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THE SERIES IS INDEXED IN SCOPUS

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Anisotropic Isoperimetric Problems and Related Topics

 Springer

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Preface

The international INdAM workshop “*Anisotropic Isoperimetric Problems and Related Topics*” has been held in Rome in September 2022, under the aegis of: INdAM, the Italian National Institute for Advanced Mathematics; Foundation Compositio Mathematica; and the Universities of Padova, Pisa and Trento. This book is the natural follow-up and it showcases contributions from invited speakers and their collaborators.

Nowadays, the classical isoperimetric problem despite its stunningly simple formulation (“which is the shape with least perimeter enclosing a given area?”), captivated mathematicians throughout history, dating back to the ancient Greeks. Despite Greeks guessing the correct solution, the full (and in general dimension) proof came two millennia later. In the twentieth century, efforts to solve the problem gave birth to a new branch of mathematics, geometric measure theory, and theories that we now consider standard, as that of sets of finite perimeter and that of currents.

What is fascinating is that it also explains various physical phenomena, such as the shape of soap bubbles, the underlying reason being that, in such physical systems, the leading energy term is driven by cohesion forces which translate into surface energy terms, thus, in mathematical terms by the perimeter. Variations of this problem are gaining attention in the mathematical community, that is now investigating weighted and/or anisotropic variants, and their interaction with nonlocal energies. This directions of research have an impact on the understanding of many physical phenomena, chemical, and biological systems such as the behavior of crystals; the formation of liquid drops; the microphase separation for diblock copolymers, ferromagnetic domains, and colloidal systems; atomic nuclear models.

The workshop in Rome provided a platform for cutting-edge scientific discussions on these topics and the book features contributions stemming from discussions and interactions of participants at the workshop location. At large, we can group the contributions in the following three thematic clusters:

- isoperimetric problems with density (contributions by Bevilacqua, Lussardi, and Marzocchi; Comi and Stefani);

- crystals and periodic structures (contributions by Alama, Bronsard, and Vriend; Bach and Ruf; Cesaroni and Novaga; Del Nin and De Luca);
- isoperimetric problems in geometric structures (contributions by Benatti and Fogagnolo; Pozzetta).

Despite not all COVID-19 restrictions being lifted at the time of the meeting, the high quality of the contributions and the involvement of numerous authors demonstrate the active and engaging nature of research surrounding anisotropic isoperimetric problems, attracting both experts and young researchers.

We express our sincere gratitude to all the authors of this book, without whom the book itself would not have appeared. Special thanks to the INdAM scientific board for providing us with the stimulating opportunity to edit this book, and to the INdAM administrative staff and the Springer publishing team for their invaluable support. Additionally, we appreciate the meticulous efforts and the quality of the reports from the reviewers.

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