Mathematisches Forschungsinstitut Oberwolfach

Report No. 04/2012

DOI: 10.4171/OWR/2012/04

Explicit Versus Tacit Knowledge in Mathematics

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January 8th – January 14th, 2012

Abstract. This workshop aims to bring together an international group of historians of mathematics to reflect upon the role played by tacit knowledge in doing mathematics at various times and places. The existence of tacit knowledge in contemporary mathematics is familiar to anyone who has ever been given an idea of how a particular proof or theory works by a verbal analogy or diagrammatic explanation that one would never consider publishing. Something of it is felt by every student of mathematics, when the process of learning mathematics often amounts to training the right reflexes. In more advanced contexts, the tacit understanding that a particular technique, instrument or approach is the one to use in a given circumstance gives another familiar instance. Tacit knowledge, a term introduced by the philosopher M. Polanyi, contrasts with the explicit knowledge that in almost all historical mathematical cultures is associated with mathematical text. The workshop invites a use of the categories of tacit and explicit knowledge to achieve a better knowledge of how mathematical creation proceeds, and also of how cultural habits play a tacit role in mathematical production. The proposed meeting offers the possibility of significant innovation and enrichment of historical method, as well as new and compelling insight into the process of creating mathematics in different times and places. The meeting will afford the opportunity for a presentation of selected case studies by leading experts and new scholars, with results that promise to be of significant interest not only to historians, but to the mathematical community more broadly.

Mathematics Subject Classification (2000): AMS-CLASSIFICATION.

Introduction by the Organisers

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[9] Ibn Ṭāhir al-Baghdādī, *at-Takmila fī l-ḥisāb ma^{c}a risāla lahū fī l-misāḥa* [The completion of Arithmetic with a tract on Mensuration]. Introduction and critical edition by A.S. Saïdan, Koweit : Manshūrāt ma^{c}had al-makhṭūṭāt al-^{c}arabīya, 1985.

Tacit versus explicit knowledge in history of mathematics: the case of Girolamo Cardano

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Girolamo Cardano, Niccolò Tartaglia, Ludovico Ferrari and Rafael Bombelli – the so-called Italian Algebraic School of the Renaissance – were the heirs of the abbacus tradition, which flourished in Italy mainly from the 14th up to the 16th century. It is difficult evaluating in detail the features of this heritage, first of all because the abbacus mathematics, transmitted essentially by manuscripts, is still largely unresearched (with some geographical exceptions). And so, some techniques, concepts and results which appear novelties at first sight, at a deeper analysis become aspects of a form of tacit knowledge, shared by practical mathematicians. The concept of number developed in the abbacus milieu, for example, deeply influenced the algebraist of the Renaissance. Cardano clearly explained this concept at the very beginning of his *Practica arithmetice* (1539) [1]: the only true number is the natural one, but positive fractions and radicals are to be considered numbers "by analogy", because defining elementary operations in each set of 'numbers' is allowed. When Cardano found square roots of negative numbers (*radices sophisticae*) in the solution formula of the third degree equation (the so-called "irreducible case", *Ars magna* 1545), he was not worried about foundational questions, but he asked himself if they behaved "by analogy" like numbers. The first step to be carried out was to establish whether they were positive or negative quantities. Although he realized that the quantities could not be considered negative or positive, but were "a third sort of thing", Cardano tried to give them a sign, even attempting to formulate a new rule of signs appropriate to his own needs. After noting the failure of this approach, Cardano tried to find a solution formula that did not contain roots of negative numbers, but his efforts were not rewarded. In his *Algebra* (1572), Bombelli, who shared the same concept of number as Cardano, reconsidered the problem of the sign of expressions having the form $b\sqrt{-1}$ and introduced the new signs – rather than imaginary numbers – "more than minus" (*più di meno*) and "less than minus" (*meno di meno*), for which he established appropriate rules of multiplication. On this basis, Bombelli founded an arithmetic of Cardano's sophistical quantities allowing him to make sense of the irreducible case of cubic equations and, in the special cases where it was easy to extract the *linked cubic roots* $\sqrt[3]{a \pm b\sqrt{-1}}$, also allowing him to solve such equations, obtaining the real roots. Furthermore the engineer Bombelli, differently from Cardano, deeply influenced by "Euclidean education", did not hesitate to provide a geometrical proof of the existence of real roots in the irreducible case, because he accepted the use of sliding squares instead of ruler and compass or, in other words, he accepted the idea of determining a point in an approximate way. In his *Ars magna*, Cardano showed the Euclidean representation of the solution formula by decomposition of a cube into other cubes and parallelepipeds, but this decomposition was possible only when the third degree equation had a non-negative discriminant. In his *De regula aliza* (1570), Cardano showed that the solution of a irreducible equation could be represented as an intersection of a parabola and a hyperbola, but he bitterly concluded that, although simple from the geometrical point of view, the construction was difficult to translate into arithmetical terms. Moreover, he added, without any real justification, that he did not find the construction fully satisfactory, probably – I suppose – because of the impossibility of using only ruler and compass. Cardano seemed to refuse abbacus heritage with respect to geometrical approach. When he and his pupil Ferrari, in the context of the famous challenge Tartaglia vs Ferrari, proved all the *Elements* using a straightedge and a fixed opening compass instead of a variable opening compass (and slightly changing the Third Postulate), he decided to publish this (relevant) result in the philosophical work *De utilitate*, thinking it was interesting from the purely mathematical point of view, but not really useful, even if a fixed opening compass was an instrument commonly used by craftsmen. While Cardano remained firmly connected to the Euclidean spirit, mathematicians like Bombelli and Tartaglia, got instruments and techniques by practical geometry. Tartaglia, for example, devoted the Fifth Part of his *General Trattato* (1560) to "geometers, draftsmen, perspectives, architects, engineers and mathematicians" and the aim of this treatise is just using ruler and fixed opening compass to prove Euclidean propositions. Moreover, Tartaglia, who translated the *Elements* into vernacular Italian (1543), was often guided in his translation by tacit knowledge based on practical experience: a comparative study of the *General Trattato* and the *Elements* is necessary to definitively describe this influence. On the other side, this case study shows that the relationship between the Renaissance Italian algebraists and their mathematical milieu, both tacit and explicit, is an issue to explore in order to deeply understand some of the main development of mathematics in 17th century.

References

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