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# Harnessing Spatial Heterogeneity in Composite Indicators through the Ordered Geographically Weighted Averaging (OGWA) Operator

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*Spatially heterogeneous weights and a non-compensatory aggregation scheme, are two important properties needed to construct a composite indicator capable of summarizing properly the multidimensional phenomenon of local spatial units. Such a composite indicator takes into account, on the one hand, the latent characteristics of the specific units related to their location in the territory, and on the other hand, the relative importance of sub-indicators highlighting both positive and negative aspects of the studied phenomena. Under these premises, this article proposes a new method called* Ordered Geographically Weighted Averaging *(OGWA), which can consider different degrees of non-compensability between sub-indicators and, at the same time, the spatial heterogeneity for continuous, ordinal, and mixed data. The properties of the method are evaluated through a simulation study. Finally, the method is applied to construct a composite indicator to map the urban public infrastructure of São Sebastião do Paraíso, a city located in the southeastern region of Brazil.*

## Introduction

Composite indicators are one-dimensional measures resulting from the mathematical aggregation of sub-indicators that reflect the different underlying elements of a multidimensional phenomenon (Kuc-Czarnecka, Lo Piano, and Saltelli, 2020). The literature on composite indicators is considerable and provides answers to many of the problems associated with their operation. (Nardo et al., 2005; OECD, 2008).

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However, most of this literature does not address the concept of heterogeneity in building the composite indicator (Sarra and Nissi, 2020). The concept of heterogeneity is associated with the idea that sub-indicators influence the composite indicator's score non-homogeneously and mainly vary from one unit to another. Let us see the example of a composite indicator built to represent the public infrastructure of a city (Libório, da Silva Martinuci, et al., 2020; Libório, Laudares, et al., 2020). The absence of asphalt in a wealthy neighborhood is more important in representing urban public infrastructure than in a poor neighborhood. The absence of other urban infrastructure is lower in wealthy neighborhoods than in poorer ones. Thus, the absence of other public infrastructure rivals the absence of asphalt, reducing the importance of asphalt in representing public infrastructure in poorer neighborhoods.

This concept of heterogeneity is even stronger when the units' geographical location is considered (Fusco, Vidoli, and Rogge, 2020; Cartone and Postiglione, 2021; Artelaris, 2022). This idea is based on the principle of the disparity of the latent characteristics that define the multidimensional phenomenon in each of the spatial units (Fusco, Vidoli, and Sahoo, 2018). Returning to the example, the concept of spatial heterogeneity in a composite indicator representing urban public infrastructure is linked with the principle of proximity whereby the presence of asphalt in a neighborhood influences and is influenced by the presence of this sub-indicator in nearby neighborhoods.

The role of the weighting approach in incorporating spatial heterogeneity into the composite indicator makes the definition of the aggregation scheme critical. The relationship between weighting and aggregation of sub-indicators has been extensively discussed by Nardo et al. (2005), Munda (2012), Becker et al. (2017b), and Greco et al. (2019). These studies show that compensatory aggregation schemes imply that the sub-indicators are substitutable, so the weights also assume a substitution function.

In compensatory aggregation, underperforming sub-indicators are compensated by overperforming sub-indicators (Ekel, Pedrycz, and Pereira Júnior, 2020). In the example, the score of the composite indicator of public infrastructure in a region with 10% of roads without asphalt (performance or score of 0.10) and 100% of roads with sewage system (performance or score of 1.00) is equal to 0.55 in compensatory aggregation.

Non-compensatory aggregation schemes do not allow compensation between poor and above-average performance sub-indicators (Pedrycz, Ekel, and Parreiras, 2011). In the example, the score of the composite indicator of public infrastructure in a region with 10% of roads without asphalt and all roads with a sewage system is equal to 0.18 in the harmonic mean non-compensatory aggregation. Thus, non-compensatory aggregation penalizes the presence of sub-indicators with poor and above-average performance.

Another advantage of non-compensatory aggregation is that the weights of the sub-indicators retain their relative importance function, which means that the sub-indicators have different levels of adherence to the concept of the multidimensional phenomenon (Becker et al., 2017a, 2017b).

Therefore, a proper incorporation of spatial heterogeneity in the composite indicators must, simultaneously, ensure that the weights of the sub-indicators assume a function of relative importance through a non-compensatory aggregation and that the weights of the sub-indicators vary between spatial units to consider their specificities.

In this regard, the Ordered Weighted Averaging (OWA) operator (Yager and Kacprzyk, 2012) is a suitable multicriteria decision-making method as it is structurally non-compensatory and weights the sub-indicators according to the characteristics of the analysis units (Yager, 1988). Besides, implementing linguistic quantifiers makes it possible to regulate the sub-indicators' compensation levels (Ekel, Pedrycz, and Pereira Júnior, 2020) and the heterogeneity of the sub-indicators' weights (Libório, Martinuci, Ekel, et al., 2022).

However, composite indicators constructed by OWA disregard the spatial effects in the weighting, not fully fulfilling the requirements indicated in the literature for considering the concept of spatial heterogeneity.

The objective of this research is to explore and contribute to the state of the art in the construction of composite indicators with multicriteria approaches, by proposing a new method called Ordered Geographically Weighted Averaging (OGWA), able to take into account the non-compensability between simple indicators and the spatial heterogeneity. In fact, to the best of the authors' knowledge, the OGWA is the first method, in a multicriteria setting, that simultaneously considers different levels of compensation between sub-indicators, positive or negative aspects of the phenomenon, and spatial heterogeneity in the composite indicator.

The study is also a pioneer in considering spatial heterogeneity in the representation of urban public infrastructure (Narbón-Perpiñá and De Witte, 2018a, 2018b). In fact, the research results fill the gap for studies that can provide better representations of urban public infrastructure, contributing to urban planning (Coutinho-Rodrigues, Simão, and Antunes, 2011) and its subfields, such as the optimization of public budgets (D'Inverno et al., 2023), public resource management (Narbón-Perpiñá & De Witte, 2018b), prioritization of public investments (Ziara et al., 2002) and equitable distribution of urban public infrastructure in the city (Pandey, Brelsford, and Seto, 2022).

Furthermore, it is necessary to point out that the presented results can be extended for their utilization in the processing of ordinal data (e.g., Likert scales – Godo and Torra, 2000; Ahn & Choi, 2012) as well as mixed ordinal and continuous data. Considering this, the findings of the present article can potentially be helpful in different domains of social studies.

The article proceeds as follows. Section 2 presents the fundamental aspects of building composite indicators, highlighting the issues of spatial heterogeneity and non-compensatory aggregation. The OGWA method is presented in Section 3, and simulations illustrating the properties and accuracy of the OGWA are reported in Section 4. Section 5 focuses on an empirical application to public urban infrastructure, with details on the study area, selection, and data collection criteria, the construction steps of the composite indicators, and the results. Conclusions, including research limitations, future work, and final considerations, are presented in Section 6.

## Literature review

Nardo et al. (2005) and OECD (2008) extensively treat building composite indicators' fundamental concepts and stages. These studies highlight that the most important sources of uncertainty related to building composite indicators occur during scale normalization, weighting, and aggregation of sub-indicators. This conclusion is confirmed in several subsequent studies (Dialga and Le Giang, 2017; Becker et al., 2017b).

Normalization is a mathematical operation that transforms the different sub-indicator scales into a single, comparable scale. The main normalization techniques transform the data for the interval [0,1] by the max–min function and for mean zero and standard deviation one by the z-scores function (Carrino, 2017; Walesiak, 2018; Cinelli et al., 2021).

Researchers have considerable concerns about sub-indicator weighting (Becker et al., 2017a, 2017b; Bruggemann and Carlsen, 2021). According to these scholars, weighting is a procedure that allows relative importance to be assigned to sub-indicators. The weights representing this relative importance can be obtained through the data-driven or participatory weighting approach. Data-driven weights are obtained statistically from the data and are free from expert judgment bias or evaluation errors influenced by personal experiences, ideologies, and viewpoints. Participatory weights are obtained from expert opinions and are free from statistical determinations that could produce weights inconsistent with the multidimensional phenomenon's conceptual structure.

There is a vast literature on aggregation schemes, which provides a good description and application examples of these methods (Munda, 2012; Greco et al., 2019, 2021; El Gibari et al., 2021; Blancas and Lozano-Oyola, 2022). According to this literature, aggregation is the procedure that combines the normalized scaled and weighted sub-indicators into a one-dimensional measure to represent the multidimensional concept.

### Non-compensability issue in composite indicators

Sub-indicators can be combined through compensatory or non-compensatory aggregation schemes. The first one is the most employed and can be defined as a linear aggregation in which sub-indicators with poor and above-average performance compensate each other. The second one is less used but more advantageous when a poor performance in one sub-indicator is not conceptually compensable by an above-average performance in another sub-indicator.

Examples of compensatory aggregation schemes are the arithmetic mean function, the Principal Components Analysis (PCA), or the Benefit-of-the-Doubt (BoD) in DEA-like models. In contrast, examples of non-compensatory aggregation schemes are the mean-min function (Casadio Tarabusi & Guarini, 2013) and BoD extensions with weight constraints or penalties according to the different mix of simple indicators (Vidoli and Mazziotta, 2013) or an explicit or implicit preference structure between simple indicators (Fusco, 2015; Vidoli, Fusco, and Mazziotta, 2015; Fusco, 2023).

### Spatial heterogeneity in composite indicators

The operationalization of the heterogeneity concept in composite indicators is closely linked to the weighting of the sub-indicators. The heterogeneity concept is considered when the weight of the sub-indicators varies according to the unit of analysis. Three classes of methods are capable of operationalizing the heterogeneity concept[1]:

1. BoD and OWA that operationalize the heterogeneity concept of the analyzed units disregarding the influence of distance in defining the sub-indicators' weights (Libório, Martinuci, Ekel, et al., 2022; Libório, Martinuci, Machado, et al., 2022);
2. Spatial BoD (Fusco, Vidoli, and Sahoo, 2018) and the Geographically Weighted Principal Components Analysis (GWPCA – Cartone and Postiglione, 2021) that operationalize the heterogeneity considering the latitude and longitude of units in defining, providing statistical information on the spatial context;
3. Multiscale analysis that are based on aggregating scale-weighted sub-indicators (Doeffinger and Hall, 2021) and neglect the mutual influence of the units of analysis in defining the sub-indicators' weights subjectively.

