in GRIN MMF [9] and >10 Pbit s^{-1} data transmission (i.e. a 100-fold increase in data capacity relative to conventional SMFs) has been successfully demonstrated with MM, MCFs (albeit over just 10–20 km distance scales) [67]. Significantly, the submarine telecoms industry has just started to employ SDM technology to meet the growing demand for transoceanic cable capacity under the constraint of fixed total electrical power supply to the wet plant [68]. and the world's first SDM submarine cables have recently been deployed incorporating an increased number of fiber pairs to improve overall cable capacity (e.g. 12 fiber pairs in the Dunant cable and 16 fiber pairs in the 2 Africa cable as compared to the 6 fiber pairs traditionally used) [69]. Indeed, SDM technology is now becoming the new paradigm in the submarine cable industry and is attracting widespread attention from both academia and industry.

Fully integrated SDM fibers amplifiers (i.e. MM and/or MCF amplifiers) are potentially one of the most important subsystems required for the ultimate realization of long-haul SDM transmission and are proven to generate significant cost, space and energy saving benefits. Figure 6 shows the comparison between $N\times$ parallel SMF amplifiers and a fully integrated SDM amplifier. In parallel SMF amplifier systems the architecture is simply a duplication of multiple SMF amplifiers requiring a large number of optical components, electronic and heat management devices. However, in the case of SDM amplifiers based on multiple modes or multiple cores it is possible to use individual components across multiple, (ideally all), spatial channels, greatly reducing the overall component count. Therefore, the cost, energy and space saving benefits of SDM amplifiers are significant compared to use of an array of parallel SMF amplifiers and provide great potential to support the more practical and sustainable growth of future high capacity fiber networks.

Current and future challenges

Figure 7 shows the state-of-the-art in SDM fiber amplifiers reported to date [70]. Remarkable research progress has been made in increasing the number of amplified spatial channels and up to 32, 10 and 42 spatial channel amplifiers have
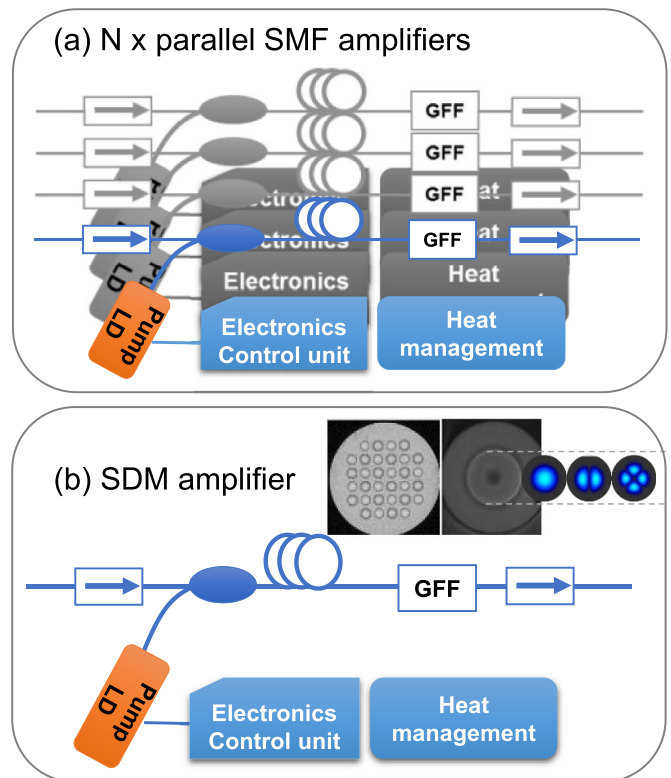

Figure 6. Comparison between (a) $N\times$ parallel SMF amplifiers and (b) a fully integrated N-channel SDM amplifier.

been reported for MCF [71], MMF [72] and multimode-multicore fiber (MM-MCF) amplifiers [73], respectively. In these amplifiers, mode (or core) dependent gain is the most important parameter in determining the overall performance for amplified SDM transmission and various approaches (e.g. control of dopant distribution and pump mode profile control etc) have been used to equalize the gain between the individual spatial channels. However, the design complexity/sensitivity steadily grows with increasing number of amplified spatial channels (e.g. due to core-to-core variation, intercore crosstalk, MDL/gain) and it is challenging to realize massive SDM amplifiers supporting >100 spatial channels in a single optical amplifier. For example, the cladding pumped configuration has been frequently employed in high spatial density SDM amplifiers due to its design simplicity and cost/space saving benefit. However, these amplifiers are generally less energy efficient and have higher noise figures than their core-pumped counterparts due to the low population inversion resulting from the relatively low brightness of the MM pump light. This may be acceptable for terrestrial long-haul transmission systems but it is challenging to apply this concept to submarine networks that need to be highly energy efficient due to the strong electrical power supply constraints associated with these systems. The core-pumped approach is particularly important for energy efficient SDM amplifiers and high power single mode pump source development and efficient pump light sharing/distribution become critical in such applications.
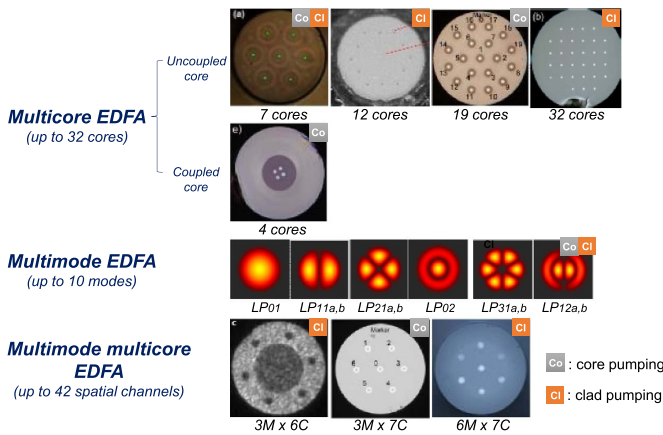
Figure 7. Current state of the art in SDM fiber amplifiers.

Advances in science and technology to meet challenges

- *Massive SDM amplifiers (>100 spatial channels):* Using a MM-MCF approach, it may be possible to realize massive SDM amplifiers (e.g. 10 modes × 12 cores = 120 channels) but it would be difficult to achieve good mode (or core) dependent gain in a single amplifier of such a scale. Therefore, it may be worth to consider multiplexing a moderate number of lesser spatial channel amplifiers, each offering good multichannel performance, in order to realize high spatial density SDM amplifier systems (e.g. 10 cores × 12 fibers = 120 channels).

- *Power efficient pump light source development:* For massive SDM amplifiers, the development of efficient optical pumping solutions is also very important. If we assume that a single amplifier requires 50 mW of pump power, then 5 W (or 50 W) pump power will be required for 100 (or 1000) spatial channel amplification. This can be achievable in a cladding pumped configuration but will be challenging for core pumped amplifiers. Therefore, power efficient pump light source development and efficient pump split/distribution methods into multiple spatial channels is an important topic, with the potential to enable the replacement of tens-to-hundreds of pump laser diodes with a single/far fewer number of pump lasers.

- *Power efficient SDM amplifiers:* Current inline optical amplifiers are optimized to operate at high input signal power levels (0–3 dBm) but these should be re-designed to operate with low input signal levels to enable power efficient transmission. In this way, optical amplifiers may be operated in the non-saturated regime which can lead to enhancements in the amplifier gain efficiency. This new amplifier design regime will require optimization of the refractive index profile, dopant distribution, glass composition, gain bandwidth, the incorporation of efficient ASE suppression filters, low loss fiber components etc.

- *Exploring other application areas:* The independent multiple spatial amplifier channels provided by SDM fibers offer attractive opportunities to develop new types of fiber laser e.g. offering multiple laser outputs or multiple laser wavelengths. This can be applied from continuous wave through to ultrashort pulses regimes of operation, offering unprecedented control of spatial and temporal properties of the output beam(s). In addition, SDM amplifiers can provide a practical platform to realize an array of multiple amplifiers and coherent beam combination in SDM amplifiers offers many advantages over the conventional coherent combination of separate amplifiers. In particular, it provides an integrated multi-channel architecture that drastically reduces system complexity by decoupling the component count from channel count. Moreover, the emission from multiple dopants (e.g. erbium, ytterbium, thulium, holmium etc) can be combined into a single broadband emission and the multi-dopant SDM approach can be applied to broadband or multi-band optical amplifier/laser development. Finally, SDM amplifiers and associated fiber components supporting multiple spatial channels in a single optical fiber can also be applied to new optical sensor and/or imaging systems to integrate multiple modalities (e.g. multi-parameter sensors, 3D shape sensing or multimodal optical microscopy).

Concluding remarks

MM and/or MCF amplifiers offer significant cost, space and energy saving benefits and are one of the most important subsystems for successful implementation of long-haul SDM transmission. Major advances in both MM and MCF amplifiers have been made in terms of the number of amplified spatial channels but further energy efficient SDM amplifiers need to be developed to facilitate practical applications. It is also to be appreciated that SDM amplifier technology is not only an important technology for the next generation of optical communications but that it also offers exciting new avenues for future fiber lasers, sensors and imaging systems.

Acknowledgments

7. Nonlinear optics in multimode fibers

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Status

Although single-mode nonlinear fiber optics has been widely studied over the last four decades, one of the first demonstrations of any kind of fiber nonlinear optics (NLO), back in 1974, involved intermodal four-wave mixing between HOMs [74] of a MMF. In addition to energy conservation between four participating photons in a $\chi^{(3)}$ medium (such as glass fibers), momentum conservation, or phase matching, is a key requirement for NLO. While dispersion engineering in optical fibers was yet to be perfected in 1974, the diversity of phase matching possibilities offered by the participation of different spatial modes readily provided a platform for enabling NLO. This, in fact, points to the dramatically larger degree of freedom offered by even a simple step-index MMF, as illustrated in figure 8. In analogy with bulk crystal (mostly $\chi^{(2)}$) based NLO, where phase matching for different wavelengths is achieved by tuning the crystal with respect to the light-ray angles (figure 8(a)), the multiple modes of a fiber essentially represent a plethora of ray bounce angles, yielding diverse phase matching possibilities (figure 8(b)). Another important parameter of interest in fiber NLO is group-velocity dispersion (GVD), the control of which can help tailor NLO spectra. As figure 8(c) shows, modal selection in an optical fiber can help achieve a wide variety of magnitudes of GVD of either sign [75]. Additionally, the periodic beating patterns that modes produce inside a fiber result in a grating due to the intensity-dependent refractive index, and this can lead to nonlinear mixing due to geometric parametric instabilities [76] (figure 8(d)). Finally, fiber modes carrying OAM have been shown to yield additional nonlinear selection rules arising from angular momentum conservation considerations [77].

