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Controlling the character of the superfluid-supersolid quantum phase transition

N. ANTOLINI([1])([2])(*)

([1]) *CNR-INO, Sede di Pisa - 56124 Pisa, Italy*
([2]) *LENS, University of Florence - 50019 Sesto Fiorentino, Italy*

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Summary. — The recent observation of supersolidity in trapped quantum gases of magnetic atoms has raised much interest about the properties of the so called dipolar supersolid. Besides the combination of superfluidity and crystalline order peculiar to this new state of matter, a fundamental question regards the nature of the quantum phase transition leading to supersolids. In this work we study both experimentally and theoretically the superfluid-supersolid transition, observing that its character changes smoothly between continuous and discontinuous depending on the atom number and transverse confinement. Starting from the Landau theory of phase transitions, we explain this phenomenon as a dimensional crossover between 1D- and 2D-like structures of the supersolid.

1. – Introduction

The supersolid is a new quantum state of matter where atoms arranged in a periodic crystal-like structure can still flow coherently, as they do in a superfluid. Supersolids were proposed theoretically more than 50 years ago [1], thinking about solid helium as the natural system able to combine properties of both crystals and superfluids. However, supersolidity was observed only recently in trapped dipolar quantum gases of magnetic atoms [2], resulting from the crystallization of a superfluid. Indeed, starting from a Bose Einstein condensate (BEC) of neutral magnetic atoms, and changing the interactions within the system, one can drive a quantum phase transition (QPT) towards a state which is still coherent, but shows a clustered structure.

The spontaneous breaking of the continuous translational symmetry arise from the attractive part of the long range dipole-dipole interactions, which overcome the repulsive contact interactions [3]. The new ground state is a system where the dipoles, aligned by an external magnetic field, stack one on top of the other forming clusters separated by weak density links, which ensure coherence. Thus, the crystalline structure develops in the plane either in 1D (stripes) or in 2D (triangular lattice), while a vertical confinement prevents the system from collapsing, also giving the length scale of the spacing of the density modulation [2]. In experiments with dipolar gases, we use a harmonic cigar-shaped trapping potential, therefore the clusters develop along the weak axis of the trap realizing the single-row supersolid shown in the insets of fig. 1(a).

We now discuss the character of the superfluid-supersolid QPT combining the results of numerical simulations with experimental evidence as discussed in refs. [4, 5].

(*) E-mail: `nicolo.antolini@unifi.it`

2. – Dimensional crossover in the superfluid-supersolid QPT

2˙1. *Order parameter and control parameter.* – Having both phase and spatial orders, atoms in the supersolid share the same macroscopic wavefunction ψ, with density $\rho(\boldsymbol{r}, z) = |\psi|^2$, which is modulated in the x-y plane. The modulation can be described as $\rho(\boldsymbol{r}) = \rho_0\left[1 + C\sum_i \cos(\boldsymbol{k}_i \cdot \boldsymbol{r})\right]$, where $\boldsymbol{k_i}$ are the lattice vectors and C is the contrast. In the experiment we drive the phase transition from a uniform superfluid ($C = 0$) to the supersolid ($C \neq 0$) by lowering the scattering length a_s, thus increasing the relative strength of dipole-dipole interactions [6]. Therefore, it would be natural to associate the order parameter and control parameter to C and a_s respectively. Note that, since in experiments we exploit time of flight absorption images, instead of C, we assume the control parameter to be its counterpart in momentum space $\tilde{C}$.

2˙2. *Infinite systems.* – Considering infinite systems, the Landau free energy $\Delta E(\tilde{C})$, *i.e.,* the energy difference between superfluids and supersolids, can be expanded in powers of $\tilde{C}$ and we can assess the nature of the phase transition by looking at the relative intensities and signs of different terms. In one dimension, all the odd terms in the expansion vanish: changing the sign of the contrast we are in practice shifting our system by one lattice site, hence the free energy is unchanged. With this symmetry, the system can only undergo second-order QPTs, where $\tilde{C}$ changes smoothly from zero to a finite quantity by lowering a_s [7]. On the other hand, when the system is two-dimensional, the $\tilde{C} \to -\tilde{C}$ symmetry is lost, since odd terms survive in the expansion and it is possible to have a barrier in the free energy, resulting in a first-order transition where $\tilde{C}$ jumps discontinuously from zero to a finite value [8]. The free energy in the two cases is sketched in fig. 2(e).

2˙3. *Trapped systems.* – In a trapped system we should expect the nature of the phase transition to change accordingly with its dimensionality. Indeed, depending on the transverse size of the system we observe two different types of transitions that are reminiscent of the first- and second-order QPTs expected in 2D and 1D discussed in the previous section. In the case of a rigid system the separation between the two regimes would be a sharp line. Instead, as shown in fig. 1(a), we find that the character of the QPT for our cluster supersolid changes smoothly as expected for a dimensional crossover.

