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Water dynamics and environmental social practice in a differential game

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ABSTRACT

This paper investigates how socially and environmentally responsible practice influences the dynamics of a common aquifer and its impact on social welfare. We analyze a differential game between two firms, profit seeking (PS) and environmental corporate social responsibility (ECSR), that pump water to sell it to farmers. The profit of the PS firm is composed of revenues, extraction and taxation cost. Conversely, the ECSR maximizes an objective function composed of profit, consumer surplus, and environmental damage. From the analysis of the model, it emerges that only a balance of the social and the environmental concern can preserve the water table and improve the social welfare.

1. Introduction

Over-exploitation of aquifers is a serious problem in many regions of the world. In fact, the intensive use of groundwater leads to a wide array of social, economic and environmental consequences such as land subsidence, increased agricultural vulnerability and strained use of other necessary water applications due to water increased in pumping costs. In particular, it has been estimated that at least the 20% of the aquifers are over-exploited in the world [1]. The water demand for agriculture represents the main pressure to groundwater and it is expected to increase due to population growth and consequently an increase of food production [2]. In such a context, it is essential a careful resource management as well as alternative managerial strategies. One of them is the corporate social responsibility (CSR) practice. By this term we refer to a firm that take into account social aspects of its market activity. Regarding water use, some scholars [3] add to the social issues also the environmental ones, from whose union we have the environmental corporate social responsibility (ECSR). Many scholars recognize that the ECSR approach should be pursued from ecological and social point of view (see, among others, [4,5]).

The aim of this paper is to investigate theoretically the effect of ECSR practice in the groundwater management context following the seminal dynamic hydro-economic model of Gisser and Sanchez [6]. This context has been applied to several issues related to groundwater management. Negri [7], Provencher and Burt [8], Rubio and Casino [9,10], and Biancardi and Maddalena [11] have studied the effects of competition on water dynamics. Roseta-Palma [12] and Erdlenbruch et al. [13] have emphasized the interaction between quality and quantity while Pereau et al. [14] pointed out the effects of water management on food security. Moreover, Esteban and Albiac [15] have introduced the ecosystem damage while heterogeneity among agents has been studied by Biancardi et al. [16]. The main approach used in this strand of the literature is the differential game approach, although Pereau and Pryet [17], Pereau et al. [18] and Pereau [19] have highlighted the differences between the optimal control technique and the viability kernel one. Finally, Biancardi et al. [20,21,22,23] have analyzed the effects of illegal extraction on water dynamics from different points of view.

Economic theory defines a CSR firm a firm who maximizes an objective function that takes into account profit and consumer surplus (see, among others, [24–27]). The main result of this strand of the literature is that the CSR firm is more aggressive and it makes higher profits than profit seeking rivals if the market is large enough. The interest on the analysis of CSR that takes into account environmental concerns is quite recent in the economic literature. As the name suggests, the ECSR is a CSR firm that adds to the social concern the environmental one. Therefore, the objective function of a ECSR firm takes into account profit, consumer surplus, and the environmental damage (see, among others, [28–31]). The presence of ECSR is viewed by its supporters as a self-regulating tool, as it leads firms to internalize the environmental effects caused by production. On the one hand, a firm may anticipate that environmental regulation will become stricter and therefore may create a competitive advantage. Secondly, shareholders ask managers to follow environmental concerns. In addition, green consumers may penalize firms without environmental preferences concerns.

The presence of environmental concern may counterbalance the negative effect due to intensive production in order to satisfy the consumer surplus.

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The theoretical approaches to the analysis of the CSR is usually proposed in a static setting and few are the results about dynamic analysis in the presence of CSR or ECSR.

To best of our knowledge, our paper is the first that uses the ECSR approach in a context of groundwater exploitation. We consider a dynamic duopoly composed of a profit seeking and a ECSR firm that pump water from a common aquifer. The water extraction is afterwards sells to the farmers. Both types of firms have to pay a water tax on withdrawals and the maximization problem is subject to the dynamics of the aquifer. From the analysis of the model it emerges that the ECSR firm pumps always more than the profit seeking one and only a combination of both social and environmental concern can mitigate the negative effects of water extraction on the ecosystem.