Significant impacts of MMF-NLO have been threefold: (a) enhanced degrees of freedom offered by the diversity of phase matching possibilities have enabled accessing a wider array of spectral ranges than that typically covered with SMFs—examples include in-fiber third harmonic generation (THG) [79], supercontinua extending to the blue, violet and beyond [80], and multiple octaves of discrete frequency conversion [81], amongst many others. A resultant exciting emerging possibility is to use MMFs to tailor diverse single-photon sources for quantum applications [82]. In almost all these cases, phase matching in SMF requires control of the chromatic dispersion of the mode, whereas access to multiple modes in MMFs significantly relaxes dispersion-zero requirements of SMFs. This is because, as the equation embedded in figure 8(b) shows, phase matching (achieving $\Delta\beta \approx 0$) is obtained by the choice of participating modes. (b) HOMs are
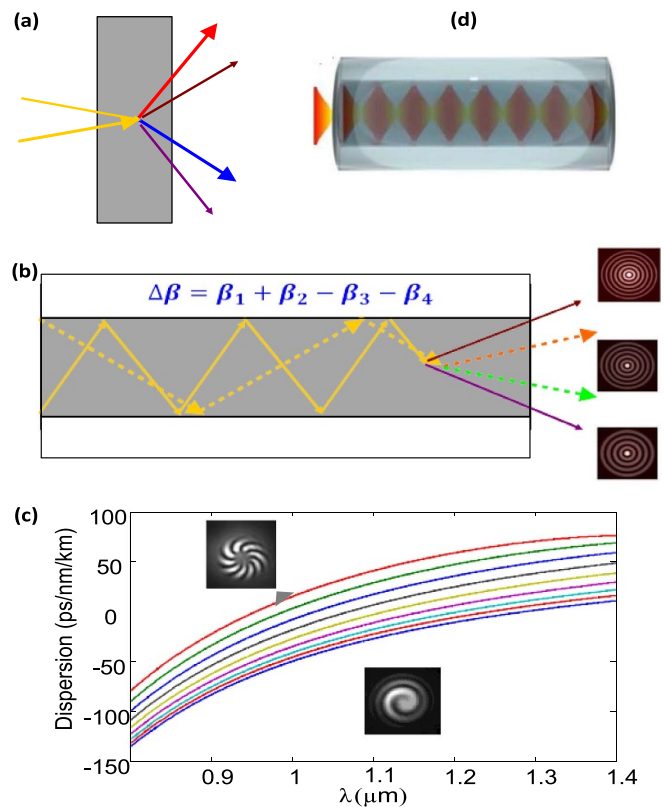

Figure 8. (a) Schematic illustration of NLO with $\chi^{(2)}$ crystals—pumps in different directions (depicted by yellow solid and dashed arrows) yield emissions at new colours and angles (corresponding solid and dashed coloured arrows) due to phase matching. (b) Similar flexibility in angular phase matching effectively available from spatial modes of MMFs. (c) Group-velocity dispersion vs. wavelength shows that different modes (depicted here are different OAM modes, ranging from $L = 1$–9) have their own characteristic GVDs, providing for modal control of GVD. (d) Periodic mode beating yields geometric parametric instabilities. Reproduced from [78]. CC BY 4.0.

naturally large in mode area. Whereas dispersion tailoring necessarily requires constraining mode areas in SMFs, this trade-off is broken for HOMs [75], enabling power-scalable nonlinear frequency generation and conversion [83]. This is illustrated in figure 8(c), which shows that, for the same fiber (and hence core size), shifting dispersion-zero now becomes a matter of choosing the desired mode order (in this case mode with the desired OAM) rather than changing fiber size. This capability promises the development of an integrated fiber platform to achieve high-power source engineering that currently remains the purview of bulk optical parametric oscillators. (c) Perhaps most excitingly, MM nonlinear interactions give rise to new nonlinear effects, selection rules and phenomena not found in single-mode systems. Examples include forward Brillouin scattering [84] due to the ability of the sound wave to yield phase matching between two forward propagating modes, non-dissipative beam clean-up [85] due
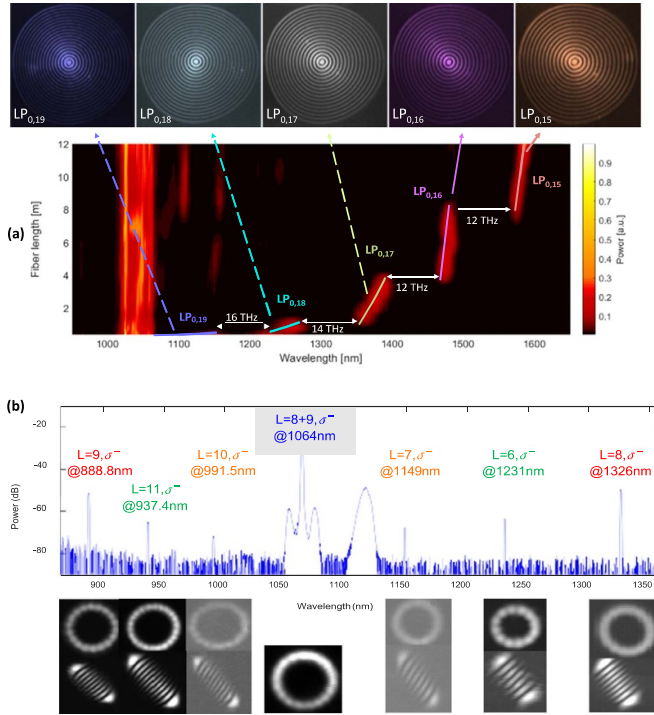
Figure 9. Select examples of nonlinear transformations unique to multimode fibers. (a) Soliton self-mode conversion [86] preferentially selects the growth of discrete ultrafast pulses from quantum noise even in the presence of an alternative seeded pathway. Enables scaling to record MW-level peak powers in the biologically crucial 1300 nm spectral range. (b) Angular momentum conservation rules provide a plethora of four-wave mixing pathways while also enabling controlled selectivity due to spin-orbit interactions in fibers [77].

to selective mode Kerr-nonlinearity-induced transformations favouring the fundamental mode, soliton self-mode conversion [86] (figure 9(a)) due to the ability to obtain selectivity in Raman interactions only between modes (and colours) that are group-velocity matched, and new modal selection rules for Raman scattering [87] arising from unique chiral behaviour of OAM modes in fibers, which in turn, enable Raman interactions only between modes (and colour) possessing the same chirality.

Current and future challenges

There are two pathways for MMF-NLO that are currently being pursued. The first involves the combination of linear and nonlinear mode mixing, typically in GRIN fibers [88]. GRIN fibers provide for similar group velocities of multiple modes, and this is beneficial in walk-off between photons in different modes. However, since linear mode mixing arises from stochastic perturbations due to temperature fluctuations, vibrations, and fiber non-uniformities, including coiling conditions, MM NLO with GRIN fibers has to contend with variable nonlinear responses that are sensitive to the ambient. Emerging work suggests that, in some cases, nonlinear coupling actually stabilizes, and at times, minimizes, linear coupling, but

exploiting the entire toolbox of available modes remains a challenge.

The second pathway involves exploiting the distinct dispersions, angular momenta (including *spin* angular momenta, a.k.a. polarizations), and modal areas of different spatial modes. This, however, requires that linear mode mixing be controlled or minimized so that the nonlinear interaction between modes is maximized and is deterministic, hence controllable. While conventional wisdom has posited that spatial modes in an optical fiber mix due to the aforementioned linear perturbations, specialized designs (and, given symmetry considerations, even the simple step-index fiber for select modes) have shown, to yield a subset of spatial modes to be stable enough to achieve even km-length transmission with negligible mode mixing [89]. This has enabled several of the controlled interactions, yielding higher-power, greater spectral ranges or new effects mentioned in the previous section. However, the number of modes found to be relatively stable in an optical fiber remain a small subset of all available modes, even in specialized RCF designs that have recently yielded OAM propagation. As such, the MM NLO space with stable modes of a fiber remains highly constrained.

As mentioned above, the majority of MMF-NLO work has occurred in conventional GRIN or step-index fibers, notwithstanding some recent work with RCFs. The field has not fully exploited the vast design capabilities offered by microstructured fibers, including photonic crystal and photonic bandgap fibers. Likewise, a majority of work in MMF-NLO has dealt with Silica fibers, and extending the linear and nonlinear manipulation of modes with mid-IR glass fibers remains a nascent field. An important challenge here is related to the aforementioned linear mode-mixing problem—non-Silica glasses have had only limited success in achieving sufficient circularity or longitudinal control to achieve stable propagation of HOMs, to date.

The final challenge for the field is that exploiting the unique nonlinear MM interactions necessarily requires low-loss, high-purity and *on-demand* mode excitation of desired HOMs. The mode multiplexing and demultiplexing task remains far more challenging than simply splicing SMFs or focussing light into them, for two reasons—device technologies to shape modes, such as phase plates, spatial light modulators and in-fiber couplers and mode transformers, remain a research specialty, and the alignment tolerances for achieving pure mode excitation or reception remain far tighter than for SMFs. As a general rule of thumb, for several of the demonstrations that exploit the unique attributes of specific ensembles of modes mentioned in the previous section, excitation mode purities in excess of 10 dB, often 15 dB, are needed. This is because even though nonlinear effects are significant only at high(er) powers, low power leakage into undesired modes represents 'seed' power in undesired modes, which may lead to efficient nonlinear mixing pathways other than the desired pathway one may be wishing to exploit. That said, desired mode purities are highly process and application specific, since some nonlinear effects actually rely on the excitation of a large ensemble of modes, and so would require mode purity levels as low as ~0 dB across a subset of modes.

Advances in science and technology to meet challenges

Non-Silica glass fiber development would need to progress towards the control of circularity, symmetry, longitudinal uniformity and low interfacial scattering that decades of development in Silica fibers have achieved. This would yield the mode stabilities with which to exploit MMF-NLO in non-standard wavelengths such as the crucial mid-IR spectral range.

Combining the capabilities of structured matter, through photonic-crystal or bandgap fiber designs, with structured light (spatial modes) is already technologically feasible, though work in this field is nascent. One speculates, for instance, that such combinations may solve the aforementioned problem of increasing the subset of spatial modes that are available for MMF-NLO.

Finally, several device methodologies for mode excitation and reception in/out of fibers already exist, and it is likely that the growing field of nano-optics will only yield more options. However, to address the tight alignment tolerances, advances in quick, low-cost, easy-to-assemble mode-purity assessment and feedback methodologies are critically needed. Several techniques, based on interferometry, mode-sorting, etc, exist today, but none match the simplicity of optimizing power at the output of an SMF. One interesting idea could be to shape light at the input of the fiber using the desired nonlinear output signal itself for adaptive feedback [90]. Another related interesting development is detectors that are preferentially sensitive to the structure of (OAM) modes [91].

Concluding remarks

In summary, MMF NLO is a field that, although older than single-mode nonlinear fiber optics, has experienced resurgent interest in the past decade. The field promises to offer attractive solutions for applications ranging from MW peak power frequency conversion to single-photon sources in integrated formats. Moreover, new nonlinear effects, not seen in single-mode waveguides or in bulk media, appear evident, especially with the emergence of angular momentum as an additional 'knob' with which to control nonlinear interactions. The one important caveat to all this excitement, however, is that it is significantly more challenging to excite or detect controlled ensembles of modes in fibers, than it is to power-optimize the coupling of light into SMFs. However, this is a technological, rather than fundamental, bottleneck that is sure to be addressed in due course, allowing the widespread adoption of MMF-NLO in the years to come.

