SeMA Journal
https://doi.org/10.1007/s40324-026-00443-w

Modern weather forecasting: mathematical foundations, tools and current state of the art

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Received: 30 December 2025 / Accepted: 19 August 2026

Abstract
Modern weather forecasting relies on the integration of observational systems, numerical modeling, data assimilation, high-performance computing, and increasingly artificial intelligence techniques. This paper reviews the scientific and technological foundations of contemporary weather prediction, with particular emphasis on the mathematical and computational tools that support operational forecasting. Weather prediction is based on the numerical solution of the nonlinear partial differential equations governing atmospheric dynamics. The accuracy of these forecasts critically depends on the availability of observations collected from heterogeneous sources, including ground-based networks, weather radars, satellites, aircraft, and other remote sensing platforms. These data are combined with model forecasts through advanced data assimilation techniques, which provide dynamically consistent estimates of the atmospheric state and improve forecast quality. Special attention is devoted to nowcasting, one of the most challenging areas of modern meteorology. At lead times of a few minutes to several hours, rapidly evolving phenomena such as severe thunderstorms, hailstorms, and flash floods require the integration of high-frequency observations, radar extrapolation methods, ensemble prediction techniques, and machine learning approaches. Artificial intelligence is increasingly used to enhance the detection, tracking, and short-term prediction of convective systems, complementing traditional physics-based methodologies. The paper also discusses the requirements of a modern national forecasting system and presents the high-resolution ICON-2I modeling framework operational at the ItaliaMeteo Agency. Particular emphasis is placed on the role of high-performance computing infrastructures, ensemble forecasting systems, and kilometer-scale data assimilation methods. Finally, current challenges and future developments are examined, including convective-scale prediction, uncertainty quantification, exascale computing, digital twins of the atmosphere, and the growing integration of artificial intelligence with numerical weather prediction.

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Published online: 02 September 2026

Keywords Weather forecasting · Numerical weather prediction · Nowcasting · Data assimilation · Ensemble forecasting · High performance computing · Artificial intelligence · Machine learning · Atmospheric modeling · Probabilistic forecasting · Severe weather events

Mathematics Subject Classification 86A10 · 76U05 · 65M06 · 65N06

Contents

1 Weather forecasting: historical evolution, current state of the art, and future perspectives

Predicting whether precipitation will occur at a given location, how much rainfall may accumulate, what temperatures will be reached, or more generally what weather conditions will prevail anywhere on Earth requires the ability to understand and forecast the evolution of atmospheric processes. These processes include the motion of air masses, radiative interactions between the atmosphere and incoming solar energy, cloud formation and evolution, exchanges between the atmosphere and land or ocean surfaces, and the physical mechanisms that lead to precipitation through condensation and the growth of raindrops, snowflakes, or hailstones. Consequently, forecasting surface weather conditions is inseparable from predicting the complex physical dynamics governing the atmosphere as a whole.

Many of these processes are strongly nonlinear and not yet fully understood in all their details, making accurate forecasts challenging even several days—and in some cases only a few hours—in advance. Modern weather forecasting relies on a common set of core components: atmospheric observations (including measurements of temperature, humidity, wind, and pressure), numerical forecast models, and post-processing techniques such as statistical correction and verification. The relative importance of these components and the way they are combined depend strongly on the forecast time range. Figure 1 schematically illustrates the meteorological forecasting chain, showing how different tools dominate at different forecast horizons.

Forecast skill also depends on the spatial and temporal scale of the phenomenon: as a general rule, smaller and shorter-lived features are more difficult to predict, even when using very high-resolution models.

For very short-term forecasts on time scales of approximately 1–3 h, commonly referred to as *nowcasting*, observations play a dominant role. Rapid data acquisition, fast data transmission, optimal integration of multiple observing systems, and short-term extrapolation

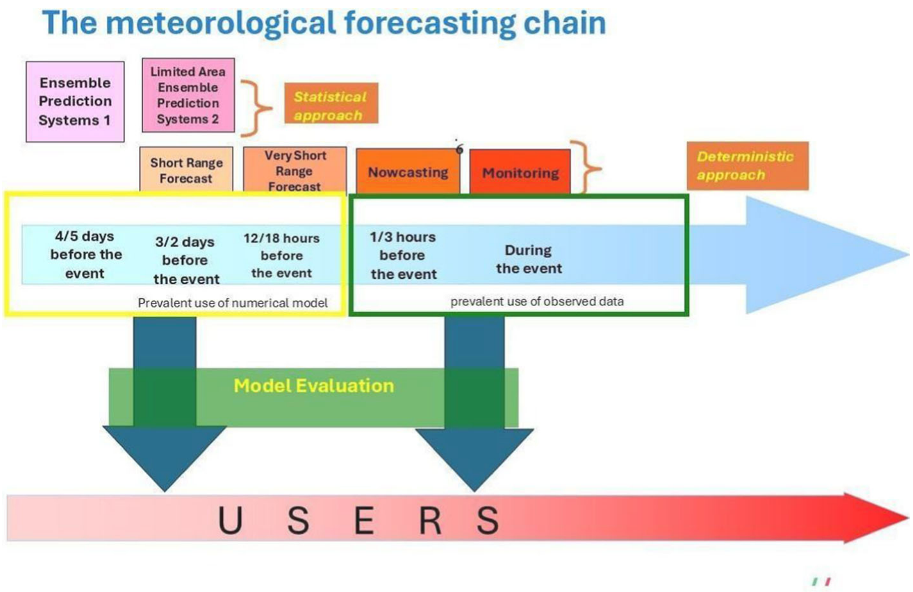

Fig. 1 The meteorological forecasting chain. Figure courtesy of Tiziana Paccagnella

techniques are the key elements of this approach [37]. Beyond this range, direct extrapolation becomes ineffective, and forecasting must rely on physical knowledge of how atmospheric processes evolve in time and space.

This physical knowledge is expressed through the fundamental laws governing fluid motion, since the atmosphere behaves as a compressible, rotating fluid. These laws can be written as systems of highly nonlinear partial differential equations. Following the formulation presented by Kasahara [21], these laws can be written in Cartesian coordinates (x, y, z) (eastward, northward, upward) and time t as:

$$\frac{\partial u}{\partial t} = -\mathbf{V} \cdot \nabla u - w \frac{\partial u}{\partial z} + f v - f' w - \frac{1}{\rho} \frac{\partial p}{\partial x} + F_x, \tag{1.1}$$

$$\frac{\partial v}{\partial t} = -\mathbf{V} \cdot \nabla v - w \frac{\partial v}{\partial z} - f u - \frac{1}{\rho} \frac{\partial p}{\partial y} + F_y, \tag{1.2}$$

$$\frac{\partial w}{\partial t} = -\mathbf{V} \cdot \nabla w - w \frac{\partial w}{\partial z} + f' u - g - \frac{1}{\rho} \frac{\partial p}{\partial z} + F_z, \tag{1.3}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{V}) - \frac{\partial (\rho w)}{\partial z}, \tag{1.4}$$

$$\frac{\partial s}{\partial t} = -\mathbf{V} \cdot \nabla s - w \frac{\partial s}{\partial z} + \frac{Q}{T}, \tag{1.5}$$

$$s = C_v \ln p - C_p \ln \rho, \tag{1.6}$$

$$T = \frac{p}{\rho R}. \tag{1.7}$$

Here u, v, w are the velocity components in the x, y, z directions, respectively, and $\mathbf{V} = (u, v)$ denotes the horizontal velocity vector. The operator ∇ is the horizontal gradient,

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right), \qquad \mathbf{V} \cdot \nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}.$$

The scalars ρ, p, T, and s denote density, pressure, temperature, and specific entropy. The terms F_x, F_y, F_z are the components of frictional (or subgrid-scale) forcing per unit mass, Q is the diabatic heating rate per unit mass, g is gravitational acceleration, C_p and C_v are the specific heats at constant pressure and volume, and $R = C_p - C_v$ is the gas constant for dry air. The Coriolis parameters are defined by

$$f = 2\Omega \sin\varphi, \qquad f' = 2\Omega \cos\varphi,$$

where Ω is the Earth's rotation rate and φ is latitude.

Because these equations cannot be solved analytically, they must be integrated numerically using powerful computers. The resulting systems constitute the basis of Numerical Weather Prediction (NWP) models, which may be either *global models* (GM), covering the entire Earth, or *limited-area models* (LAM), focusing on specific regions. See, respectively, Tables 1 and 2.

In operational forecasting, global models are typically employed for medium-range predictions (beyond approximately three days), while limited-area models are used for shorter forecast ranges and operate at higher spatial resolution. The enhanced resolution of LAM allows a more detailed representation of small-scale atmospheric features. However, unlike global models, limited-area models are not self-contained and require lateral boundary conditions along the edges of their computational domains. Any errors introduced through these boundary conditions can rapidly propagate inward and degrade the forecast quality. For this reason, regional models are generally nested within coarser-resolution global or regional models that provide the necessary boundary information.

Operational NWP forecasts are produced routinely and in real time, under strict time and reliability constraints. This operational framework is essential to ensure timely daily

Table 1 Some examples of global model

Organization	Model name (acronym)	Spatial resolution	Vertical levels	Temporal resolution
ECMWF	IFS	9 km	137 levels	Every 6 h
NCEP	GFS	13 km	127 levels	Every 6 h
MeteoFrance	ARPEGE	12 km	105 levels	Every 6 h
DWD	ICON	13 km	90 levels	Every 3 h

Table 2 Some example of LAM models

Organization	Model name	Spatial resolution	Vertical levels
MeteoFrance	AROME	1.3 km	60 levels
DWD	ICON-D2	2.2 km	65 levels
UK Met Office	UKV	1.5 km	70 levels
ARPAE-SIMC	COSMO-2I	2.2 km	65 levels

forecasts, enable rapid response during severe weather events, maintain continuity and consistency of forecasting services, and support decision-making in sectors such as agriculture, transportation, energy production, and civil protection.

