MIND, BRAIN, AND EDUCATION

Training Executive Functions Within the Mathematical Domain: A Pilot Study with an Integrated Digital-Paper Procedure in Primary Second-Grade

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ABSTRACT— Interventions targeting cognitive control processes, such as Executive Functions (EF) have recently been experimented to enhance early math skills. This pilot study explored the feasibility and effectiveness of an intervention integrating EF activities into the mathematical domain among second-grade students. One hundred and four typically-developing-children were assigned to either a group that underwent the intervention (Trained Group; $n = 58$) or a group that continued with daily didactic activities (Control Group; $n = 46$). The training lasted for 8 weeks and included both home-based digital and school-based paper activities. According to teachers' feedback, the intervention was highly appreciated by children and compatible with classical school curricula. The Trained Group improved in behavioral self-regulation, math abilities and problem-solving in comparison to the Control Group. Notably, within the Trained Group, benefits of the training were higher in children with high working memory. This training offers a model to support math learning in primary school, considering inter-individual differences in EF.

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Numeracy encompasses the ability to master math facts, concepts, and procedures in all aspects of daily life. It involves understanding and reasoning with data and processes, solving problems, evaluating situations and making decisions (Baker, Street, & Tomlin, 2003; Brooks & Pui, 2010). The acquisition of mathematical skills must be supported and empowered, especially during primary school ages, when the fundamental foundations of arithmetic and problem solving are established (Fischer, Moeller, Cress, & Nuerk, 2013).

Despite humans, like many other living beings, being predisposed to perceive approximate quantities, they need to develop a complex multi component brain circuit for math (Dehaene, 2011). This circuit underpins a series of processes, some domain-specific, and others domain-general (Vogel & De Smedt, 2021). Domain-specific functions pertain to cognitive functions specifically related to a given disciplinary competence to be acquired. Thus, in the context of mathematics, these functions include the representation of large numerical quantities and ordinal relationships, the acquisition of counting skills up to infinite numbers, and the ability to carry out operations. Conversely, domain-general functions are transversal and reflect mental operations crucial for information processing across different learning domains (e.g., emotional regulation, math, language, reading, and writing etc.) and input/output modalities (e.g., verbal, visuo-spatial etc.). Executive Functions (EF) are considered one of the most important domain-general processes underpinning math acquisition (Cragg & Gilmore, 2014; Vogel & De Smedt, 2021).

Given the complementary role that math-specific skills and EF play in math, it is acknowledged that, in order to support math learning, both domain-general and domain-specific functions must be taken into account.

Executive Functions

EF refers to a set of skills that regulate and control thoughts, actions, and emotions to achieve a defined goal. EF are involved in new or complex situations where impulsive responses are inadequate, and behavioral control and action planning are necessary. They are also implicated in behaviors requiring the initiation of a new sequence of actions, situations where habitual responses need interruption in favor of non-automatic actions and tasks demanding constant behavioral monitoring (Diamond, 2013; Miyake & Friedman, 2012).

While several EF dimensions have been described in children by different studies (Lee, Bull, & Ho, 2013; Miller, Giesbrecht, Müller, McInerney, & Kerns, 2012; Scionti & Marzocchi, 2021; Usai, Viterbori, Traverso, & De Franchis, 2014), the most widely accepted theoretical model of EF in children defined three basic EF components (Diamond, 2013). In Diamond's model (2013), the three main basic components are inhibition, working memory, and cognitive flexibility. Inhibition can be divided into two subcomponents: the ability to focus attention on relevant data while ignoring distractors (interference control) and the ability to inhibit inadequate or impulsive responses (response inhibition) (Gandolfi, Viterbori, Traverso, & Usai, 2014; Traverso, Viterbori, Gandolfi, Zanobini, & Usai, 2022). Working memory is the ability to maintain and work on thoughts, plans, and other mental contents temporarily held in verbal or visuo-spatial memory, including the crucial process of updating old mental contents with newer ones. Cognitive flexibility involves flexibly switching attention and processing between different task requirements, mental rules, or strategies according to the context.

EF development occurs progressively from early childhood to adolescence. In preschool age, where the greatest development is recorded, a two-factor structure emerges, with the two main executive components, inhibition and working memory, distinct but interrelated (Lee et al., 2013; Monette, Bigras, & Lafrenière, 2015; Usai et al., 2014). The third basic EF, cognitive flexibility, becomes more prominent in later school age (Buttelmann & Karbach, 2017). Inhibition, working memory and cognitive flexibility form the bases for more complex EF components such as abstract reasoning, problem-solving, and planning (Diamond, 2013).