Literature review[2] shows that scholars consider only the second class of methods for operationalizing the concept of spatial heterogeneity. None of the main works on BoD (Cherchye, Moesen, and Van Puyenbroeck, 2004; Nardo et al., 2005; Cherchye et al., 2007; OECD, 2008) consider this method compatible with the spatial heterogeneity concept. Only Libório, Martinuci, Ekel, et al. (2022) and Libório, Martinuci, Machado, et al., 2022 argue that the BoD definition of weights operationalizes the concept of spatial heterogeneity. On the contrary, researchers agree that the Spatial BoD and the GWPCA operationalize the concept of spatial heterogeneity. Seven out of ten articles mention the Spatial BoD (Fusco, Vidoli, and Sahoo, 2018; Fusco, Vidoli, and Rogge, 2020; Artelaris, 2022) and the GWPCA (Saib et al., 2015; Trogu and Campagna, 2018; Cartone and Panzera, 2021; Cartone and Postiglione, 2021) simultaneously with the terms "composite indicators" and "spatial heterogeneity"[1]. Only Doeffinger and Hall (2021) mention the Multiscale analyses among the 10 articles[1] that simultaneously mention the terms "composite indicators" and "spatial heterogeneity". Besides, none of the reference works in the composite indicator's literature (Nardo et al., 2005; OECD, 2008) recognize Multiscale analysis as a method of building composite indicators.

Spatial BoD identifies the sub-indicators' weights through "spatial conditioning", that is, by comparing nearby units to consider external factors common to neighboring units, which directly or indirectly affect the sub-indicators in constructing the composite indicator. This process assigns greater weights to the above-average performance sub-indicators in each neighborhood, that is, locally, resulting in a composite indicator able to consider local territorial differences (Fusco, Vidoli, and Sahoo, 2018; Fusco, Vidoli, and Rogge, 2020).

GWPCA defines the sub-indicators' weights to maximize the variance of the sub-indicators in each spatial unit. This maximization assigns greater weights to the most correlated sub-indicators. Thus, the composite indicator built by the GWPCA reflects the most similar sub-indicators for each spatial unit (Harris, Brunsdon, and Charlton, 2011; Demšar et al., 2013).

Spatial BoD and GWPCA have at least two limitations in common. First, the sum of the weights of the sub-indicators of the spatial units is different from one. Methods popularly used in building composite indicators have weights that add up to one (El Gibari, Gómez, and Ruiz, 2019). This difference in the sum of weights makes it impossible to compare composite indicators built by the Spatial BoD and GWPCA with other composite indicators (Nardo et al., 2005).

Second, the Spatial BoD and GWPCA aggregate the sub-indicators by the arithmetic mean, allowing maximum compensation between poor and above-average performance sub-indicators. This compensatory aggregation removes the relative importance property of the weights (Greco et al., 2021) and distorts the conceptual function of the Spatial BoD and GWPCA weights.

That is why the full consideration of spatial heterogeneity in composite indicators depends concomitantly on the attribution of differentiated weights of a sub-indicator to each unit and on the guarantee that these weights assume their function of relative importance through the non-compensatory aggregation of the sub-indicators.

**Spatial heterogeneity and non-compensatory aggregation in composite indicators**

The literature shows a limited number of studies that simultaneously address the heterogeneity of sub-indicator weights through non-compensatory aggregation. A broader search[3] shows that only Fusco, Vidoli, and Rogge (2020) mention the terms "composite indicators", "non-compensatory aggregation," and "spatial heterogeneity" simultaneously. However, the multicriteria decision-making literature offers a method that allows simultaneous consideration of heterogeneity and non-compensatory aggregation in composite indicators and is suitable for extension to the case of spatial heterogeneity: the OWA (Yager, 1988).

An OWA operator of dimension $n$ and function $[0,1]^n \rightarrow [0,1]$ (Pedrycz, Ekel, and Parreiras, 2011) aggregates a given number of normalized sub-indicators $a_1, a_2, \dots, a_n$ as follows:

$$OWA\left(a_1, a_2, \dots, a_n\right) = \sum_{i=1}^{n} w_i b_i, \tag{1}$$

where $b_i$ is the best-performing sub-indicator between the sub-indicators $a_1, a_2, \dots, a_n$ and the set of weights $w_1, w_2, \dots, w_n$ meets the condition $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Therefore, each $w_i$ weight is assigned to a position that corresponds to the performance of a sub-indicator and not to a particular sub-indicator. The definition of which sub-indicators are weighted with $[0,1]$ depends on the performance of each sub-indicator in their respective spatial units and the Orness Degree (Ekel, Pedrycz, and Pereira Júnior, 2020; Silva et al., 2020). The Orness Degree or $\theta$ corresponds to the degree of optimism or risk adopted by the decision maker (Yager, 1988). The decision maker can adjust $\theta$ to emphasize the positive aspects of the phenomenon and reflect an optimistic decision. In this case, worse-performing sub-indicators are disregarded, biasing the composite indicator scores upward. The emphasis on the negative aspects reflects a pessimistic stance by the decision maker, whereby higher performance sub-indicators are not aggregated, skewing the composite indicator scores downward.

The adjustment of $\theta$ allows biasing the results upward (emphasis on positive aspects) or downward (emphasis on negative aspects). The following expression gives its definition:

$$\theta = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i \tag{2}$$

This flexible weighting approach is consistent with the heterogeneity concept, as the weights of sub-indicators may vary between units, making it possible to capture the particularities of each unit.

Additionally, the fuzzy linguistic quantifiers "More than $j$" and "At least $j$" criteria allow the decision maker to regulate the compensation levels between sub-indicators and to emphasize the positive or negative aspects of the multidimensional phenomenon (Bernardes et al., 2022).

The "More than $j$ criteria" quantifier emphasizes the negative aspects ($OWA^-$ in the rest of the article) by preventing sub-indicators with poor performance from being compensated by sub-indicators with above-average performance (Pereira Jr. et al., 2016). The adjustment of the degree of emphasis is carried out by setting $j$, which is reflected through $\alpha$ in the value of $\theta$ using the following expression:

$$Q(r) = \begin{cases} 0 & \text{if} \quad 0 \leq r \leq \alpha \frac{r}{1-\alpha}, \\ 1 & \text{if} \quad \alpha < r \leq 1 \end{cases} \tag{3}$$

where $Q(r)$ is a fuzzy set reflecting the level at which the portion of $r \in [0,1]$ sub-indicators meets the concept of the multidimensional phenomenon given by $Q$, which must satisfy three conditions: $Q(0) = 0$, $Q(1) = 1$, and if $r_1 > r_2$, then $Q\left(r_1\right) > Q\left(r_2\right)$, $\alpha = j/n$, and $\theta = (1-\alpha) \div 2$.

The composite indicator scores that result using Equation (3) are biased downward, which is advantageous in social and environmental analysis because the trade-off between sub-indicators of poor and above-average performance masks areas of vulnerability (Zabihi et al., 2019; Libório, Martinuci, Ekel, et al., 2022).

In contrast, the "At least $j$ criteria" quantifier emphasizes the positive aspects ($OWA^+$ in the rest of the article) of the multidimensional phenomena (Pedrycz, Ekel, and Parreiras, 2011) by the following expression:

$$Q(r) = \begin{cases} \frac{r}{\alpha} & \text{if} \quad 0 \leq r \leq \alpha \\ 1 & \text{if} \quad \alpha < r \leq 1 \end{cases}, \tag{4}$$

where $\alpha = j/n$ and $\theta = 1 - (\alpha \div 2)$.

The composite indicator scores constructed from this quantifier are upward biased. This upward bias, similar to the BoD, is an advantageous property for analyses comparing the economic performance of countries as it avoids disputes over the sub-indicator weighting scheme (Cherchye et al., 2007).

After choosing the appropriate linguistic quantifier for the problem, it is possible to set the OWA operator weights by applying the following expression:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad i = 1, \ldots, n. \tag{5}$$

As presented, the OWA assigns different weights to a sub-indicator according to the performance of each unit of analysis, aggregates the sub-indicators in a non-compensatory way, and allows emphasizing the positive or negative aspects of the multidimensional phenomenon.

In the literature, several articles constructing composite indicators through OWA have been proposed (Rocco et al., 2009; Badea, Tarantola, and Bolado, 2011; Hernández-Perdomo, Rocco, and Ramirez-Marquez, 2016; Almoghathawi et al., 2017; Libório, Martinuci, Ekel, et al., 2022; Parsons et al., 2021; Bernardes et al., 2022; Shu et al., 2022). However, there is no recognition of OWA as a method for operationalizing the concept of spatial heterogeneity.

Therefore, the consideration of the concept of spatial heterogeneity in the OWA through the incorporation of the influence of the distances between spatial units in the definition of the weights of the sub-indicators is the basis of the new method proposed and implemented in Section 3 of this research: the *Ordered Geographically Weighted Averaging* (OGWA).

## The OGWA method

In Trogu and Campagna's (2018) stream, the need to apply different and distinct weights to different spatial units, aiming to account for local variations (non-stationary spatial conditions) on the latent contextual conditions, is also realistic in the OWA case.

Therefore, to consider the influence of spatial heterogeneity (variability in space) in the composite indicator building, by following Fotheringham et al. (2003) idea, the OWA can be modified as follows:

$$OGWA\left(a_1, a_2, \ldots, a_n\right) = \sum_{i=1}^{n} w_i Sb_i, \tag{6}$$

where $Sb_i$ is the best-performing spatially weighted sub-indicator among the spatially weighted sub-indicators $Sa_i, i = 1, \ldots, n$, and $S$ is a matrix of spatial weights of dimension $m \times m$ ($m$ is the number of spatial units), where each element $s_{uv}$ in $S$ represents the distance between the spatial units $u$ and $v$, where $u, v = 1, 2, \ldots, m$.

The OGWA formulation, concerning OWA, considers spatial heterogeneity assuming that observations close to the spatial unit $u$ (neighborhood) have more influence over the weights than more remote observations. Therefore, the geographical weights decrease[4] with the distance to the spatial unit $u$ and $v$ and can be determined, without loss of generality, by using a kernel function, for example, the Gaussian[5] as in Cartone and Postiglione (2021):

$$s_{uv} = \exp\left(-\frac{1}{2}\left(\frac{d_{uv}}{\gamma}\right)^2\right), \tag{7}$$

where $d_{uv}$ is the geographical distance between the units $u$ and $v$, and $\gamma$ is the bandwidth.[6]

In particular, the spatial weights matrix obtained from Equation (7) is also called the spatial continuity matrix (Brandão and Brandão, 2017). This spatial continuity matrix is associated with the influence and mutual relationship between spatial units (Fotheringham and Brunsdon, 1999).