Acknowledgments

8. Broadband nonlinear interactions in multimoded fibers

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Status

With the advent of space-division-multiplexing in the early 2000s MM fibers have been the subject of a renewed interest, boosted by novel precise fabrication methods as well as by novel devices that allow mode control with unprecedented modal purity. Although SDM is based upon the propagation of independent information channels over distinct (non-interacting) spatial modes in the linear regime, however in the last decade the interest has spread to include nonlinear phenomena involving the interaction among different spatial modes. Indeed the interplay among different modes gives rise to unexplored, complex nonlinear dynamics expanding the capabilities of the single mode platform.

In a simplified scenario, one can see the intermodal nonlinear dynamics as the result of the interaction among several and distinct couples of modes. This approach—despite not exhaustive and not capturing the whole complexity [92]—however offers a simple understanding and design rules in view of broadband operation. Among the set of different nonlinear processes—which in silica fibers include stimulated Raman scattering (SRS), stimulated Brillouin scattering, THG and Kerr-driven four-wave-mixing (FWM), the latter is the most versatile. Indeed, the intermodal FWM interaction among two modes can give rise to light amplification/generation in a gain band centred approximately at angular frequencies $\omega_c \sim \omega_p \pm \Delta IVG/\beta 2_{avg}$ [92], where ωp is the pump frequency, ΔIVG is the differential inverse group velocity between the modes whereas $\beta 2_{avg}$ is the average group velocity dispersion. Therefore, differently from SMFs, by tailoring the dispersion properties of the modes we can generate or amplify light in several different spectral bands of choice and far away from the pump wavelength.

In a relatively low pump power regime, the FWM interactions can be exploited for all-optical MM signal processing—e.g. ultrafast amplification or wavelength conversion (figures 10(a)–(c))—that may become a cornerstone of the envisaged SDM platform [93]. When the pump power increases, the interplay among FWM and other processes (e.g. SRS and THG) leads to broadband visible to mid-infrared supercontinuum generation (figure 10(d)) [94, 95]. The use of large core-area MM fibers allow to substantially increase the output power compared to the case of SMFs.

Current and future challenges

FWM interactions offer the unique opportunity to generate gain bands centred at frequencies $\omega_c \sim \omega_p \pm \Delta IVG/\beta 2_{avg}$.
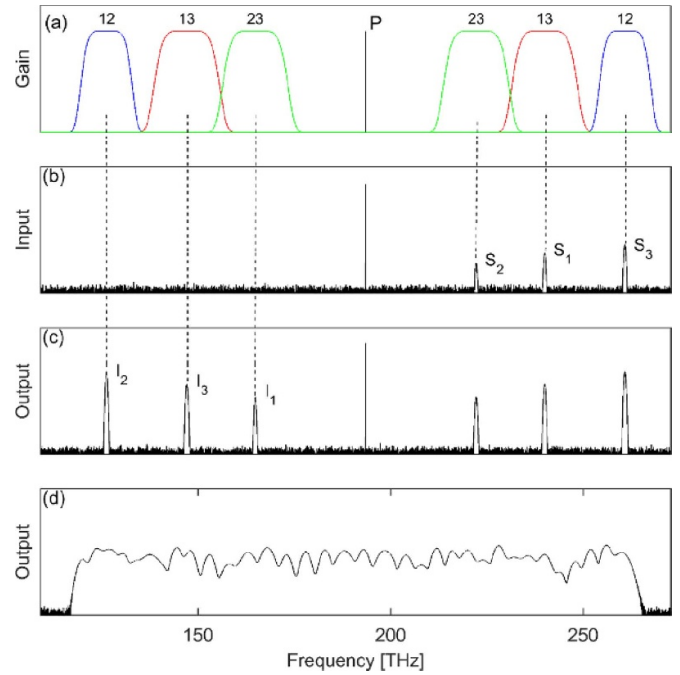

Figure 10. Illustration of intermodal FWM interactions. (a) In this example three spatial modes give rise to three distinct gain bands related to the interaction between different couples of modes (12, 13, 23). (b), (c) By properly coupling the input pump (P) and seeds (s1, s2, s3) over the spatial modes, the seeds are amplified (panel c, light amplification) and one or more idler copies (i1, i2, i3) are generated (panel (c), light generation/conversion) at different wavelengths far from the pump [92]. (d) for large pump power, flat and high-power supercontinuum generation is achieved even if not seeded.

The possibility to span a broad range of frequencies is therefore linked to the ability to shape the dispersion coefficients ΔIVG and $\beta 2_{avg}$ for the different couples of modes into play. In a simple step-index fiber only two free parameters are available—namely core radius and core-to-cladding refractive index difference—which limits the range of values for ΔIVG and $\beta 2_{avg}$. Having more free parameters at disposition (figure 11) would add flexibility in shaping the MD coefficients and so, ultimately, would extend the range of frequencies ω_c that can be achieved. On the other hand, the requirement of several free parameters adds complexity to the fiber design and fabrication [96], as it may impose tight (and perhaps unfeasible) fabrication tolerances to achieve the required precision on the MD coefficients.

A further challenge is represented by random fiber perturbations introduced at manufacturing stage that result in random longitudinal variations of the MD coefficients. This finally causes random variations of the central frequency ω_c, which prevents a constructive gain condition along the fiber length. Realistic variations of $\pm 1\%$ of the core radius may drastically impair the amplification process in km-long fibers [97]. Here the notion of *perturbation length* is central, which is the length scale over which the random perturbations occur. In a fiber shorter than the perturbation length the uniformity is preserved
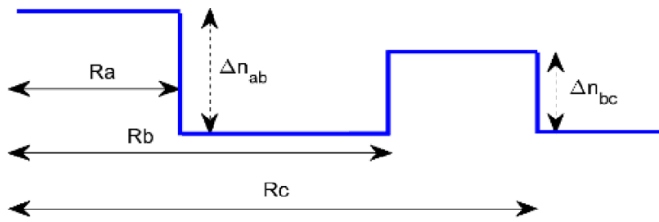
Figure 11. (a) Refractive index radial profile of standard step-index fibers, where a core of radius R is present and Δn is the core-to-cladding refractive index difference. (b) Example of complex refractive index radial profile. Differently from the standard case in figure (a), here three radii R_a, R_b, R_c and two refractive index steps Δn_{ab}, Δn_{bc} are present. This makes a total of five free parameters at disposition to shape the modal dispersion coefficients.

and this removes the issue above. However, the FWM gain is proportional to the product among pump power and fiber length. Reducing the latter comes therefore at the expenses of a larger pump power, which may be inconvenient if not unfeasible.

Finally, it is worth noting that both the bandwidth and the gain of the intermodal nonlinear processes is related to the power coupled over each mode [92]. In order to fully exploit and control the novel opportunities offered by the intermodal nonlinear interactions, there is therefore urgent need for flexible mode multiplexing techniques that allow controlling the amount of power coupled over each mode at the fiber input. Current commercial solutions are designed and optimized for one specific fiber on-demand, but modal purity is strongly reduced when they are used with different fibers [98].

Advances in science and technology to meet challenges

Whether SDM will represent the next big step in optical communication systems, this will depend on the cost effectiveness and performance with respect to the actual single-mode architecture. There seem to be clear indications that this is likely to happen within a not so distant future [5]. If this will be the case, then it is reasonable to assume that this will represent a massive incentive to address the challenges previously mentioned. Better techniques to fabricate uniform MM fibers will be implemented, along with flexible mode multiplexing systems to achieve extraordinary control on the input modal content. Moreover, novel and complex MM fibers—which nowadays fiber suppliers are reluctant to fabricate on demand due to the high development cost—could become routinely implemented to diversify the market of MM fibers.

If on the one side SDM will help addressing the challenges as outlined above, it is also true that it may benefit directly

from the use of intermodal nonlinear interactions. Indeed, future SDM systems will require wavelength converters and amplifiers capable of fast processing of several spatial modes over a broad bandwidth, for which these interactions could represent an ideal solution as outlined in figure 10. Along with SDM, another growing market that may substantially impact the development of broadband nonlinear interactions in MM fibers is that of mid-infrared optical fibers. Take for example chalcogenide fibers. In the last decade they have undergone tremendous improvements bringing losses down to fractions of dB m^{-1} [99]. With a transparency window that extends beyond 10 μm, these fibers represent the ideal platform to exploit the idea illustrated in figure 10: a single pump coupled to several modes may generate light in several mid-infrared spectral regions for different applications (e.g. sensing of different gases). Moreover, the high mode confinement along with the large nonlinear index $n2$ allow reducing the fiber length several order of magnitudes with respect to the silica counterpart, without impairing the effectiveness of the nonlinear interactions nor increasing the pump power. This may finally reduce the overall length down to the sub-meter scale, where fiber uniformity may be preserved.

Concluding remarks

Nonlinear interactions in MM fibers open the way to unprecedented opportunities, from the simultaneous light generation and amplification in several different spectral bands of choice up to high power broadband supercontinuum in large core-area fibers. Both SDM and the mid-infrared optical fiber market may benefit from these interactions and, at the same time, they may contribute significantly to address the related challenges. Meanwhile, lot of academic research worldwide is contributing more and more to a better understanding and exploitation of nonlinear MM fibers, which includes among the other the development of numerical and theoretical models to study and mitigate the impact of random perturbations [97, 100], and the design of novel fibers [96] and mode multiplexers. Therefore, it is reasonable to expect that the field of nonlinear interactions in MM optical fibers will experience significant growth and will represent one of the hot-topics of this new decade.

Acknowledgments

9. Spatial beam self-cleaning in multimode optical fibers

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Status

Multimode optical fibers (MMFs) were introduced in telecommunications in the 1980's, but they were quickly dismissed in favour of SMFs. This was because of their large MD, leading to loss of temporal information. Moreover, using MMFs for imaging applications, e.g. in endoscopy, is limited by modal interference, which leads to loss of spatial information. With the advent of practical coherent systems enabled by DSP and electronic dispersion compensation, MMFs came roaring back for mode-division multiplexing. Whereas, the recovery of spatial beam quality at the output of MMFs became possible with the emergence of adaptive wavefront shaping, yet remaining challenging because of the complexity and environmental-sensitivity of this time-consuming technique.

We have shown that the intensity-dependent contribution to the refractive index, or Kerr effect, automatically solves this problem, when using GRIN MMFs [76, 85]. The principle of beam self-cleaning is shown in figure 12. Above a certain input power threshold, the transverse pattern of the beam intensity self-organizes along the fiber: the highly speckled beam progressively transforms itself into a bell-shaped, high-quality beam, sitting on a low intensity background. A similar result is obtained for the beam at the output of a MMF of a given length, when the input power is increased above the threshold value [85].