The key components of an operational NWP system are:

- **Observations**: acquisition from heterogeneous sources (surface stations, satellites, radars, aircraft, marine platforms), followed by preprocessing and quality control;
- **Data assimilation (DA)**: optimal merging of observations with model information to generate initial conditions;
- **Numerical models (NM)**: time integration of the governing equations starting from the analyzed initial state;
- **Computing infrastructure**: High Performance Computing (HPC) systems enabling real-time simulations and data storage;
- **Forecast production**: automation, post-processing, and verification;
- **Dissemination**: data distribution platforms, visualization tools, and real-time access for users;
- **Applications**: downstream models (e.g., air quality, hydrology, oceanography) and decision-support systems.

An NWP model can be viewed as a numerical simulator of atmospheric evolution. It is grounded in the ability to observe the atmosphere, to develop a conceptual understanding of its behavior, and to translate that understanding into mathematical equations based on physical laws. Once an atmospheric state has been defined, the model uses it as the initial condition for time integration. Determining this initial condition is a complex and computationally expensive process that relies heavily on observational data.

Because the atmosphere is a highly complex system involving strong interactions with the Earth's surface, its governing equations must be discretized before they can be solved numerically. This discretization introduces approximations and, inevitably, errors. Meteorological variables such as temperature, wind, and humidity are represented on a three-dimensional grid that may cover the entire globe or a limited region. This reflects an intrinsic trade-off between spatial resolution, computational cost, and forecast accuracy [20].

The forecasting procedure can be illustrated using the Integrated Forecasting System (IFS) of the European Centre for Medium-Range Weather Forecasts (ECMWF), one of the most advanced global NWP models currently in operation [9]. The first step consists of collecting all available observational data into a centralized database. These data originate from surface stations, radiosondes, satellites, ships, aircraft, radar networks, and GNSS-based systems. After transmission through international communication networks, observations undergo preprocessing and quality control to ensure consistency and reliability.

This processed dataset is then used to construct a physically consistent estimate of the atmospheric state through data assimilation. Historically referred to as *objective analysis*, this step is now recognized as a cornerstone of modern NWP. Data assimilation combines observations with a short-range model forecast (the *background* or *first guess*) within a unified mathematical framework [4]. Because observations may disturb the dynamical balance of the model, they must be incorporated carefully to avoid introducing spurious oscillations or shocks. Figure 2 illustrates the modern data assimilation (DA) process.

The third major phase of the forecasting chain is the numerical integration of the governing equations. This step requires immense computational resources and explains why operational NWP became feasible only with the advent of electronic computers in the mid-20th century [6, 26]. Since then, forecast quality has steadily improved, largely due to advances in computing power and model resolution.

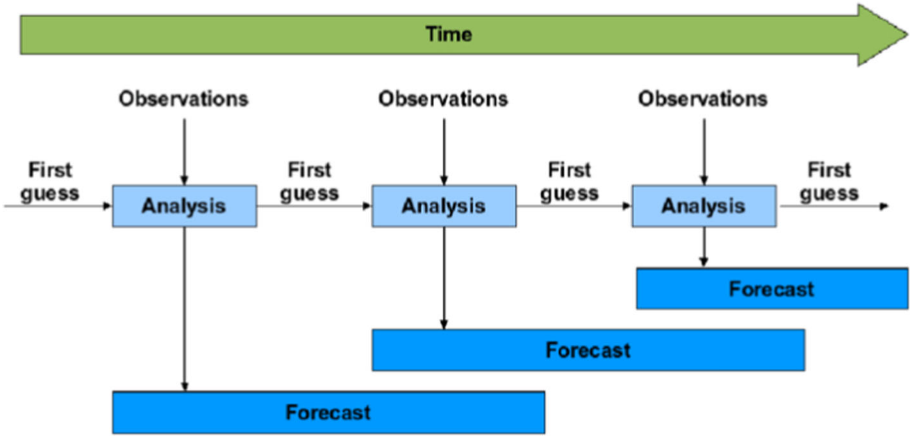

Fig. 2 The process of data assimilation, which consists of a consistent merge between observed data and the numerical model simulating the atmosphere

Only a few decades ago, global models operated with horizontal grid spacings of several hundred kilometers. Today, ECMWF's operational global model runs at a horizontal resolution of approximately 9 km, allowing a much finer representation of atmospheric processes. Increased resolution reduces representativeness errors by making grid cells more homogeneous and enables explicit simulation of a wider range of physical phenomena.

Processes occurring at spatial scales smaller than the grid spacing cannot be explicitly resolved by numerical weather prediction (NWP) models and must instead be represented through physical parameterizations. During a simulation, the model computes the temporal evolution of meteorological variables—such as temperature, wind, humidity, and precipitation—only at the discrete grid points. Consequently, all atmospheric and surface processes occurring within a single grid cell are represented by a single set of prognostic variables.

If the horizontal resolution is relatively coarse (for example, on the order of 100 km), each grid cell may encompass terrain and surface characteristics that are highly heterogeneous in reality, including complex orography, land–sea contrasts, urban areas, and diverse vegetation types. The model, however, is forced to average these features into a single representation. When forecasts are compared with observations at specific locations within such a grid cell, substantial discrepancies may therefore arise, not necessarily because the model physics are incorrect, but because the resolved scale is too coarse to capture local variability.

As spatial resolution increases, each grid cell represents a smaller and more internally homogeneous portion of the Earth's surface. This allows the model to describe atmospheric structures, surface properties, and their interactions with greater realism. In the limit of increasing resolution, the simulated atmospheric fields progressively converge toward a representation that more closely resembles the real atmosphere. Higher resolution also enables a larger fraction of atmospheric processes—such as deep convection, orographic precipitation, and mesoscale circulations—to be explicitly resolved rather than parameterized.

Nevertheless, any processes occurring at scales smaller than the grid spacing remain unresolved and must still be treated through parameterization schemes that approximate their net effect on the resolved flow. Reducing grid spacing therefore decreases, but does not eliminate, the reliance on parameterizations. This reduction generally leads to improved forecast realism and accuracy, provided that the underlying physical schemes and numerical methods are appropriate for the chosen resolution.

An example of such high-resolution operational forecasting is provided by the ICON limited-area model operated by the ItaliaMeteo Agency, which runs at a horizontal resolution of approximately 2.2 km over the Italian territory. Simulations at this scale require substantial computational resources and are supported by the high-performance computing infrastructure of the CINECA supercomputing center.

Despite these technological and scientific advances, forecast errors remain unavoidable because the atmosphere is a chaotic dynamical system, highly sensitive to initial conditions [23–25]. Even very small inaccuracies in the specification of the initial atmospheric state can grow rapidly during the model integration, ultimately limiting deterministic predictability. This intrinsic sensitivity places a fundamental constraint on the accuracy of single deterministic forecasts, particularly at small spatial and temporal scales.

Recognition of this limitation has led to the development of ensemble forecasting, in which multiple model simulations are performed to sample uncertainties associated with initial conditions, model physics, and numerical approximations [22]. Ensemble forecasting represents a conceptual shift from deterministic prediction toward probabilistic guidance. Rather than producing a single forecast scenario, the model generates a range of plausible outcomes, allowing forecasters to quantify forecast uncertainty and assess weather-related risk.

From a user perspective, this approach requires a change in mindset: deterministic statements are replaced by probabilistic ones, in which forecast information must be interpreted in terms of likelihoods and potential impacts. Such probabilistic guidance is particularly valuable for decision-making in weather-sensitive applications, where cost–loss considerations play a crucial role.

Looking ahead, future developments in numerical weather prediction are expected to focus on the more effective exploitation of observational data, including advanced satellite sensors, radar measurements, and high-frequency monitoring networks. Continued increases in spatial resolution, improved representation of surface–atmosphere interactions, and wider adoption of probabilistic forecasting frameworks are likely to further enhance forecast quality and reliability.

At the same time, weather forecasting is becoming increasingly important for sectors that depend strongly on atmospheric conditions. Beyond its traditional role in civil protection and risk management, accurate forecasts are now essential for transportation, agriculture, water-resource management, and especially renewable energy production. Reliable predictions of solar radiation, cloud cover, wind speed, and precipitation allow operators to estimate electricity generation from photovoltaic and wind power systems, improving grid management and the efficiency of energy markets.

These developments illustrate how modern weather forecasting has evolved into a strategic scientific and technological infrastructure whose value extends far beyond meteorology itself.

2 The role of monitoring data

In recent decades, the availability and diversity of meteorological observations have increased dramatically, driven by advances in sensor technology, telecommunications, and data processing. Modern numerical weather prediction (NWP) systems rely on a heterogeneous set of monitoring data, including conventional ground-based and upper-air networks, weather radar systems, satellite remote sensing platforms, and specialized observational networks such as lightning detection systems. These data sources provide complementary information across a wide range of spatial and temporal scales and play a fundamental role in constraining atmospheric models.

