PROBABILISTIC GEOTECHNICAL STABILITY ANALYSIS OF VEGETATIVE CRIB WALLS: INITIAL INSIGHTS

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Synopsys

Nature-based solutions (NBS) provide sustainable technical means to achieve sustainable geotechnical engineering design following soil and water bioengineering principles. Vegetative crib walls represent one of the most effective NBS for the stabilization of slopes or banks. Geotechnical systems involving NBS are pervaded by significant aleatory and epistemic uncertainties. Despite the ongoing regulatory evolution which shifts design paradigms towards non-deterministic formats, the design of vegetated crib walls typically relies on empiricism and experience. The paucity of quantitative engineering design methods compatible with current design formats hinders the diffusion of geotechnical NBS. This paper aims to contribute to the development of code-compatible geotechnical design methods for vegetated crib walls by addressing the probabilistic analysis of external stability and by comparing the results with the outputs of deterministic approach for an example scenario.

1. Introduction: performance and sustainability

Within the practice of soil and water bioengineering (SWBE), nature-based solutions (NBS) provide a promising technical means to overcome diverse challenges in soil and slope stabilization, erosion control, and ground improvement. NBS are "solutions that are inspired and supported by nature, which are cost-effective, simultaneously provide environmental, social and economic benefits and help build resilience. Such solutions bring diverse natural and natural features and processes into cities, landscapes, and seascapes through locally adapted, resource-efficient, and systemic interventions" (Nature-based solutions 2023). Many NBS stem from ancient practices, including dams and embankments constructed using natural aggregates, vegetated riverbanks, and wooden pile foundations for buildings. These solutions have been rediscovered and are again being implemented in modern engineering interventions. Slope stability and erosion control are currently the most popular domains of application of NBS in geotechnics. Nature-based engineering solutions commonly used for slope stabilization include live grids (particularly effective against erosive phenomena) and live crib walls. The latter, which lie at the focus of this study, are generally realized by arranging wooden elements (timber logs, bamboo, etc.) to form a box which is subsequently filled with soil. These structures are typically placed against the slope in a similar way to what is done with gabion walls. Plant species can be inserted into the soil and in these cases these systems take the name of live crib walls. Over time, the natural growth of the roots will replace the resistance provided by the trunks which in the meantime will deteriorate. The role and positive impact of vegetation on slope stability are receiving increasing attention on the part of the geotechnical community. It is well known that geotechnical systems are pervaded by significant aleatory and epistemic uncertainties. Aleatory uncertainty stems from the “truly existing” spatial and temporal variability of physical elements. Epistemic uncertainty includes: (a) measurement uncertainty, stemming from the invariably limited precision and accuracy of testing data; (b) statistical uncertainty, resulting from the limited size of data samples from testing and measurements; and (c) transformation uncertainty, surging from the inevitable approximations and simplifications of the complex physical phenomena occurring in geotechnical systems. The inclusion of NBS further increases the magnitude of aleatory and epistemic uncertainty. A very limited number of studies focusing on geotechnical design using NBS are available, and no specific provisions related to green solutions are currently included in geotechnical design codes. This paper aims to contribute to the diffusion of NBS among geotechnical researchers and practitioners by providing a comparative example of deterministic and probabilistic geotechnical design of vegetative crib walls, with specific reference to external stability as described in the following.

2. Geotechnical design of vegetative crib wall

2.1 Reference model

The external stability of live crib walls is investigated with respect to sliding and bearing capacity of the foundation and overturning of the wall in static conditions by means of analytical models taken from a more comprehensive approach proposed by Acharya (2018), which also includes internal stability checks. In external stability assessment methods, live crib walls are assumed to behave as monolithic gravity structures subjected to external loads.

The safety factor with respect to foundation sliding is calculated as

$$FS_{sl} = \frac{V tan\delta}{H} \geq 1.5 \tag{1}$$

where V and H are the resultants of the vertical and horizontal components of the forces and δ is the friction angle between the base and the soil. The safety factor towards bearing capacity was evaluated with reference to Eurocode 7 Part 1 §9 (CEN, 2004) as:

$$FS_{bc} = \frac{\sigma_b}{\sigma_e} \geq 1.5 \tag{2}$$

where σ_b is the permissible soil stress (assumed) and σ_e is the maximum pressure at the bottom of the wall. The safety factor against overturning is:

$$FS_{ot} = \frac{M_{pas}}{M_{act}} \geq 1.5 \tag{3}$$

where M_{pas} is the moment of the resultant of the stabilizing forces and M_{act} is the moment of the resultant of the un-stabilizing forces. Details of the calculation approaches are not provided here due to