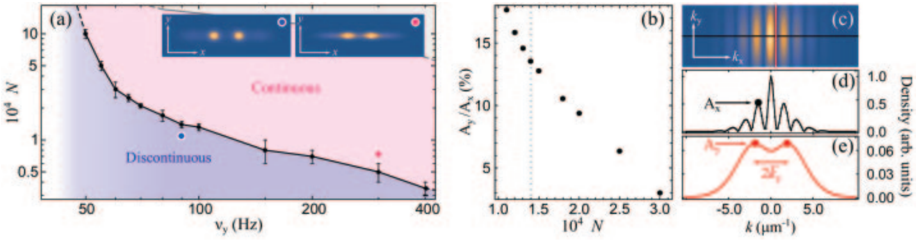

Fig. 1. – (a) Character of the superfluid-supersolid phase transition from numerical simulations in the N-ν_y plane. The two samples of single-row supersolid density in the insets correspond to continuous (magenta) and discontinuous (blue) transitions. Black dots correspond to the boundary between the two regimes. (b) Two-dimensional structure of the density background across the dimensional crossover for $\nu_y = 90$ Hz. The ratio of the amplitudes A_x and A_y in the Fourier space (c) is extracted from the cuts along x (d) and y (e) as a function of N. The vertical dotted line marks the continuous-discontinuous boundary. Adapted from [4].

The boundary between the two regimes depends on N and ν_y, which directly affect the system dimensionality. As shown in fig. 1(b), a two dimensional structure is still present in single-row supersolids featuring discontinuous transitions and is suppressed on the continuous side of the crossover.

3. – Experimental observations

We explored the crossover by choosing two trapping potentials with similar atom number, in the two opposite regions: potential V_C has frequencies $(\nu_x,\, \nu_y,\, \nu_z) = (15.0,\, 101.0,\, 93.9)$ and, tightly confining the system along y, leads to a continuous transition, while potential V_D, with frequencies (21.8, 67.0, 102.0), provides a weak confinement along y which allows for discontinuous transitions. The measured contrast as a function of a_s is reported in fig. 2(a), (b).

From the BEC side ($\tilde{C} = 0$) we first lower the scattering length studying the crystallization process. While for potential V_C the transition is smooth, for potential V_D the contrast shows strong fluctuations and a rapid growth at the transition point. Looking at the fluctuation spectrum of the order parameter in fig. 2(c), (e), for potential V_D we see a double peak structure in the region of scattering lengths just before the transition, which directly reveals the double minimum structure of the free energy typical of discontinuous transitions. The same analysis for potential V_C shows instead a single peak, as we expect when the free energy has no barrier.

In a different experiment, we first prepare a supersolid by crossing the QPT, then we go back to the BEC side observing the melting process of the supersolid. As shown in fig. 2(a), (b), the system smoothly returns a superfluid for potential V_C, while a modulated state persists for V_D. This happens because crossing a discontinuous transition the system gains excitation energy since the process is intrinsically non-adiabatic and the final state both in the supersolid and BEC sides is an excited state. A qualitative characterization of these excited states can be done by following the dynamics of the contrast over time. In fig. 2(f), (g) we see that in the supersolid state the contrast oscillates around a non-zero value while, after melting, oscillation touches zero. In qualitative

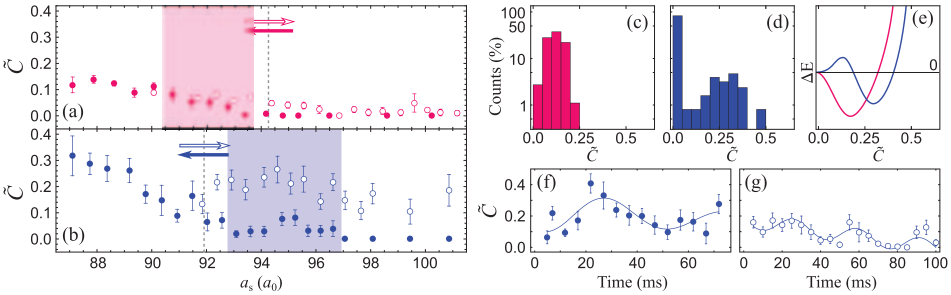

Fig. 2. – Observations of continuous and discontinuous superfluid-supersolid QPT. Contrast as a function of a_s for potential V_C (a) and V_D (b) in the crystallization (BEC→ supersolid) and melting (BEC→supersolid→BEC) experiments are represented as dots and circles. Vertical dashed lines mark the transition points. The shaded regions show the intervals of a_s corresponding to the fluctuations spectra in panels (c),(d). (e) Cartoon of the free energy curves associated to the spectra. Dynamics of $\tilde{C}$ for potential V_D at $a_s = 87.3\, a_0$ after crystallization (d) and at $a_s = 100.3\, a_0$ after melting (e), respectively. Adapted from [4].

agreement with numerical simulations of the dynamics [4], we interpret these excitations as high amplitude oscillations around the equilibrium state of the system, which has $\tilde{C} = 0$ in the BEC side and a finite contrast in the supersolid.

4. – Conclusions

In conclusion, we observe a smooth change from discontinuous to continuous transitions in the superfluid-supersolid quantum phase transition. This behaviour can be explained in terms of a dimensional crossover, which we explored experimentally by tuning the character of the phase transition by changing experimental parameters. The possibility of exploiting continuous transitions to produce excitation-free supersolids is very appealing for future experiments to study superfluidity and quantum entanglement.

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