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Analysis of Masonry Block Structures with Unilateral Frictional Joints

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Abstract. Masonry constructions dry-assembled and composed of rigid blocks, such as most of the ruins in archaeological sites, can be categorized into two groups: structures in which equilibrium is mainly assured by the compression force in correspondence to the joints and structures that rely on the presence of friction between blocks. In this paper a mechanical model and a numerical procedure are proposed to analyze both types of structures, but the second ones are emphasized. The numerical examples described prove the reliability of the approach proposed.

Introduction

The stability of masonry constructions mostly relies on their capacity to transfer compression forces among the blocks that form the load-bearing structure. However, it is worth noting that much of this capacity is the consequence of the occurrence of friction forces acting in correspondence to the contact joints between the brick or stone blocks.

In some structural elements, such as the masonry arch, friction does not play a significant role because such structures are conceived to work only under compression forces. This behavior is the consequence of a peculiar constructive geometry characterized by a targeted arrangement and shape of blocks in the masonry brickwork that, in the case of an arch, is the result of the disposition of blocks along radial joints so as to keep the shear forces low along the contact joints. In this way, the shear forces are much lower than the axial forces and the resultant force in a joint is not much inclined as respects to the straight line orthogonal to the joint.

In the case of a mortar joint, the mortar is aimed at transferring the low shear forces between blocks. Instead, in the case of contact joints, i.e. dry-assembled blocks, a rather low friction factor is sufficient to justify a stable equilibrium condition of the structure.

For this reason, the analysis of masonry arches can be dealt with relying on the J. Heyman's well known hypotheses [1] on masonry behaviour. These hypotheses are corroborated by the following deductions:

1) Brick and stone deformability is very restricted (especially if it is compared to that of mortar joints);
2) Given its actual inability of transferring even low tensile forces, mortar does not play a fundamental role in the equilibrium of the structure (especially in the case of historical heritage buildings), exactly as if the construction was dry-assembled without mortar;
3) Inner compression stresses provoked by the actual loads are limited in most cases;
4) Friction that is necessary to prevent the sliding of blocks along the joints is easily conceivable in almost all the real cases.

Under these hypotheses, the external actions can lead the structure to instability activating the only possible collapse mechanism historically known as "flexural collapse mechanism" described by Claude Antoine Couplet [2]. This mechanism is activated when in four distinct joints of the arch the rotational dislocations around the intrados or extrados point of two subsequent blocks occur. Such a point is considered to be an inner hinge in the structural model.

The sufficient and necessary condition of a masonry arch to be stable under any load condition derives from the static theorem of the limit analysis that requires the existence of an inner stress state in equilibrium with the external loads. This equilibrium condition can be represented by a "line of thrust" entirely contained in the profile of the arch.

It is worth noting that, except in special cases, the sides of the funicular polygon of equilibrium are always slightly inclined with respect to the lines orthogonal to the joints. Considering that the straight line of thrust is the so-called "funicular polygon of subsequent resultants" [3], it follows that if each resultant of all forces that precede a certain joint is decomposed into shear and axial forces, the first one has an intensity negligible as respects to the second (Fig. 1a).

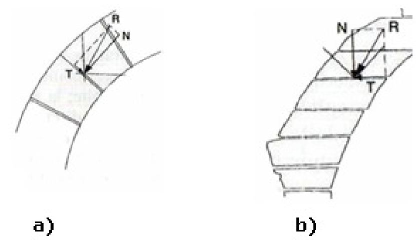

Figure 1. The role of friction in the case of an arch (a) and a corbel arch (b).

However, in some structural typologies the shear force in one joint could also be greater and, consequently, the sliding of two subsequent blocks along the joint could also occur. In this specific case, a shear-sliding collapse mechanism could also be activated if such a phenomenon occurs in a meaningful number of joints. Usually the reason for this is strictly connected to the peculiar disposition of blocks in the framework and to the consequent slope of joints. The overall behaviour of these types of structures and their degree of stability are strongly affected by the role of friction (Fig. 1b). Targeted examples of structures belonging to this category are the flat arch, the stocky arch, the corbel arch, the corbel dome, the wall subjected to horizontal actions and, generally, all masonry structures in which joints are mostly horizontal.

