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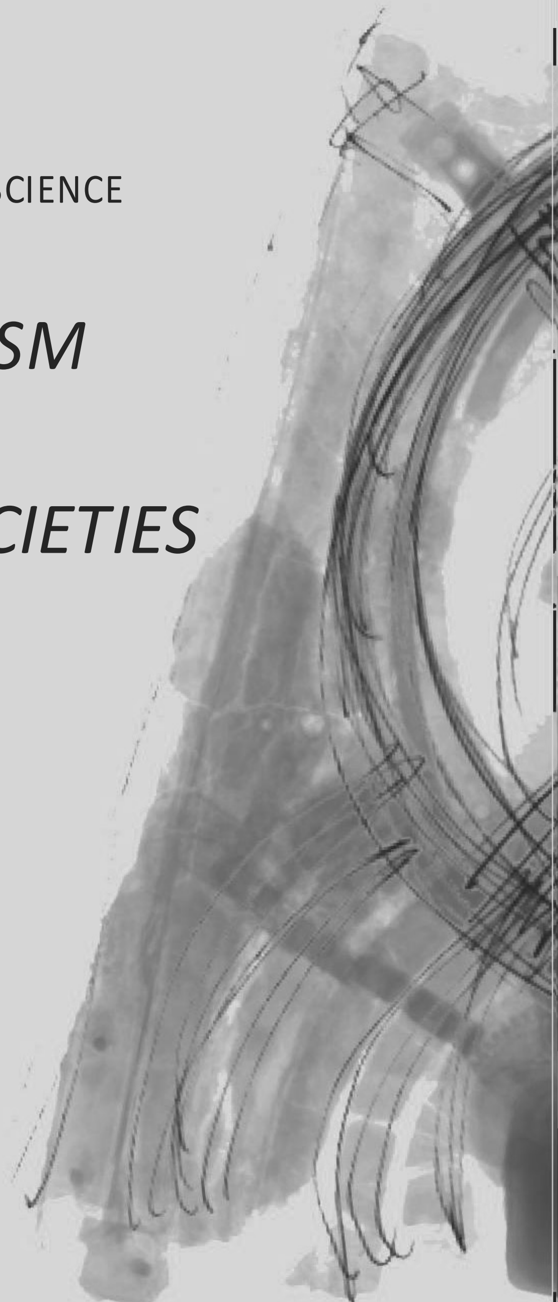

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The Euclidean Tradition at the Renaissance Courts: the Case of Federico Commandino

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Federico Commandino (1509-1575), the founder of the so-called Urbino School, lived under patronage of important Renaissance families, such as della Rovere and Farnese. This permitted him access to the most valuable libraries, enabled him to maintain close contact with humanistic circles, and facilited his pursuit of a major program of research in the renaissance of mathematics. He published and commented the works – to mention the most important ones -- of Apollonius (1566), Archimedes (1558), Pappus (posthumous 1588) and Euclid, both in Latin (1572) and in vernacular (1575). Commandino's edition of the Elements, based on Greek sources, combined philological rigour together with mathematical exactness and soon became the reference edition up to the nineteenth century. Furthermore the Urbinate scholar enriched the Euclidean text with comments and additions based on both classical and contemporary sources. This edition epitomizes Commandino's idea of restitutio or the re-appropriation of ancient sources in light of integrated, early modern scientific knowledge.

Paraphrasing the classic essay of Paul Lawrence Rose (Rose 1975), we can recognize in "the Italian renaissance of mathematics" at least two main issues: the restoration of Greek mathematics and the development of rhetorical and syncopated algebra. Usually, the activities of study, translation and edition of Classics were conceived within the most important humanistic circles or the most prestigious Schools – as the Venetian San Marco School or Rialto School – or, again, in the world of the courts (rarely in the Universities). Of course the term 'court' is to be intended in a broad sense: the court was everything and everyone surrounding the public authority, that could be the Prince, the Venetian Doge or even the Pope. On the other side, algebra mainly developed within the abacus environment, inasmuch as it was conceived as a useful tool for solving problems [1].