EF development is characterized by high inter-subject variability and can be influenced by both risk and protective environmental factors through complex and probabilistic interactions. Factors, such as individual developmental history (Zelazo, 2020), parenting style (Fay-Stammbach, Hawes, & Meredith, 2014), socio-economic status (Farah et al., 2006; Noble, McCandliss, & Farah, 2007; Noble, Norman, & Farah, 2005), and school curricula (Diamond, 2012) contribute to differences in EF development. Thus, to equip individuals with strong EF, early interventions have been proposed to promote EF development and consequently support long-term cascade effects on learning and adaptation (Diamond & Lee, 2011). Despite being cognitive functions that cut across various domains, recent evidence suggests that EF interventions may face challenges in transferring improvements to skills not directly trained (meta-analyses: Bombonato et al., 2023; Kassai, Futo, Demetrovics, & Takacs, 2019; Scionti, Cavallero, Zogmaister, & Marzocchi, 2020; Takacs & Kassai, 2019).

The Involvement of Executive Functions in Math Development

Due to their domain-general nature, EF are involved in and support learning, potentially influencing acquisitions, behaviors, and competences across several specific domains, from infancy to adulthood (Korzeniowski, Ison, & Difabio de Anglat, 2021). EF play a crucial role in school adjustment and are predictive of academic achievement and success, supporting learning prerequisites (Ruffini, Marzocchi, & Pecini, 2021; Traverso, Viterbori, & Usai, 2019). They are essential for learning math, as well as reading and writing (De Franchis, Usai, Viterbori, & Traverso, 2017; Miller, Müller, Giesbrecht, Carpendale, & Kerns, 2013; Moffitt et al., 2011; Ruffini, Osmani, Martini, Giera, & Pecini, 2023; Usai, Viterbori, & Traverso, 2018; Viterbori, Usai, Traverso, & De Franchis, 2015). The highest rate of EF development occurs between preschool and the first years of primary school, concurrent with the acquisition of basic learning skills and school adjustment, when learning and behavior require cognitive control processes due to new tasks and challenges. Indeed, the pre-literacy and pre-math tasks in the preschool context, along with school activities related to reading, writing and arithmetic skills acquisition, are new and complex tasks for children, potentially requiring several cognitive control processes (Blair & Raver, 2015; Miyake & Friedman, 2012). All EF basic components are involved in math skills (Bull & Lee, 2014; Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013; Yeniad, Malda, Mesman, Van IJzendoorn, & Pieper, 2013) with roles that vary based on children's age and math competences. Inhibition supports general math achievement and arithmetic skills among 5th–7th graders (Gómez, Jiménez, Bobadilla, Reyes, & Dartnell, 2015). Its importance in math tasks may vary depending on the age of the students. Younger children may rely more heavily on inhibition when engaged in math procedural tasks as they struggle with the suppression of learned strategies to adopt more advanced or sophisticated ones that are not yet automatized (Cragg & Gilmore, 2014). Working memory is crucial in supporting calculation, arithmetic and math problem solving in primary-school children (meta-analysis: Friso-Van den Bos, Van der Ven, Kroesbergen, & Van Luit, 2013). Cognitive flexibility is related to general, conceptual, and procedural math as well, with a greater influence in younger children compared to older ones (for a review: de Santana, Roazzi, & Nobre, 2022).

However, there is agreement that the nature of math-EF relationship is reciprocal (Ellis et al., 2021; Raghubar, Barnes, & Hecht, 2010) with dynamics that may change according to children's age (Cragg & Gilmore, 2014; Peng & Kievit, 2020; Van der Ven, Kroesbergen, Boom, & Leseman, 2012). For instance, engaging in a high-quality mathematics education contributes to the development of EF processes (Clements, Sarama, & Germeroth, 2016). Moreover, EF seem to predict math acquisition with a directional path, meaning that variations in EF impact the acquisition of mathematical skills (Schmitt, Geldhof, Purpura, Duncan, & McClelland, 2017).

Interventions on Math Skills

Children's underachievement in math represents a significant problem (Suan, 2014), prompting the development of various interventions (Dowker, 2009). Interventions aiming to promote math skills should target diverse math abilities, recognizing the multi componential nature of the math domain (Burns, Kanive, & DeGrande, 2012; Kilpatrick, Swafford, & Findell, 2002). However, the majority of interventions involve isolated educational strategies focusing on specific math skills, such as the use of schemes and tables for categorization, identifying relevant information in problems, and promoting self-regulation strategies for problem solving (Kaufman & von Aster, 2012; Meltzer, 2018a; Steinberg & Roditi, 2018).