This paper aims to contribute both on the literature on ECSR and, in particular, about the management of groundwater when asymmetric firms (ECSR and profit seeking firms) are involved. The main contributions obtained are in economic context. In fact, we have obtained results about the level of the water table and withdrawals of the two types of firms considering the social and environmental concerns. With respect to water tax, it influences the level of aquifer instead it has no effects on the withdrawals.

The paper is structured as follows. Section 2 introduces the description of the model and its analysis. The main economic results are presented in Section 4 and Section 5 concludes. All proofs are collected in the Appendix.

2. The model

Let assume an asymmetric duopoly composed of two firms, a profit seeking firm (PS) and an environmental corporate social responsibility (ECSR) one. We denote the PS firm with subscript p and the ECSR firm with subscript s. Enterprises compete in pumping water that sell to farmers for irrigation. For the sake of simplicity, the water demand function is linear:

$$W = g - kP$$

where $g > 0$ is the intercept, $k > 0$ is the slope of the water demand, and $P > 0$ is the water price. Since the market is composed of only two firms, we can define the total water pumped as $W = \omega_p + \omega_s$. Hence, the inverse water demand becomes:

$$P = \alpha - \left(\omega_p + \omega_s\right) \beta$$

where $\alpha = \frac{g}{k}$ and $\beta = \frac{1}{k}$. According to the seminal work of Gisser and Sanchez [6], the dynamics of the water table is:

$$\dot{H} = \frac{R - (1-\gamma)\left(\omega_p + \omega_s\right)}{AS} \tag{1}$$

where $R > 0$ is the natural recharge, $\gamma \in (0,1)$ is the constant return flow and $AS > 0$ is the aquifer area times storativity. Eq. (1) implies that the water table increases if rainfall increases and decreases if pumping increases. The return flow parameter γ denotes that part of the water used for irrigation goes back to the aquifer.

To avoid over-exploitation and to reduce the negative environmental externalities that may arise, we assume that both firms have to pay a tax ($\tau > 0$) on individual water withdrawals. Therefore, the pumping costs are a function of hydrological parameters and taxation:

$$C = c_0 - c_1 H + \tau$$

where $c_0 > 0$ and $c_1 > 0$ represent the fixed and the variable cost with respect to water level. Notice that $C = 0$, namely pumping water has zero cost, if $H = c_0/c_1$. Therefore, the ratio $H = H_{\max} := c_0/c_1$ represents the maximum level of the aquifer [9]. This means that the higher is the water table, the less it costs to pump water.

The optimization problem of the PS firm is to maximize the profit, composed of revenues minus pumping costs, choosing the water to pump under the dynamic constraint of the water table:

$$\max_{\omega_p \geqslant 0} \pi_p = \int_0^{+\infty} \left\{ [\alpha - (\omega_p + \omega_s)\beta]\omega_p - (c_0 - c_1 H + \tau)\omega_p \right\} e^{-\rho t}\, dt$$
$$\text{s.t. } \dot{H} = \frac{R - (1-\gamma)\left(\omega_p + \omega_s\right)}{AS}$$

the parameter $\rho > 0$ denotes the discount rate. Differently from the PS firm, the ECSR does not choose the level of water in order to maximize the profit function, but in order to maximize its objective function, composed of revenues, pumping costs, consumer surplus, and ecosystem damage. The maximization problem of the ECSR firm is:

$$\max_{\omega_s \geqslant 0} O_s = \int_0^{+\infty} \left\{ [\alpha - (\omega_p + \omega_s)\beta]\omega_s - (c_0 - c_1 H + \tau)\omega_s + \eta CS - \phi ED \right\} \times e^{-\rho t}\, dt$$
$$\text{s.t. } \dot{H} = \frac{R - (1-\gamma)\left(\omega_p + \omega_s\right)}{AS}$$

The parameter $\eta \in (0,1)$ represents the sensitivity to social issues (also called as social concern) while $\phi \in (0,1)$ denotes the share of the environmental damage internalized (also called as environmental concern). Due to the linearity of the water demand, the consumer surplus is:

$$CS = \frac{(\omega_p + \omega_s)^2}{2}$$

namely it is a function of the total water pumped. The introduction of the consumer surplus in the objective function is the main difference between a profit-seeking firm and a CSR one (see the seminal works of Goering [24] and Kopel and Brand [25]).