Thus, the OGWA method in Equation (6) makes it possible to capture local latent contextual conditions in constructing the composite indicator, thus emphasizing local positive and negative aspects of the analyzed phenomena, which might be less evident or masked.

Based on this proposed approach, a Monte Carlo significance test, in the stream of Fotheringham et al. (2002) and Harris, Brunsdon, and Charlton (2011), is adapted to our procedure to investigate spatial non-stationarity and justify the use of OGWA instead of OWA. It tests whether the results are significantly different from those obtained by randomly varying the data (taking sample locations in pairs), that is, whether the results vary significantly across space.

This is done by comparing the actual (observed) standard deviation of the results with the standard deviation of several random distributions of the data, assuming that each pattern in the data occurs by chance and that each permutation of the data is equally likely to occur. Therefore, the null hypothesis is the spatial stationarity of OGWA scores (whatever the permutation of the data, the variability in scores does not change) and the alternative hypothesis is the spatial non-stationarity of OGWA scores (different permutations of the data give different variability in scores).

The proposed test proceeds as follows:

1. Using the sample data, calculate the actual standard deviation (SD) of OGWA;
2. Randomly select a permutation of data;
3. Calculate the SD of the OGWA by using the randomly generated data;
4. Repeat steps 2 and 3, for example, 999 times (the more, the better);
5. Rank the 999 simulated OGWA's SDs and the actual OGWA's SD (the existence of significant spatial variation in the calculated OGWA scores is related to its position in this ranked distribution);
6. Find out where the actual SD of the OGWA is on this ranked scale of 1000 values;
7. If the actual SD lies in the upper or lower 2.5% tail of this ranking distribution, then the actual OGWA's SD is "significantly" different (at the 95% level) from the simulated OGWA's SDs.

Finally, an important issue to discuss is the bandwidth choice for OGWA. The optimal bandwidth can be selected using a *leave-one-out* cross-validation (CV) procedure (Harris, Brunsdon, and Fotheringham, 2011). Leave-one-out CV removes each observation (e.g., $u$) one at a time for a potential bandwidth and provides a measure of how well the method with that bandwidth determines the value of each removed data point. The differences between values are evaluated by using the following Root Mean Square Error (RMSE) formula:

$$\text{RMSE} = \sqrt{\left(\frac{1}{m}\right)\sum_{u=1}^{m}\left(Sb - Sb_{-u}\right)^2} \tag{8}$$

Note that the bandwidth is found before the OGWA score is computed. This is to avoid including another type of heterogeneity given by the OGWA operator weights $w_i$ and to focus only on spatial latent contextual conditions.

The method and the elaborations have been implemented in R software and will be included in the R package Compind (Vidoli and Fusco, 2023).

## A spatial point-pattern simulation study to evaluate OGWA properties

In order to test and demonstrate the properties and accuracy of the OGWA method, a spatial point-pattern simulation has been performed. In order to stress the method and show how it performs in case of presence of clusters[7], the chosen Data Generating Process (DGP) is a Matern cluster process (Matern, 1986) with a constant density of $\mu = 300$ points per area (intensity) and a radius of 0.2 (Fig. 1). Note that clusters are overlapped to account for the influence of all points in the data.

Once the patterns of spatial points are determined, in order to appreciate the OGWA data sorting and comparison process, three simple indicators (namely I1, I2, and I3) are constructed in such a way as to obtain spatial clusters with homogeneous observations for high values of one indicator and low values of another (Fig. 2). In particular, with random differences between units with a mean of zero and a constant variance of 0.02, clusters A and D have high values for indicators I1 and I3, cluster B has high values for I2, and cluster C has medium-high values for all the indicators.

Then, OGWA was implemented for different values of the bandwidth $\gamma$ from 0.1 to 1 by 0.01 and fixed "More than $j$" and "At least $j$" criteria equal to two indicators.

The results for $OGWA^{-}$ and $OWA^{-}$ are plotted in Figs. 3 and 4. Fig. 3 is a lines plot (Dykes and Brunsdon, 2007) where the bandwidths are placed on the x-axis and the $OGWA^{-}$ and $OWA^{-}$ on the y-axis. Each line corresponds to the results of a single unit obtained at different bandwidths, and the color is given by the $OGWA^{-}$ values. $OWA^{-\prime}$s results are plotted at $\gamma = 1$ to show that $OGWA^{-}$ converges to $OWA^{-}$ when the bandwidth is large.

Fig. 4 shows the maps of the pattern of the results as the bandwidth changes (for simplicity, the results are presented with bandwidths 0.1, 0.5, 1), exhibiting that at lower bandwidths, units are compared locally, and as the bandwidth increases, the comparison becomes more global, tending toward the $OWA^{-}$ pattern.

For ease of reading, since the same findings in terms of convergence of $OGWA^{+}$ to $OWA^{+}$ are observed, Figs. A1 and A2 for $OGWA^{+}$ can be found in the Appendix A.

## Real data: Mapping the spatial heterogeneity of public urban infrastructure through OGWA

### Public urban infrastructure

Urban infrastructure is a multidimensional concept that reflects visible urban structures such as roads and buildings and invisible systems such as public services, culture, and the environment

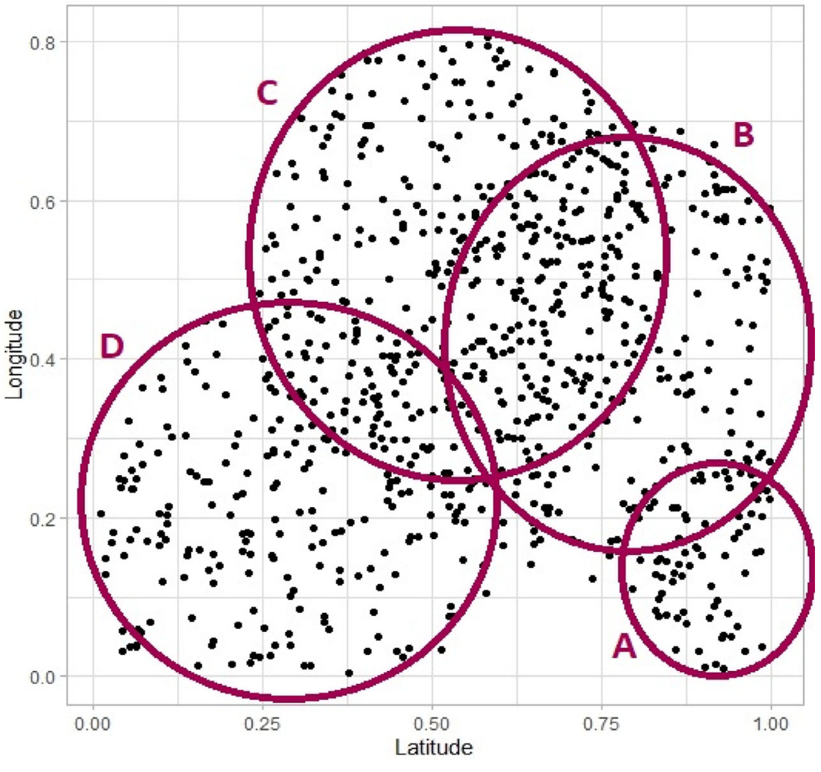


**Figure 1.** Simulated matern cluster process.

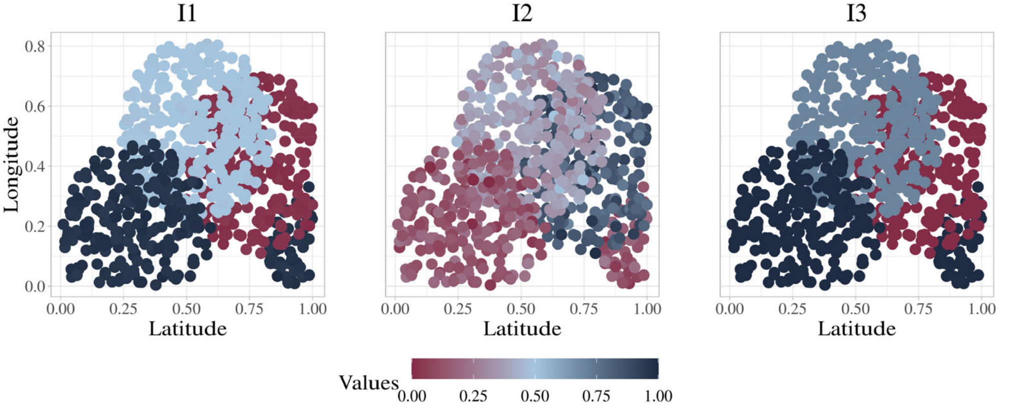


**Figure 2.** Simulated data: simple indicators values.

(Brandão and Brandão, 2017). The literature on urban infrastructure is quite broad, encompassing the disciplines of urban studies, geography, sociology, philosophy, and economics (Steele and Legacy, 2017) and addressing issues of public investment, environmental impacts, climate change, urban infrastructure, and social vulnerability (Ferrer, Thomé, and Scavarda, 2018).

In particular, the results of this research are exclusively related to physical infrastructure and public services to the extent that the definition of urban infrastructure includes private construction (Pandey, Brelsford, and Seto, 2022).

