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In the vast literature on logical paradoxes, there seems to be a growing trend of researches aiming at isolating unifying characters that may disclose similarities between otherwise different examples of logically grounded contradictions, and may also provide scholars with new tools for research in the field. The paper under review is an attempt to contribute to both directions in the case of semantic paradoxes (i.e. paradoxes involving properties related to that of truth for sentences).

In a previous work by the second author, [J. Symb. Log., 69 (2004), no. 3; MR2078920], a restricted language $\mathcal{L}_{\mathsf{P}}$ was introduced that allows to study patterns of sentential sentences that generate semantic paradoxes. The language features infinitely many sentence names, a falsity predicate and conjunction (and the possibility of building infinitely-long conjunctions is permitted). Denotation functions associating sentence names with well-formed formulas of $\mathcal{L}_{\mathsf{P}}$, along with the dependency relations between names that naturally arise, suggest a natural definition of paradoxicality for (sets of) sentence names within this setting. This allows to prove a variety of results from particular, like, e.g., that the construction leading to Yablo's paradox is indeed paradoxical in this technical sense, to more general ones.

The paper under review aims at finding new and stronger results of this general kind by relying on the notion of *strong homomorphism* (SH, henceforth). In short, given any two sets of sentence names of $\mathcal{L}_{\mathsf{P}}$, a function f is an SH between the two (relatively to two denotation functions defined over the two sets) if (i) f is a map from the first set to the second that preserves the mutual dependency relations and (ii) for every sentence name s such that $f(s)$ depends upon any given s', there exists s'' that s depends upon such that $f(s'') = s'$. The main result, named the strong $\mathcal{L}_\mathsf{P}$-homomorphism theorem ($\mathsf{S}\mathcal{L}_\mathsf{P}\mathsf{H}$ theorem, henceforth), shows that strong homomorphisms between $\mathcal{L}_\mathsf{P}$ constructions preserve paradoxicality. The authors then show that, by reconstructing some of the machinery of $\mathcal{L}_\mathsf{P}$ within the category of sets, it is possible to obtain a simple category-theoretic proof of the $\mathsf{S}\mathcal{L}_\mathsf{P}\mathsf{H}$ theorem, as well as to disclose its relations to other, apparently independent results.

The rest of the paper is concerned with a (two-fold) application of the previous results. First, it is shown that the $\mathcal{L}_\mathsf{P}$ counterpart of (the dual of) McGee's construction from [J. Philos. Log.. 14 (1969), no. 4; MR0816243] is paradoxical. Then, the same result is (more easily) obtained via the $\mathsf{S}\mathcal{L}_\mathsf{P}\mathsf{H}$ theorem by introducing a novel construction, called the Infinite Flower (IF, henceforth), and by showing: (i) that IF is paradoxical and (ii) that there is an SH between IF and the dual of McGee's construction.