It turns out that the waist of the central bell-shaped beam is very close to that of the fundamental mode of the GRIN MMF. Therefore, spatial beam self-cleaning has been interpreted as a manifestation of a self-organized instability [101], which is a universal phenomenon leading to spontaneous pattern formation in Nature. Since the transverse beam dynamics evolves in a two-dimensional space, modal instabilities lead to a nonlinearity-induced re-organization of their distribution, analogous to 2D-turbulence in hydrodynamics [102]. As a result, there is an irreversible flow of energy into the fundamental mode of the GRIN MMF, or condensate, accompanied by energy transfer into higher order modes [103], so that the average mode number remains a constant, as experimentally demonstrated in [102]. Thus, spatial beam cleaning in MMFs is physically analogous to Bose–Einstein condensation [103, 104], and it corresponds to a state of thermal equilibrium for the MM field [105, 106]. Note that the power threshold for self-cleaning is orders of magnitude lower than
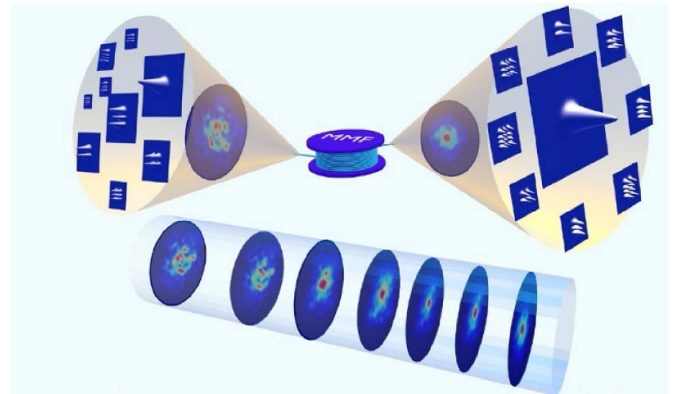

Figure 12. Illustrative demonstration of the transformation of the initial (or low power) speckled output beam composed of numerous transverse modes into a nearly bell-shaped beam of fundamental mode sitting of a low intensity multimode background.

the value required for catastrophic self-focusing and spatial solitons.

Current and future challenges

Although self-cleaning is a spatial beam shaping effect, for understanding its mechanism it is important to consider the accompanying pulse reshaping, which occurs in the frequency and temporal domains. In figure 13, we show that the beam clean-up that occurs at the output of a GRIN MMF when the input power of sub-nanosecond pulses grows larger, is not accompanied by any significant pulse broadening in the wavelength domain [85]. Spectral broadening due to self-phase-modulation (or cross-phase modulation, in case that multiple beams were present) only occurs for power levels that are significantly higher than the threshold for beam cleaning. This observation rules out the possibility that beam self-cleaning (which is time-averaged by the infrared camera) is due to the incoherent combination of the several spectral components of the output pulse. On the other hand, when measuring the power-induced reshaping of the output pulses (see an example in figure 13), one obtains different waveforms when moving the position of a point-like detector across the transverse output plane [107]. From this, one can infer that the central bell-shaped beam is mainly generated at the high-power top portion of the pulses, whereas the pulse wings carry the MM background.

Therefore, a current challenge is to make use of beam self-cleaning with ultrashort pulses, which is important for its applications to different technologies, such as nonlinear microscopy based on multiphoton imaging, and mode-locking in MMF lasers. When operating in the normal dispersion regime, the fiber length should be kept relatively short, in order to avoid significant pulse broadening (accompanied by a decrease of the peak power) due to modal and chromatic dispersion [108]. On the other hand, in the anomalous dispersion regime one may exploit the generation of MM solitons, whereby intra-MD and modal walk-off are both compensated by the Kerr effect. Indeed, very recent experiment demonstrated the self-cleaning of femtosecond MMF soliton
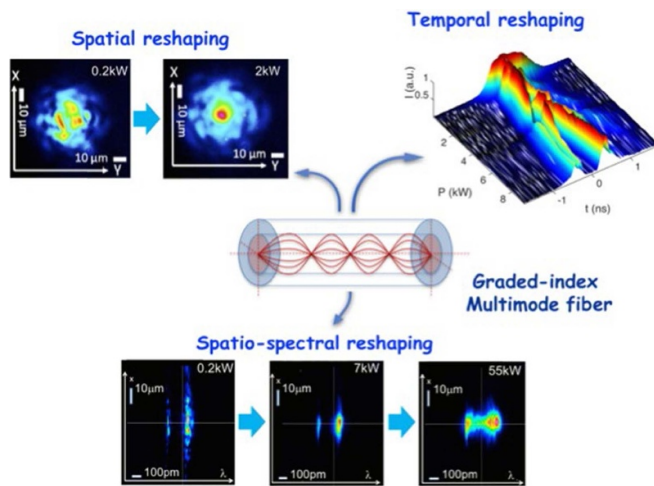
Figure 13. Self-shaping of pulses at the output of a nonlinear GRIN MMF; upper left inset: beam cleaning in the transverse x and y dimensions as the input peak power grows from 0.2 kW up to 2 kW. Reprinted figure with permission from [107], Copyright 2018 by the American Physical Society. Bottom inset: beam cleaning in the x transverse dimension and spectral broadening in the wavelength dimension [85]; upper right inset: input power-dependent temporal reshaping at beam center [107].

beams propagating over km distances [109]. In this case, the solitons are subject to Raman-induced self-frequency shift: the combined action of Raman and Kerr effects in beam reshaping towards the fundamental fiber mode is a topic of hot investigation.

Advances in science and technology to meet challenges

A major technological advance can be provided by the combination of self-cleaning with linear amplification in MMF rare-earth doped fibers, which are a key component for both amplifier and laser systems based on MMFs. Kerr induced beam clean-up has been demonstrated in both unpumped and in pumped Ytterbium doped MMFs [110]. However, linear gain leads to a dramatic acceleration of beam-cleaning, which cannot be simply explained with the increased path-average power. Therefore, the mechanism of spatial self-cleaning in active fibers remains poorly understood. The technology of active MMFs could permit the transverse shaping of both linear index and doping profiles. Combining a GRIN profile with a suitably engineered transverse beam profile, could lead to the engineering of on-demand transverse mode generation with MM fiber oscillators.

A different approach for exploiting Kerr nonlinearity in generating nonlinear structured light beams from MMFs involves the use of the adaptive wavefront shaping [111]. Although this technology has been well developed for the control of beam propagation in random media, its application has remained so far essentially limited to the linear wave propagation regime. In combination with the novel beam shaping effect associated with self-cleaning, wavefront shaping driven by electrooptical feedback and machine learning techniques may lead to environmentally stable and robust generation of higher-order beam patterns, which again may find application in fiber lasers.

The full potential of using nonlinear MMFs for controlling the spatio–temporal properties of light beams will only be released when suitable methods for characterizing the input and output modal content will be developed. Among these methods, all-optical mode decomposition techniques based on phase-only liquid crystal spatial light modulators appears to be particularly promising, in particular when large (>50) numbers of transverse modes are involved [106, 112].

Key photonic technologies where spatial beam cleaning may provide a breakthrough advance, we may cite coherent beam combining [113], all-optical beam switching, ultrashort pulse generation based on distributed Kerr-lens mode locking [114], and high-resolution nonlinear imaging in microscopy and endoscopy [115].

Concluding remarks

Over the past 50 years, we have experienced tremendous advances in our ability to generate and control ultrashort pulses via nonlinear optical fibers. However, these developments have been essentially limited to SMF systems, thus overlooking the spatial degrees of freedom of light beams. The recently discovered effect of spatial beam self-cleaning shows that nonlinear MM interactions may lead to unexpected and beautiful phenomena, which are still in an early stage of research, but could potentially lead to significant technological progress in the different areas of photonics.

Acknowledgments

10. Imaging through multimode fibers

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Status

Endoscopes are widely used imaging devices enabling visualization of internal cavities for biomedical applications and industrial inspection. Conventional endoscopes rely on bundles of optical fibers, each fiber guiding a single pixel of the transmitted image. However, there is a limit to how closely these individual fibers may be packed before light couples between them, blurring the resulting image. This in turn limits both the minimum cross-sectional area and the minimum angular resolution of traditional endoscopic technology.

Single MMFs can support tens-of-thousands of spatial light modes within a footprint comparable to that of a strand of human hair. Each spatial mode, recognizable as a particular optical field pattern, forms an independent information channel. Therefore, the number of resolvable features in images that can be transmitted through MMFs is proportional to the number of supported fiber modes per polarization. This high information density has generated much interest in the use of MMFs as micro-endoscopes, which have the potential to relay diffraction-limited images through fibers that are about an order of magnitude thinner than conventional endoscopes [116–118]. Current developments are focussed on biomedical imaging, where MMF-based micro-endoscopy promises a new window into the body, allowing sub-cellular resolution imaging at the tip of a needle. For example, this technology could be used to identify cell types for the guidance of keyhole surgery and the navigation of biopsy needles, to provide feedback for targeted drug delivery, and also to observe neuronal activity on the cellular level deep inside the brain [117, 118].

More broadly, the task of transmitting image information through MMFs is closely related to the concept of space division multiplexing in fibers, that has the potential to increase the bandwidth of fiber based optical communications. Recent work has also begun to explore the potential of MMFs as mixing elements for classical and quantum information processing and for secure quantum communications [40].

However, the compact form factor of MMFs also comes at a cost: optical signals propagating along MMFs are subject to MD, as the phase velocity of a signal is dependent upon its spatial mode. This means that an image projected onto one end of a MMF is typically unrecognizably scrambled into a speckle pattern at the other end (see figure 14). This problem was first considered in the 1970s, but progress has accelerated in the last decade with the development of high-speed digital light shaping techniques and sophisticated computational algorithms. Using these technologies, several ways to overcome the scrambling of stationary MMFs have recently been developed.
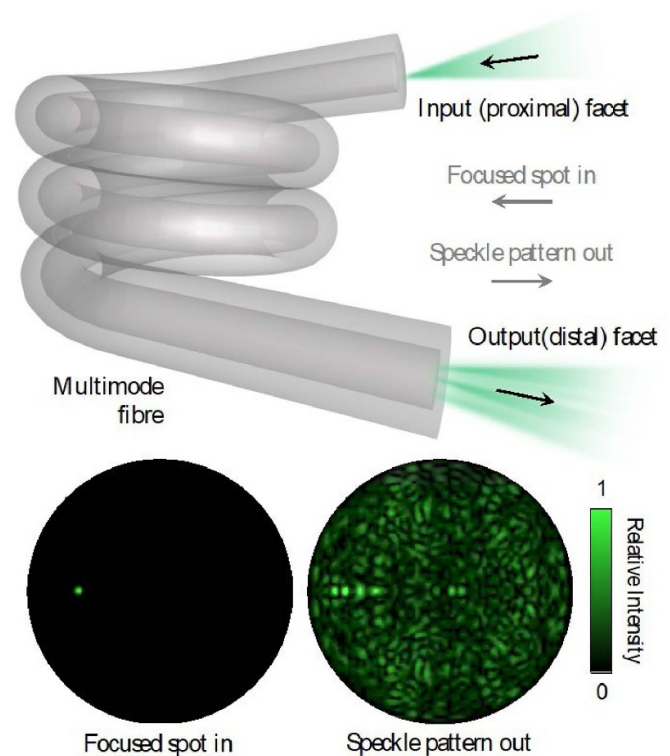

Figure 14. *The challenge: overcoming scrambling of optical signals by MMFs*. A focused spot incident on one end of a MMF results in a speckle pattern exiting at the other end that is strongly dependent on fiber configuration. Insets show a simulation of the input focus and the resulting speckle pattern created across the core of the output facet of an ideal MMF, which in this case supports 754 spatial modes.