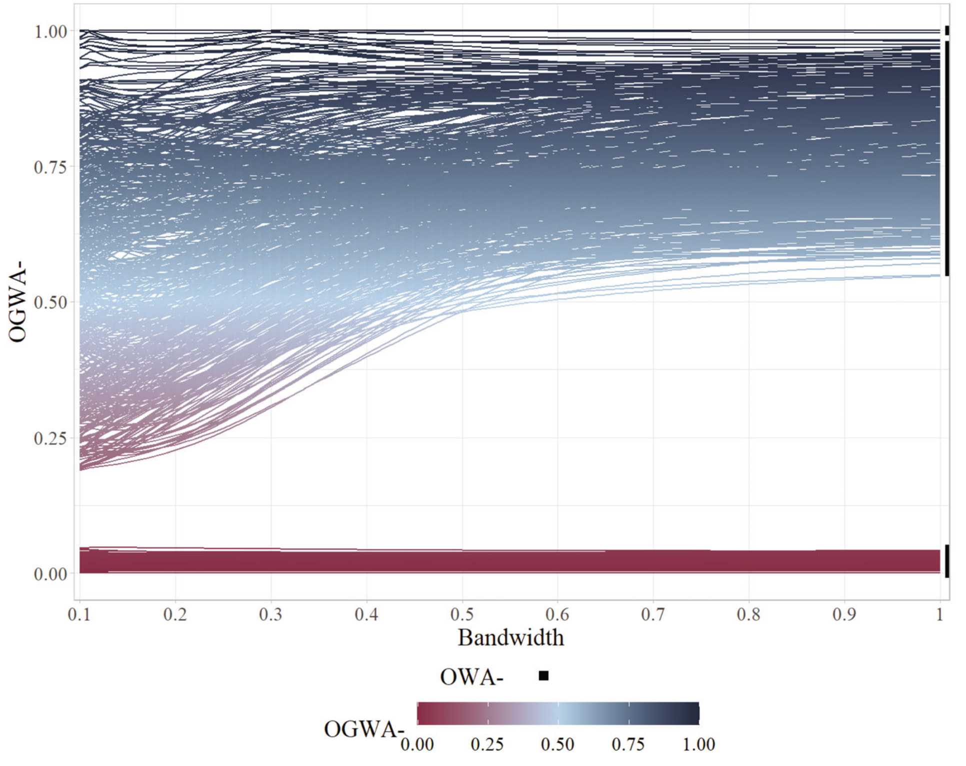


**Figure 3.** Simulated data: Scalogram of $OGWA^{-}$ and $OWA^{-}$.

Narbón-Perpiñá and De Witte (2018a, 2018b) provide a complete literature review on physical infrastructure and public services in cities. Public service infrastructure includes waste collection, sewerage systems, water supply, and electricity. Urban public physical infrastructure can be divided into two types. The first type of public physical infrastructure is related to the provision of services such as administration, recreation, health, education, social, and public safety. The second type of public physical infrastructure includes roads, traffic lights, and stormwater networks.

The fundamental difference between these two types of physical infrastructure is related to their spatial representation. Hospitals, schools, and police stations are geographic points with latitude and longitude, whereas polygons represent roads and stormwater networks with complex geometries and have multiple geographic coordinates.

This distinction is particularly relevant in this research, as polygons are continuous spatial representations compatible with the principle of spatial continuity in which the spatial behavior of the phenomenon is not defined by discrete boundaries (Fotheringham and Brunsdon, 1999; Brandão and Brandão, 2017).

From an application point of view, the consideration of heterogeneity and spatial continuity has been ignored in the construction of composite indicators of urban infrastructure. Current approaches are limited to the compensatory aggregation of physical infrastructure and services such as, for example, education, health, roads, social programs, sports and culture, waste collection, and water supply in a composite indicator (Afonso & Fernandes, 2006, 2008; Nijkamp and Suzuki, 2009; Yusfany, 2015).

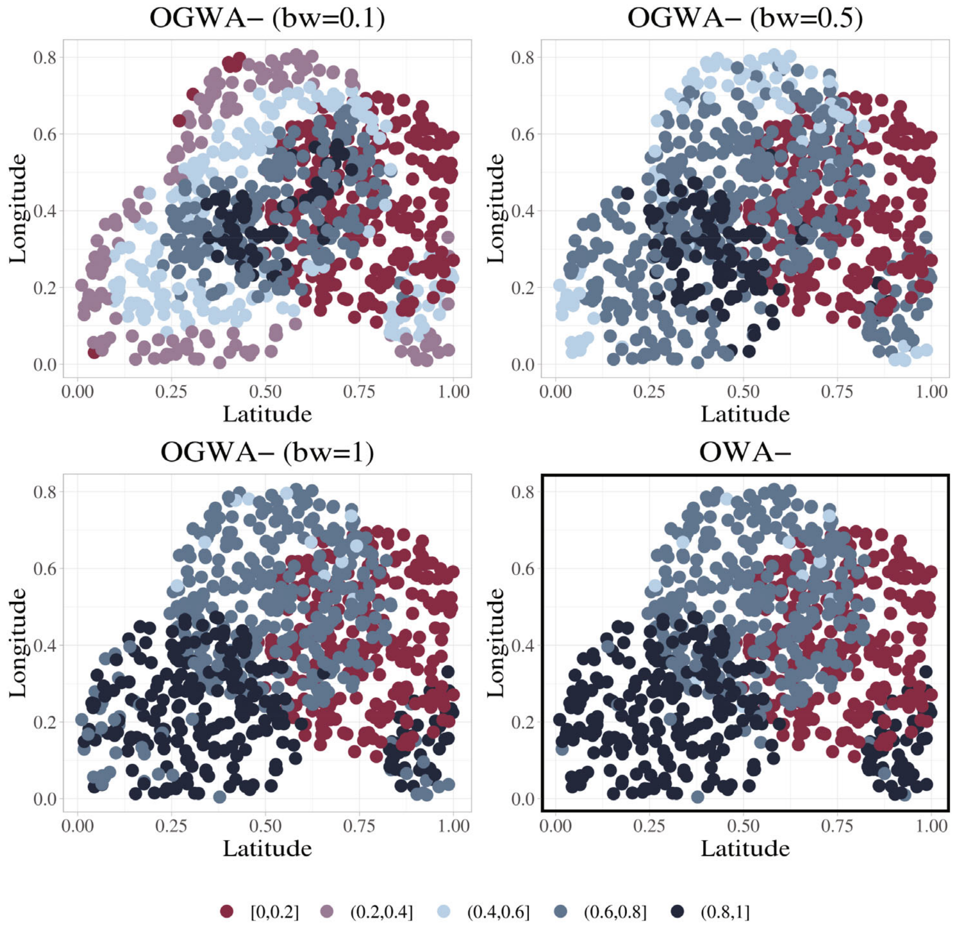


**Figure 4.** Simulated data: $OGWA^{-}$ and $OWA^{-}$ maps by bandwidth.

Therefore, the simultaneous consideration of multidimensionality, non-compensability, heterogeneity, and spatial continuity is an innovative approach that fills the gap for studies that provide better representations of urban public infrastructure (Narbón-Perpiñá and De Witte, 2018a, 2018b).

Consequently, the results of this approach have a high degree of applicability in urban planning (Coutinho-Rodrigues, Simão, and Antunes, 2011) and can be used to optimize public budgets (D'Inverno, Vidoli, & De Witte et al., 2023), reduce inequalities in the distribution of urban infrastructure (Pandey, Brelsford, and Seto, 2022), prioritize investments in infrastructure, taking into account the multidimensional nature of the problem (Ziara et al., 2002). As an effect, it is expected to increase the efficiency in the management of public resources (Narbón-Perpiñá and De Witte, 2018a, 2018b) and stimulate the economic development of the city through the economies of scale, specialization, and economies of agglomeration generated by public infrastructure (Collier and Venables, 2016).

## Study area, selection, and data collection

This research maps the urban public infrastructure of São Sebastião do Paraíso, a city located in the southeastern region of Brazil. The city's urban infrastructure is distributed among the

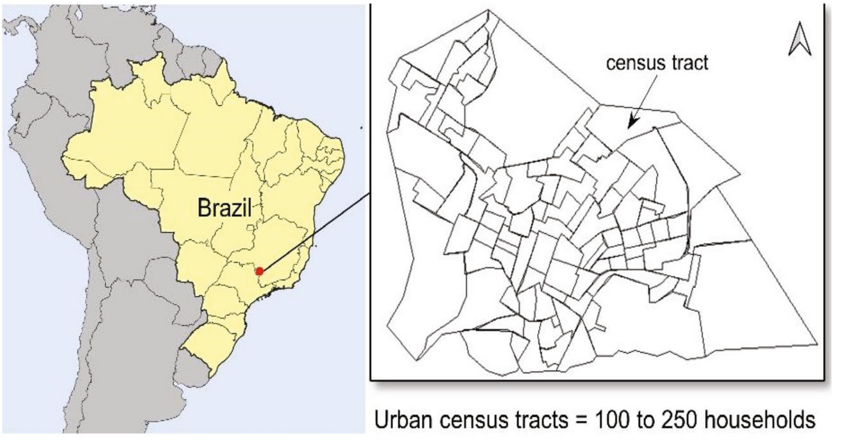


**Figure 5.** Location map and urban census tracts of São Sebastião do Paraiso.

94 urban census tracts shown in Fig. 5 (IBGE, 2010b). Census tracts are the smallest unit of census information collection and are the most reliable data source on Brazilian cities' urban infrastructure (Libório, da Silva Martinuci, et al., 2020; Libório, Laudares, et al., 2020).

The literature shows the existence of different frameworks for representing urban infrastructure and that the selection of sub-indicators should consider the availability of data and the identification of services provided to the population according to the local government (Narbón-Perpiñá and De Witte, 2018a, 2018b).

In this sense, the sub-indicators in the survey "Surroundings of domiciles" (IBGE, 2010a) were selected. This survey is based on urban infrastructure managed directly or indirectly by local governments and provides information on the number of households in the census area served by one of the two types of urban public infrastructure listed below:

- Physical infrastructure: S-1. Street identification, S-2. Public lighting, S-3. Asphalt on the street, S-4. Sidewalk, S-5. Curbstone, S-6. Sidewalk curb ramps, S-7. Afforestation, and S-8. Rainwater harvesting (storm drain).
- Service infrastructure: S-9. Sewerage system, S-10. Water distribution network, and S-11. Waste collection service.

Data for the urban public infrastructure sub-indicators were collected from the latest available demographic census (IBGE, 2010b). The proportion of households served by public urban infrastructure was calculated considering the total number of households in each census tract. Thus, the sub-indicator scores vary within the [0.1] interval and reflect the performance of the sub-indicator in each census tract. The descriptive statistics of the proportion of households served by the 11 selected urban public infrastructure sub-indicators are presented in Table 1.

