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It is a well-known fact nowadays that self-referential constructions depend upon different things, such as the theory they are built relative to and its language, the coding mechanism, how the notions they involve are defined, and possibly some other features. This paper aims at investigating the conditions that produce "genuine" self-reference as opposed to cases in which the self-referential nature of the construction is "accidental" only. As a matter of fact, fixed-point refutable sentences can be built for known self-referential constructions and coding functions, even by respecting the Kreisel-Henkin criterion for self-reference (which corresponds to having, for a term t and a formula $\varphi(x)$, $\Sigma \vdash t = \ulcorner \varphi(t) \urcorner$, relative to a given coding function $\ulcorner \cdot \urcorner$ and a given theory Σ).

The first goal of the paper is to make the distinction between accidental and genuine self-reference precise in the first place. This is addressed in section 3 by taking the uniformity character of the diagonalization procedure to be the mark of genuineness. Uniformity is defined in terms of recursive and finite combinations of basic operations involving the naming function $\ulcorner \cdot \urcorner$ (which is treated as a parameter of the definition) over syntactical objects.

Attention is then focused on uniform diagonalization operators whose outputs are terms satisfying the Kreisel-Henkin criterion. In sections 3.1-3.3, it is shown that the canonical diagonalization methods that can be found in literature are uniform in the sense of the paper. Then, the question about whether the definition of uniformity rules out accidental constructions is tackled. A positive answer to it is shown to depend upon excluding that constructions obtained by a uniform diagonal operator d are such that the diagonal term $d(\varphi)$ for a given formula $\varphi(x)$ does not occur in $\varphi(x)$ itself. As to what additional assumptions must be made to grant this latter condition, these are extracted from diagonal constructions that lead to refutable fixed-point formulas of provability predicates even in the case of uniform diagonal operators (section 4). The result is then summarized by proposition 5.2 of section 5: if the relation of dependence between expressions given by a certain coding function (the naming relation) is not circular, i.e. is irreflexive, and d is uniform, then $d(\varphi)$ cannot occur in φ. In order to support the claim that this condition is natural, section 6 shows that the consequence of it, i.e. assuming the naming relation to be well-founded, is a necessary condition for the naming function to play the role of a quotation device. Then, section 7 presents some more Kreisel-like constructions and shows that, while uniformity and non-circularity are not enough to rule them out, uniformity and well-foundedness of the naming relation grant that this can be achieved instead (although this does not exclude that there are limitations to these two constraints, as shown in Section 7.1). Finally, Section 8 presents some applications of the results.