According to Esteban and Albiac [15] and Biancardi et al. [23], we assume that the ecosystem damage is represented by the volume depleted from the aquifer in each period, namely how much water is pumped by the two firms minus the rainfall:

$$ED = (1-\gamma)(\omega_p + \omega_s) - R$$

Since the ECSR firm internalizes ED, the parameter η denotes the cost of damage to ecosystem from each cubic meter of aquifer depletion.

3. Analysis of the model

The Hamiltonian functions of the PS and ECSR firms are:

$$\Omega_p = \left[\alpha - (\omega_p + \omega_s)\beta - c_0 + c_1 H - \tau\right]\omega_p + \frac{\lambda_p}{AS}\left[R - (1-\gamma)(\omega_p + \omega_s)\right]$$
$$\Omega_s = \left[\alpha - (\omega_s + \omega_p)\beta - c_0 + c_1 H - \tau\right]\omega_s + \frac{\eta}{2}\left(\omega_s + \omega_p\right)^2 - \phi\left[(1-\gamma)\left(\omega_s + \omega_p\right) - R\right] + \frac{\lambda_s}{AS}\left[R - (1-\gamma)\left(\omega_s + \omega_p\right)\right]$$

where λ_p and λ_s are the adjoint variables. Applying the maximum principle, we get the following dynamical system:

$$\begin{aligned}
\dot{H} &= \frac{1}{AS}\left[R - (1-\gamma)(\omega_p + \omega_s)\right] \\
\dot{\omega_p} &= \frac{1}{AS\beta(3\beta-\eta)}\{AS\rho c_1(\eta-\beta)H - c_1(2\beta-\eta)(1-\gamma)\omega_s \\
&\quad + \beta[AS\rho(3\beta-\eta) \\
&\quad + c_1(1-\gamma)]\omega_p - (\beta-\eta)[(\alpha - c_0 - \tau)\rho AS - c_1 R] - \phi(1-\gamma)\beta\rho AS\} \\
\dot{\omega_s} &= \frac{1}{AS\beta(3\beta-\eta)}\{-AS\rho c_1(\beta+\eta)H - 2\beta(1-\gamma)c_1\omega_p + [AS\beta\rho(3\beta-\eta) \\
&\quad + c_1(1-\gamma)(\beta-\eta)]\omega_s - (\beta+\eta)[(\alpha - c_0 - \tau)\rho AS - c_1 R] \\
&\quad + 2\phi\beta AS\rho(1-\gamma)\}
\end{aligned} \tag{2}$$

The following proposition holds.

Proposition 1. *The unique steady state open loop Nash equilibrium of the dynamic system (2) is:*

$$\begin{aligned} H^* &= \frac{Rc_1(1-\gamma)+\rho AS[R(3\beta-\eta)-2(\alpha-c_0-\tau)(1-\gamma)+\phi(1-\gamma)^2]}{2\rho c_1 AS(1-\gamma)} \\ \omega_p^* &= \frac{AS\rho[(1-\gamma)^2\phi+R(\beta-\eta)]+Rc_1(1-\gamma)}{2(1-\gamma)[AS\beta\rho+c_1(1-\gamma)]} \\ \omega_s^* &= \frac{AS\rho[R(\beta+\eta)-(1-\gamma)^2\phi]+Rc_1(1-\gamma)}{2(1-\gamma)[AS\beta\rho+c_1(1-\gamma)]} \end{aligned} \tag{3}$$

Notice that it is necessary to restrict some parameter values to guarantee that the steady state water table is included between zero and its maximum level ($H^* \in [0, H_{\max}]$) and the steady state withdrawals are always positive ($\omega_p^*, \omega_s^* \geqslant 0$). The following corollary holds.

Corollary 1. *The steady state $(H^*, \omega_p^*, \omega_s^*)$ admits values if*

$$\eta \in \left[\underline{\eta}, \overline{\eta}\right]$$

$$\phi \in [\underline{\phi}, \overline{\phi}]$$

where

$$\underline{\eta} = \max\left\{ \frac{(1-\gamma)\left\{Rc_1+\rho AS[3\beta R-2(\alpha-c_0-\tau)+\phi(1-\gamma)]-2\rho ASc_0\right\}}{R\rho AS}, \frac{\rho AS[\phi(1-\gamma)^2-R\beta]-Rc_1(1-\gamma)}{AS\rho R} \right\}$$