### Composite indicator building

Six composite indicators have been constructed to analyze the benefits of considering spatial heterogeneity, non-compensatory aggregation, and highlighting the positive and negative aspects of the phenomenon in the representation of urban public infrastructure. The steps of the procedure were as follows:

1. Normalization of the sub-indicators for the interval [0, 1] through the function (observed value – minimum value)/(maximum value – minimum value);

**Table 1.** Descriptive Statistics

| | S-1 | S-2 | S-3 | S-4 | S-5 | S-6 | S-7 | S-8 | S-9 | S-10 | S-11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Average | 0.82 | 0.98 | 0.95 | 0.92 | 0.96 | 0.96 | 0.79 | 0.18 | 0.98 | 0.99 | 0.94 |
| Minimum | 0.21 | 0.78 | 0.32 | 0.21 | 0.40 | 0.20 | 0.30 | 0.00 | 0.80 | 0.82 | 0.61 |
| Maximum | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.91 | 1.00 | 1.00 | 1.00 |
| SD | 0.17 | 0.04 | 0.11 | 0.14 | 0.09 | 0.10 | 0.16 | 0.23 | 0.04 | 0.02 | 0.09 |

Note: N = 94 observations.

2. Construction of the Euclidean distances matrix between the census tracts of the city by the latitude and longitude of the centroids of the respective polygons;
3. Definition of the optimal bandwidth (value of $\gamma$ of the function (7)) through the leave-one-out cross-validation method (Harris, Brunsdon, and Fotheringham, 2011);
4. Building of the matrix of weights (spatial continuity) through the application of function (7) based on the matrix of distances;
5. Adoption of an intermediate degree of emphasis on negative aspects ($\theta = 0.73$) applying $j = 8$ in the function (3);
6. Adoption of an intermediate degree of emphasis on positive aspects ($\theta = 0.27$) applying $j = 3$ in the function (4);
7. The construction of the six composite indicators of urban public infrastructure is detailed below:

   7.1. $OWA^{-}$ that disregards spatial heterogeneity and emphasizes the negative aspects, applying the results of steps 1 and 5 to functions (1) and (2);
   7.2. $OWA^{+}$ disregards spatial heterogeneity and places emphasis on the positive aspects, applying the results of steps 1 and 6 to functions (1) and (2);
   7.3. $OWA^{=}$ that disregards spatial heterogeneity and non-compensation between sub-indicators, applying the results of step 1 with a $j = 11$ to functions (1) and (2);
   7.4. $OGWA^{-}$ that considers spatial heterogeneity and emphasizes the negative aspects, applying the results of steps 1–5 to function (6);
   7.5. $OGWA^{+}$ considers spatial heterogeneity and places emphasis on the positive aspects, applying the results of steps 1–4 and 6 to function (6);
   7.6. $OGWA^{=}$ considers spatial heterogeneity and disregards non-compensation between sub-indicators, applying the results of steps 1–5 with $j = 11$ to function (6).

An important step, as discussed in Section 3 and highlighted in Section 4 is the bandwidth choice for OGWA on the current data. In fact, in Section 4, it was seen that small bandwidths lead to greater spatial variability of results, while large bandwidths lead to results closer to OWA.

The optimal bandwidth, equal to 0.03, has been selected using the procedure exposed in Section 3. Fig. 6 shows the leave-one-out CV function (in terms of RMSE – Equation 8) for different bandwidths, with a global minimum at 0.03.

Then, the geographical weights were obtained using the GWmodel R package (Gollini et al., 2015; Lu et al., 2014), calculating the distance between any GW model calibration points and the data points and using the chosen optimal bandwidth.

Finally, the composite indicators are compared to analyze the benefits of OGWA in increasing the accuracy and amount of information on problems of a multidimensional nature.

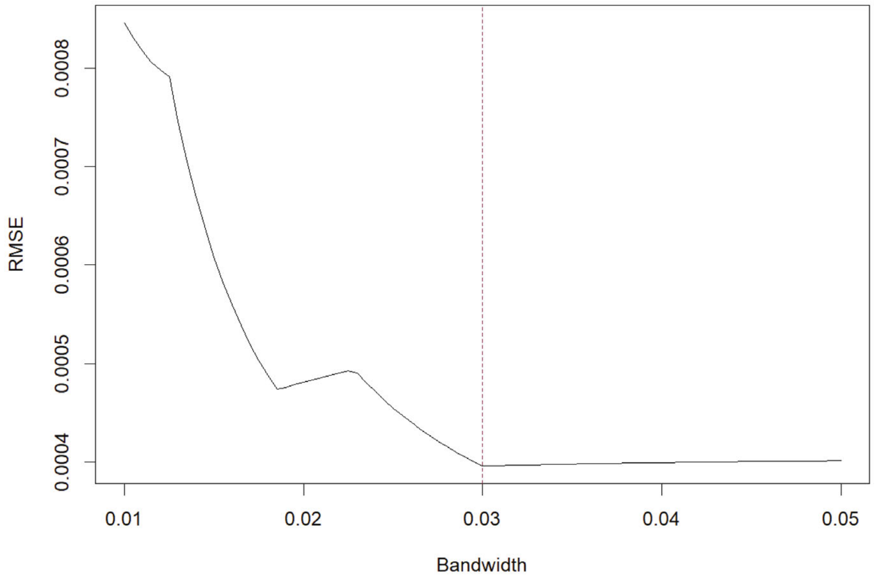


**Figure 6.** Leave-one-out cross-validation (CV) function.

**Table 2.** Statistics of Composite Indicator Scores of Urban Public Infrastructure

| | $OWA^-$ | $OWA^+$ | $OWA^=$ | $OGWA^-$ | $OGWA^+$ | $OGWA^=$ |
|---|---|---|---|---|---|---|
| Average | 0.76 | 0.95 | 0.82 | 0.65 | 0.83 | 0.71 |
| Minimum | 0.29 | 0.57 | 0.41 | 0.22 | 0.45 | 0.33 |
| Maximum | 0.92 | 1.00 | 0.94 | 0.87 | 0.99 | 0.90 |
| SD | 0.12 | 0.07 | 0.09 | 0.14 | 0.13 | 0.13 |

## Results and discussions

Considering spatial heterogeneity in the representation of urban infrastructure builds composite indicators with scores 11% lower on average. This result indicates a negative relationship between a census tract's urban public infrastructure and neighboring areas' public infrastructure (Table 2).

The standard deviation of the composite indicator scores $OGWA^-$, $OGWA^+$, and $OGWA^=$, on average, is 1.4 times greater than that of the composite indicators $OWA^-$, $OWA^+$, and $OWA^=$, indicating a better differentiation of the urban public infrastructure when spatial heterogeneity is considered. This finding is even more evident in Fig. 7 where the composite indicators $OGWA^-$, $OGWA^+$, and $OGWA^=$ show a greater variation of colors (scores), which helps to distinguish between areas with deficient (census tracts colored in dark red) and satisfactory urban public infrastructure (census tracts colored in dark blue).

The maps in Fig. 7 also show how the emphasis on the positive and negative aspects of the multidimensional phenomenon increases the informational power over the spatial dynamics of the urban public infrastructure. The mapping of urban public infrastructure by $OWA^-$ and $OGWA^-$ makes it possible to identify priority areas for public investment. This identification is especially useful in decision-support systems that prioritize investments in public infrastructure (Ziara et al., 2002) in a perspective of rebalancing and equalizing the territory.

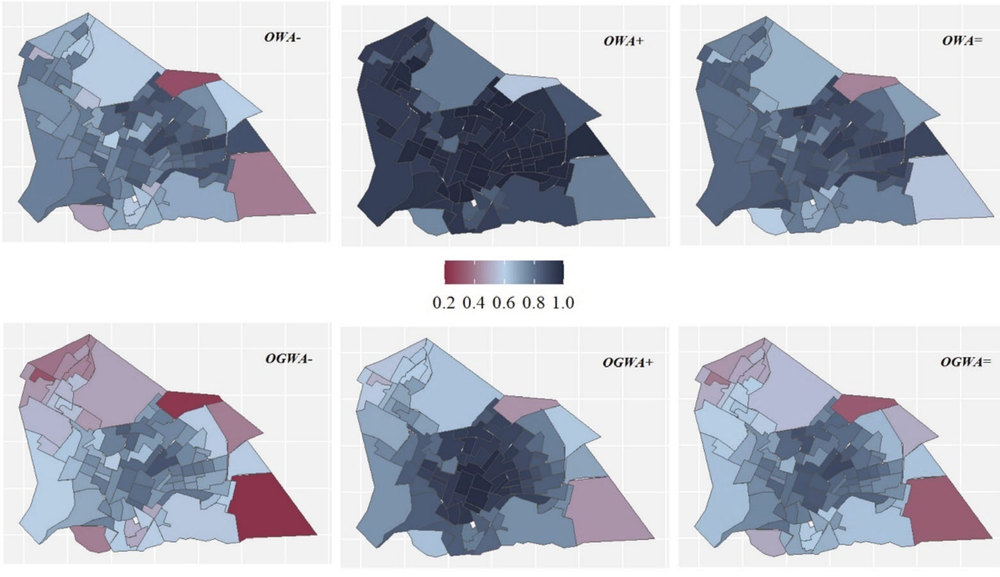


**Figure 7.** Urban infrastructure mapping of composite indicators $OWA^{-}$, $OWA^{+}$, $OWA^{=}$, $OGWA^{-}$, $OGWA^{+}$, and $OGWA^{=}$.

The incorporation of spatial heterogeneity in $OWA^{-}$ reflected in $OGWA^{-}$ offers public policy planners a better distinction between areas with a deficit of urban public infrastructure. In addition, the consideration of the negative effect of the neighborhood on the representations of the public infrastructure of each area through the $OGWA^{-}$ signals that the number of city areas with deficient infrastructure is greater when spatial heterogeneity is considered. This is a significant finding, as it impacts urban planning, especially in defining the public budget. In particular, the $OGWA^{-}$ indicates that the public budget for infrastructure investments should be greater to reduce inequalities in city distribution of public infrastructure.

Similarly, the mapping of urban public infrastructure by $OGWA^{+}$ favors the distinction of areas that most benefit from the appreciation of property and the benefits generated by urban public infrastructure. This better distinction makes it possible for governments to adjust property taxes, considering the urban public infrastructure available in each area.