It is only thanks to his numerous patrons – the Duke Ottavio and the Cardinal Ranuccio Farnese, Marcello Cervini (the future Pope Marcello II), the Duke Guidobaldo of Urbino -- that Federico Commandino, one the most important scholar of the Renaissance Italy, could succeed in his program for the renaissance of mathematics. Following, in some sense, Regiomontanus's *Program*, based on the restoration of the whole Greek mathematical *corpus*, Commandino published the works of Archimedes, Apollonius, Serenus, Ptolemy, Pappus and, of course, Euclid.

[1] Of course these worlds weren't so definitively divided. For examples, Luca Pacioli and Niccolò Tartaglia, teachers in the abacus schools were also editor of Euclidean and Archimedean works. They were interlocutors of both the worlds and, by means of their work, they tried to unify these two cultural environments.

His various patronages secured him not only financial support, but access to the most important humanistic libraries and acquaintance with intellectual circles[2]. After the death of Ranuccio, his last patron of the Farnese family, Commandino returned to his native Urbino, where the patronage of mathematics was a long established tradition and where the Duke's library included a large number of mathematical manuscripts. He established the so-called *Urbino School*, that trained Guidobaldo dal Monte, Bernardino Baldi, and other distinguished mathematicians[3]. Strictly connected to the foundation of the Urbino School, is Commandino's teaching activity in his native town, which probably spanned the years from 1568 to 1574[4]. Commandino's lectures were influenced by the interests of Francesco Maria II della Rovere (1549-1631), who in 1574 succeeded his father Guidobaldo II as Duke of Urbino. Since Francesco Maria was particularly interested in Euclid's *Elements*, the question of getting a satisfactory edition of this text immediately arose.
In the first decades of the sixteenth century the editions of the *Elements* available to the scholars were, essentially 1) the first Latin printed edition (*editio princeps*), which appeared in Venice in 1482 by Erhard Ratdolt and was based on the medieval version of Campanus from Novara; and 2) the Venetian edition of 1505, based instead on the translation of a Greek code, made by the humanist Bartolomeo Zamberti. The medieval *recensio* showed additions, changing of definitions or differences in numbering propositions, whereas the humanist translation, very careful to the linguistic aspect, mercilessly highlighted the very poor geometrical talent of Zamberti. Even if several editions followed, none of them were so correct from a mathematical and linguistic viewpoint to become a shared and trustworthy edition of the *Elements*, the reference point for European scholars. Actually, most of the sixteenth-century *Elements* simply embraced Campanus' or Zamberti's approach with only marginal changes[5].
Commandino's Latin edition of the *Elements,* appeared in 1572 and soon became the reference edition for the scholarly community up until the early nineteenth century[6]. Since he encountered many problems in printing this edition, Commandino set up a press in his household, where, in 1575, the Italian translation was printed by his son-in-law Valerio Spaccioli (because of Commandino's death).[7]

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[2] In this respect, (Rose 1973) publishes two letters illustrating the extent to which Commandino depended on Farnese patronage.
[3] Gamba, Montebelli 1988 and Frank 2013 provide an overview of the cultural and scientific climate in the Duchy of Urbino in the late Renaissance.
[4] I would like to thank Martin Frank for having provided me his draft on Commandino's teaching activity (Frank, forthcoming).
[5] For an overview of the Euclidean tradition in the Renaissance, see Gavagna 2009; 2012, whilst Gavagna 2010 is mainly focused on the Venetian environment.
[6] On Commandino's role in the Renaissance mathematics, see Napolitani 1997; 2000; on Commandino's edition of the *Elements*, see Gamba 2009 and Gavagna, forthcoming.
[7] Among the Commandino's extant drafts, there is the manuscript copy of Book II, whereas Book III finishes at Proposition 10. Remark that in this redaction, very close to the 1572 published text, Commandino's comment are lacking, This and others manuscripts are available in the webpage devoted to *Federico Commandino Manuscript Collection*,