$$\overline{\eta} = \min\left\{ \frac{Rc_1(1-\gamma)+\rho AS[3\beta R-2(\alpha-c_0-\tau)(1-\gamma)+\phi(1-\gamma)^2]}{R\rho AS}, \frac{\rho AS[\phi(1-\gamma)^2+R\beta]+Rc_1(1-\gamma)}{AS\rho R} \right\}$$

$$\underline{\phi} = \frac{Rc_1(1-\gamma)+AS\rho R\beta}{AS\rho\phi(1-\gamma)^2}$$

$$\overline{\phi} = \frac{(1-\gamma)\{Rc_1+\rho AS[3\beta R-2(\alpha-c_0-\tau)]-2\rho ASc_0\}}{\rho AS(1-\gamma)^2}.$$

Proposition 2. *The steady state $(H^*, \omega_p^*, \omega_s^*)$ is a saddle point if and only if $3\beta-\eta>0$. The optimal trajectories are:*

$$\begin{aligned} H(t) &= H^* + (H^0 - H^*)e^{rt} \\ \omega_p(t) &= \omega_p^* + c_1 e^{rt}(H^0 - H^*)\Gamma \\ \omega_s(t) &= \omega_s^* + c_1 e^{rt}(H^0 - H^*)\Theta \end{aligned} \tag{4}$$

where

$$r = -\frac{\sqrt{\Delta} - AS\rho(3\beta-\eta)+c_1(1-\gamma)}{2AS(3\beta-\eta)}$$

$$\Delta = AS^2\rho^2(3\beta-\eta)^2 + 6ASc_1\rho(3\beta-\eta)(1-\gamma)+c_1^2(1-\gamma)^2$$

$$\Gamma = \frac{(2\beta-\eta)\sqrt{\Delta}+AS\rho\eta^2-\eta[3AS\rho\beta+c_1(1-\gamma)]+2\beta c_1(1-\gamma)}{(3\beta-\eta)\{\beta\sqrt{\Delta}+3\rho AS\beta^2+\beta[7c_1(1-\gamma)-\rho AS\eta]-2c_1\eta(1-\gamma)\}}$$

$$\Theta = \frac{2AS\rho\eta(3\beta-\eta)+2\beta[c_1(1-\gamma)+\sqrt{\Delta}]}{(3\beta-\eta)\{\beta\sqrt{\Delta}-\eta[AS\beta\rho+2c_1(1-\gamma)]+\beta[7c_1(1-\gamma)+3AS\rho\beta]\}}$$

and H^0 denotes the water table initial conditions.

Fig. 1 shows the optimal trajectories using Western La Mancha data (see Table 1), widely used in the literature (see, among others, [17,18,22,32]). In Fig. 1(a) we can see the graph of the optimal trajectory of the water table from its maximum ($H_{\max} = c_0/c_1 = 665$ m using La Mancha data) to the steady state value, while Figs. 1(b) and 1(c) represent the optimal trajectories of the withdrawals of PS and ECSR firm, respectively. Finally, Fig. 1(d) shows the optimal trajectory in the phase box (H, ω_p, ω_s).

Table 1
Parameter values.

Parameters	Description	Units	Value
g	Intercept of the water demand	€/Mm^3	4400.73
k	Slop of the water demand	€/Mm^3	0.097
c_0	Intercept of the pumping cost	€/Mm^3	266 000
c_1	Slope of the pumping cost	€/Mm^3 m	400
γ	Return flow coefficient	–	0.2
AS	Aquifer area	Mm^2	126.5
R	Natural recharge	Mm^3	360
H^0	Maximum water level and initial condition	m	665
ρ	Farmers discount rate	–	0.05
τ	Water tax	€/Mm^3	2500
η	Social concern	–	0.3
ϕ	Environmental concern	–	0.15

4. Economic results

We derive now some economic implications that emerge from the analysis of the model.

Proposition 3. *At the steady state, the ECSR firm pumps always more water than the PS firm.*

This results should not be surprising due to the presence of the consumer surplus in the objective function, although counterbalanced by the internalization of the ecosystem damage.