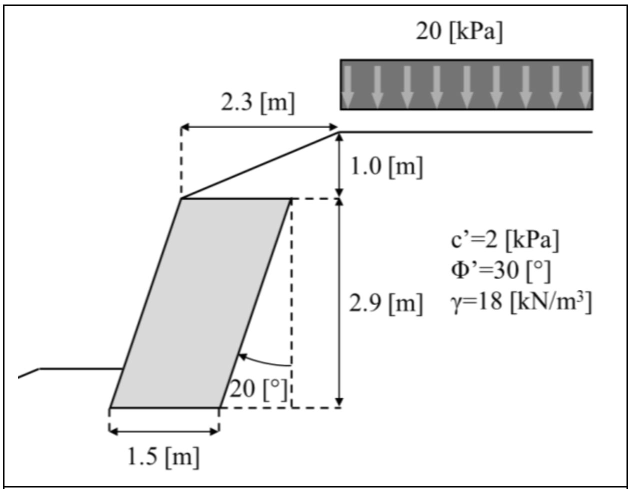

Figure 1. Reference cross-section of the live crib wall with soil deterministic parameters (modified from Acharya 2018)

paper length limitations. While dynamic conditions can be addressed by means of a pseudo-static approach, only the static case is considered in this paper.

3. Case-study application

3.1 Description of reference scenario

The case study analysed by Acharya (2018) and adopted in this work involves a single order of live crib walls with a 2.3 m sloping bank which then extends upslope with a horizontal planar surface subjected to a uniform distributed load of 20 kPa. The wall is made with bamboo elements and has a height of 2.9 m with an inclination with respect to the vertical of 20° while the lower and upper bases are horizontal with an extension of 1.5 m (Figure 1). The soil is characterized by an effective cohesion c'=2 kPa, a friction angle ϕ'=30° and a unit weight γ=18 kN/m^3. The ground pressure inclination angle is assumed to be equal to $\delta = 2/3\varphi$ =20°. No groundwater is present. In Acharya (2018), these single-valued parameters provide inputs to the deterministic analysis.

3.2 Probabilistic design

All geotechnical and geometric parameters entering the reference model described above are affected by aleatory and epistemic uncertainties. In this paper, only the aleatory uncertainty related to external loading and the epistemic uncertainty in geotechnical parameters (unit weight, effective cohesion, friction angle, and angle of inclination of earth pressure) are modeled as random variables. Geometric variables of the system and physical parameters of wooden elements are assumed to be deterministic as, specifically for the purpose of external stability checks, their uncertainties can be assumed to be of lesser magnitude and influence.

Statistics and probability provide a wide range of well-established methods of varying complexity for non-deterministic analysis of geotechnical systems. This paper implements the probabilistic estimation of the external stability of vegetated live crib walls (as parameterized by the factors of safety against sliding, overturning, and bearing capacity failure) using Monte Carlo simulation (MCS). MCS entails the generation of sampling distributions for all random variates and the subsequent repeated implementation of the Acharya (2018) design method using inputs sampled from such distributions. Sampling distributions are generated stepwise through: (1) the selection of a suitable distribution type; (2) the assignment of suitable distribution parameters; (3) the modeling of correlation between random variable. This process can rely on subjective and/or objective criteria. In this paper, Gaussian distributions are assumed for all random variates. In this paper, the "nominal" deterministic values used in the example provided in Acharya (2018) are taken as the Gaussian distribution means μ of the respective distributions, while standard deviations are calculated for each random variate from the mean and the respective coefficient of variation (CoV) as $\sigma = \mu \cdot CoV.$ The CoV is often used in the subjective definition of sampling distributions as it effectively parameterizes the expert-assigned level of uncertainty in the form of "degree of relative dispersion around the mean value". According to a well-established "rule of thumb" widely adopted in structural and geotechnical engineering studies, a value of CoV<0.10 is indicative of "low" scatter, a value between 0.10 and 0.30 indicates "medium" scatter, and a value above 0.30 represents a scenario of "high" scatter. CoVs can also be assigned objectively

Table 1. Parameters used in the generation of Gaussian sampling distributions for random parameters and model factors

Parameter	**units**	**mean**	**CoV**
q	[kPa]	20	0.30
γ	[kN/m^3]	18.0	0.10
ϕ'	[°]	30	0.15
c'	[kPa]	2	0.15
δ/ϕ'	[-]	2/3	0.30
MF_{sl}	[-]	1.00	0.10
MF_{ot}	[-]	1.00	0.10
MF_{bc}	[-]	1.00	0.10

through dedicated statistical and probabilistic analyses which lie beyond the scope of this paper.