From an urban planning point of view, analyzing the positive and negative aspects of urban public infrastructure offers valuable information for prioritizing investments and adjustments in property tax collection (Coutinho-Rodrigues, Simão, and Antunes, 2011). Besides, considering spatial heterogeneity favors this analysis even more, making it possible to adjust the collection of property taxes, generating resources to reduce inequality in the distribution of public infrastructure in cities (Pandey, Brelsford, and Seto, 2022).

From a statistical point of view, the need to consider spatial heterogeneity is also confirmed by the test of non-stationarity (1,000 replications) carried out on the results of $OGWA^{-}$, $OGWA^{+}$, and $OGWA^{=}$ (Fig. 8).

Plots in Fig. 8, and the corresponding $P$-value, suggest that $OGWA^{-}$'s (subgraph a), $OGWA^{+}$'s (subgraph b), and $OGWA^{=}$'s (subgraph c) results are non-stationary with a $P$-value $<0.001$, 0.003 and $<0.001$, respectively.

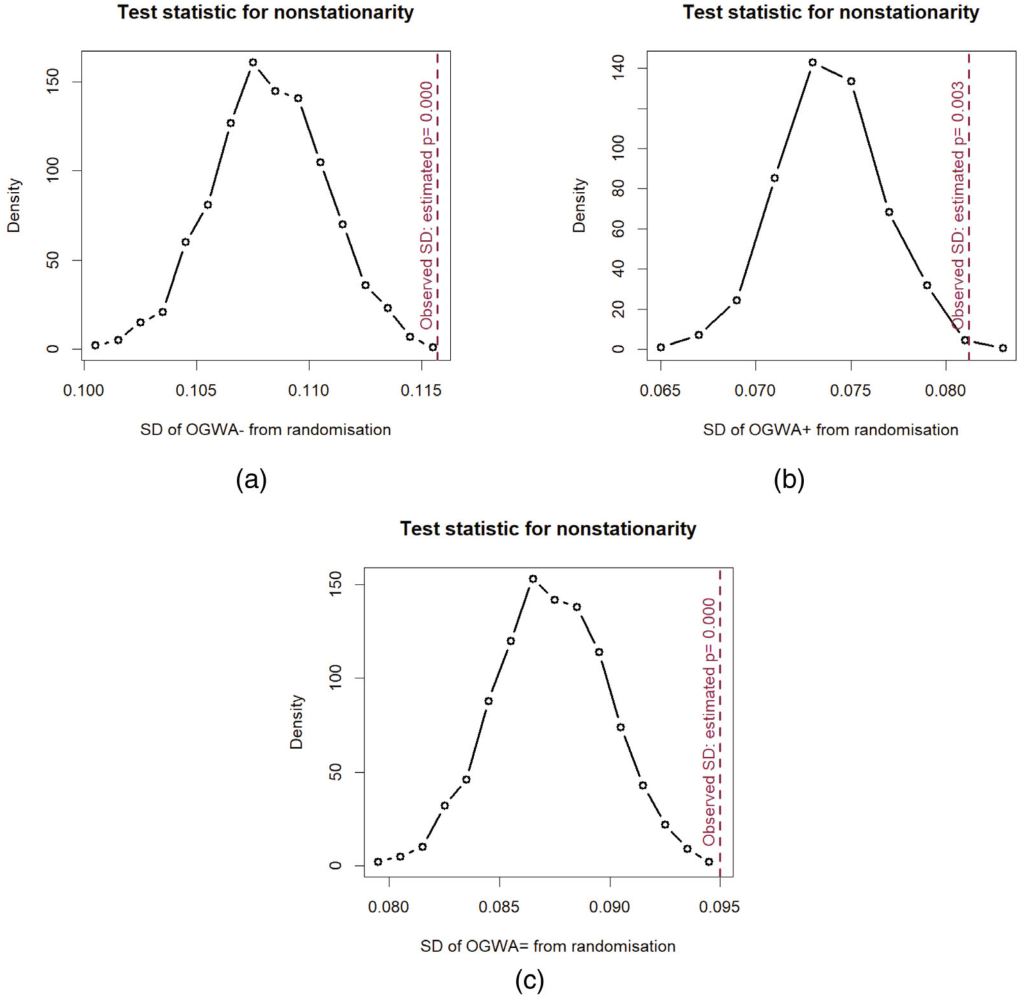


**Figure 8.** Non-stationarity test of composite indicators (a) $OGWA^{-}$, (b) $OGWA^{+}$, and (c) $OGWA^{=}$.

## Conclusions

This article extends the literature on the construction of composite indicators through multicriteria approaches, improving existing methods by allowing for spatial heterogeneity and providing evidence on its impact on the representation of multidimensional phenomena.

Indeed, the concept of spatial heterogeneity has not been widely explored in the field of composite indicators (Sarra and Nissi, 2020), as confirmed by the literature review. Although widely used methods consider the heterogeneity of sub-indicator weights, the literature recognizes that the operationalization of the spatial heterogeneity concept occurs only when the sub-indicator weights are defined considering the matrix of spatial weights.

Among the methods that consider the matrix of spatial weights in the heterogeneous definition of sub-indicator weights, the Spatial BoD and the GWPCA were found to be the most popular. However, it was noted that the Spatial BoD and the GWPCA are methods based on compensatory aggregation, which makes the sub-indicator weights assume a function of substitution rather than relative importance. Thus, the fundamental idea of spatial heterogeneity, that the influence of a sub-indicator on the multidimensional concept varies according to the spatial unit is not adequately considered.

The research discusses the compatibility of the mathematical properties of OWA, that is a multicriteria method able to take into account non-compensability among simple indicators, with the concept of spatial heterogeneity. It proposes a new method, the "Geographically Ordered Weighted Averaging" (OGWA), capable of operationalizing the spatial heterogeneity concept in the OWA. The mathematical properties of OGWA make it an innovative method to simultaneously account for heterogeneity and spatial continuities in the construction of the composite indicator, in addition to the non-compensatory aggregation of sub-indicators.

Furthermore, the proposed method may be useful in several areas of social research, as it can be extended to ordinal data and mixed ordinal and continuous data.

## Acknowledgements

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## Data Availability Statement

Martinuci, O. D. S., A. Machado, and M. Libório. (2021). "Data for: Time-in-space analysis of multidimensional phenomena." *Mendeley Data*, V4, doi: 10.17632/m3y4jncvch.4

## Notes

1 It is important to note that all of the methods mentioned are capable of accounting for spatial heterogeneity, but are not designed to test for spatial association. Local spatial association indicators for evaluating the existence of spatial patterns in a multivariate context have also been very poorly addressed in the literature. The most relevant is the extension of the Local Geary's c statistic (Anselin, 1995) proposed by Anselin (2019), called the Multivariate Local Geary Statistic, which collapses the squared distances associated with variables into a weighted sum by providing a single value for the statistic at each location.

2 Search in the Scopus and Web of Science databases, in the title, abstract, or keywords on February 19, 2023.

3 Search in the Google Scholar, Scopus, and Web of Science databases, in the title, abstract, or keywords on February 19, 2023.

4 Another possibility would have been to use a spatial weights matrix as in Fusco, Vidoli, and Sahoo (2018) with element 1 if the unit is within the inclusion circle (given by a chosen distance radius) and 0 otherwise. In this case, we would have calculated the score considering the units in the cluster, but this would have resulted in completely discarding more distant units that have an influence, albeit small, on the performance of the other observations.

5 Gaussian presents the advantage to weight all the observations, with a weight that tends toward zero if the distance increases.

6 Bandwidth is a chosen distance beyond which the geographical weight of the observations is set to the value 0. The larger the bandwidth, the higher the number of observations to which the kernel gives a nonzero weight.

7 Despite the different nature and purpose of the two methods, a test of the underlying spatial association patterns was performed with the Multivariate Local Geary Statistic method (Anselin, 2019), varying the bandwidth, as a robustness check of our method. A high positive correlation is found for group C. In fact, as pointed out in Anselin (2019), “the generalization of the Local Geary c statistic to multiple variables is a way to formalize the combination of attribute similarity and locational similarity” and C is the only group with similar values for all indicators. As expected, the correlation is less positive for A, B, and D, and the larger the bandwidth, the lower the value.