Proposition 4. *At the steady state, we have that:*

(i) An increase of the water tax τ causes a rise of the water table while it has no effects on the withdrawals.
(ii) An increase of the social concern η generates a decrease of the water table and of the PS withdrawal, while it generates a rise of the ECSR withdrawal.
(iii) An increase of the environmental concern ϕ causes a rise of the water table and of the PS withdrawal, while it causes a decrease of the ECSR withdrawal.

As one might expect, a higher water tax preserves the aquifer level but it has no effects on pumping decisions. This happens because the taxation, as well as hydrological cost c_0 and the intercept of the inverse water demand function α, is a multiplicative constant of the withdrawals, and so, when we derive with respect to time the dynamic system (2), it disappears (see the Proof of Proposition 1 in the Appendix). Hence, τ does not affect neither the optimal trajectory nor the steady state value. In line with the ECSR literature, an increase of the social concern rises the ECSR withdrawal and decreases the PS one. Since the ECSR firm pumps more than the PS one, an increase of the social concern reduces the water table elevation. The opposite occurs if the environmental concern rises. We can derive that a balance between the two issues, social and environmental, is essential to preserve the aquifer level.

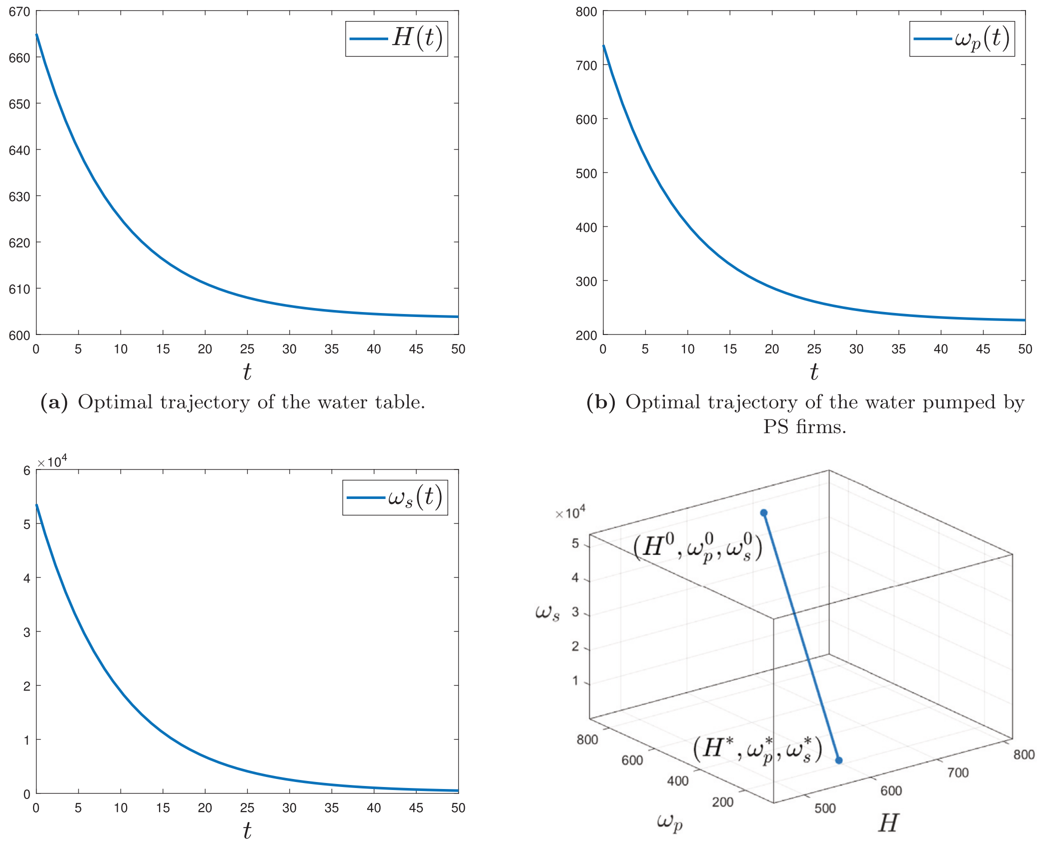
Denoting the social welfare as

$$SW = \pi_p^* + \pi_s^* + CS^* - (ED^*)^2 + (\omega_s^* + \omega_p^*)\tau \tag{5}$$

namely, as the sum of profits realized by ECSR and PS firms, the consumer surplus CS, the water tax collected by the public agency minus the quadratic ecosystem damage costs ED, the following proposition holds.