Mean values and CoVs are assigned to random variables as well as to the analytical models used to calculate the factors of safety against sliding (FS_{sl}), overturning (FS_{ot}), and bearing capacity failure (FS_{bc}). Coefficients of variation are assigned on the basis of existing research and subjective reasoning as given in Table 1. Note that assigning unit distribution means to all model factors parameterizes the hypothesis of unbiasedness.

The modelling of correlation (or lack thereof) between random variates is an important step in the definition of sampling distributions. Geotechnical data are almost invariably multivariate. Past research has highlighted the mutual physical dependencies between numerous geotechnical parameters and has quantified these dependencies for statistical and probabilistic analysis in the form of correlation coefficients ranging between -1 (perfect inverse correlation) and 1 (perfect direct correlation). Accounting for such dependencies entails that the sampling distributions of the various parameters are more adherent to the mechanical properties of real soils. Correlation parameters are assigned on the basis of literature findings and subjective reasoning. Details are not provided here due to length limitations. No correlation is hypothesised between the three model factors and between these and geotechnical input parameters. The resulting correlation matrix is shown in Table 2.

4. Analysis and discussion of results

Sampling distributions of size 10,000 were generated through purposely compiled Python routines to obtain equally sized samples of FS_{sl}, FS_{ot}, and FS_{bc} through MCS. Figure 2a-2h show the Gaussian probability density functions of input variables and the respective relative frequency histograms. Figures 2i-2k plot the relative frequency histograms of the three output factors of safety. Deterministic input values are also reported in each subplot.

The effects and implications of uncertainty modelling and processing can be assessed both qualitatively and quantitatively. It is good practice to perform both types of assessments to obtain a more comprehensive and critical insight. The qualitative assessment relies on visual inspection of Figure 2. A number of observations can be made. For instance, while all probability density functions of input

Parameter	q	γ	ϕ'	c'	δ/ϕ'	MF_{sl}	MF_{ot}	MF_{bc}
q	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
γ	0.00	1.00	0.30	0.10	0.00	0.00	0.00	0.00
ϕ'	0.00	0.30	1.00	-0.50	0.00	0.00	0.00	0.00
c'	0.00	0.10	-0.50	1.00	-0.15	0.00	0.00	0.00
δ/ϕ'	0.00	0.00	0.00	-0.15	1.00	0.00	0.00	0.00
MF_{sl}	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
MF_{ot}	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
MF_{bc}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