From a mathematical perspective, observations enter the forecasting system primarily through data assimilation, which can be formulated as an inverse problem. Let $\mathbf{x}(t) \in \mathbb{R}^n$ denote the (discretized) atmospheric state vector at time t, evolving according to a nonlinear dynamical system

$$\frac{d\mathbf{x}}{dt} = \mathcal{M}(\mathbf{x}), \tag{2.1}$$

where $\mathcal{M}$ represents the numerical model. Observations $\mathbf{y}_k \in \mathbb{R}^{m_k}$, acquired at times t_k, are related to the true state through an observation operator $\mathcal{H}_k$,

$$\mathbf{y}_k = \mathcal{H}_k(\mathbf{x}(t_k)) + \boldsymbol{\varepsilon}_k, \tag{2.2}$$

where $\boldsymbol{\varepsilon}_k$ denotes observation errors, typically modeled as random variables with known or estimated covariance matrices $\mathbf{R}_k$.

The integration of observations from multiple monitoring systems therefore requires accurate modeling of the operators $\mathcal{H}_k$ and of the associated error statistics. Effective data management infrastructures are essential to ensure redundancy, traceability, secure dissemination, and visualization of these data streams. In recent years, machine learning and artificial intelligence techniques have been increasingly employed to assist in quality control, bias correction, and data fusion, complementing traditional statistical approaches.

3 Ground-based observations

Ground-based meteorological networks provide direct measurements of near-surface atmospheric variables and constitute the backbone of real-time weather monitoring and long-term climate analysis. Typical observations include temperature, humidity, pressure, wind, and precipitation, which can be interpreted as time series $\{y(t_i)\}_{i=1}^{N}$ and analyzed using statistical and stochastic methods for trend detection and variability assessment.

Only a subset of these observations is disseminated internationally via the World Meteorological Organization (WMO) Global Telecommunication System (GTS), currently evolving into the WMO Information System (WIS). Stations contributing to these international datasets must satisfy strict WMO standards regarding instrumentation, siting, calibration, and data archiving [38]. These standards are crucial to ensure homogeneity and comparability of observations across space and time.

In addition to GTS data, a large volume of non-GTS observations remains within national and regional networks. In Italy, these networks are exceptionally dense, comprising over four thousand stations and more than ten thousand sensors. While many stations do not fully comply with all WMO specifications—often measuring a limited set of variables—their spatial density provides valuable high-resolution information. Data from these networks are collected in near real time by centralized systems managed by the Administrative Regions, the Department of Civil Protection, and the ItaliaMeteo Agency.

From a data assimilation standpoint, the high density of ground observations introduces challenges related to representativeness error and spatial correlation of observation errors. These issues are typically addressed by introducing appropriate observation error covariance models or through thinning and superobbing techniques [2, 7]. Amateur networks, such as MeteoNetwork, further enrich the observational dataset and, when properly quality-controlled, can significantly enhance the spatial characterization of meteorological fields.

4 Satellite observations

Satellite remote sensing provides global and quasi-continuous coverage of the atmosphere and the Earth's surface, making it indispensable for modern meteorology and climatology. Satellite instruments measure electromagnetic radiation across visible, infrared, and microwave spectral bands, yielding indirect observations of atmospheric and surface variables.

Mathematically, satellite observations are characterized by highly nonlinear observation operators $\mathcal{H}$ that map model variables (e.g., temperature and humidity profiles) to radiances. These operators typically involve solutions of the radiative transfer equation,

$$I_\nu(\infty)e^{-\tau_\nu(\infty)} + \int_0^\infty B_\nu(T(s)), e^{-\tau_\nu(s)} \frac{d\tau_\nu}{ds}, ds, \tag{4.1}$$

where I_ν is the observed radiance at frequency ν, B_ν is the Planck function, and τ_ν denotes optical depth [32]. For simplicity, in many meteorological applications this term is neglected or prescribed through suitable boundary conditions.

Italy is continuously monitored by both geostationary and polar-orbiting satellites operated by EUMETSAT, ESA, NOAA, NASA, JAXA, and other agencies. These platforms provide information on cloud properties, precipitation, wind fields, atmospheric composition, surface temperature, sea state, and cryospheric variables. Satellite data are operationally assimilated into NWP models using variational and ensemble-based methods and are a dominant contributor to forecast skill at medium and global scales [3, 20].

5 Radar observations

Weather radar systems provide high-resolution observations of precipitation and cloud structures, with temporal resolutions on the order of minutes and spatial resolutions down to hundreds of meters. Radar measurements are inherently indirect, relating observed reflectivity Z to rainfall rate R through empirical relationships of the form

$$Z = aR^b, \tag{5.1}$$

where the coefficients a and b depend on the microphysical characteristics of hydrometeors [27]. This dependence introduces uncertainty and nonlinearity into the observation operator.

In Italy, the national radar network includes C-band and X-band systems managed by regional agencies, ENAV, the Italian Air Force, and the Department of Civil Protection. Radar data are combined into national composite products and play a crucial role in monitoring severe weather, issuing warnings for hydrogeological and hydraulic risks, and supporting agricultural applications.

From a modeling perspective, radar observations are particularly valuable for high-resolution data assimilation, where they help constrain convective-scale dynamics. Assimilating radar reflectivity and radial velocity requires specialized techniques, including nonlinear filtering and ensemble-based approaches, to account for strong non-Gaussian error characteristics [10, 36].

Note that the integration of ground-based, satellite, radar, and specialized observations represents a high-dimensional data fusion problem, combining heterogeneous data with different error structures, spatial footprints, and temporal sampling. Ensemble-based data assimilation methods provide a mathematically consistent framework for addressing these challenges by explicitly representing uncertainty through probability distributions [22].

Continued progress in monitoring technologies, observation operators, and assimilation methodologies is expected to further enhance forecast accuracy, particularly at high spatial resolution. However, these advances also demand sustained investment in computational resources, theoretical research, and the development of robust data infrastructures capable of handling increasingly complex observational datasets.

6 Nowcasting: the major challenge of modern weather forecasting

Among all components of modern weather forecasting, nowcasting is arguably the most challenging one. While medium- and long-range forecasts have benefited enormously from advances in numerical weather prediction (NWP), high-performance computing, and data assimilation techniques, the prediction of atmospheric phenomena at very short lead times remains an open scientific problem. Severe thunderstorms, hailstorms, flash floods, wind gusts, and convective precipitation systems evolve on spatial scales of only a few kilometers and temporal scales of tens of minutes, making their prediction extremely difficult. In this context, nowcasting refers to the forecasting of weather conditions from a few minutes up to approximately 1–3 h ahead. At these lead times, observational systems often contain more useful predictive information than numerical models themselves. Consequently, nowcasting can be regarded as a bridge between real-time monitoring and numerical weather prediction, combining observational data, mathematical modeling, statistical inference, and increasingly artificial intelligence techniques [31, 37]. From a mathematical perspective, nowcasting can be formulated as a short-term state estimation problem. Let

$$X(t) \in \mathbb{R}^n$$

denote the atmospheric state vector at time t, while

$$Y(t)$$

represents the set of available observations collected from radar systems, satellites, lightning detection networks, GNSS sensors, aircraft reports, and ground stations. The forecasting problem consists of estimating the future atmospheric state

$$X(t + \Delta t)$$

or, more generally, the conditional probability distribution

$$P\Big(X(t + \Delta t) \mid Y(s),\ s \leqslant t\Big),$$

using all available observations up to the current time. The most widely used operational nowcasting systems are based on the extrapolation of radar observations. Let

$$R(x, y, t)$$

denote radar reflectivity or precipitation intensity. Assuming that precipitation structures are transported by an underlying velocity field

$$\mathbf{v}(x, y, t) = (u(x, y, t), v(x, y, t)),$$

their evolution can be approximated by the advection equation

$$\frac{\partial R}{\partial t} + \mathbf{v} \cdot \nabla R = 0. \tag{6.1}$$

The corresponding characteristic equations are

$$\frac{dx}{dt} = u, \qquad \frac{dy}{dt} = v, \tag{6.2}$$

implying that reflectivity remains approximately constant along particle trajectories,

$$\frac{dR}{dt} = 0.$$

This approach forms the basis of many operational radar nowcasting systems currently used by meteorological services worldwide. Operational radar nowcasting systems generally rely on two classes of extrapolation techniques. The first class consists of Eulerian optical-flow methods, which estimate the motion field directly from consecutive radar images by assuming conservation of radar reflectivity. These techniques treat radar observations as a sequence of images and compute displacement vectors that are projected forward in time, typically providing useful forecasts up to 2–3 h ahead [5, 18].

A second class consists of Lagrangian cell-tracking methods, in which individual convective cells are identified as coherent objects and followed through time. Representative examples are TITAN (Thunderstorm Identification, Tracking, Analysis and Nowcasting) and SCIT (Storm Cell Identification and Tracking). These systems estimate cell velocity, growth, decay, volume, and intensity, allowing forecasts not only of displacement but also of the potential evolution of severe thunderstorms [8, 19].