Proposition 5. *An increase of the water tax and of the environmental concern increases the Social Welfare. Conversely, an increase of the social concern decreases the Social Welfare.*

A change in the taxation increases the Social Welfare because it rises the water table reducing the pumping costs, and so the profits of both firms are higher and the ecosystem damage decreases. Analogously, a



(a) Optimal trajectory of the water table.

(b) Optimal trajectory of the water pumped by PS firms.

(c) Optimal trajectory of the water pumped by ECSR firms.

(d) Phase box.

Fig. 1. Optimal trajectories.

change in the environmental concern rises the water table because it reduces the withdrawals of the ECSR firm which is the one that pumps more water. Conversely, a change in the social concern reduces the Social Welfare because it increases the withdrawals of the ECSR firm and so the aquifer level decreases.

5. Conclusions

The environmental corporate social responsibility practice (ECSR) is recognized as one of the main strategy to manage a common aquifer. However, there are no models that investigate from a theoretical point of view the effect of a ECSR strategy on water dynamics. To fill this gap in the literature, we built a differential game between a profit seeking (PS) and a ECSR firm. The PS firm maximizes its profit under the aquifer dynamics choosing how much water to pump. Conversely, the ECSR firm maximizes its objective function composed of profit, a share of the consumer surplus, and a share of the ecosystem damage, always under the water table dynamics choosing how much water to pump.

We derive the unique open loop Nash equilibrium and the optimal trajectory that approaches asymptotically to it. From the analysis of the model, it emerges that the ECSR firm pumps always more than the PS one, in line with the theoretical economic literature that considers the CSR firm more aggressive. Computing the partial derivatives of the optimal values of the control variables and of the state variable at the steady state, we get some economic implications. For instance, the taxation has not direct effects on optimal withdrawals, although it preserves the water table. Moreover, an increase of the share of the consumer surplus reduces the water table while it rises if the share of the environmental damage increases. Analyzing the Social Welfare, it emerges that it increases at change of the water tax and of the environmental concern. However, it decreases if the social concern increases, due to a higher pressure on the water table.

We can conclude that the environmental concern plays a key role to counterbalance the negative effect of the social concern on the aquifer level and on the Social Welfare. Therefore, the Nash equilibrium obtained can be interpreted as a tool to determine regulatory aspects from a centralized perspective. In fact, the water agency, could use the water tax to increase the social welfare thanks to raise of the water table.

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CRediT authorship contribution statement

Marta Biancardi: Conceptualization, Formal analysis. **Gianluca Iannucci:** Conceptualization, Formal analysis. **Giovanni Villani:** Conceptualization, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix

Proof of Proposition 1. Applying the maximum principle, we have the following optimality conditions:

$$\frac{\partial \Omega_p}{\partial \omega_p} = \alpha - 2\beta\omega_p - \omega_s\beta - c_0 + c_1 H - \tau - \frac{\lambda_p}{AS}(1-\gamma) = 0$$
$$\frac{\partial \Omega_s}{\partial \omega_s} = \alpha - 2\beta\omega_s - \omega_p\beta - c_0 + c_1 H - \tau + \eta(\omega_s + \omega_p) - \phi(1-\gamma) - \frac{\lambda_s}{AS}(1-\gamma) = 0$$
$$\dot{\lambda}_p = \rho\lambda_p - c_1\omega_p$$
$$\dot{\lambda}_s = \rho\lambda_s - c_1\omega_s$$
$$\lim_{t\to+\infty} e^{-\rho t}\lambda_p = 0$$
$$\lim_{t\to+\infty} e^{-\rho t}\lambda_s = 0$$

From the first order conditions we obtain:

$$\omega_p = \frac{(1-\gamma)[\lambda_s\beta - \lambda_p(2\beta-\eta) + \phi AS\beta] + AS(\alpha - c_0 - \tau + c_1 H)(\beta - \eta)}{AS\beta(3\beta-\eta)}$$
$$\omega_s = \frac{(1-\gamma)[\lambda_p(\beta-\eta) - 2\lambda_s\beta - 2\phi AS\beta] + AS(\alpha - c_0 - \tau + c_1 H)(\beta + \eta)}{AS\beta(3\beta-\eta)}$$
$$\lambda_p = \frac{AS\left(\alpha - c_0 - \tau + c_1 H - 2\beta\omega_p - \beta\omega_s\right)}{1-\gamma}$$
$$\lambda_s = \frac{AS\left[\alpha - c_0 - \tau + c_1 H - \phi(1-\gamma) - (2\beta-\eta)\omega_s - (\beta-\eta)\omega_p\right]}{1-\gamma}$$