Transmission matrix-based methods represent a powerful way to achieve light control through scattering media. A static MMF scrambles light in a deterministic way, which can be represented as a linear operator, known as a 'Transmission Matrix' (TM), that relates input optical fields at one end of the fiber to those emerging at the other end [119]. The TM of a MMF can be measured using digital holography. It can then be used to calculate how to pre-aberrate a sequence of input fields such that they transform into focussed spots raster scanned across the output—thus creating a micro-endoscopic scanning imaging system. So far, such TM based methods have been successfully used to image live cells through MMFs inside biological systems [117, 118] (see figure 15).

More recently, machine learning based techniques have also been explored to 'learn' the relationship between patterns of light at either end of the fiber. A neural network can be trained to interpret output intensity speckle patterns and reconstruct artificially synthesized input images that share the same general features as the training set [120]. Although imaging of arbitrary scenes has been established [121] (see figure 15), application of machine learning to micro-endoscopy is still to be demonstrated. Emerging machine learning protocols have the potential to deliver high-speed single-shot image transfer through longer lengths of fiber than TM based methods, since they are less susceptible to, or may even be able to account for, dynamic changes in fiber configuration.
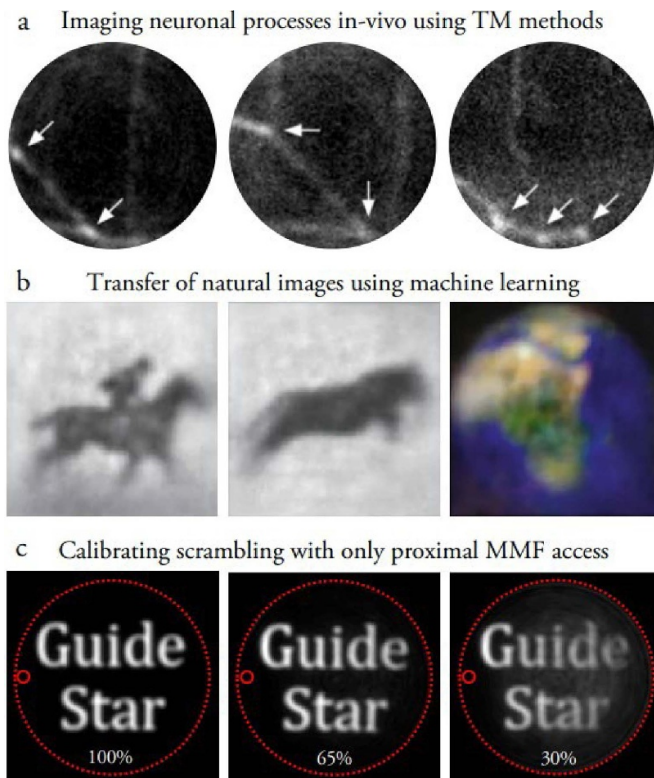
Figure 15. Current solutions to image through MMFs: (a) imaging neuronal processes (shown by arrows) *in-vivo* through a micro-endoscope using TM-based methods. Field-of-view is 50 μm wide. Reproduced from [118]. CC BY 4.0. (b) Reconstructed images of a rider on horseback, a jumping tiger, and a globe depicted in colour, transmitted through a MMF using machine learning. Reproduced from [121]. CC BY 4.0. (c) Simulations of imaging quality when a MMF TM is calibrated using a guide-star (solid red circle) combined with memory effects, as the level of knowledge provided by the memory effects progressively reduces (given by figure at bottom of each panel). Reproduced from [123]. CC BY 4.0.

Current and future challenges

Despite these recent successes, there are several outstanding challenges preventing full optical control of light propagating through MMFs. A key issue is the fragility of the measured scrambling operator (such as the TM or trained neural network) needed to link the light fields at the proximal (input) and distal (output) fiber facets. The way MMFs scramble light is highly dependent upon their configuration, meaning that once calibrated, MMFs must be held completely static during their operation. Any small bends, twists, or temperature fluctuations change how light is scattered through the fiber, thus rendering the initial calibration no longer valid and causing loss of optical control and failure of imaging. While short rigid micro-endoscopes that are mounted inside needles minimize these issues, longer more flexible MMFs have a much wider range of applications. The development of new methods to control light and continuously image through flexible MMFs will drive the uptake of this technology in the future.

From the perspective of imaging applications, most current MMF-based endoscopes rely upon point scanning which limits the frame rate that images can be acquired to a few Hz. A future goal is the creation of high-frame rate micro-endoscopes that can capture incoherent images of arbitrary distal scenes in a single-shot, akin to current conventional endoscopes. The development of hyperspectral imaging capabilities that can deliver *in-situ* diagnostic information about the health of biological systems is also a key step for future applications.

A longer-term challenge is the miniaturization of the optical systems used to calibrate the scrambling of MMFs and prepare the structured light fields transmitted through them. Current set-ups are bulky and often require interferometric stability. Finding new ways to scale down these systems will open pathways to many new on-chip applications.

Advances in science and technology to meet challenges

Several technological developments are required to meet these challenges. Successful image and data transmission through flexible MMFs requires new ways to actively monitor changes in the scrambling operator as a fiber bends—and in the case of micro-endoscopic applications, relying only on access to the proximal end. Recent work has shown that the cylindrical symmetry of MMFs means that the scrambling operator of short lengths of MMF is predictable if the configuration of the fiber is known [122]. Emerging solutions to this problem have also been inspired by adaptive optics and 'guide-star' based methods first developed for observing the stars through atmospheric turbulence in ground-based astronomy [123]. These techniques take advantage of subtle spatial or spectral correlations in the scrambling operator. Indeed, several new forms of optical memory effect—correlations linking changes in the fields at either end of the fiber—have recently been uncovered in MMFs, with exciting potential to facilitate optical control (see figure 15). These advances also point towards physically inspired algorithmic solutions that encompass ideas from transmission matrix theory, machine learning, compressive sensing and optimization to computationally unscramble optical signals through MMFs. We also expect to see numerical and AI techniques provide opportunities for correcting for fiber-bending dynamically and 'on the go'.

A complementary route to image through flexible fibers is the development of new fiber technology that is more resilient to bending. For example, recent work hints that it may be possible to create new types of MMF that possess scrambling operators which are much less sensitive to changes in fiber configuration [124].

Another area of development is light shaping technology itself. Current solutions use phase-only spatial light modulators, or high-speed digital micromirror devices, both born from the enormous global investment in display technologies over the last decades. Moving beyond these technologies, steps are now being taken to develop new forms of on-chip light

modulation which could be significantly more compact and deliver much higher modulation rates in a more versatile way than current light shaping technology. Prototype devices in this area are based on meshes of waveguide interferometers on-chip. While very promising, the number of spatial modes they can control are currently far below the thousands needed for imaging applications.

Concluding remarks

In summary, our ability to transmit images through ultra-thin MMFs and arbitrary scattering media has seen great strides forward over the last decade. While the challenge of imaging through static MMFs has been solved, extending this to flexible MMFs is essential to realize the full promise of MMF-based micro-endoscopy. The technology needed to achieve this goal is mutually beneficial to a number of other arenas, including spatially multiplexed optical fiber communications, and classical and quantum optical information processing. As the pace of development continues to accelerate, we predict a bright future for these exciting areas of technology in the years to come.

Acknowledgments

11. Optical manipulation in optofluidic multimode hollow-core photonic crystal fibers

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Status

Since its invention by Arthur Ashkin in the 1970s, optical trapping has been widely applied in fields such as biology and life science, fundamental physics, and precision metrology. Hollow-core photonic crystal fibers (HCFs) have recently emerged as an attractive platform for optical manipulation of micro- and nanoparticles, offering the major advantages of well-defined modes and thus optical forces, manipulation over long distances, and shielding of particles from transverse flows. For example, standing-wave optical potentials, created by either co- or counter-propagating modes in HCF, have been used in microparticle transport [125] and to study cooperative effects in cold atomic gases [126].

Suitably designed, HCF supports a large number of HOMs (figure 16) that can be excited using spatial light modulation (SLM) techniques in, for example, liquid-filled 'optofluidic' HCFs (figure 16) [127]. Superpositions of co-propagating modes can create a beating pattern that can be used as a mode-based 'optical conveyor belt' in HCF [125] (figure 17), allowing their use in 'flying particle' sensing schemes [128]. Modal interference can also create optical binding between particles trapped near microfibers [129] or within HCF [130]. In such experiments, the first particle acts as a mode converter that creates a beat pattern in which subsequent particles can become trapped. The bound particles move as a single array along the waveguide, allowing the study of collective dynamics in this optomechanical system [130] (figure 17).

The use of HOMs in HCF adds new degrees of freedom for optical trapping, optomechanical interaction, and optical detection. While recent work has focused on linearly polarized modes, there are exciting opportunities in the use of vector modes with complex intensity and polarization profiles. For example, radially- and azimuthally polarized modes may selectively excite TE and TM Mie resonances of optically-trapped microparticles [131]. Furthermore, the use of modes carrying circular polarization, which is robustly maintained if HCF is drawn chiral by spinning the preform during the draw, will enable the manipulation and study of chiral particles.

