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Suppose that a jury with three members has to reach a verdict about two facts: that (A) John committed the crime, and (B) that the crime was premeditated. Judge no. 1 is positive about both facts. Judge no. 2 thinks John is guilty, but that the crime was not premeditated. Judge no. 3 thinks the crime was premeditated, but that John is not responsible for it. It should be clear that, when it comes to determine what verdict of the jury the convictions of its members sum up to, it makes a lot of difference how the three judges' opinions are collected: for, if the votes of each judge are paired first and then combined with those of others, then there is no majority in favour of the conjunction of A and B (which gets one vote only); however, if votes in favour of A and B are collected first and then group together, the situation changes (as both A and B gets two votes each). The resulting dilemma is known in social choice theory as the *doctrinal paradox* (from L.A. Kornhauser [Modeling collegial courts I: path-dependence, *Int. Rev. Law Econ.*, 12, 1992]).

The paper under review aims at introducing a new way of aggregating votes, and at comparing it with the two others that are known in the literature as the *conclusion-based* (*Conc*, henceforth) rule and the *premise-based* rule (*Prem*), respectively. The new, "path-based" (*Path*) rule reads as follows: A & B is proclaimed if the number of votes for it are greater than those against A, and greater than those against B, separately (so, if supporters of A & B form a majority against detractors of A, no matter what their position toward B is, and similarly for detractors of B).

To measure the performance of the three rules the authors apply to this case a methodology based on the *Reicever Operating Characteristics* plots (ROC), and on the concepts of *true* and *false positive/negative rates.* ROC is commonly at use to compare binary decision rules in signal detection and is commonly used in those fields where binary decisions are taken under conditions of uncertainty, like medicine and machine learning theory (see T. Fawcett [An introduction to ROC analysis, *Pattern Recogn. Lett.*, 27, 2006]). A decision rule in signal detection theory is evaluated according to the ability of receving a bit of information correctly, i.e., to receive it as $\hat{\mathbf{1}}$ when $\mathbf{1}$ is the bit that has been sent, and as $\hat{\mathbf{0}}$ when $\mathbf{0}$ has been sent. The *true positive* (*negative*) *rate* (TPR and TNR, respectively) is the probability of receiving $\hat{\mathbf{1}}$ ($\hat{\mathbf{0}}$) when $\mathbf{1}$ ($\mathbf{0}$) is the bit sent. The *false positive* (*negative*) *rate* (FPR and FNR, respectively) is the probability of receiving $\hat{\mathbf{1}}$ ($\hat{\mathbf{0}}$) when $\mathbf{0}$ ($\mathbf{1}$) is the bit sent. The ROC *space* is the unit square $[0,1] \times [0,1]$ representing the coordinates (FPR, TPR) of a decision rule calculated from sample data. Good performance of a decision rules is granted when TPR is close to 1 and FPR to 0, which corresponds to points in the upper left corner of the ROC space.

After translating the ROC analysis vocabulary to the probabilistic framework of the decision rules under investigation and in such a way to account for the logical form of the proposition to be assessed, the authors state their main results. It turns out that, under a "straight" assessment (in which the roles of the error rates FPR and FNR are symmetric), the rule *Prem* is strictly better than rules *Path* and *Conc* when the number n of members of the jury is equal to, or greater than 3, and that the rule *Path* is at least as good as *Conc* for $3 \leq n \leq 6$ and strictly better for $n \geq 7$. The results remain exactly the same under a refined, "weighted" assessment in which the roles of the error rates FPR and FNR are assigned different weighs, but it can be also found a threshold for the values involved in the calculation of the performance of the rules over which the said relations of relative goodness break down.

The paper provides examples to further illustrate the significance of these results, as well as a detailed discussion of them.