In the Preface to his Latin edition, the Urbinate scholar presented an historical essay of the development of mathematics and clarified the two most common misunderstandings related to the *Elements*[8]. In fact, most humanist mathematicians believed that Euclid, wrongly identified with the philosopher Euclid of Megara, wrote only the statements for the propositions in the *Elements*. They thought that the demonstrations were the work of Theon, who wrote a new edition of the *Elements* several hundred years later. One of the few voices questioning the attribution of the proofs to Theon was the mathematician Jean Borrel, who wrote an appendix to his 1559 work on the quadrature of the circle noting the most common mistakes in interpreting Euclid's *Elements* (*Ioannis Buteonis annotationum liber in errores Campani, Zamberti, Orontii, Peletarii, Ioannis Penae intepretum Euclidis*). Borrel explained that ancient authors, and Proclus in particular, unanimously attributed the *Elements* to Euclid alone and that the misunderstandings arose from an incorrect interpretation of the title found in the Greek codices. Commandino completely agreed with Borrel's interpretation and, furthermore, stated that the author of the *Elements* could not be Euclid of Megara, but rather Euclid of Alexandria. So, when he published his Latin translation of the *Elements*, Commandino eliminated these mistakes, and because of his undisputed authority they disappeared from every following essay on Euclid and his *Elements.*

Even from a design viewpoint, the layout of both 1572 and 1575 editions was completely different from the previous editions. The Urbinate humanist, in fact, decided to visually distinguish the Euclidean text (in roman typeface) from the editor's comments (in italics). Furthermore, the references to previous propositions or definitions or postulates – absent in the Greek codices but usually enclosed in the text of the Renaissance editions – were shifted to the margin of the page. In other words, the Euclidean text had to be clearly separated by further interpolations and comments thereby allowing Commandino to return to the "original purity" (*pristinum nitorem*) of the text, as remarked the Jesuit Christopher Clavius in the Preface of his 1574 edition of the *Elements*. Nevertheless, even if has become commonplace to attribute to Commandino a philological accuracy very close to the modern accuracy, we still don't know which was or were the Euclidean codices used by the scholar to constitute the critical text. He seems to have used the Greek text published in Basel in 1533, but – as far as I know -- we don't have any information about manuscript sources: so we can't determine how rigorous Commandino's philological approach was.

Anyway, comparing the main Renaissance Euclidean editions, we can remark that – at least concerning the case study considered[9] -- Commandino's approach towards the construction of the text aimed to combine mathematical exactness with philological accuracy. To take an example, we could consider the following case

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Contenitore 120, cartella 1, ff.116r-132v, http://echo.mpiwg-berlin.mpg.de/content/mpiwglib/urbino.

[8] For a detailed study about the history of mathematics in the Renaissance, mainly focused on the figures of Pierre de La Ramée and Henry Savile and with some references to Commandino, see Goulding 2010 .

[9] See for example Gavagna, forthcoming.

related to Book VI. According to the Renaissance editions belonging to the Greek tradition, we find, in sequence, the following:

Proposition VI.19 (P19): Similar triangles are to one another in the duplicate ratio of the corresponding sides.

Corollary (Cor19): If three straight lines are proportional, then the first is to the third as the *figure* described on the first is to that which is similar and similarly described on the second (italics mine).

Proposition VI.20 (P20): Similar polygons are divided into similar triangles, and into triangles equal in multitude and in the same ratio as the wholes, and the polygon has to the polygon a ratio duplicate of that which the corresponding side has to the corresponding side

Corollary I (Cor20I): Similar rectilinear figures are to one another in the duplicate ratio of the corresponding sides.

Corollary II (Cor20): […] so it is evident that if three straight lines are proportional, then the first is to the third as the figure described on the first is to that which is similar and similarly described on the second.

First of all, it sounds strange that Cor19 is almost the same as Cor20 and, secondly, the reference to a polygon (*figure*) in Cor19 is not correct, since P19 deals with triangles, not polygons. A problem arose. What was the editors' approach regarding the correction of these anomalies? Let us consider some examples. Bartolomeo Zamberti simply ignored the problems and published the clearly incorrect sequence; Campanus (we include this author even if he does not follow the Greek tradition) remarked that Cor19 should be moved after P20, but left the sequence unchanged; Jacques Peletier expunged Cor19[10]. Niccolò Tartaglia, who used both Zamberti and Campanus for his Italian translation of the *Elements* appeared in 1543, was convinced that Cor19 was in the wrong position and criticized the two editors, but did not make major changes. Finally, Commandino found the most elegant solution. He simply substituted the word *figure* with *triangle* (as some Theonine manuscripts actually read) thus justifying the mathematical meaning of Cor19 and the whole sequence of propositions and corollaries.