In summary, the shear-sliding problem can be dealt with using three different approaches [4]:

1) It can be neglected in the case in which the structure is conceived in such a way that sliding is prevented (the Heyman's hypothesis); this choice consists of assuming that the shear force in a joint can reach very high values, until the fictitious value of infinite: $T = \infty$;

2) The *"sliding-friction"* behaviour (the Coulomb's hypothesis) is assumed, according to which when the shear force reaches the threshold value, the block undergoes a sliding that provokes the shear force in the joint to become zero: $T = 0$ (Fig. 2a);

3) The *"plastic shearing"* behaviour (the Drucker's hypothesis) is assumed, according to which the shear force can never overcome the threshold value, which is a function of the axial force N in the joint and the friction coefficient *f*. When the threshold value is reached, the block does not move, as occurs in the second approach, but the joint undergoes a plasticization phenomenon and the value of the shear force that can be transferred from one block to the subsequent one across the joint remains constant and equal to the threshold value (Fig. 2b).

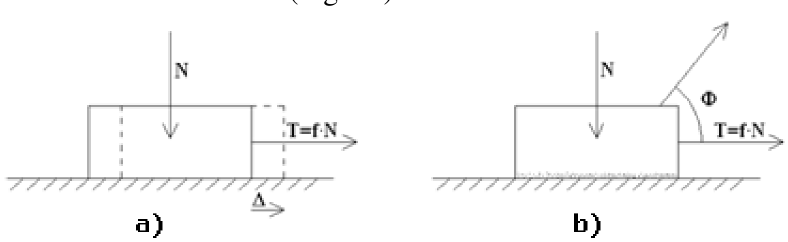

Figure 2. a) Sliding-friction behaviour; b) plastic shearing behavior.

The Mechanical Model of the Contact Joint

Sliding of blocks along the joints is the consequence of the shear force that overcomes the friction reaction. In turn, the friction reaction of the joint is proportional to the axial force N (that is compression). For this reason, sliding generally occurs along the (horizontal) joints that are located at the highest levels of the masonry construction and can be described through the relation: *Friction* = *Friction(N)*.

Shear force T wins the friction reaction when its value exceeds the plastic threshold value that can be computed considering the strength criterion expressed by the *Coulomb's friction cone* [5].

According to this criterion, T cannot exceed the friction reaction in any joint, that is $T \leq f \cdot N$, in which $f = tg(\Phi)$ is the tangent of angle Φ of the Coulomb's friction cone (Fig. 3).

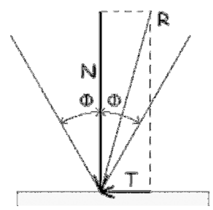

Figure 3. Coulomb's friction cone.

In order to investigate the shear-sliding behaviour in a joint of a masonry structure, in this paper the reference mechanical model illustrated in Fig. 4a has been developed.

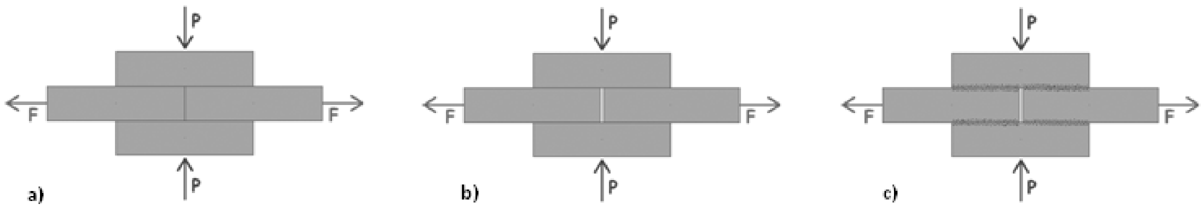

Figure 4. a) Reference mechanical model; b) crack in the vertical joint; c) horizontal joints plasticization.

The model describes the set of four blocks (taken from a masonry framework) subjected to a vertical compression force P. The aim of the investigation is to compute the value of the horizontal shear force F that provoke the two central blocks to slip. In turn, the value of force F, capable of pulling these two blocks out from the other part of the structure, is a function of the compression force P, that is always present in a masonry construction: $F = F(P)$.

Let us envision to use the reference mechanical model to simulate a simple test, consisting of the increasing of the force F step by step, little by little. When the force F overcomes the tensile strength of the vertical joint, a crack occurs in that joint (Fig. 4b). Subsequently, when force F equals the value of the friction reaction, horizontal joints undergo a plasticization process and the structure fails due to the loss of adherence that provokes the loss of equilibrium (Fig. 4c).

In order to numerically describe the structural behaviour of the mechanical model, it has been represented by an assemblage of rigid blocks linked through fictitious devices capable of simulating a contact joint (Fig. 5a). These devices are modeled through a set of *n* rigid links orthogonal to the joint (capable of transferring only the compression axial force) and *n* rigid links along the interface (capable of transferring the shear force).