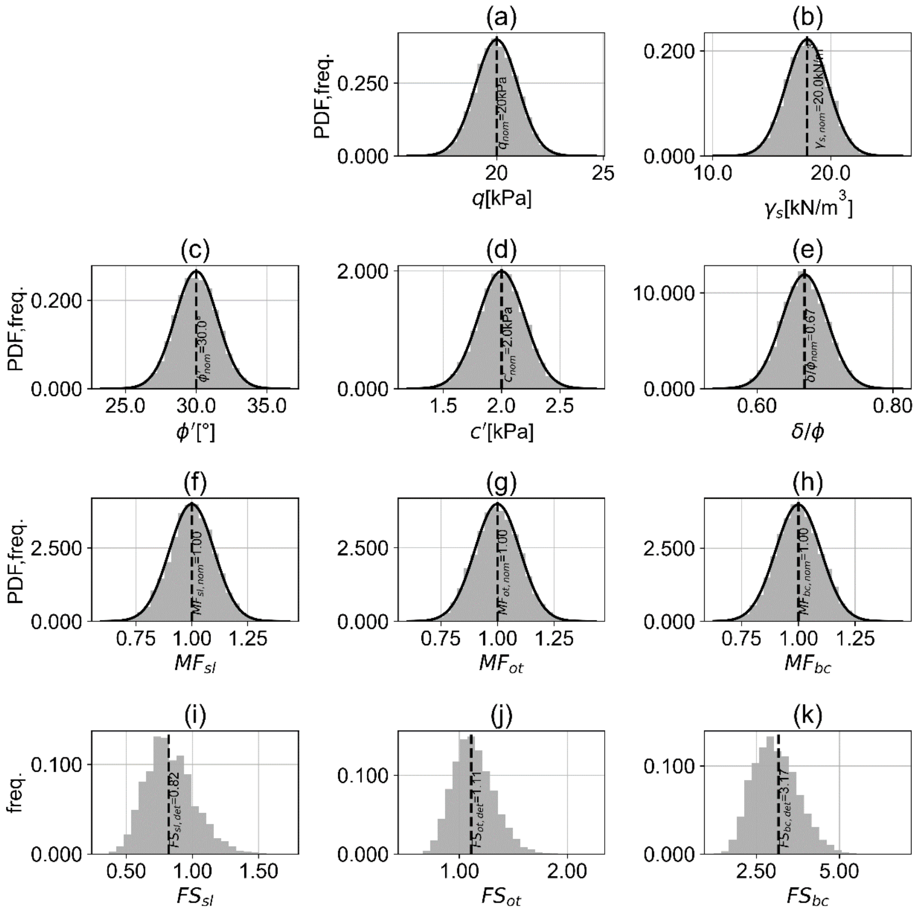

Figure 2. Frequency histograms (input variables and outputs) and probability density functions of sampling distributions (input variables only)

variables are Gaussian and, thus, symmetric, output samples of FS_{sl}, FS_{ot}, and FS_{bc} are not symmetric but are instead, left-skewed. Moreover, sample modes do not coincide with – and are smaller than – deterministic values. Quantitative assessment relies on the comparison of sample statistics of model outputs and the respective deterministic values.

Table 3 reports sample the deterministically model-calculated values and selected statistics for FS_{sl}, FS_{ot}, and FS_{bc}. Sample statistics include mean, coefficient of variation, mode, minimum, and maximum. The frequentist probability of stability P_{safe}, given by the ratio of the number of "performing" instances (i.e., the number of samples for which the factor of safety exceeds unity) to the total number of simulations. is also provided.

Examination of Table 3 allows several observations. First, sample means of FS_{sl} and FS_{ot} are very similar to the respective deterministic values while the difference is somewhat larger, albeit still small, for FS_{bc}. Second, sample modes and sample means are almost coincident for FS_{sl} and FS_{bc}, while the difference is slightly larger for FS_{ot}. Third, sample CoVs suggest "medium" levels of scatter around sample means, attesting to the relevance of uncertainties as assigned in this example. This observation is further supported by the large sample ranges (given by the difference between the maximum and

Table 3. Deterministic outputs and sample statistics from probabilistic analysis

Output	determ.	mean	CoV	mode	min	max	P_{safe}
FS_{sl}	0.82	0.83	0.22	0.77	0.30	1.85	0.170
FS_{ot}	1.11	1.13	0.17	1.08	0.56	2.04	0.738
FS_{bc}	3.17	3.07	0.22	3.09	1.29	5.94	1.000

minimum values) of the three factors of safety. The most relevant outcome of the study lies perhaps in the possibility to quantify the level of reliability in design. Such information is not available from a deterministic analysis, which only allows to compare output factors of safety with the respective target values given, for instance, by regulatory prescriptions. While deterministic outputs only allow the binary assessment of performance (yes/no), probabilistic analysis allows to quantify design reliability by calculating the frequentist probability of stability. Note that in this case it is not necessary to define arbitrary target deterministic factors of safety such as 1.5 as these values exceed unity as they are meant to account for the “lumped” effect of uncertainties in deterministic inputs.

5. Concluding remarks

This paper presented an initial insight into the quantitative comparison of deterministic and probabilistic assessment of the external stability of a live crib wall with respect to sliding, overturning, and bearing capacity failure mechanisms. The comparison between deterministic and sample statistics of the respective factors of safety allowed the qualitative and quantitative analysis of the effects of the modelling and propagation of uncertainties within the analytical models used to calculate them.

The scope of this paper is limited to the comparative assessment of the effects of probabilistic modelling of parameter and model uncertainties for a single design scheme (given by a specific set of geometric and geotechnical parameters) and a single uncertainty modelling scenario (as given by distribution statistics and correlation matrix). Notwithstanding this intentional restriction which impedes their generalization, the results of this study can be framed within the vast corpus of literature which increasingly demonstrates and exemplifies the many ramifications of probabilistic outputs and the numerous opportunities offered by non-deterministic analyses. The future extension of this approach to a broad set of design schemes and uncertainty modelling scenarios will allow the generalisation of the results and the conduction of important analyses aimed at the possible inclusion of NBS-specific provisions in geotechnical design codes through, for instance, the calibration of partial factors to achieve a uniform level of reliability in design in a LRFD format.

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