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Title: Intuitionistic epistemic logic with two modal operators.

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The paper provides a comprehensive expansion of the intuitionistic doxastic logic $\mathbf{IEL}^-$ and intuitionistic epistemic logic **IEL**, originally introduced by Artemov and Protopopescu [*The Rev. of Sym. Log.*, 9, 266298 (2016);MR3506909]. The authors primary contribution is the addition of a diamond modal operator to the previously box-based propositional language, resulting in the development of two new systems in particular, among others, $\mathbf{L}_{dox}$ (intuitionistic doxastic logic) and $\mathbf{L}_{epi}$ (intuitionistic epistemic logic). Recognizing that $\Box$ and $\Diamond$ are generally not interdefinable in an intuitionistic context, the work justifies this addition by adopting an interpretation of $\Diamond$ *la* Přenosil [*Advances in Modal Logic. Vol. 10*, College Pub., 423–438 (2014);MR3329835], which allows the logics to remain faithful to the original frame classes without requiring additional confluence assumptions.

A significant portion of the work is dedicated to the relational semantics for these systems. The author considers various classes of frames–such as basic, visionary, futuristic, founded and the like–by identifying the specific modal formulas that define them. To establish the formal validity of $\mathbf{L}_{dox}$ and $\mathbf{L}_{epi}$, the paper puts forward Hilbert-style axiomatizations and provides rigorous completeness proofs for $\mathbf{L}_{dox}$ and $\mathbf{L}_{epi}$ utilizing a canonical model construction based on prime proper theories.

Furthermore, the paper evaluates these logics against several essential formal features expected of intuitionistic modal systems. The author shows that both $\mathbf{L}_{dox}$ and $\mathbf{L}_{epi}$ are conservative over intuitionistic propositional logic, possess

the Disjunction Property, and ensure that the $\Box$ and $\Diamond$ operators remain independent. The study concludes by outlining several open research problems, including the computational complexity of the membership problem and the possibility of extending these systems to include multi–agent distributed knowledge, or alternative diamond interpretations.