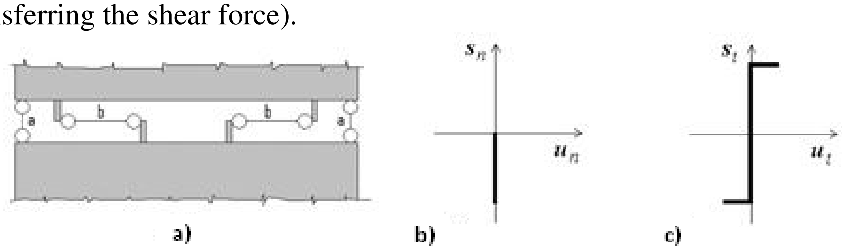

Figure 5. a) Devices of the contact joints in the reference mechanical model; b) behaviour of the links orthogonal to the joints; c) behaviour of the links along the joints.

According to the assumption of contact joints, the tensile strength of joints is zero and the shear force cannot exceed the threshold value governed by the presence of friction. Therefore, a rigid-brittle behaviour of the links orthogonal to the joints and a rigid-plastic behaviour of the links along the joints have been assumed (Fig. 5b, 5c).

Coherently with the above hypotheses, the structural problem is governed only by the equilibrium equations and can be expressed in the matrix form of Eq. 1:

$$\mathrm{AX} + \mathrm{F} = 0 \tag{1}$$

subjected to the sign conditions (Eq. 2):

$$\begin{cases} X_n \leq 0 \\ X_t \leq f \cdot X_n \\ \delta \geq 0 \end{cases} \tag{2}$$

where *{F}* is the vector of the loads acting on the structure, *{X}* is the vector of the unknown forces in the links of the joints and *[A]* is the equilibrium matrix whose coefficients are a function of the shape of the blocks and the slope of the joints. The assumptions on the behaviour of the joints, described in Fig. 5, are expressed by the sign conditions on the vector *{X}*. Vector *{X}* can be opportunely partitioned into the two sub-vectors $\{X_n\}$ (forces of the links orthogonal to the joints) and $\{X_t\}$ (forces of the links arranged along the joints). Vector $\{\delta\}$, the "impressed distortion vector" [6], is the key to oblige the respect of the first and second inequality in Eq. 2. About this, the iterative numerical technique to implement the automatic calculation of the structure, have been described in [7, 8] and illustrated in several case studies [9, 10].

The numerical technique formulated for the solution of a problem subjected to sign constraints, as that of the masonry rigid-block construction under investigation, needs the preliminary computation of an initial solution [11] (the trial vector $\{X_0\}$ in Eq. 3 that corresponds to the linear-elastic solution of a deformable structure). Successively, the procedure checks if the coefficients of such a vector are coherent with the constrains of Eq. 2 and, finally, if not, the initial solution is modified by the superposition of the new solution (expressed by vector $\{X_N\}$ in Eq. 3) of the same structure subjected to the external actions of the distortions in place of the loads. The sum of vector $\{X_0\}$ and vector $\{X_N\}$ provides the final solution (vector *{X}*) expressed by the third of Eq. 3:

$$\begin{aligned} X_0 &= \tilde{A}(A\tilde{A})^{-1} \cdot F \\ X_N &= \left(I - \tilde{A}\left(A\tilde{A}\right)^{-1} A\right) \cdot \delta \\ X &= X_0 + X_N \end{aligned} \tag{3}$$

At the end of the iterative procedure, the structure can be unstable or stable as a function of its load condition. In this last case the final solution vector is expressed by the following partitioned vector (Eq. 4):

$$X = \begin{bmatrix} 0 \\ X_n^{(-)} \\ X_t \end{bmatrix} \tag{4}$$

Eq. 4 clearly shows that the procedure uses the distortions to nullify all tensile forces in the links orthogonal to the joints and to lower, until equal to their threshold value, all the shear forces in the links along the joints that are greater than the corresponding friction reactions.

Numerical Examples

In order to validate the reference mechanical model (Fig. 4) and check if numerical results are in line with reality, the model has been analyzed twice by means of the numerical procedure proposed.

The analysis has been performed under a vertical compression action of 100N plus two horizontal forces of 50N that simulated the shear action along the horizontal joints of the model. Initially the self-weight of the blocks has been neglected, subsequently it has been inputted in order to assess its contribution. In both cases the procedure has shown that the horizontal force is not able to provoke the sliding of the two central blocks. The structural solution has highlighted a friction factor equal to 0.5

in the first case, to which the friction angle of 26.56° corresponds, and a friction factor equal to 0.25 in the second case, to which the friction angle of 14.03° corresponds.