In operational practice, many modern nowcasting systems combine optical-flow advection with cell-based tracking algorithms in order to improve forecasts of rapidly evolving convective phenomena. However, precipitation systems do not simply move. They continuously grow, decay, merge, and split as a consequence of nonlinear atmospheric dynamics. A more realistic model therefore includes source and sink terms,

$$\frac{\partial R}{\partial t} + \mathbf{v} \cdot \nabla R = S(R, x, y, t), \tag{6.3}$$

where S represents unresolved microphysical and convective processes. Estimating the velocity field itself constitutes a nontrivial inverse problem. Modern systems frequently employ optical-flow techniques. Assuming brightness conservation between two consecutive radar images,

$$R(x, y, t) = R(x + \delta x, y + \delta y, t + \delta t),$$

a first-order Taylor expansion yields

$$R_t + uR_x + vR_y = 0. \tag{6.4}$$

Since this equation alone is insufficient to determine both velocity components, regularization is introduced through the minimization of the variational functional

$$J(u, v) = \int_\Omega (R_t + uR_x + vR_y)^2 \, d\Omega + \lambda \int_\Omega (|\nabla u|^2 + |\nabla v|^2) \, d\Omega. \tag{6.5}$$

The corresponding Euler–Lagrange equations provide a smooth estimate of the motion field and constitute the mathematical foundation of many contemporary tracking algorithms [18]. A further difficulty arises from the chaotic nature of atmospheric dynamics. Let $\delta X(t)$ denote a perturbation of the atmospheric state. Following the classical analysis of Lorenz, its growth can be approximated by

$$\|\delta X(t)\| \approx \|\delta X(0)\| e^{\lambda t}, \tag{6.6}$$

where λ is the dominant Lyapunov exponent. For convective-scale phenomena, the associated predictability horizon may be only one or two hours [23, 24]. For this reason, modern nowcasting increasingly adopts probabilistic methodologies. Instead of producing a single deterministic forecast, one generates an ensemble

$$\left\{X^{(1)}, X^{(2)}, \ldots, X^{(N)}\right\},$$

whose mean

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X^{(i)}$$

provides the best estimate, while the ensemble variance

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(X^{(i)} - \bar{X})^2$$

quantifies forecast uncertainty. From a Bayesian perspective, the problem can be formulated as

$$p(X_k|Y_{1:k}) \propto p(Y_k|X_k)p(X_k|Y_{1:k-1}), \tag{6.7}$$

which naturally connects nowcasting with the data assimilation techniques discussed in the following sections. More recently, artificial intelligence and machine learning techniques have become an important component of modern nowcasting systems. Deep neural networks, recurrent neural networks, convolutional neural networks, transformers, and neural operators are increasingly used to learn directly the evolution operator of radar and satellite fields. Rather than solving explicitly a system of partial differential equations,

$$\frac{\partial X}{\partial t} = \mathcal{F}(X),$$

one seeks a nonlinear operator

$$\mathcal{G}_\theta$$

such that

$$X(t + \Delta t) = \mathcal{G}_\theta(X(t)). \tag{6.8}$$

These approaches have demonstrated remarkable skill in the prediction of convective precipitation and rapidly evolving weather systems [29, 30, 35]. Current operational systems increasingly combine observational extrapolation, ensemble prediction, data assimilation, and numerical weather prediction into a unified forecasting framework. One of the earliest and most influential examples of this philosophy was the NIMROD system developed by the UK Met Office, which demonstrated the operational benefits of blending radar-based nowcasting with numerical weather prediction guidance [16]. A schematic representation of such hybrid systems is

$$F(t + \Delta t) = \alpha(\Delta t)F_{\mathrm{obs}} + (1 - \alpha(\Delta t))\, F_{\mathrm{NWP}}, \tag{6.9}$$

where the weighting coefficient $\alpha(\Delta t)$ decreases as forecast lead time increases. Such hybrid approaches are currently regarded as the most promising pathway toward seamless forecasting, enabling a smooth transition from observation-driven nowcasting to physics-based numerical prediction. The future of nowcasting is expected to rely on the integration of high-frequency radar and satellite observations, advanced data assimilation methods, ensemble prediction systems, digital twins of the atmosphere, and artificial intelligence. Despite remarkable progress over the last decade, nowcasting remains one of the most active and challenging research areas in modern meteorology.

7 Requirements for a national weather forecasting modelling system

A national weather forecasting modelling system designed to support operational meteorology, environmental monitoring, and decision-making across multiple sectors must satisfy a set of interrelated structural, computational, and methodological requirements. From a mathematical and computational standpoint, such a system can be viewed as a large-scale, high-dimensional dynamical system constrained by heterogeneous observational data and subject to stringent operational constraints.

At the structural level, national coverage is a fundamental requirement. The forecasting domain must include the entire national territory together with surrounding maritime areas, ensuring spatial continuity of the simulated atmospheric fields. In mathematical terms, this corresponds to defining a computational domain $\Omega \subset \mathbb{R}^3$ that adequately captures boundary interactions between land, sea, and atmosphere. Equally critical is the rapid availability of forecasts: numerical solutions of the governing equations must be produced, post-processed, and disseminated within strict operational deadlines. This introduces constraints not only on model formulation but also on numerical solvers and parallel scalability.

From the perspective of computing and data distribution, reliability and resilience are essential properties. Operational continuity requires fault-tolerant architectures and redundancy, particularly for high-performance computing (HPC) systems used to integrate the governing equations of atmospheric motion. Let $\mathcal{M}_{\Delta t}$ denote the numerical time-advancement operator; its repeated application over an operational forecast window must be robust to hardware failures and network interruptions. Timely processing further implies that the computational complexity of $\mathcal{M}_{\Delta t}$, combined with data assimilation cycles, remains compatible with real-time constraints. Efficient access to both input observations and output products is therefore a necessary condition for sustained operational performance.

Forecast methodology requirements are closely tied to the mathematical formulation of the prediction problem. High forecast quality demands the use of state-of-the-art numerical models, whose discretizations of the primitive equations balance accuracy, stability, and computational efficiency. Frequent forecast updates require short assimilation windows, leading to a sequence of inverse problems in which observational data are continuously incorporated into the evolving model state.

The integration of key observations is particularly critical for intense or high-impact events. Radar reflectivity, lightning detection, satellite-derived precipitation, and surface information such as snow cover or soil moisture provide essential constraints on small-scale processes. Their assimilation constitutes a high-dimensional data fusion problem, combining heterogeneous observations with distinct spatial footprints, temporal sampling, and error statistics. Ensemble-based data assimilation methods offer a mathematically consistent framework for addressing this challenge by representing forecast uncertainty through probability distributions rather than single trajectories [22]. In this setting, the atmospheric state is modeled as a random variable, and forecast products naturally acquire a probabilistic interpretation.

Quantification of forecast uncertainty is therefore a core requirement of a modern national system. Deterministic forecasts alone are insufficient, particularly at high spatial resolution where predictability limits are more severe. Ensemble forecasting systems propagate uncertainty through the nonlinear model dynamics, providing estimates of forecast spread that are essential for risk-based decision-making. Continuous development of both deterministic and probabilistic components is necessary to address evolving user needs and to incorporate advances in physical parameterizations, numerical methods, and observational capabilities.

A further requirement is multiscale capability. Accurate forecasts must be provided in complex environments such as urban areas, mountainous regions, and coastal zones, where strong surface–atmosphere interactions dominate the dynamics. Mathematically, this implies resolving or appropriately parameterizing processes across a wide range of spatial and temporal scales, often necessitating grid resolutions approaching the kilometer or sub-kilometer range. Objective verification systems play a crucial role in this context, enabling systematic evaluation of forecast skill, identification of model deficiencies, and the definition of the so-called "useful scale" at which forecasts retain practical value.

Ensuring high overall performance requires adequate computing resources and strong operational management. Kilometer-scale data assimilation systems impose significant computational demands, particularly when integrating remote-sensing data and non-conventional observations such as lightning networks, GNSS-derived water vapor, lidar, or crowd-sourced measurements. The assimilation framework must therefore be modular and extensible, allowing new observation types and operators to be incorporated without fundamental redesign.

Advanced modeling capabilities further depend on sustained collaboration within large, interdisciplinary scientific teams embedded in the international research community. Expertise must span numerical analysis, turbulence and convection theory, cloud microphysics, radiation, land-surface processes, cryosphere dynamics, ocean–atmosphere coupling, and urban meteorology. In addition to deterministic solvers, ensemble methods must be fully integrated to provide probabilistic forecasts that support informed operational decisions.

The operational value of forecasts ultimately depends on their integration into end-user applications. Post-processing, statistical calibration, verification of forecast value, and continuous feedback between model developers, forecasters, and users form a closed loop that drives systematic improvement. At the same time, the transition toward exascale computing requires a thorough revision and optimization of model codes and workflows. Numerical algorithms, data structures, and parallelization strategies must be adapted to modern HPC architectures, making specialized computational expertise an integral part of modeling teams.

Looking forward, national forecasting systems are expected to evolve along three complementary directions: higher spatial resolution, tighter integration of Earth-system components, and seamless forecasting across temporal scales ranging from nowcasting to seasonal prediction. Achieving these objectives requires the coordinated development of numerical models, data assimilation methodologies, observational networks, and high-performance computing infrastructures.

The societal value of these systems extends well beyond weather services. Modern forecasts support risk management, transportation, agriculture, water resources, and the operation of renewable energy systems. As the share of weather-dependent energy sources continues to increase, accurate forecasts will play an increasingly important role in balancing electricity production and demand, thereby contributing to both economic efficiency and energy security.