## Appendix A

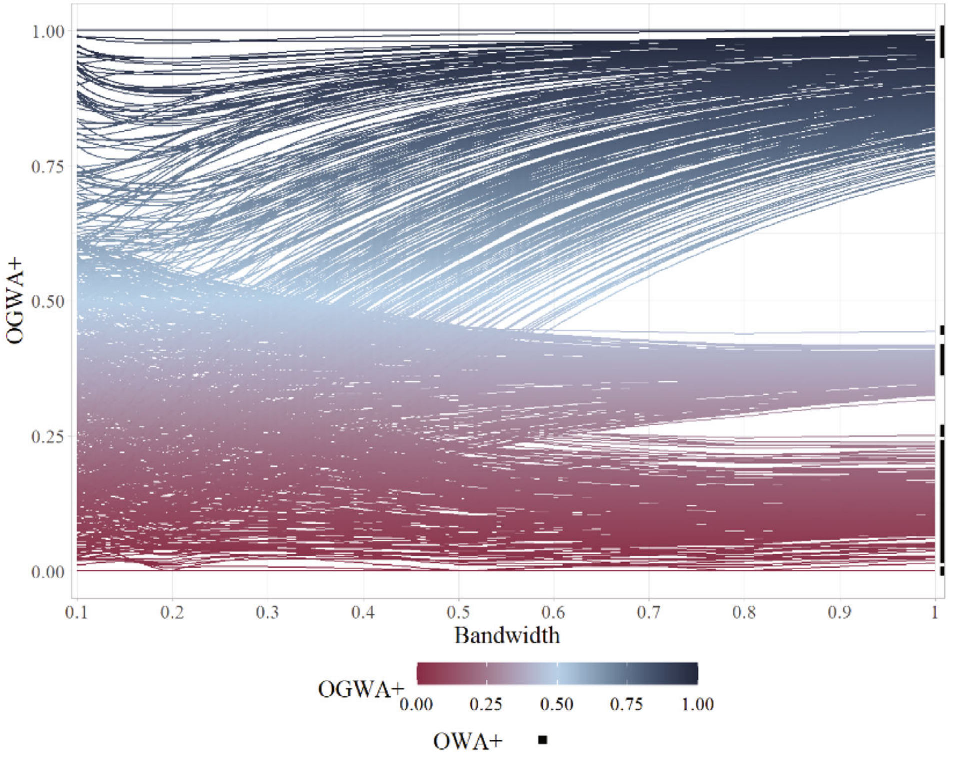


**Figure A1.** Simulated data: Scalogram of $OGWA^+$ and $OWA^+$.

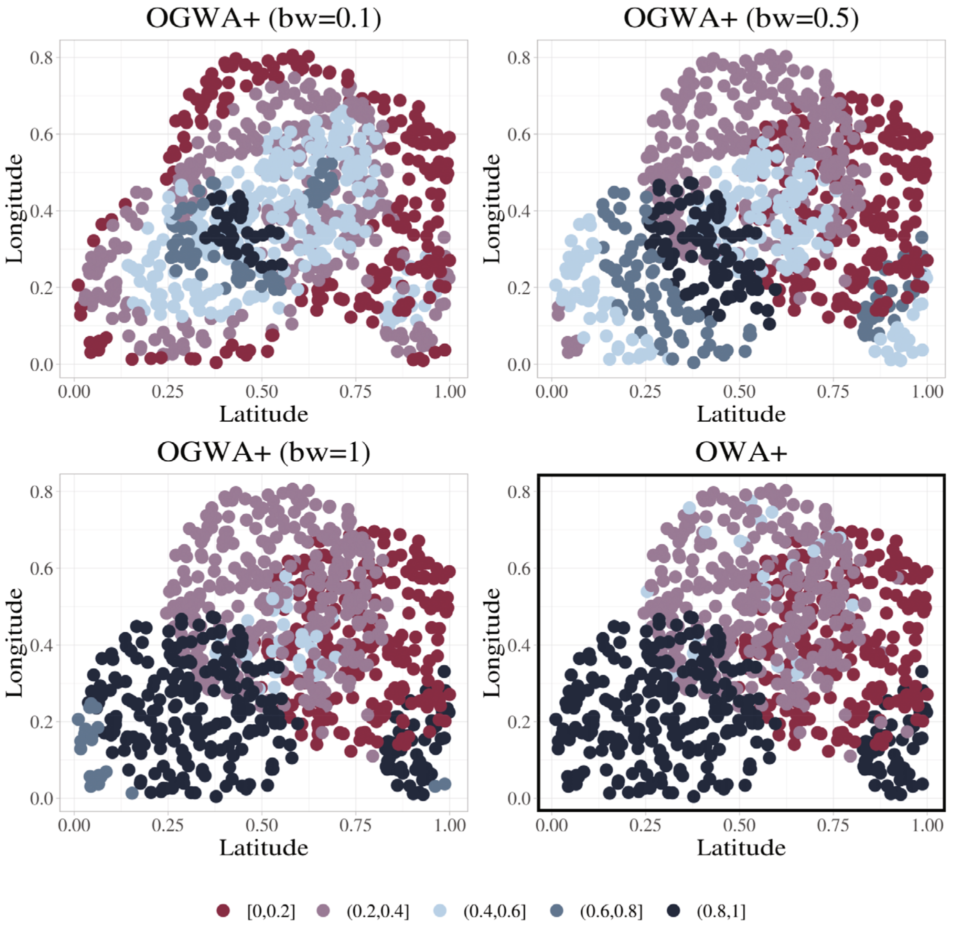


**Figure A2.** Simulated data: $OGWA^{+}$ and $OWA^{+}$ maps by bandwidth.

## References

Afonso, A., and S. Fernandes. (2006). "Measuring Local Government Spending Efficiency: Evidence for the Lisbon Region." *Regional Studies* 40(1), 39–53.

Afonso, A., and S. Fernandes. (2008). "Assessing and Explaining the Relative Efficiency of Local Government." *The Journal of Socio-Economics* 37(5), 1946–79.

Ahn, B. S., and S. H. Choi. (2012). "Aggregation of Ordinal Data Using Ordered Weighted Averaging Operator Weights." *Annals of Operations Research* 201, 1–16.

Almoghathawi, Y., K. Barker, C. M. Rocco, and C. D. Nicholson. (2017). "A Multi-Criteria Decision Analysis Approach for Importance Identification and Ranking of Network Components." *Reliability Engineering & System Safety* 158, 142–51.

Anselin, L. (1995). "Local Indicators of Spatial Association LISA." *Geographical Analysis* 27(2), 93–115.

Anselin, L. (2019). "A Local Indicator of Multivariate Spatial Association: Extending Geary's c." *Geographical Analysis* 51, 133–50.

Artelaris, P. (2022). "A Development Index for the Greek Regions." *Quality & Quantity* 56(3), 1261–81.

Badea, A. C., S. Tarantola, and R. Bolado. (2011). "Composite Indicators for Security of Energy Supply Using Ordered Weighted Averaging." *Reliability Engineering & System Safety* 96(6), 651–62.

Becker, W., P. Paruolo, M. Saisana, and A. Saltelli. (2017a). "Weights and Importance in Composite Indicators: Mind the Gap." In *Handbook of Uncertainty Quantification*, 1187–216. Cham: Springer.

Becker, W., M. Saisana, P. Paruolo, and I. Vandecasteele. (2017b). "Weights and Importance in Composite Indicators: Closing the Gap." *Ecological Indicators* 80, 12–22.

Bernardes, P., P. I. Ekel, S. F. L. Rezende, J. G. Pereira Júnior, A. C. G. dos Santos, M. A. R. Costa, and R. L. Carvalhai. (2022). "Cost of Doing Business Index in Latin America." *Quality & Quantity* 56(4), 2233–52.

Blancas, F. J., and M. Lozano-Oyola. (2022). "Sustainable Tourism Evaluation Using a Composite Indicator with Different Compensatory Levels." *Environmental Impact Assessment Review* 93, 106733.

Brandão, A. L., and P. Brandão. (2017). "Public Space, Infrastructure, Landscape: An Interdisciplinary Matrix for Urban Spatial Continuity." *International Journal for Crime, Justice and Social Democracy* 2(1), 123–34.

Bruggemann, R., and L. Carlsen. (2021). "Uncertainty in Weights for Composite Indicators Generated by Weighted Sums." In *Measuring and Understanding Complex Phenomena*, 45–62, edited by R. Brüggemann, L. Carlsen, T. Beycan, C. Suter, and F. Maggino. Cham: Springer.

Carrino, L. (2017). "The Role of Normalisation in Building Composite Indicators. Rationale and Consequences of Different Strategies, Applied to Social Inclusion." In *Complexity in Society: From Indicators Construction to their Synthesis*, 251–89. Cham: Springer.

Cartone, A., and D. Panzera. (2021). "Deprivation at Local Level: Practical Problems and Policy Implications for the Province of Milan." *Regional Science Policy & Practice* 13(1), 43–61.

Cartone, A., and P. Postiglione. (2021). "Principal Component Analysis for Geographical Data: The Role of Spatial Effects in the Definition of Composite Indicators." *Spatial Economic Analysis* 16(2), 126–47.

Casadio Tarabusi, E., and G. Guarini. (2013). "An Unbalance Adjustment Method for Development Indicators." *Social Indicators Research* 112, 19–45.

Cherchye, L., W. Moesen, N. Rogge, and T. V. Puyenbroeck. (2007). "An Introduction to 'Benefit of the Doubt' Composite Indicators." *Social Indicators Research* 82(1), 111–45.

Cherchye, L., W. Moesen, and T. Van Puyenbroeck. (2004). "Legitimately Diverse, Yet Comparable: On Synthesizing Social Inclusion Performance in the EU." *JCMS: Journal of Common Market Studies* 42(5), 919–55.

Cinelli, M., M. Spada, W. Kim, Y. Zhang, and P. Burgherr. (2021). "MCDA Index Tool: An Interactive Software to Develop Indices and Rankings." *Environment Systems and Decisions* 41(1), 82–109.

Collier, P., and A. J. Venables. (2016). "Urban Infrastructure for Development." *Oxford Review of Economic Policy* 32(3), 391–409.

Coutinho-Rodrigues, J., A. Simão, and C. H. Antunes. (2011). "A GIS-based Multicriteria Spatial Decision Support System Model to Handle Health Facilities Resources. Case of Crisis Management in Batna, Algeria." *Geographia Technica* 51(3), 720–6.

Demšar, U., P. Harris, C. Brunsdon, A. S. Fotheringham, and S. McLoone. (2013). "Principal Component Analysis on Spatial Data: An Overview." *Annals of the Association of American Geographers* 103(1), 106–28.

Dialga, I., and T. H. Le Giang. (2017). "Highlighting Methodological Limitations in the Steps of Composite Indicators Construction." *Social Indicators Research* 131(2), 441–65.

D'Inverno, G., F. Vidoli, and K. De Witte. (2023). "Sustainable Budgeting and Financial Balance: Which Lever Will you Pull?" *European Journal of Operational Research* 309(2), 857-871.

Doeffinger, T., and J. W. Hall. (2021). "Assessing Water Security across Scales: A Case Study of the United States." *Applied Geography* 134, 102500.

Dykes, J., and C. Brunsdon. (2007). "Geographically Weighted Visualization: Interactive Graphics for Scale-Varying Exploratory Analysis." *IEEE Transactions on Visualization & Computer Graphics* 13(6), 1161–8.

Ekel, P., W. Pedrycz, and J. Pereira Júnior. (2020). *Multicriteria Decision-Making under Conditions of Uncertainty: A Fuzzy Set Perspective*. Newark: Wiley.

El Gibari, S., J. M. Cabello, T. Gómez, and F. Ruiz. (2021). "Composite Indicators as Decision Making Tools: The Joint Use of Compensatory and Noncompensatory Schemes." *International Journal of Information Technology & Decision Making (IJITDM)* 20(3), 847–79.

El Gibari, S., T. Gómez, and F. Ruiz. (2019). "Building Composite Indicators Using Multicriteria Methods: A Review." *Journal of Business Economics* 89(1), 1–24.

Ferrer, A. L. C., A. M. T. Thomé, and A. J. Scavarda. (2018). "Sustainable Urban Infrastructure: A Review." *Resources, Conservation and Recycling* 128, 360–72.

Fotheringham, A. S., and C. Brunsdon. (1999). "Local Forms of Spatial Analysis." *Geographical analysis* 31(4), 340–58.

Fotheringham, A. S., C. Brunsdon, and M. Charlton. (2003). *Geographically Weighted Regression: The Analysis of Spatially Varying Relationships*. New York: Wiley.