Some numerical simulations were also carried out on recurrent masonry structures representative of historical and monumental heritage. The automatic calculations have been performed by implementing this procedure in the computer program *BrickWORK* [12].

As expected, in the case of a *flat arch* (Fig. 6), the slope of the contact joints between blocks governs the problem of the equilibrium relying on the friction. In the examined case, the structure with blocks along radial joints (Fig. 6b) has a friction angle 23 times lower than the one with blocks along vertical joints (Fig. 6a) and it is, consequently, safer.

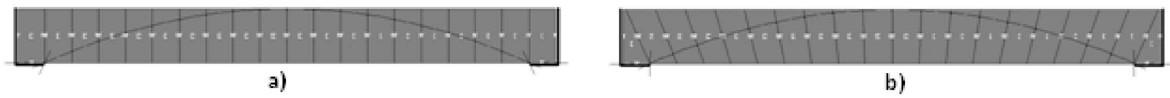

Figure 6. Flat arch: structural response as a function of the slope of the joints.

In the case of a stocky arch, the problem of equilibrium is affected by the presence of friction between the joints, as highlighted in Fig. 7a. The corbel arch is a structure even more affected by the presence of friction and its equilibrium condition is completely governed by it (Fig. 7b, 7c).

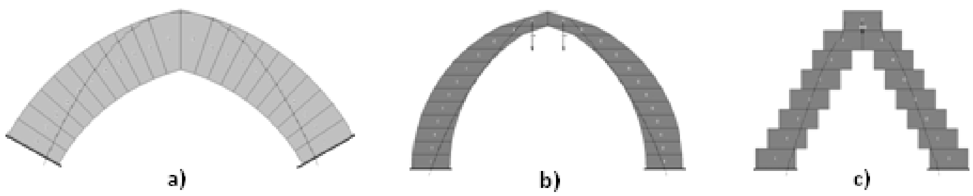

Figure 7. a) Stocky and b,c) corbel arch patterns.

The last numerical example (Fig. 8) is a real case study in the archaeological site of Civitavecchia di Arpino (FR, Italy). It is an arched gate with dry-assembled stone blocks. Under the assumption of the unitary weight of 20kN/m3, the analysis has provided the friction angle of 21.80° (i.e. a friction factor equal to 0.40) that clearly states that this monument can stand up thanks to the friction between the stone blocks (Fig. 8a). It is finally worth noting that, during construction, stones were not assembled along horizontal joints but along joints slightly inclined towards the inside (Fig. 8b). This feature proves that ancient builders were very knowledgeable about the problem of sliding blocks and the beneficial role of friction.

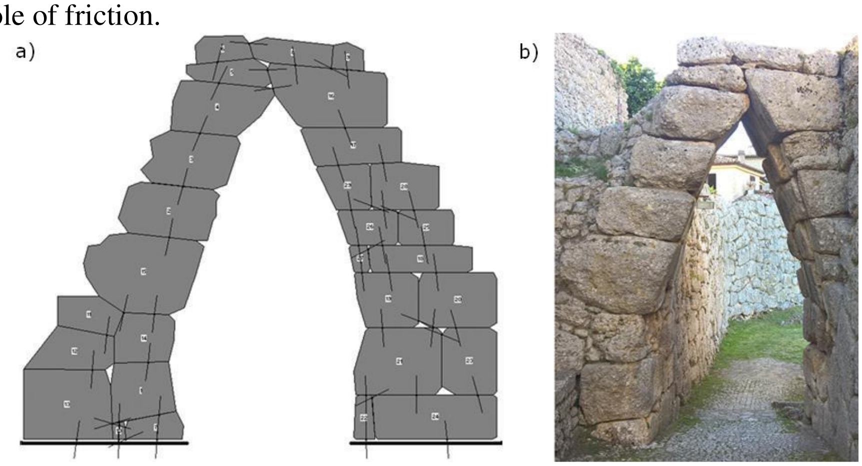

Figure 8. Arched gate of Arpino (FR, Italy): a) numerical model; b) photo.

Summary

In this paper a reference mechanical model and the numerical procedure for the nonlinear analysis of rigid-block masonry structures is presented. In particular, the case of structures in which the equilibrium is based, above all, on the friction in the contact joints between blocks is examined. Shear-sliding failure is investigated considering both the Coulomb's and Drucker's hypotheses, and the possibility of rotational failure is also taken into account, especially in the case of an arch. Results of the numerical examples are coherent with reality and, therefore, this tool seems to be reliable for the analysis of these types of structures. Nevertheless, further investigations based on laboratory experimental in-scale models should be carried out and tests should be compared to numerical results, to assure the effective reliability of this tool.

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