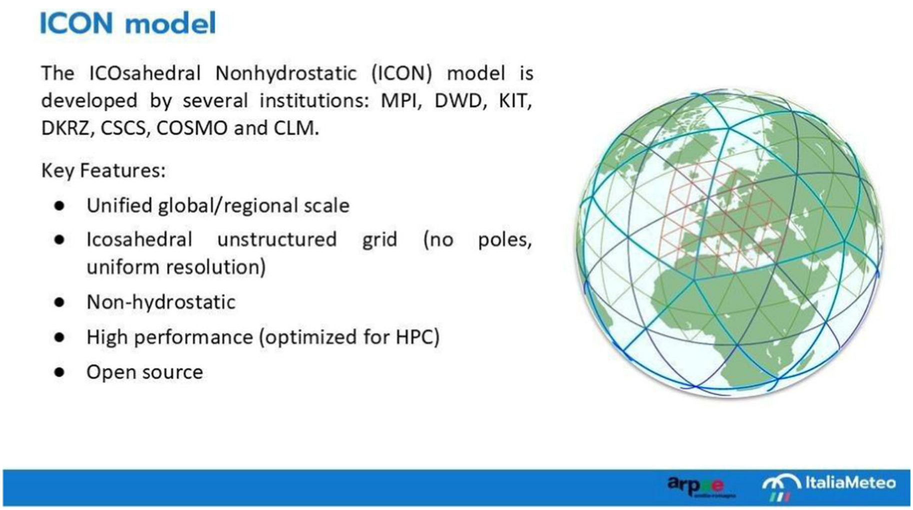

Fig. 3 The ICON modelling system

8 The high-resolution limited-area model: the ICON system operational at the ItaliaMeteo Agency

The ItaliaMeteo Agency relies on the ICON-2I modeling system as its primary operational numerical weather prediction (NWP) framework. ICON (Icosahedral Nonhydrostatic) is a unified global and regional atmospheric model jointly developed by the German Meteorological Service (DWD) and the Max Planck Institute for Meteorology (MPI-M), with the explicit objective of enabling seamless non-hydrostatic simulations across a wide range of spatial and temporal scales [40].

A distinctive feature of ICON is its use of an icosahedral grid on the sphere, which provides quasi-uniform spatial discretization and avoids the coordinate singularities inherent to traditional latitude–longitude grids. A schematic representation of the ICON grid structure and overall modeling architecture is shown in Fig. 3, highlighting the global tessellation and the absence of polar grid convergence.

From a mathematical perspective, ICON solves the fully compressible, non-hydrostatic Navier–Stokes equations on the rotating sphere. In conservative form, the governing equations for mass, momentum, and thermodynamic energy can be expressed as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{8.1}$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) + \nabla p + \rho \mathbf{g} + 2\rho \boldsymbol{\Omega} \times \mathbf{v} = \nabla \cdot \boldsymbol{\tau}, \tag{8.2}$$

$$\frac{\partial (\rho \theta)}{\partial t} + \nabla \cdot (\rho \theta \mathbf{v}) = Q_\theta, \tag{8.3}$$

where ρ denotes air density, $\mathbf{v}$ the velocity field, p the pressure, θ the potential temperature, $\boldsymbol{\Omega}$ the Earth's rotation vector, $\boldsymbol{\tau}$ the viscous stress tensor, and Q_θ represents diabatic heating associated with unresolved physical processes [33, 40].

Spatial discretization is performed on a triangular icosahedral grid obtained through successive refinements of a base icosahedron projected onto the sphere. This approach ensures nearly uniform cell areas and favorable numerical properties for global and regional simulations. In the standard global configuration, the mean grid-cell area is approximately 173 km^2, while limited-area configurations such as ICON-2I achieve substantially finer resolutions. All prognostic variables, including wind components, thermodynamic variables, moisture species, and hydrometeors, are defined as cell-averaged quantities, thereby preserving discrete conservation properties [40].

Vertical discretization employs a terrain-following hybrid coordinate system extending from the surface to approximately 75 km altitude. The global ICON configuration typically uses up to 90 vertical levels, while the limited-area ICON-2I setup adopts 65 vertical levels, with enhanced resolution in the planetary boundary layer and lower troposphere, where surface forcing and strong gradients dominate atmospheric dynamics [40].

ICON is coupled to a land-surface model that prognostically evolves soil temperature and soil moisture through diffusion-type equations across multiple soil layers. In snow-covered regions, additional prognostic variables describe snow water equivalent and snow density. Over ice-free ocean areas, sea surface temperature is prescribed from daily analyses, whereas in ice-covered regions a simplified thermodynamic sea-ice model predicts ice thickness and surface temperature.

Subgrid-scale physical processes not explicitly resolved by the dynamical core are represented through physical parameterizations. ICON provides two main physics suites: the AES physics package, derived from the ECHAM climate model for climate applications, and the NWP physics package optimized for short-range forecasting. These parameterizations introduce additional source terms into the governing equations and can be interpreted mathematically as closure operators acting on unresolved spatial and temporal scales [14].

Since January 2024, ICON has been released under a permissive BSD-3-Clause open-source license, fostering transparent development and broad community adoption. In operational intercomparisons, ICON is widely recognized as one of the leading global and regional NWP systems, often achieving forecast skill comparable to ECMWF-IFS and GFS, particularly over Europe and regions characterized by complex orography [3, 17].

The ItaliaMeteo Agency operates a dedicated limited-area configuration known as ICON-2I, covering the entire Italian territory at a horizontal resolution of 2.2 km. Lateral boundary conditions are provided by the ECMWF Integrated Forecasting System (IFS). The model is currently executed on the *Galileo100* and *Leonardo* supercomputing systems at the CINECA center (Fig. 4), which provide CPU/GPU hybrid architectures optimized for large-scale numerical simulations. A migration toward a dedicated operational cluster named *Marco Polo* is planned starting in 2026.

Operationally, the ICON-2I forecasting chain comprises deterministic, rapid-update, and ensemble configurations. The deterministic system produces twice-daily forecasts with a horizon of up to 72 h. The Rapid Update Cycle (RUC) emphasizes frequent re-initialization to optimally exploit high-resolution observational data. In addition, a 20-member ensemble system is run daily to quantify forecast uncertainty, consistent with the probabilistic interpretation of atmospheric predictability [22, 28].

The three operational ICON-2I configurations are schematically summarized in Fig. 5.

Deterministic run. The deterministic configuration covers the Italian domain at 2.2 km resolution, with a forecast horizon of up to three days. It is executed twice daily at 00 and 12 UTC and includes a dedicated kilometer-scale data assimilation system, described in detail in a subsequent section.

Rapid Update Cycle (RUC). The Rapid Update Cycle is executed six times per day, each run producing forecasts up to 24 h ahead. This configuration is specifically designed to maximize the impact of frequent high-resolution observations and the most up-to-date initial conditions.

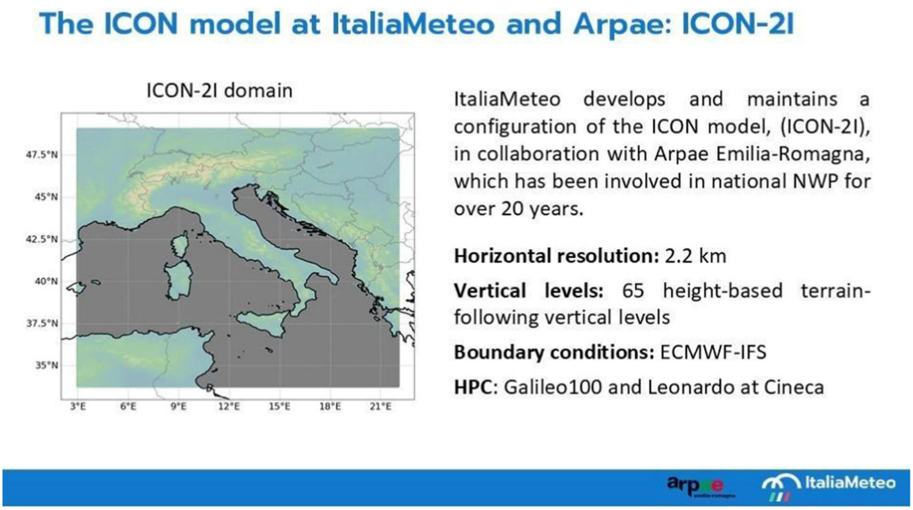

Fig. 4 The ICON-2i modelling system used at the ItaliaMeteo Agency in cooperation with Arpae Emilia-Romagna meteorological service

ICON-2I: operational forecasts

Model	Forecast type	Forecast range	Initial time (UTC)	Main application
ICON-2I	deterministic	+72h	00, 12	weather forecast, civil protection warning, input for other models
ICON-2I-RUC	deterministic	+24h	03, 06, 09, 15, 18, 21	monitoring during high impact weather events
ICON-2I-EPS	ensemble, 20 members	+51h	21	weather forecast, assess forecast confidence

Boundary conditions: IFS (ECMWF)
Initial conditions: generated with LETKF scheme

ItaliaMeteo

Fig. 5 The ICON-2i operational forecasting chain

Ensemble run. The ensemble configuration consists of 20 members and is run once daily at 21 UTC, with a forecast range extending up to 51 h. This setup provides probabilistic guidance and supports quantitative assessment of forecast uncertainty.

Figure 6 presents representative forecast fields produced by the ICON-2I deterministic run, illustrating key near-surface meteorological variables.