Deriving them with respect to time, we get:

$$\dot{\omega}_p = \frac{(1-\gamma)[\dot{\lambda}_s\beta - \dot{\lambda}_p(2\beta-\eta)] + ASc_1(\beta-\eta)\dot{H}}{AS\beta(3\beta-\eta)}$$
$$\dot{\omega}_s = \frac{(1-\gamma)[\dot{\lambda}_p(\beta-\eta) - 2\beta\dot{\lambda}_s] + ASc_1(\beta+\eta)\dot{H}}{AS\beta(3\beta-\eta)}$$

Substituting the value of $\dot{H}$, $\dot{\lambda}_p$, $\dot{\lambda}_s$, we get the dynamic system (2). Solving it, we obtain the values of (3). □

Proof of Corollary 1. It occurs $H^* \geqslant 0$ if

$$\eta \leqslant \frac{Rc_1(1-\gamma) + \rho AS[3\beta R - 2(\alpha - c_0 - \tau)(1-\gamma) + \phi(1-\gamma)^2]}{R\rho AS}$$

while $H^* \leqslant H_{\max}$ if

$$\eta \geqslant \frac{(1-\gamma)\left\{Rc_1 + \rho AS[3\beta R - 2(\alpha - c_0 - \tau) + \phi(1-\gamma)] - 2\rho ASc_0\right\}}{R\rho AS}$$

Notice that
$$\frac{(1-\gamma)\left\{Rc_1 + \rho AS[3\beta R - 2(\alpha - c_0 - \tau) + \phi(1-\gamma)] - 2\rho ASc_0\right\}}{R\rho AS} \geqslant 0 \text{ if}$$
$$\phi \leqslant \frac{Rc_1(1-\gamma) + AS\rho R\beta}{AS\rho\phi(1-\gamma)^2}$$

Conversely, it occurs that $\omega_p^* \geqslant 0$ if

$$\eta \leqslant \frac{\rho AS[\phi(1-\gamma)^2 + R\beta] + Rc_1(1-\gamma)}{AS\rho R}$$

while $\omega_s^* \geqslant 0$ if

$$\eta \geqslant \frac{\rho AS[\phi(1-\gamma)^2 - R\beta] - Rc_1(1-\gamma)}{AS\rho R}$$

Notice that $\frac{\rho AS[\phi(1-\gamma)^2 - R\beta] - Rc_1(1-\gamma)}{AS\rho R} \geqslant 0$ if

$$\phi \geqslant= \frac{(1-\gamma)\{Rc_1 + \rho AS[3\beta R - 2(\alpha - c_0 - \tau)] - 2\rho ASc_0\}}{\rho AS(1-\gamma)^2}$$

Therefore, $H^* \in [0, H_{\max}]$, $\omega_p^* \geqslant 0$, $\omega_s^* \geqslant 0$ if $\eta \in [\underline{\eta}, \overline{\eta}]$ and $\beta \leqslant \overline{\beta}$, where

$$\underline{\eta} = \max\left\{\frac{(1-\gamma)\left\{Rc_1 + \rho AS[3\beta R - 2(\alpha - c_0 - \tau) + \phi(1-\gamma)] - 2\rho ASc_0\right\}}{R\rho AS}, \frac{\rho AS[\phi(1-\gamma)^2 - R\beta] - Rc_1(1-\gamma)}{AS\rho R}\right\}$$
$$\overline{\eta} = \min\left\{\frac{Rc_1(1-\gamma) + \rho AS[3\beta R - 2(\alpha - c_0 - \tau)(1-\gamma) + \phi(1-\gamma)^2]}{R\rho AS}, \frac{\rho AS[\phi(1-\gamma)^2 + R\beta] + Rc_1(1-\gamma)}{AS\rho R}\right\}$$
$$\underline{\phi} = \frac{Rc_1(1-\gamma) + AS\rho R\beta}{AS\rho\phi(1-\gamma)^2}$$
$$\overline{\phi} = \frac{(1-\gamma)\{Rc_1 + \rho AS[3\beta R - 2(\alpha - c_0 - \tau)] - 2\rho ASc_0\}}{\rho AS(1-\gamma)^2}.$$