Concerning Commandino's comments on Euclidean propositions, even if a complete analysis is still lacking, a quick overview shows that his commentary drew mainly from classical authors and commentators like Archimedes, Apollonius, Pappus, Proclus and Eutocius. The purpose of the Urbinate humanist was to establish a network of relationships and crossed references that made Greek mathematics an organic *corpus*. In the same time, he did not forget to relate the ancient wisdom with the contemporary one and this is the reason why we find some references, for example, to Regiomontanus' works or to Pierre de la Ramée and Jean Borrel's writings and even to Girolamo Cardano's *De regula aliza libellus*, a puzzling algebraic work published only two years before.

Except for some marginal remarks, Campanus' medieval edition of the *Elements* – indeed very far from the humanistic canons – seems to be completely neglected by Commandino. A more careful – but still partial -- study shows that Campanus'

[10] In 1557 Peletier published *In Euclidis Elementa geometrica demonstrationum libri sex.*

influence is probably deeper than it seems. As example we could consider the following case study.
At the end of Book V, devoted to the general theory of proportions, Commandino added eight propositions about inequality between ratios, explaining that they were tools commonly used by ancient mathematicians like Archimedes, Apollonius and Pappus. So, even if they weren't in Euclid's treatise, they belonged to the body of Greek mathematical knowledge and this was a good reason to add the 'lacking' propositions. Commandino says he has drawn the eight propositions from Pappus' *Mathematical Collection*, but commuting their original order and modifying them just a little when necessary. Apparently Commandino totally ignored Campanus, who also added nine propositions on the same issue at the end of his own version of Book V. But comparing the propositions carefully, it is clear that, even if the proofs are actually shaped on the Pappus' ones, they are ordered following Campanus. Furthermore, Commandino considered four propositions not belonging to Pappus' *Collection*, but included in the Campanus' medieval redaction of the *Elements*, as we can check in the correspondence table:

Commandino	Campanus	Pappus
V.26	V.26	VII.7
V.27	V.27	VII.5
V.28	V.28	VII.3
V.29	V.29	
V.30	V.30	VII.6
V.31	V.32	
V.32	V.33	
V.33	V.31	
	V.34	

Table 1

Of course the previous case study does not prove anything, but it offers a clue to the (hidden) influence of Campanus on Commandino.
A very interesting aspect of Commandino's Euclid concerns its diagrams and its relationship to the idea of the restoration of classic mathematics[11]. The Greek-Latin tradition differed from the Arabic-Latin one not only with respect to Euclid's text (number and order of propositions, proofs and so on) but also with respect to the geometrical diagrams, especially those of solid figures, which are very difficult to

[11] This issue is developed in Sorci 2001.

draw. For example, in Campanus' Arabic-Latin redaction, most of the figures of Book XV were omitted, whereas Zamberti, following the Greek-Latin tradition, supplied all the figures[12]. But despite the differences, the drawings of solids were based on the same criterion (a faithful representation of the step-by-step geometrical proof), because the purpose was to illustrate geometrical properties of the figure, not to represent the solid in a realistic way. The resulting diagrams were extremely complex and it was very hard to recognize them as a representation of solid figures. Commandino broke with this iconographic tradition and provided perspective drawings in his printed editions. At first sight, this could seem surprising, because the Urbinate scholar, as we have said, aimed to restore the 'original purity' of classical texts (and diagrams), while perspective was a discipline re-discovered in the Renaissance. Or maybe it's not so surprising? Actually, in the new Renaissance classification of the sciences we find *scenographica*, a classical discipline whose aim was to represent reality as it appears. Unfortunately the extant fragments (attributed to Geminus of Rhodes) described only the purpose of the *scenographica* but not the technics used, and this is probably the reason that convinced Commandino in using the contemporary perspective technics to represent solid figures according to the Greek science. In other words, even Commandino's perspective drawings are to be interpreted in the light of his program of restoration. In order to attribute to Commandino the right role in the development of Renaissance mathematics – an essential prerequisite for the Scientific Revolution – it will be useful to deepen or explore several aspects of his life, career and scientific production. On one side a careful and complete reconstruction of the scientific environment where he worked is required; on the other side, scholars should analyze Commandino's published and unpublished writings in order to identify the sources he used (even the hidden ones) and how he used them. This contribution is just a small step in this last direction.

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[12] The Renaissance versions of the *Elements* included the apocryphal Book XIV and XV, that continued Euclid's theory of regular solid.

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