Panel (a) shows 24-hour accumulated precipitation, panel (b) the 2 m air temperature, and panel (c) the 10 m wind field. These baseline meteorological fields provide essential input for a wide range of downstream applications, including hydrological and hydraulic

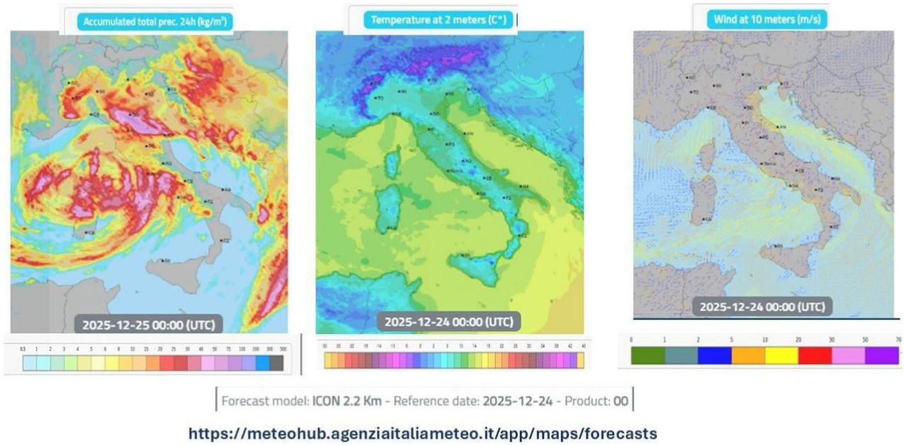

Fig. 6 Examples of numerical forecast fields produced by the ICON 2i deterministic model used at the ItaliaMeteo Agency. Panel **a**: 24hours accumulated precipitation starting from the initial time; Panel **b**: temperature at two meters over ground; Panel **c**: wind field at 10 m above the ground; Representative operational forecast example

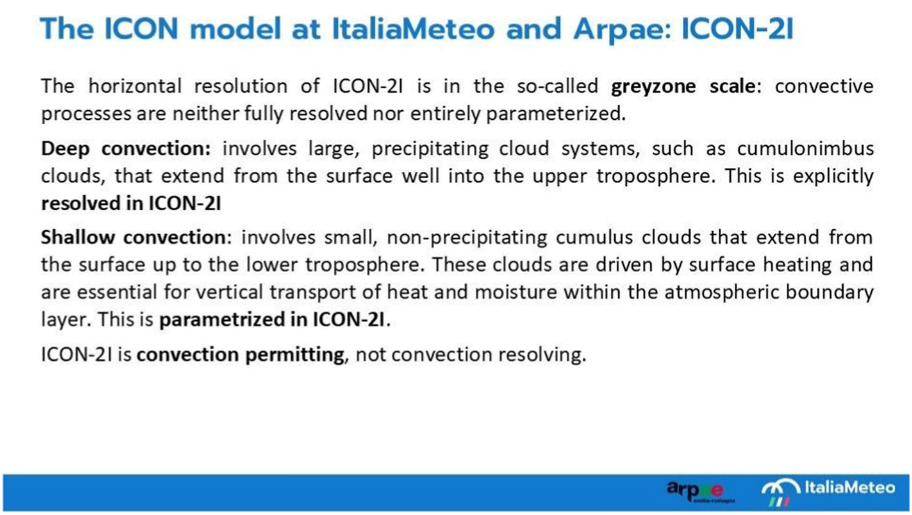

Fig. 7 The ICON-2i description of deep and shallow convection

forecasting, heat-wave risk assessment for public health, and the estimation of renewable energy production from photovoltaic and wind power systems [15].

All figures presented in this work are sourced from the Meteohub data repository maintained by the ItaliaMeteo Agency [1]. This open-access platform provides observational datasets from multiple monitoring networks as well as forecast products from several NWP models, including ICON-2I and WRF.

The 2.2 km horizontal resolution of ICON-2I places the model within the so-called *grey zone* of atmospheric convection (Fig. 7), where convective processes are neither fully parameterized nor completely resolved [13, 39].

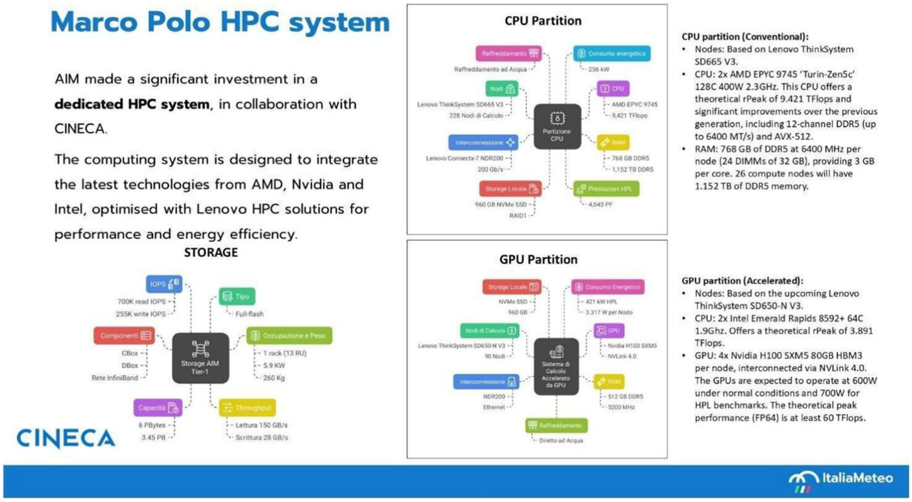

Fig. 8 The new dedicated HPC sy stem MarcoPolo of ItaliaMeteo where ICON-2i operational high-resolution forecasting chains will be executed

In this regime, deep convection, such as organized precipitation systems associated with intense cumulonimbus development, is explicitly represented by the model dynamics, whereas shallow convection, characterized by limited vertical extent and weak precipitation, remains parameterized.

From a computational standpoint, ICON-2I simulations are executed on massively parallel high-performance computing platforms using MPI and OpenMP paradigms, with increasing use of GPU acceleration and modern compiler toolchains such as Intel OneAPI. These systems are integrated with advanced storage and cloud infrastructures to support data-intensive workflows [11].

In the near future, the ICON-2I configuration will be migrated to a dedicated high-performance computing cluster named *Marco Polo* (Fig. 8), specifically designed to optimize operational forecasting workflows and to support high-resolution numerical simulations.

The computational cost of running ICON simulations on such high-performance systems is highly variable and is typically expressed in terms of CPU or GPU hours. It depends on several factors, including the spatial domain and resolution (e.g., the Italian domain at 2.2 km), the forecast horizon (e.g., 24 h versus 72 h), the forecast configuration (deterministic, ensemble, or RUC), and the underlying hardware architecture.

In operational practice, these simulations generally require tens to hundreds of processing cores running for several hours to ensure timely forecast delivery. The most influential factors affecting computation time include:

- **Grid resolution:** finer grids, such as the 2.2 km ICON-2I setup, significantly increase computational demands compared to coarser configurations;
- **Forecast horizon:** longer forecast integrations require proportionally more computational time;
- **Forecast type:** ensemble simulations and high-frequency RUC runs introduce additional computational overhead due to multiple members or frequent executions;
- **Model phases:** data assimilation, short-range forecasting, and ensemble generation exhibit different runtime characteristics;

- **Code and hardware efficiency:** algorithmic optimization (e.g., vectorization and GPU acceleration) and the performance of the target supercomputer (such as Leonardo at CINECA) play a critical role in reducing wall-clock time.

As a representative example, a standard deterministic ICON-2I simulation for Italy at 2.2 km resolution typically requires several hours of execution on a modern computing cluster, using tens to hundreds of processing cores for a 24-hour forecast. Ensemble configurations, such as the daily 21 UTC run, are substantially more demanding and may require many hours of computation, together with significant memory and CPU resources.

In summary, the performance of the ICON-2I system results from a careful balance between numerical formulation, physical parameterizations, data assimilation, and efficient exploitation of high-performance computing resources, enabling reliable high-resolution forecasts within the strict time constraints of operational meteorology.

9 Data assimilation techniques and major developments

Accurate numerical prediction of atmospheric and marine flows critically depends on an adequate representation of the initial state of the system. Since numerical weather prediction (NWP) models integrate nonlinear partial differential equations forward in time, even small errors in the initial conditions may rapidly amplify. The systematic incorporation of observational data into numerical models, known as *data assimilation*, aims to optimally estimate this initial state by combining observations with short-range model forecasts [20].

Modern operational weather services employ data assimilation frameworks that merge heterogeneous observations with prior information provided by ensemble forecasts. While the theoretical foundations of these methods are well documented in advanced textbooks, their operational implementation at high resolution poses significant mathematical and computational challenges.

At kilometer-scale resolution and below, remotely sensed observations—in particular weather radar and satellite measurements—play a central role due to their dense spatial coverage. These observations, however, do not directly measure the primary prognostic variables of NWP models (such as temperature, wind, or humidity). Instead, they provide indirect electromagnetic quantities, including satellite radiances and radar reflectivities, which are related to the model state through nonlinear and often ill-conditioned observation operators.

Historically, such observations were assimilated through inversion procedures, for example by converting radiances into temperature estimates via simplified physical laws. This approach proved problematic, as small measurement errors could be strongly amplified by the nonlinear inversion, leading to spurious corrections that degraded forecast quality. Contemporary data assimilation systems address this issue by adopting a *forward modeling* strategy: the observation operators are used to simulate the observed quantities from the model state, allowing the direct assimilation of raw observations without explicit inversion. This paradigm shift was first enabled by variational methods and has since become standard practice in both variational and ensemble-based systems.

At kilometer-scale resolution, data assimilation becomes one of the most mathematically and computationally demanding components of the forecasting chain. Within the ICON framework, the assimilation problem consists of estimating the atmospheric state vector $\mathbf{x} \in \mathbb{R}^n$ that best reconciles short-range forecasts with a large set of heterogeneous observations unevenly distributed in space and time.