The proof is concluded. □

Proof of Proposition 2. The Jacobian matrix of the system (2): J is given in Box I.
The determinant is:

$$Det(J) = -\frac{2c_1\rho(1-\gamma)[c_1(1-\gamma) + \rho\beta AS]}{\beta AS^2(3\beta-\eta)}$$

that is negative if and only if $3\beta - \eta > 0$. Moreover, the trajectories (4) converge to the steady state only if $r < 0$, that holds only if $\Delta > 0$, that in turns is occurs only if $3\beta - \eta > 0$. □

Proof of Proposition 3. From (3) it emerges that $\omega_s^* > \omega_p^*$ if $\eta > \hat{\eta}$, where

$$\hat{\eta} = \frac{(1-\gamma)^2\phi}{R}$$

Notice that $\hat{\eta} < \underline{\eta}$ if

$$\phi > \frac{-R[AS\rho\beta + c_1(1-\gamma)]}{AS\rho(1-\gamma)^2}$$

that it is always true. □

Proof of Proposition 4. The first order partial derivatives are:

$$(i)\quad \frac{\partial H^*}{\partial \tau} = \frac{1}{c_1} > 0, \quad \frac{\partial \omega_p^*}{\partial \tau} = 0, \quad \frac{\partial \omega_s^*}{\partial \tau} = 0.$$
$$(ii)\quad \frac{\partial H^*}{\partial \eta} = -\frac{R}{2c_1(1-\gamma)} < 0,$$
$$\frac{\partial \omega_p^*}{\partial \eta} = -\frac{\eta R\rho AS}{2(1-\gamma)[AS\beta\rho + c_1(1-\gamma)]} < 0,$$
$$\frac{\partial \omega_s^*}{\partial \eta} = \frac{R\rho AS}{2(1-\gamma)[AS\beta\rho + c_1(1-\gamma)]} > 0.$$
$$(iii)\quad \frac{\partial H^*}{\partial \phi} = \frac{(1-\gamma)}{2c_1} > 0, \quad \frac{\partial \omega_p^*}{\partial \phi} = \frac{\rho AS(1-\gamma)}{2[AS\beta\rho + c_1(1-\gamma)]} > 0,$$
$$\frac{\partial \omega_s^*}{\partial \phi} = -\frac{\rho(1-\gamma)AS}{2[AS\rho\beta + c_1(1-\gamma)]} < 0.$$

The proof is concluded. □

Proof of Proposition 5. The value of SW computed at the equilibrium $(H^*, \omega_p^*, \omega_s^*)$ is:

$$SW^* = \frac{R\{Rc_1(1-\gamma) + \rho AS[R(\beta + 1 - \eta) + (1-\gamma)(2\tau + \phi(1-\gamma))]\}}{2AS\rho(1-\gamma)^2}$$

With the following first order partial derivatives:

$$\frac{\partial SW^*}{\partial \tau} = \frac{R}{1-\gamma} > 0; \quad \frac{\partial SW^*}{\partial \eta} = -\frac{R^2}{2(1-\gamma)^2} < 0; \quad \frac{\partial SW^*}{\partial \phi} = \frac{R}{2} > 0$$

The proof is concluded. □

$$J = \begin{bmatrix} \frac{\rho AS(3\beta-\eta)+c_1(1-\gamma)}{AS(3\beta-\eta)} & -\frac{c_1(1-\gamma)(2\beta-\eta)}{AS\beta(3\beta-\eta)} & -\frac{\rho c_1(\beta-\eta)}{\beta(3\beta-\eta)} \\ -\frac{2c_1(1-\gamma)}{AS(3\beta-\eta)} & \frac{3\rho\beta^2 AS-\eta c_1(1-\gamma)+\beta[c_1(1-\gamma)-\rho\eta AS]}{AS\beta(3\beta-\eta)} & -\frac{\rho c_1(\beta+\eta)}{\beta(3\beta-\eta)} \\ -\frac{1-\gamma}{AS} & -\frac{1-\gamma}{AS} & 0 \end{bmatrix}$$

Box I.

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