With the continuous development of new types of HCFs and advances in SLM techniques, such experiments are approaching real-world applications, such as schemes for advanced particle metrology [132], and optical chromatography, in which particles are separated based on their optical resonances and/or their optical chirality. MM optical fields
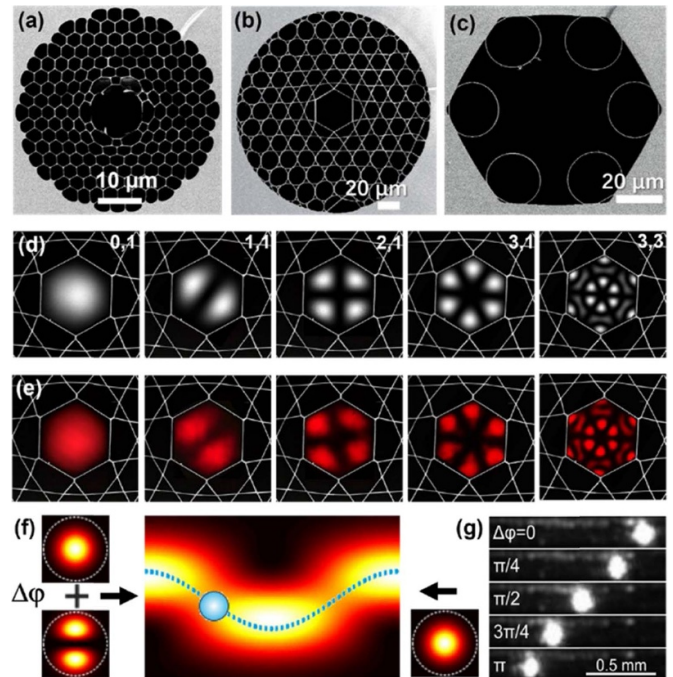

Figure 16. (a)–(c) Scanning electron micrographs of different styles of HCF: (a) photonic bandgap HCF, (b) kagomé HCF, (c) single-ring HCFs. Calculated (d) and experimentally excited (e) LP*nm* modes in kagomé-style HCF. Numbers *n, m* indicate the azimuthal and radial mode order of each mode. Reproduced from [127]. CC BY 4.0. (f) Optical trapping of microparticles in HCF with a combination of LP01 and LP11 modes (schematic). (g) Experimental demonstration of a mode-based 'conveyor belt'. Reproduced from [125]. CC BY 4.0.

in HCF may also find applications in measuring the mechanical properties of soft particles and living cells, and in light-controlled self-assembly of nanoparticles.

Current and future challenges

To make further progress in this research area and achieve multidimensional optical manipulation of particle motion within HCF, it will be essential to fully exploit the vectorial nature of HOMs. We identify the following experimental and theoretical issues that will need to be addressed:

One of the key issues is that state-of-the-art HCFs are typically optimized for guidance of the fundamental core mode, resulting in relatively high propagation losses for HOMs. This limits the range of particle manipulation as well as the strength of long-range optomechanical interactions. The difficulty here is to design HCFs that offer stable and low-loss guidance of vector modes, and novel fiber designs and advances in fabrication techniques are necessary to meet these challenges. A key ingredient will be chiral HCF, which robustly preserves circular polarization- [133] and OAM states [134].

A second major challenge is the efficient excitation of high-purity HOMs, ideally at a fast switching speed. In particular, dynamic control of the angular momentum of HOMs, including their polarization and the OAM degrees of freedom, is hard to achieve using current holographic mode excitation schemes.
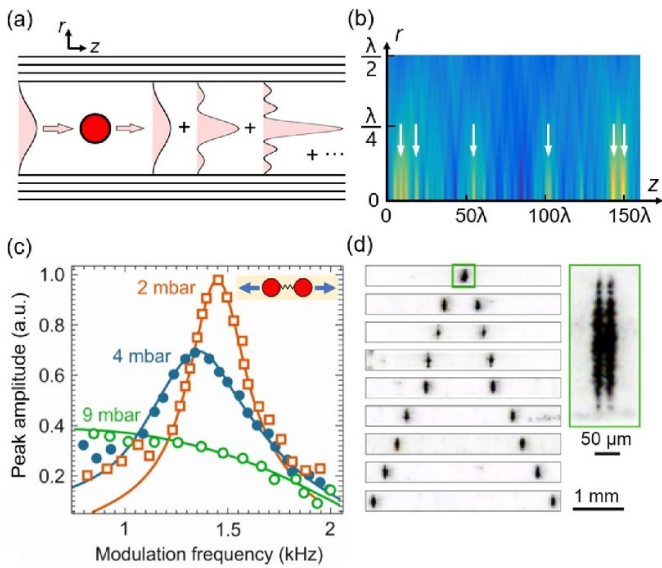
Figure 17. (a) A particle trapped in the centre of the hollow core converts an incident fundamental mode (LP01) into LP0*m* modes (schematic). (b) Example of a calculated intensity beat pattern excited just after a trapped polystyrene microparticle (diameter 1.25λ, core diameter 10λ, trapping wavelength λ). The white arrows mark possible trapping sites for second particles. (c) Measured out-of-phase collective motion for two bound polystyrene particles at three different gas pressures, driven by modulating the trapping power. Reproduced with permission from [130]. © 2021 Optical Society of America. (d) Reconfigurable binding of polystyrene microparticles with bond-lengths between 42 μm and 3.01 mm. Inset: zoom into a bound pair with $d = 42$ μm. Reproduced with permission from [130]. © 2021 Optical Society of America.

A third challenge is to accurately track the 3D particle position within the HCF core. The position of optically trapped particles along the fiber is typically observed by side-imaging, or by Doppler velocimetry [128]. The transverse motion of the particle, however, is much more difficult to measure and has so far only been estimated by monitoring the drop in light intensity transmitted through the fiber [128]. It could potentially be correlated with changes in the HOM content of the transmitted light.

Finally, the physics and observable features of optomechanical interactions strongly depend on the modal content in the hollow core. A good understanding particle-related intermodal scattering is essential both for precise detection of particle motion and for modelling local optical forces and long-range particle-particle interactions. Recent work has demonstrated that analytical expressions can be derived to obtain the optical forces on spherical particles interacting with an arbitrary mode [131], and a major challenge is to extend such methods to include the coupling of the scattered fields to other waveguide modes and to incorporate interactions between multiple particles.

Advances in science and technology to meet challenges

A promising approach to optimizing the HCF design for HOM guidance is the use of recently-reported 'single-ring' HCF that guides by anti-resonant-reflection (figure 16(a)). The simple structure, consisting of a ring of thin walled capillaries surrounding the central hollow core, makes it much easier to optimize the design by adjusting the capillary wall thickness and diameter. The use of such fibers opens up the possibility for relatively simple and repeatable design and fabrication of HCFs targeting low-loss HOM propagation. The recent development of chiral HCFs, formed by spinning the preform during fiber drawing, on the other hand, offers a promising route to controlling and maintaining the azimuthal momentum and spin of the core modes. Circularly birefringent chiral HCF has been recently used to drive rotational motion of optically-trapped birefringent particles within the hollow core [133].

In recent years, new SLM techniques have been developed that can fully characterize the transmission matrix of a MM optical fiber. This has, for example, enabled the use of coherent superpositions of many modes emerging from a high-NA MMF to create 3D optical traps near the fiber tip [135]. Similar techniques could be employed to fully characterize the eigenmode basis of HCFs and to optimize the design of holograms for efficient excitation of the desired optical modes. For example, advanced hologram generation algorithms have recently been exploited to achieve 14%–37% launching efficiency across seven high-purity HOMs of an HCF without mechanical realignment [136].

Trapped particles act as mode convertors for incident light, and conversely are typically deflected sideways from the core centre by antisymmetric HOMs. In both cases the incident and transmitted mode mixtures are different, the relative modal amplitudes strongly depending on the transverse particle position. It may therefore be possible to monitor the transverse particle position by using off-axis holography to precisely measure the phase and amplitude of the modes exiting the fiber. This will require the development of fully analytical theory for fast and accurate calculation of the particle scattering matrix, and the resulting optical forces within the HCF, taking into account the full vectorial nature of the core modes and optomechanical interactions between particles.

Concluding remarks

MM HCFs offer a unique and rich playground for both fundamental science and applications of optical trapping, optomechanics, and optofluidic sensing. During the past decade much progress has been made in SLM techniques, HCF design and underlying theory, which lays the ground-work for new experiments that take full advantage of the new features offered by HCF-based optical manipulation, compared to its free-space counterpart. We foresee the field developing towards long-range optomechanical interactions, ultra-precision particle metrology, and optomechanical manipulation of soft materials including living cells. The emerging field of chiral HCF offers further exciting opportunities, through circular birefringence and dichroism [133] and stable OAM mode guidance [134].

12. Multimode silicon photonics for space division multiplexing

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Status

SDM technology provides a promising way to improve the capacity of optical interconnects by using multicore or MM optical waveguides/fibers [66, 137]. Particularly, MM SDM (i.e. mode-division multiplexing, MDM) has attracted intensive attention in recent years. In this case, it becomes vital to manipulate more than one mode simultaneously, which is totally different from the traditional case with the fundamental mode only, as shown in figure 18. The key is how to achieve mode-selective or mode-transparent manipulation of light, which fortunately is feasible because of the strong mode-dispersion in MM silicon photonic waveguides. In the past years, great efforts have been made by many groups, and significant progresses of MM silicon photonic devices and circuits for MDM systems have been achieved [138]. It is also shown that HOMs have been playing a key role for some special silicon photonic devices, which cannot be realized with the fundamental mode only. Furthermore, some emerging applications have been enabled by MM photonics with the assistance of HOMs. Therefore, MM silicon photonics beyond the singlemode regime paves the way for new functionalities, high performances, and emerging applications [138].

Current and future challenges

MM silicon photonics brings new opportunities by breaking the singlemode condition. Various MM silicon photonic devices have been demonstrated [138], including mode (de)multiplexers, mode converters, multimode waveguide crossings (MWCs), multimode power splitters (MPSs), MM photonic switches, MM chip-fiber couplers. Some special photonic devices assisted with HOMs have also been demonstrated, such as polarization-handling devices and photonic filters [138]. Furthermore, MM photonics has been extended for some emerging applications [139, 140], like quantum photonics, nonlinear photonics, etc.

On-chip mode (de)multiplexers

Currently multi-channel mode (de)multiplexers have been developed with mode-selective coupling in asymmetric directional couplers or supermode-selective evolution in adiabatic dual-core tapers [138]. They mostly have <10 channels and exhibit decent performances in a large bandwidth,
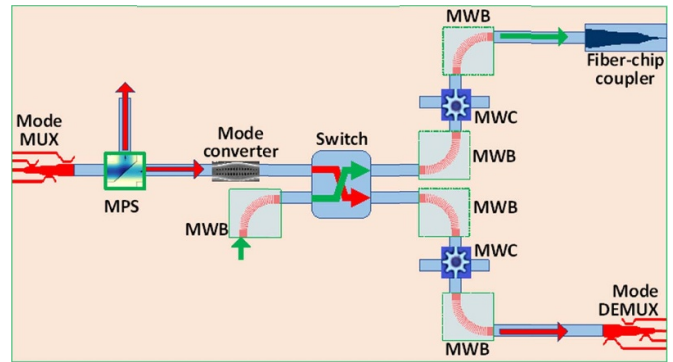
Figure 18. A configuration of on-chip MDM links with various multimode photonic devices, including mode (de)multiplexers, mode converters, multimode waveguide bends (MWBs), multimode waveguide crossing (MWCs), multimode power splitters (MPSs), multimode photonic switches, and fiber-chip couplers.

and can work compatibly with WDM/PDM for hybrid multiplexing. It is promising to develop silicon MM transceivers for on-chip/off-chip optical interconnects by monolithically integrating mode (de)multiplexers with high-speed modulators/photodetectors. One of the challenges is to realize compact and high-performance mode (de)multiplexers with >10 channels due to reduced mode dispersion and weakened mode-selectivity.