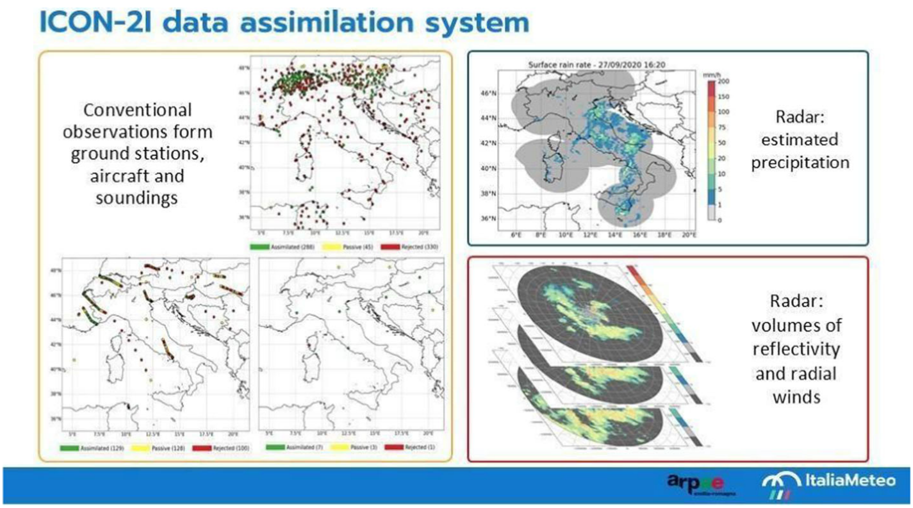

Fig. 9 Main observational datasets assimilated by the KENDA system operational at the ItaliaMeteo Agency

Formally, this problem can be cast as a Bayesian inverse problem. The posterior probability density function of the state is proportional to the product of a prior distribution, representing the forecast background, and a likelihood function, representing observational constraints. In variational form, this leads to the minimization of the cost functional

$$J(\mathbf{x}) = \frac{1}{2}\|\mathbf{x} - \mathbf{x}_b\|^2_{\mathbf{B}^{-1}} + \frac{1}{2}\sum_k \|\mathcal{H}_k(\mathbf{x}) - \mathbf{y}_k\|^2_{\mathbf{R}_k^{-1}}, \tag{9.1}$$

where $\mathbf{x}_b$ denotes the background state, $\mathbf{B}$ the background error covariance matrix, $\mathbf{y}_k$ the observation vectors, $\mathbf{R}_k$ the observation error covariance matrices, and $\mathcal{H}_k$ the (generally nonlinear) observation operators.

At kilometer-scale resolution, the dimensionality n of the state vector typically exceeds 10^8, making explicit representations of the covariance matrix $\mathbf{B}$ infeasible. ICON therefore relies on ensemble-based approaches, in which background error statistics are estimated implicitly from a finite ensemble of forecasts. In ensemble Kalman filter (EnKF) formulations, the analysis update can be written as

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}\left(\mathbf{y} - \mathcal{H}(\mathbf{x}^f)\right), \qquad \mathbf{K} = \mathbf{P}^f\mathcal{H}^T\left(\mathcal{H}\mathbf{P}^f\mathcal{H}^T + \mathbf{R}\right)^{-1}, \tag{9.2}$$

where $\mathbf{x}^f$ is the forecast state and $\mathbf{P}^f$ is the forecast error covariance estimated from the ensemble [10].

The ICON modeling system employs the KENDA ensemble data assimilation framework (see [12, 34]), which combines short-range ensemble forecasts with radar, satellite, and lightning observations processed through advanced forward operators. This approach effectively propagates observational information from measurement locations to dynamically relevant regions of the model domain via flow-dependent covariance structures. Operationally, observations such as radar reflectivities or satellite radiances are combined with ensemble forecasts to update model variables through the Kalman filter (Fig. 9). This effectively “transports” observational information from the measurement locations to the model grid points, leveraging the statistical properties inferred from the ensemble.

High-resolution data assimilation places particular emphasis on controlling sampling errors, representativeness errors, and non-Gaussian error distributions. Localization and covariance inflation techniques are therefore essential to stabilize the filter and maintain physical consistency of the analysis [2, 36].

Despite improvements in resolution and data assimilation, atmospheric predictability remains fundamentally limited by the chaotic nature of the governing equations. Small perturbations in the initial state grow exponentially in time, a phenomenon first identified by Lorenz and later formalized within the theory of dynamical systems [23, 24].

Let $\delta\mathbf{x}(t)$ denote an infinitesimal perturbation to the system state. In the linearized regime, its evolution satisfies

$$\|\delta\mathbf{x}(t)\| \approx \|\delta\mathbf{x}(0)\|\, \mathrm{e}^{\lambda t}, \tag{9.3}$$

where λ is the leading Lyapunov exponent. At synoptic scales, the associated error-doubling time is on the order of several days, whereas at convective scales relevant for kilometer-resolution models it may be as short as one to two hours.

The rapid amplification of uncertainty has two important consequences. First, deterministic forecasts at kilometer-scale resolution inevitably lose skill at relatively short lead times. Second, probabilistic approaches based on ensemble prediction become essential for quantifying forecast confidence and supporting risk-based decision making.

The transition toward kilometer- and sub-kilometer-scale forecasting also poses major computational challenges. As horizontal resolution increases, the number of grid points and the frequency of assimilation cycles grow rapidly, leading to a substantial increase in computational cost. Ensemble-based methodologies further amplify these requirements, since multiple realizations of the atmospheric state must be integrated simultaneously. More in detail: the transition toward kilometer and sub-kilometer resolutions dramatically increases computational cost. For a model with horizontal grid spacing Δx, the number of grid points scales approximately as $O(\Delta x^{-2})$, while stability constraints impose a time-step scaling of approximately $O(\Delta x)$. As a result, the total computational cost for a fixed forecast length typically scales between

$$C \sim O(\Delta x^{-3}) \quad \text{and} \quad C \sim O(\Delta x^{-4}), \tag{9.4}$$

depending on the model configuration. The lower estimate assumes a fixed number of vertical levels, whereas the higher estimate accounts for the increase in vertical resolution that often accompanies horizontal grid refinement in modern atmospheric models.

Including ensemble forecasting with N_e members further increases the computational cost to

$$C_{\text{ens}} \sim N_e\, \mathcal{O}(\Delta x^{-3}\text{–}\Delta x^{-4}), \tag{9.5}$$

where the precise exponent depends on the relationship between horizontal and vertical grid refinement and on the specific model implementation.

For this reason, modern forecasting systems increasingly rely on petascale and emerging exascale computing infrastructures. Within the ICON framework, efficient parallelization, advanced numerical algorithms, and optimized data management strategies are critical for maintaining operational feasibility while preserving forecast quality [3].

Future developments are expected to focus on tighter integration between data assimilation, ensemble forecasting, and artificial intelligence techniques. In this context, the migration of ICON-2i toward dedicated HPC infrastructures such as the forthcoming *Marco Polo* system represents a necessary step to sustain further increases in resolution, ensemble size, and forecast sophistication within realistic computational budgets. Machine learning methods can support quality control, observation processing, surrogate modeling, and post-processing applications, complementing rather than replacing physics-based approaches. The combination of advanced observations, probabilistic forecasting, and AI-assisted methodologies is likely to play a central role in the next generation of operational weather prediction systems.

A particularly important open question concerns the future relationship between artificial intelligence and traditional physics-based numerical weather prediction systems. Current AI-based forecasting models have demonstrated impressive forecasting capabilities and, in some cases, forecast performance comparable to that of leading operational NWP systems. However, it is important to recognize that most existing AI forecasting frameworks are not entirely data-driven in the strict sense. Their training datasets are typically based on atmospheric reanalyses, such as ERA5 produced by ECMWF, which themselves are generated through the combination of observations, physical models, and advanced data assimilation techniques.

Consequently, present-day AI systems inherit a substantial amount of physical knowledge indirectly embedded within the reanalysis products used during training. Whether future generations of AI models will be able to operate successfully without any dependence on traditional numerical models remains an open scientific question. Some researchers argue that sufficiently large datasets, increasing computational power, and more advanced machine learning architectures may eventually enable fully data-driven forecasting systems. Others remain more cautious, noting that atmospheric observations alone provide only a partial and irregular sampling of the true atmospheric state. Furthermore, observations are sparse in both space and time and do not directly provide a complete description of the atmospheric state. Numerical models and data assimilation systems currently play a fundamental role in reconstructing dynamically consistent atmospheric fields from incomplete measurements, while physical laws offer essential constraints that help ensure consistency, stability, and extrapolation capability under previously unseen conditions.

For this reason, hybrid approaches combining physical models, data assimilation, and artificial intelligence currently appear to be the most promising pathway. Such systems may exploit the strengths of both paradigms: the physical consistency and interpretability of numerical models together with the pattern-recognition and computational efficiency of machine learning techniques. At present, however, the long-term balance between purely data-driven and physics-informed forecasting remains uncertain. Given the rapid pace of current developments in artificial intelligence, the balance between purely data-driven, physics-informed, and hybrid forecasting systems may evolve significantly over the next few years. but, at present, no clear consensus exists regarding the long-term architecture of future operational forecasting systems.

10 Conclusions and perspectives

Modern weather forecasting represents one of the most successful applications of mathematics, physics, and high-performance computing to a real-world problem of enormous societal relevance. The combination of increasingly sophisticated observational systems, advanced data assimilation techniques, and numerical weather prediction models has transformed forecasting from a largely empirical activity into a quantitatively rigorous scientific discipline.

A central feature of contemporary forecasting systems is the integration of heterogeneous observations with dynamical models. Ground-based networks, weather radars, satellites, aircraft observations, and emerging monitoring technologies provide an unprecedented description of the atmosphere, while data assimilation methods combine these measurements with model forecasts to produce dynamically consistent estimates of the atmospheric state. Together with ensemble prediction systems, these techniques allow not only the prediction of future weather conditions but also the quantification of forecast uncertainty.

The operational experience of the ItaliaMeteo Agency, based on the high-resolution ICON-2i modelling framework, illustrates how modern forecasting systems rely on the close interaction between numerical modelling, observational data, and advanced computing infrastructures. Such systems provide increasingly detailed information on precipitation, temperature, wind, and severe weather phenomena, supporting both routine forecasting activities and risk-management applications.