Fusco, E. (2015). "Enhancing Non-compensatory Composite Indicators: A Directional Proposal." *European Journal of Operational Research* 242(2), 620–30.

Fusco, E. (2023). "Potential Improvements Approach in Composite Indicators Construction: The Multi-Directional Benefit of the Doubt Model." *Socio-Economic Planning Sciences* 85, 101447.

Fusco, E., F. Vidoli, and N. Rogge. (2020). "Spatial Directional Robust Benefit of the Doubt Approach in Presence of Undesirable Output: An Application to Italian Waste Sector." *Omega* 94, 102053.

Fusco, E., F. Vidoli, and B. K. Sahoo. (2018). "Spatial Heterogeneity in Composite Indicator: A Methodological Proposal." *Omega* 77, 1–14.

Godo, L., and V. Torra. (2000). "On Aggregation Operators for Ordinal Qualitative Information." *IEEE Transactions on Fuzzy Systems* 8(2), 143–54.

Gollini, I., B. Lu, M. Charlton, C. Brunsdon, and P. Harris. (2015). "GWmodel: An R Package for Exploring Spatial Heterogeneity Using Geographically Weighted Models." *Journal of Statistical Software* 63(17), 1–50.

Greco, S., A. Ishizaka, M. Tasiou, and G. Torrisi. (2019). "On the Methodological Framework of Composite Indices: A Review of the Issues of Weighting, Aggregation, and Robustness." *Social Indicators Research* 141(1), 61–94.

Greco, S., A. Ishizaka, M. Tasiou, and G. Torrisi. (2021). "The Ordinal Input for Cardinal Output Approach of Non-compensatory Composite Indicators: The PROMETHEE Scoring Method." *European Journal of Operational Research* 288(1), 225–46.

Harris, P., C. Brunsdon, and M. Charlton. (2011). "Geographically Weighted Principal Components Analysis." *International Journal of Geographical Information Science* 25(10), 1717–36.

Harris, P., C. Brunsdon, and A. S. Fotheringham. (2011). "Links, Comparisons and Extensions of the Geographically Weighted Regression Model when Used as a Spatial Predictor." *Stochastic environmental Research and Risk assessment* 25, 123–38.

Hernández-Perdomo, E., C. M. Rocco, and J. E. Ramirez-Marquez. (2016). "Node Ranking for Network Topology-Based Cascade Models – An Ordered Weighted Averaging operators' Approach." *Reliability Engineering & System Safety* 155, 115–23.

IBGE. (2010a). Características urbanísticas do entorno dos domicílios (Dataset). Accessed July 15, 2023. https://www.ibge.gov.br/geociencias/organizacao-do-territorio/tipologias-do-territorio/24702-caracteristicas-urbanisticas-do-entorno-dos-domicilios.html

IBGE. (2010b). Data From: Demographic Census: 2010 (Dataset). Accessed May 21, 2021. https://censo2010.ibge.gov.br

Kuc-Czarnecka, M., S. Lo Piano, and A. Saltelli. (2020). "Quantitative Storytelling in the Making of a Composite Indicator." *Social Indicators Research* 149(3), 775–802.

Libório, M. P., O. da Silva Martinuci, R. M. Hadad, and J. F. de Melo Silva. (2020a). "Procedimento para compatibilizar mapas em diferentes escalas eo problema da mudanca de suporte espacial." *Caminhos de Geografia* 21(74), 126–41.

Libório, M. P., S. Laudares, J. F. D. Abreu, P. Y. Ekel, and P. Bernardes (2020b). Property Tax: Dealing Spatially with Economic, Social, and Political Challenges. Urbe. Revista Brasileira de Gestão Urbana, 12.

Libório, M. P., O. D. S. Martinuci, P. I. Ekel, R. M. Hadad, R. D. M. Lyrio, and P. Bernardes. (2022a). "Measuring Inequality through a Non-compensatory Approach." *GeoJournal* 87(6), 4689–706.

Libório, M. P., O. D. S. Martinuci, A. M. C. Machado, P. I. Ekel, J. F. D. Abreu, and S. Laudares. (2022b). "Representing Multidimensional Phenomena of Geographic Interest: Benefit of the Doubt or Principal Component Analysis?" *The Professional Geographer* 74(4), 758–71.

Lu, B., P. Harris, M. Charlton, and C. Brunsdon. (2014). "The GWmodel R Package: Further Topics for Exploring Spatial Heterogeneity Using Geographically Weighted Models." *Geo-spatial Information Science* 17(2), 85–101.

Matern, B. (1986). *Spatial Variation*. *2*. New York: Springer-Verlag.
Munda, G. (2012). "Intensity of Preference and Related Uncertainty in Non-compensatory Aggregation Rules." *Theory and Decision* 73(4), 649–69.
Narbón-Perpiñá, I., and K. De Witte. (2018a). "Local governments' Efficiency: A Systematic Literature Review – Part I." *International Transactions in Operational Research* 25(2), 431–68.
Narbón-Perpiñá, I., and K. De Witte. (2018b). "Local governments' Efficiency: A Systematic Literature Review – Part II." *International Transactions in Operational Research* 25(4), 1107–36.
Nardo, M., M. Saisana, A. Saltelli, and S. Tarantola (2005). "Tools for Composite Indicators Building". European Commission, ISPRA, 15(1), 19–20.
Nijkamp, P., and S. Suzuki. (2009). "A Generalized Goals-Achievement Model in Data Envelopment Analysis: An Application to Efficiency Improvement in Local Government Finance in Japan." *Spatial economic analysis* 4(3), 249–74.
OECD. (2008). *Handbook on Constructing Composite Indicators: Methodology and User Guide*. Paris: OECD publishing.
Pandey, B., C. Brelsford, and K. C. Seto. (2022). "Infrastructure Inequality Is a Characteristic of Urbanization." *Proceedings of the National Academy of Sciences* 119(15), e2119890119.
Parsons, M., I. Reeve, J. McGregor, P. Hastings, G. R. Marshall, J. McNeill, R. Stayner, and S. Glavac. (2021). "Disaster Resilience in Australia: A Geographic Assessment Using an Index of Coping and Adaptive Capacity." *International Journal of Disaster Risk Reduction* 62, 102422.
Pedrycz, W., P. Ekel, and R. Parreiras. (2011). *Fuzzy Multicriteria Decision-Making: Models, Methods and Applications*. Chichester: Wiley.
Pereira Jr., J., P. I. Ekel, E. V. S. Moreno, I. S. Silva, C. M. C. da Silva, and F. H. L. Mascarenhas. (2016). "Decision Making in a Fuzzy Environment as Applied to Analyzing and Prioritizing Industrial Districts." *Anais do XLVIII SBPO Simpósio Brasileiro de Pesquisa Operacional*, 292–302.
Rocco, S., S. Tarantola, A. Costescu Badea, and R. Bolado Lavin. (2009). "Composite Indicators for Security of Energy Supply in Europe Using Ordered Weighted Averaging." In *Reliability, Risk and Safety: Theory and Applications*, 3, 1737–44, edited by C. Radim Bris, S. Guedes, and M. Sebastian. Boca Raton, FL: CRC Press
Saib, M. S., J. Caudeville, M. Beauchamp, F. Carré, O. Ganry, A. Trugeon, and A. Cicolella. (2015). "Building Spatial Composite Indicators to Analyze Environmental Health Inequalities on a Regional Scale." *Environmental Health* 14(1), 1–11.
Sarra, A., and E. Nissi. (2020). "A Spatial Composite Indicator for Human and Ecosystem Well-Being in the Italian Urban Areas." *Social Indicators Research* 148(2), 353–77.
Shu, Z., R. A. Carrasco, J. P. García-Miguel, and M. Sánchez-Montañés. (2022). "Multiple Scenarios of Quality of Life Index Using Fuzzy Linguistic Quantifiers: The Case of 85 Countries in Numbeo." *Mathematics* 10(12), 2091.
Silva, L. M. L., P. Ekel, D. Vieira, and G. L. Soares. (2020). "Multiattribute Spatial Decision-Making for Qualitative Information Processing as Applied to the Renewable Energy Generating Sites Prospection." *IEEE Access* 8, 137745–57.
Steele, W., and C. Legacy. (2017). "Critical Urban Infrastructure." *Urban Policy and Research* 35(1), 1–6.
Trogu, D., and M. Campagna. (2018). "Toward Spatial Composite Indicators: A Case Study on Sardinian Landscape." *Sustainability* 10(5), 1369.
Vidoli, F. and E. Fusco (2023). Compind: Composite Indicators Functions. R Package Version 2.7, https://CRAN.R-project.org/package=Compind
Vidoli, F., E. Fusco, and C. Mazziotta. (2015). "Non-compensability in Composite Indicators: A Robust Directional Frontier Method." *Social Indicators Research* 122(3), 635–52.
Vidoli, F., and C. Mazziotta. (2013). "Robust Weighted Composite Indicators by Means of Frontier Methods with an Application to European Infrastructure Endowment." *Statistica Applicata – Italian Journal of Applied Statistics* 23(2), 259–82.
Walesiak, M. (2018). "The Choice of Normalization Method and Rankings of the Set of Objects Based on Composite Indicator Values." *Statistics in Transition*. *New Series* 19(4), 693–710.
Yager, R. R. (1988). "On Ordered Weighted Averaging Aggregation Operators in Multicriteria Decision-making." *IEEE Transactions on Systems, Man, and Cybernetics* 18(1), 183–90.

Yager, R. R., and J. Kacprzyk, eds. (2012). *The Ordered Weighted Averaging Operators: Theory and Applications*. New York: Springer New York.

Yusfany, A. (2015). "The Efficiency of Local Governments and its Influence Factors." *International Journal of Scientific & Technology Research* 4(10), 219–41.

Zabihi, H., M. Alizadeh, P. K. Langat, M. Karami, H. Shahabi, A. Ahmad, M. N. Said, and S. Lee. (2019). "GIS Multi-Criteria Analysis by Ordered Weighted Averaging (OWA): Toward an Integrated Citrus Management Strategy." *Sustainability* 11(4), 1009.

Ziara, M., K. Nigim, A. Enshassi, and B. M. Ayyub. (2002). "Strategic Implementation of Infrastructure Priority Projects: Case Study in Palestine." *Journal of Infrastructure Systems* 8(1), 2–11.