On-chip mode converters

Flexible mode conversion in MDM systems is vital for data routing/exchanging, similarly to the wavelength conversion in WDM systems. Mode conversion is much easier than wavelength conversion and some high-performance mode converters with compact footprints have been reported with some special subwavelength structures. Mode converters enabling the exchange of any two of the mode-channels are useful [141], and the challenge is to have low losses and low crosstalk for all mode-channels in a broad wavelength-band.

On-chip multimode propagation

For on-chip optical interconnects, low-loss and low-crosstalk light propagation in MM waveguides is a fundamental issue. In order to realize some indispensable photonic components, such as MWCs, MWBs and MPSs, a general idea is to demultiplex the N mode-channels in the MM waveguide to the fundamental modes in N singlemode waveguides and then use those components developed for the fundamental mode. However, this makes the system very complicated and inconvenient for functional integration.

In recent years great efforts have been made and smart designs were proposed. For example, the idea of using subwavelength-structure waveguide lenses provides an excellent option for realizing MWCs [142]. For MWBs, there are several typical solutions by following the idea of minimizing the mode mismatching at the straight-bent junction

[143]. An alternative is using Euler-curve MWBs, which has a gradually-varied curvature radius and can work with dual-polarization mode-channels in a broad wavelength-band [144]. Another promising approach is to use MM waveguide corner bends, which work well in an ultra-broad wavelength-band. For MPSs, it is still challenging and there is no much work. The design with a subwavelength-grating transflector proposed recently provides a potential solution [145].

Even though some special subwavelength structures have been proposed to enable MM propagation of light, the fabrication is still challenging for standard MPW processes. It is essential to develop high-performance MM silicon photonic devices with excellent fabrication compatibility.

Multimode fiber-chip couplers

In order to realize off-chip MDM optical interconnects, one should connect few-mode fibers and silicon photonic waveguides. Thus, efficient MMF-chip coupling becomes the key. The challenge lies in the significant mismatch of the mode spot sizes and the mode field profiles of the HOMs, which is totally different from the traditional case with the fundamental mode only. There are some potential solutions using a MM inverse taper with some special on-chip polarization mode conversion, while a long taper is required to be adiabatic. It is still challenging to realize mode-transparent MMF-chip couplers with compact footprints and high coupling efficiencies.

Multimode photonic switching

Reconfigurable photonic circuits are important for flexible routing in optical networks. Currently mode-selective optical switching has been demonstrated by integrating mode (de)multiplexers and traditional photonic switches based on Mach-Zehnder interferometers or microring resonators. However, it is still a big challenge to realize mode-transparent photonic switches for switching all the mode-channels simultaneously.

Advances in science and technology to meet challenges

Significant advances should be made to realize various high-performance MM photonic devices enabling flexible manipulations of the mode-channels.

For mode (de)multiplexers/converters, one needs to significantly enhance the mode selectivity so that one can manipulate the target mode-channels as desired. Previously, the mode-selective coupling is achieved by utilizing the mode

dispersion, in which way the phase match condition can be satisfied for the target mode while there is phase mismatch automatically for the other modes. A possible approach for enhancing the mode-selectivity is to redistribute the modal field profiles by introducing some special subwavelength structures in the bus waveguide [138], in which way one can strengthen the coupling for the target mode-channel and weaken the coupling for the other mode-channels.

For MWBs, MWCs, MPSs and switches, it is often desired to be mode-transparent so that all the mode-channels can be manipulated simultaneously. Note that most photonic devices are based on two-/multi-beam optical interference, while the mode-channels have very different phase-delays due to the mode dispersion. Thus, it is challenging to realize mode-transparent photonic devices and advances in physical mechanisms are required. A possible solution is introducing some hybrid ray-/wave-optic structures for MM photonics, such as corner bends, lenses, transflectors. In order to flexibly manipulate the modal properties of MM photonic waveguides, subwavelength structures are often used and thus advances in nanofabrication technologies are required.

Concluding remarks

MM photonics with HOMs provide a new option to effectively improve the link capacity in MDM systems. Silicon photonics provides an excellent platform with high mode-selectivity due to its high mode-dispersion, and thus can effectively manipulate more than one mode. Many MM silicon photonic devices and circuits have been demonstrated and it is desired to further improve their performances for satisfying the demands in the applications. More efforts are demanded to solve the MMF-chip coupling issue so that MM silicon photonics can contribute more to MDM systems based on few-mode fibers. It becomes promising to introduce the horizontal and vertical HOMs for developing more special components, which cannot be achievable with the fundamental mode only. It is also interesting to further explore the utilization of MM silicon photonics with HOMs in the fields of e.g. quantum photonics, nonlinear photonics, etc.

Acknowledgments

13. Integrated components for multimode photonics

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Status

As an effective way to increase transmission capacity, MDM technique has attracted increasing attentions in recent years [146]. Since all mode channels share the same wavelength emitted from a laser diode, the cost and power consumption could be decreased efficiently [66]. On the other hand, high-density integration of photonic components is of significant interest in terms of link price, performance and power consumption. The silicon on insulator (SOI) is an attractive integration platform owing to its high refraction index contrast, high transparency in telecom wavelength band and compatibility with well-established CMOS processes.

The integrated MM components applied to achieve diverse functionalities are key building blocks to construct a chip-scale MDM system. In the past few years, various Silicon integrated components for MM photonics have been proposed and demonstrated, enabling coupling, multiplexing/demultiplexing, transmitting, switching, as well as modulation and detection. The optimizations of their loss and operation bandwidth have been investigated extensively. Switch and transmission circuit based on these components are also constructed. Furthermore, to build a complete MM transmission system utilized in data centre and long-haul network, the chip-fiber coupling in MM regime is a crucial technology to be developed. Coupling schemes based on grating array and edge couplers are proposed and demonstrated, achieving up to six modes coupling of FMF.

To satisfy the rapidly increasing demands for high-speed data transfer and data intensive applications, optical communication and interconnection are still promising candidates of the next-generation solution to alleviate the transmission capacity bottleneck and unfavourable power scaling. MM photonics, combined with mature multiple wavelength scheme, will play a more important role in future hybrid multiplexing system.

Current and future challenges

The major challenges in integrated MM photonics are the scalability and reconfigurability. Typically, the mode multiplexer (Mux)/demultiplexer (DeMux) determines the maximal channel number that can be utilized. The purpose of the mode Mux is to convert single mode into a set of orthogonal eigenmodes and combine those modes to a shared MM waveguide. A high-performance Mux/DeMux is expected to be lossless, broadband and low crosstalk, ensuring scalability and compatibility with WDM technology. The Mux should be able to convert one mode to the desired mode, while eliminating the excitation of any other modes. Although a variety of structures are developed, such as the asymmetric directional coupler, adiabatic coupler, Y-junction and MM interference [147], the achieved mode number is still limited.

To build a reconfigurable and highly integrated MM circuit including signal processing and switching, two major approaches could be considered. The 'demultiplexing-single mode operation-multiplexing' architecture is the most straightforward one, based on a configuration of MM transmission but single mode processing/switching, as the schematic diagram in figure 19(a) shown. The MM system can be easily implemented thanks to the well-developed single mode processing and switching components, at the cost of inefficient layout and large footprint. An 8 × 8 optical switch has been experimentally achieved based on this approach [148], shown in figure 19(b). However, the mode scalability is very limited, especially in the case of larger channel number. Alternatively, the second approach is to process/switch MM signals simultaneously via a single device, which is promising to be a more efficient way, as the schematic diagram in figure 19(c) shown. Here, the mode independent/transparent operation is crucial, and the key components should be redesigned to accommodate multiple modes, targeting mode independent characteristics. Some basic components, such as the power splitter [149], bending and crossing waveguides [150], are proposed and reported, achieving a simplified architecture and compact footprint. Figure 19(d) shows the MM 3 dB coupler based on this approach.

For a complete and flexible MM system, the reconfigurability is indispensable. It can be easily achieved for the first approach, while specific and elaborate design for MM tuning mechanism should be performed for the second one. A reconfigurable integrated MM circuit with a few modes had been verified and demonstrated [151], limited by the modal crosstalk and mode dependent tuning efficiency.

Advances in science and technology to meet challenges

To address the challenges of scalability and reconfigurability in integrated MM photonics, the major obstacles are the crosstalk and MD. The crosstalk limits the adopting mode number, while the MD hinders the consistency of tunability. It is remarkable that the supported mode number has reached >10 thanks to the effective investigations on the mode multiplexer. As a promising solution, the nanostructure assisted by inverse design is adopted to engineer the MD of the silicon waveguide, resulting the reduction of modal crosstalk while shortening the footprint of the mode Mux. Consequently, 16-channel mode Mux has been demonstrated [152].

Although large amounts of computation and optimization are adopted to support more modes, mode-independent manipulation is still hard to be achieved due to inherent and severe MD of the SOI waveguides. To further address this issue, one interesting solution inspired by the geometrical-optics concept is developed. By adopting a planar waveguide
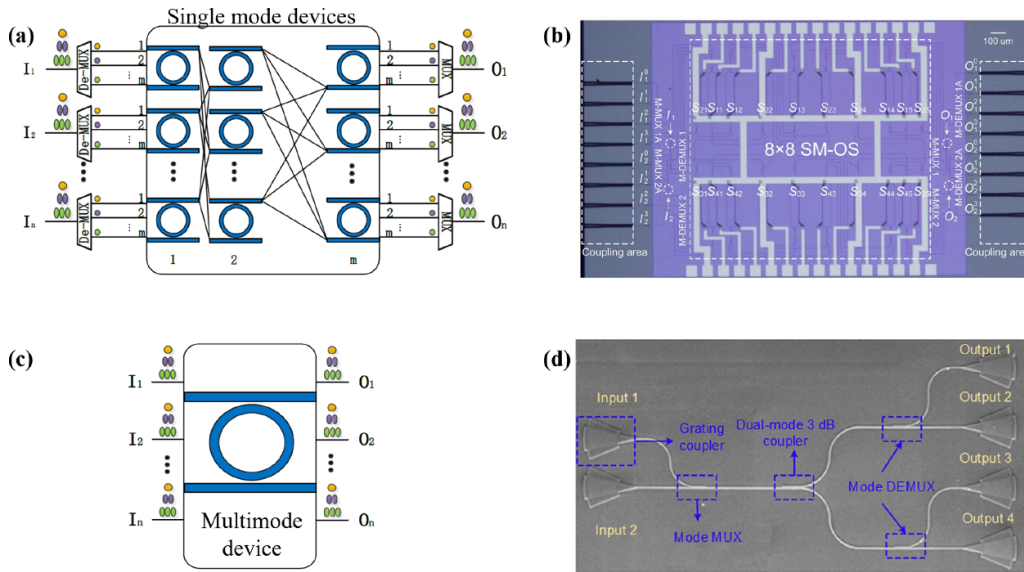
Figure 19. (a) The 'demultiplexing-single mode operation-multiplexing' architecture. (b) 8 × 8 multimode switch utilizing demultiplexing-single mode operation-multiplexing. (c) Single multimode device based architecture. (d) Multimode 3 dB power splitter.
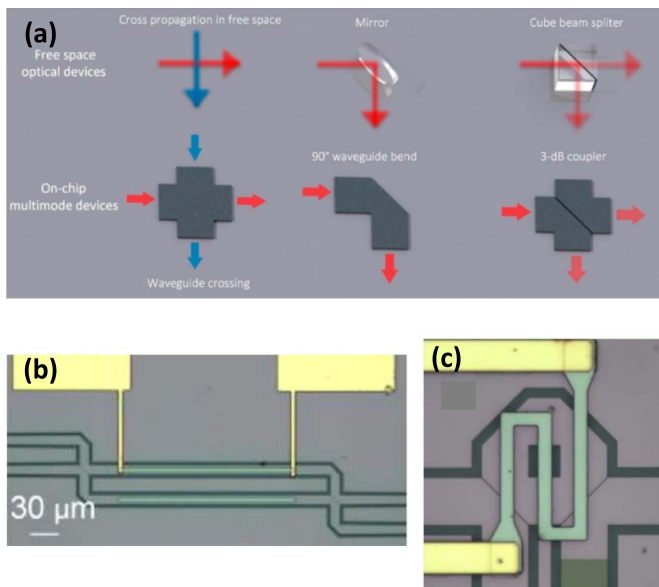