Despite remarkable progress, significant scientific challenges remain. The representation of convective processes at kilometer and sub-kilometer scales, the assimilation of rapidly growing volumes of heterogeneous observations, and the efficient exploitation of emerging exascale computing architectures continue to be active areas of research. At the same time, artificial intelligence and machine learning are opening new perspectives in data processing, nowcasting, post-processing, and uncertainty estimation, complementing traditional physics-based methodologies.

The importance of weather forecasting is also expanding beyond its traditional role in meteorology and civil protection. Modern societies increasingly depend on weather-sensitive infrastructures and services. In particular, the growing penetration of renewable energy sources has created a strong demand for highly accurate forecasts of solar radiation, cloud cover, wind speed, and precipitation. Reliable meteorological predictions make it possible to estimate electricity production from photovoltaic and wind power systems several hours or days in advance, improving grid stability, optimizing energy trading strategies, and reducing balancing costs. In this context, weather forecasting has become an essential component of modern energy systems and electricity markets.

Whether future forecasting systems will remain predominantly physics-based, become increasingly data-driven, or evolve toward hybrid architectures remains one of the most important open questions in atmospheric science.

Looking ahead, progress will depend not only on advances in mathematical modelling and computational power, but also on the ability to transform ever-growing volumes of environmental data into actionable information for decision makers. Weather forecasting therefore stands at the intersection of science, technology, and societal needs, providing a clear example of how mathematical modelling can generate direct economic, environmental, and social benefits.

Acknowledgements The authors would like to thank Romina Duro for her assistance in reviewing the article and for her support with the more technical aspects of LaTeX editing. The first author would like to thank several colleagues of the ItaliaMeteo Agency for their help in making available some figures of the paper and for many useful discussions.

Author Contributions The first author contributed to the descriptive and modeling aspects, while the second author assisted in formulating the models from a mathematical perspective.

Funding Open access funding provided by Università degli Studi di Firenze within the CRUI-CARE Agreement.

Declarations

Conflict of interest On behalf of all authors, the corresponding author declares that there is no conflict of interest.

References

1. Agenzia Italia Meteo.: MeteoHub Data Repository. https://meteohub.agenziaitaliameteo.it/app/maps. Accessed Aug 2026
2. Bannister, R.N.: A review of forecast error covariance statistics in atmospheric variational data assimilation. Part I: characteristics and measurements of forecast error covariances. Q. J. R. Meteorol. Soc. **134**, 1951–1970 (2008)
3. Bauer, P., Thorpe, A., Brunet, G.: The quiet revolution of numerical weather prediction. Nature **525**, 47–55 (2015)
4. Bouttier, F., Courtier, P.: Data assimilation concepts and methods. In: Meteorological Training Course Lecture Series. ECMWF (1999)
5. Bowler, N.E., Pierce, C.E., Seed, A.: Development of a precipitation nowcasting algorithm based upon optical flow techniques. J. Hydrol. **288**, 74–91 (2004)
6. Charney, J.G., Fjørtoft, R., von Neumann, J.: Numerical integration of the barotropic vorticity equation. Tellus **2**, 237–254 (1950)
7. Daley, R.: Atmospheric Data Analysis. Cambridge University Press, Cambridge (1991)
8. Dixon, M., Wiener, G.: TITAN: thunderstorm identification, tracking, analysis, and nowcasting—a radar-based methodology. J. Atmos. Ocean. Technol. **10**, 785–797 (1993)
9. ECMWF.: ECMWF Integrated Forecasting System Documentation CY49R1. Available at https://www.ecmwf.int/en/forecasts/documentation-and-support/changes-ecmwf-model
10. Evensen, G.: Data Assimilation: The Ensemble Kalman Filter, 2nd edn. Springer, Berlin (2009)
11. Fuhrer, O., Chadha, T., Hoefler, T., Kwasniewski, G., Lapillonne, X., Leutwyler, D., Lüthi, D., Osuna, C., Schär, C., Schulthess, T.C., Vogt, H.: Near-global climate simulation at 1 km resolution: establishing a performance baseline on 4888 GPUs with COSMO 5.0. Geosci. Model Dev. **11**, 1665–1681 (2018). https://doi.org/10.5194/gmd-11-1665-2018
12. Gastaldo, T., Poli, V., Marsigli, C., Alberoni, P.P., Paccagnella, T.: Data assimilation of radar reflectivity volumes in a LETKF scheme. Nonlinear Process. Geophys. **25**, 747–764 (2018)
13. Gérard, L.: An integrated package for subgrid convection, clouds and precipitation compatible with meso-γ scales. Q. J. R. Meteorol. Soc. **136**, 1203–1219 (2010)
14. Giorgetta, M.A., et al.: The ICON-AES model: formulation and global simulations. J. Adv. Model. Earth Syst. **10**, 1613–1637 (2018)
15. Göber, M., et al.: Enhancing the value of weather and climate services in society: identified gaps and needs as outcomes of the first WMO WWRP/SERA weather and society conference. Bull. Am. Meteor. Soc. **104**(3), E645–E651 (2023). https://doi.org/10.1175/BAMS-D-22-0199.1
16. Golding, B.W.: Nimrod: a system for generating automated very short range forecasts. Meteorol. Appl. **5**, 1–16 (1998)
17. Haiden, T., et al.: Evaluation of ECMWF forecasts, including the 2017 upgrade. ECMWF Technical Memorandum No. 832, ECMWF, Reading, UK (2018)
18. Horn, B.K.P., Schunck, B.G.: Determining optical flow. Artif. Intell. **17**, 185–203 (1981)
19. Johnson, J.T., MacKeen, P.L., Witt, A., Mitchell, E.D.W., Stumpf, G.J., Eilts, M.D., Thomas, K.W.: The storm cell identification and tracking algorithm: an enhanced WSR-88D algorithm. Weather Forecast. **13**, 263–276 (1998)
20. Kalnay, E.: Atmospheric Modeling, Data Assimilation and Predictability. Academic Press (2003)
21. Kasahara, A.: Dynamics of the upper atmosphere and numerical prediction. Adv. Geophys. **11**, 1–115 (1968)
22. Leutbecher, M., Palmer, T.N.: Ensemble forecasting. J. Comput. Phys. **227**, 3515–3539 (2008)
23. Lorenz, E.N.: Deterministic nonperiodic flow. J. Atmos. Sci. **20**, 130–141 (1963)
24. Lorenz, E.N.: The predictability of a flow which possesses many scales of motion. Tellus **21**, 289–307 (1969)

25. Lorenz, E.N.: Atmospheric predictability as revealed by Naturally occurring analogues. J. Atmos. Sci. **26**, 636–646 (1969)
26. Lynch, P.: The Emergence of Numerical Weather Prediction. Cambridge University Press, Cambridge (2011)
27. Marshall, J.S., Palmer, W.M.K.: The distribution of raindrops with size. J. Meteorol. **5**, 165–166 (1948)
28. Palmer, T.N.: The primacy of doubt: evolution of numerical weather prediction from determinism to probability. J. Adv. Model. Earth Syst. **11**, 3296–3332 (2019)
29. Pulkkinen, S., et al.: Pysteps: an open-source python library for probabilistic precipitation nowcasting. J. Open Res. Softw. **7**, 12 (2019)
30. Ravuri, S., et al.: Skilful precipitation nowcasting using deep generative models of radar. Nature **597**, 672–677 (2021)
31. Roberts, N.M., Lean, H.W.: Scale-selective verification of rainfall accumulations from high-resolution forecasts of convective events. Mon. Weather Rev. **136**, 78–97 (2008)
32. Rodgers, C.D.: Inverse Methods for Atmospheric Sounding: Theory and Practice. World Scientific, Singapore (2000)
33. Satoh, M., Tomita, H., Yashiro, H., Kajikawa, Y., Miyamoto, T., Yamada, Y., Yamada, H.: The non-hydrostatic icosahedral atmospheric model: description and development. Prog. Earth Planet Sci. **1**, 18 (2014)
34. Schraff, C., Reich, H., Rhodin, A., Schomburg, A., Stephan, K., Periáñez, A., Potthast, R.: Kilometre-scale ensemble data assimilation for the COSMO model (KENDA). Q. J. R. Meteorol. Soc. **142**, 1453–1472 (2016)
35. Shi, X., Chen, Z., Wang, H., Yeung, D.-Y., Wong, W.-K., Woo, W.-C.: Convolutional LSTM network: a machine learning approach for precipitation nowcasting. Adv. Neural. Inf. Process. Syst. **28** (2015)
36. Wang, H., Sun, J., Zhang, X., Huang, X., Auligné, T.: Radar data assimilation with WRF 4D-Var. Part I: system development and preliminary testing. Mon. Weather Rev. **141**, 2224–2244 (2013)
37. Wilson, J.W., Mueller, C.K.: Nowcasts of Thunderstorm initiation and evolution. Weather Forecast. **8**, 113–134 (1998)
38. World Meteorological Organization.: Guide to meteorological instruments and methods of observation, WMO-No. 8, 2018 edition, Geneva (2018)
39. Wyngaard, J.C.: Toward numerical modeling in the "terra incognita". J. Atmos. Sci. **61**, 1816–1826 (2004)
40. Zängl, G., Reinert, D., Rípodas, P., Baldauf, M.: The ICON (ICOsahedral Non-hydrostatic) modelling framework of DWD and MPI-M: description of the non-hydrostatic dynamical core. Q. J. R. Meteorol. Soc. **141**(687), 563–579 (2015)