Figure 20. (a) Multimode building blocks inspired by the geometrical-optics concept. (b) Multimode optical switch. (c) Multimode ring resonator.

with a width larger than the wavelength, i.e. several microns, light propagates in the plane as in free-space while still being confined vertically in the waveguide. In this regaRoad, waveguide effect tends to be negligible in the plane, and thus, various modes tend to be degenerate. Therefore, the problem of mode-independent implementation and tunability can be solved fundamentally. A set of building blocks are experimentally demonstrated as proof of concept, including the MM power splitter, bending/crossing waveguides, switches [153], polarization beam splitter and ring resonator [154], as shown in figure 20. These works promote the MM photonics research and make the MDM technique more practical.

Concluding remarks

Remarkable progress on integrated MM photonics has been made over the past 10 years, and continuous researches are still highly desired to achieve advanced MM circuits. With the development of new material, novel structure, intelligent design manner and advanced fabrication method, the improvement on key MM components, in terms of loss, crosstalk, footprint, power consumption and robustness, can be expected. Based on the breakthroughs of the devices, large-scale and high-performance MM photonic circuit, combined with wavelength and polarization multiplexed circuits, can be potentially applied in future ultrahigh bandwidth communications.

Acknowledgment

14. Nonlinear interactions in multimode integrated photonics

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Status

Silicon photonics devices have undoubtedly changed the field of integrated photonics, introducing a number of compact and interconnected devices capable of performing various operations. Typical silicon-based waveguides exhibit sub-micron cross-section dimensions, thus allowing for ultra-compact optical mode confinement. Additionally, they show low propagation loss (in the region of 2 dB cm^{-1}) [155]. These two facts have stimulated researchers to design nonlinear components based on these structures, and many impressive demonstrations of various nonlinear functionalities have been demonstrated [156]. One of the most desired functionalities is represented by wave mixing and harmonic generation, typically achieved by means of the four wave mixing (FWM) effect [156]. FWM is efficient only when the phase matching condition among the involved waves is achieved. In single mode devices, this can be done by carefully engineering the high-order dispersion coefficient [155, 156], the successful application of which however, is strongly susceptible to fabrication errors. Another possible technique involves the use of different modes to achieve phase matching in the nonlinear process. The physics behind this has been known for several years, however it has been only in the last decade or so that fabrication progresses have allowed for the effect to be efficiently exploited, both in fibers and integrated photonics. Since each mode in a MM or few-mode waveguide is characterized by a different constant the use of a higher-order dispersion coefficient to achieve phase matching is no longer mandatory. Wideband and broadband operations can also be achieved by properly designing the multimodal waveguide. Intermodal processes are set to play an important role in the development of next generation nonlinear devices, allowing to overcome historic limitations that have been limiting the performance of silicon-based nonlinear waveguides.

Current and future challenges

Intermodal wave mixing has emerged [157], in the last years, as a powerful tool to realize efficient nonlinear components. Among the full spectrum of nonlinear processes, intermodal FWM has been studied and tested by several groups, showing that the main potential of this technique is the ability to generate new frequency lines over an unprecedented wide
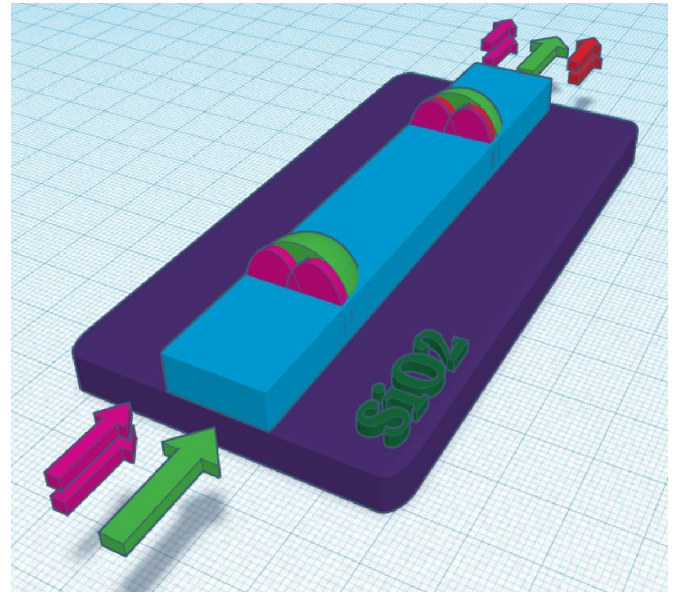

Figure 21. Three dimensional illustrative sketch of a high index contrast intermodal waveguide for FWM interaction. The green elements represent the fields on the fundamental modes, while the purple and the red elements represent the fields on the first order modes.

bandwidth, overcoming the phase matching-related limitation that inevitably affects single mode FWM systems. In our most recent work, we focused on the generation of L-band optical beams, starting from C-band telecom lasers, exploring Bragg scattering intermodal FWM [158, 159]. We showed wide and broadband frequency generation in two different technological platforms, namely SOI and silicon rich nitride on insulator. The waveguides were designed to show phase matching between wavelengths separated by 50 nm in the C- and L-bands of the telecommunications spectrum, respectively. Experimentally, two pump waves (located in the C-band) were launched to the fundamental mode while the signal was located in the L-band and was exciting the TE10 mode of the waveguide (as shown in figure 21). We observed the generation of an idler through Bragg scattering FWM, with a maximum conversion efficiency of −15 dB. More importantly, our experiments showed that, owing to the engineered dispersion characteristics of our waveguide, phase matching could be maintained and idler generation with a conversion efficiency that remained at a constant level could still be observed even after one of the pump waves was wavelength-detuned relative to the other by as much as 30 nm (limited by the available instrumentation), see figure 22. These results demonstrated that intermodal processes might be the key to enable the demonstration of many, long-desired, nonlinear functionalities, that could set to play important roles in a variety of applications, ranging from metrology to telecom uses.

Advances in science and technology to meet challenges

Our experimental results have shown that efficient nonlinear interaction among different modes is possible in both silicon
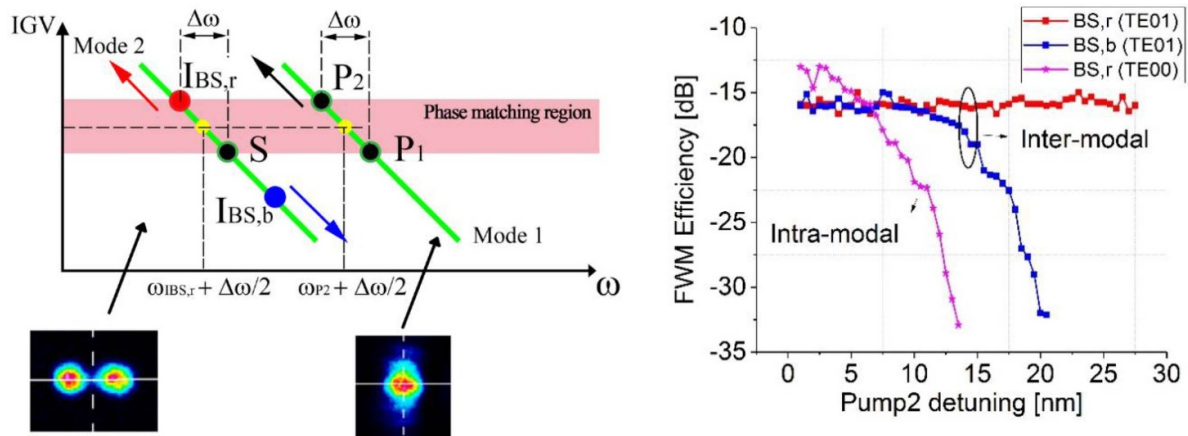
Figure 22. Left panel: intermodal four wave mixing scheme using two spatial modes. The green lines represent the inverse group velocity (IGV) curves for each mode. IBS,r and IBS,b are the red and blue scattered idlers, respectively, P1 and P2 represent the pump of the FWM process. Right panel: Bragg scattering FWM efficiency measured for intramodal and intermodal processes. Reproduced from [159]. CC BY 4.0.

and silicon rich silicon nitride platforms. It is well known that submicron silicon waveguides suffer from nonlinear losses when operated at relatively high power levels. This strongly limits the use of this platform if high efficiencies (>-10 dB) are necessary. On the other hand, experiments performed on our silicon rich silicon nitride platform [160] have shown no limitations in terms of operating pump powers (for peak power levels above 10 W TPA effects must be considered as described in [160]) opening the path to the development of efficient nonlinear components, operated with power levels in the range of 1–10 W. In order to reach high efficiency processes based on intermodal nonlinearities, the silicon nitride platform needs significant developments to be achieved in terms of loss reduction. Indeed, numerical simulations have shown propagation losses of the order of 0.1 dB cm^{-1} are needed to achieve, for example, FWM efficiency of the order of -6 dB [159]. More research breakthroughs are also needed for the development of complex and library-compatible components devoted to the management of optical modes inside the circuits. This will allow to convert and manipulate modes without the need to operate outside the integrated circuit, therefore increasing the capabilities of the final components. Lastly, intermodal wave mixing has not been fully explored in optical resonators. This possibility would allow to operate the nonlinear mixer with considerably lower pump powers, thus strongly reducing the power consumption.

Concluding remarks

In this section we discussed recent results on intermodal wave mixing in silicon and silicon rich silicon nitride waveguides. The maximum efficiency registered to date is -15 dB with a phase matching bandwidth exceeding 30 nm. We also discussed the challenges that researchers are bound to face in the next few years, in order to realize efficient and integrable MM nonlinear components.

Data availability statement

No new data were created or analysed in this study.

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