To support math development, interventions that promote underlying general-domain processes, such as EF, have been recently proposed. Consistently, several studies have shown significant effects of EF interventions on the development of mathematical skills in preschool and school-age children (Dong et al., 2022; Rosas, Espinoza, Porflitt, & Ceric, 2019; Sánchez-Pérez et al., 2018; Traverso et al., 2019), with positive effects observed across children with low, medium and high abilities (Dong et al., 2022). However, recent meta-analyses have indicated the difficulty in achieving far-transfer effects of EF interventions: trainings that target EF promote improvement in the executive domain but struggle to generalize effects to other domains (Bombonato et al., 2023; Kassai et al., 2019; Scionti et al., 2020). Embedding an intervention within a specific learning context is crucial for promoting children's school performances, linking EF exercises to the math domain through different types of activities could enhance math skills (Meltzer, 2018b). Consistently, embedding EF within the math domain is more likely to result in improvements in mathematics compared to interventions solely focusing on EF (for a review: Scerif et al., 2023). However, to our knowledge, only a recent study (Sánchez-Pérez et al., 2018) conducted a computer-based training combining working memory and mathematics activities as part of the school routines and found positive significant results in both domains. Furthermore, most studies investigating the effects of EF interventions on math competency focused on one individual EF component, often working memory, demonstrating improvement in math across different grades (Bergman-Nutley & Klingberg, 2014; Holmes & Gathercole, 2014; Söderqvist & Bergman Nutley, 2015). However, there is a gap in the literature regarding interventions that simultaneously train all basic EF components, which could be highly effective in promoting mathematics given each EF component's involvement in different mathematical processes (Bull & Lee, 2014; Kolkman et al., 2013; Yeniad et al., 2013). Moreover, EF trainings were mainly conducted at home (Klingberg et al., 2005), while conducting the training in the school context, where the learning of instrumental skills is primarily accomplished, could favor the generalization of the training effects (Carretti, Borella, Elosúa, Gómez-Veiga, & García-Madruga, 2017; Kadosh, Dowker, Heine, Kaufmann, & Kucian, 2013; Kucian et al., 2011).

Additionally, interventions on math learning skills tended to use either computerized or paper-and-pencil activities, adopting one isolated methodological approach (De Witte, Haelermans, & Rogge, 2015; Sánchez-Pérez et al., 2018). However, using computerized and paper-and-pencil activities together could merge the appeal of the new technologies with the possibility to work in groups on paper materials, thus being more powerful (Ruffini et al., 2021; Shaw & Lewis, 2005).

Furthermore, the previously mentioned studies focused exclusively on the efficacy of the implemented training, analyzing changes in measures collected before and after the training. However, it is equally important to consider the feasibility of the training, namely the assessment of the possibility and practicability of successfully conducting a program to identify significant obstacles that could make it difficult or impossible. The assessment of feasibility could provide information about the suitability of the intervention, flexibility, available time, and any potential adverse events (Bowen et al., 2009; Soneson et al., 2020). This aspect appears particularly relevant for school and school-home integrated interventions, according to which the activities of the training must be compatible with the traditional school curricula or the family's routine.

In conclusion, further studies are needed in order to verify whether interventions that embed different EF components within the math domain in both school and home settings can be effective and feasible in promoting math skills.

The Present Study

The present study attempted to overcome the methodological limitations of previous studies, with the goal of enhancing the generalizability of the training's effects. To achieve this, an intervention was developed with specific features: (1) integration of EF activities in the math domain; (2) incorporation of computerized and paper-and-pencil activities; (3) implementation of the intervention both at home and at school. The selection of activities adhered to principles of intensity, novelty, challenge, and usability (Blair, 2017; Diamond & Lee, 2011). This design aligned with the idea that intervention demands should be intensive (Klingberg et al., 2005) and escalate in tandem with progressive improvements in children's executive skills (Bergman Nutley et al., 2011; Klingberg et al., 2005). Moreover, this study focused on a specific population, second graders. This age group was chosen because it is during the second year that children begin to tackle complex numbers and quantities, constructing mathematical skills that heavily rely on cognitive control processes such as EF (MIUR, 2012).

In line with the theoretical considerations outlined above, the present study sought to assess the feasibility and, at a pilot level, measure the efficacy of defined intervention tasks on both EF and math competencies in second graders. Feasibility, in this context, is evaluated as the practicality and likelihood of successful implementation of the intervention, as assessed by the key stakeholders in the project, namely teachers and children, in their daily routines. The intervention was expected to be feasible and suitable for children in this age group, with the anticipation that EF and math skills show more improvement in the group receiving the intervention compared to the control group.

METHODS

Participants

The study involved 123 primary school children attending the second grade in 5 primary schools located in two Italian regions (Tuscany and Lazio). Each class was randomly assigned to the trained condition (4 classes; Trained Group, TG) and to the control condition (3 classes; Control Group, CG). From the total sample, the following cases were excluded from the analysis: 8 children with Neurodevelopmental Disorders (e.g., ADHD, autism…), 9 children who did not complete at least 20% of the activities and 2 children who were absent in the post-test phase. Data of children with atypical development were excluded from the analyses as the training activities conducted by them were adapted according to the special needs of each specific child; thus, results were not comparable to those of typically developing children.

All children included in the analysis ($n = 104$) had nonverbal intelligence in the normal range (within -2 *SD* from the mean of the age-matched population) as evaluated by Raven's Colored Progressive Matrices standardized for the Italian school-aged population (Belacchi, Scalisi, Cannoni, & Cornoldi, 2008). The Trained Group was composed of 58 children (Mean age = 7.24; $SD = .43$; 28 males, 30 females); the Control Group consisted of 46 children (Mean age = 7.33; $SD = .45$, 26 males, 20 females). The two groups did not differ in age ($F(1) = 1.26$, $p > .05$), gender ($\chi^2 = .27$, $p > .05$) and nonverbal fluid intelligence ($F(1,103) = 0.64$, $p > .05$).

The research was approved by the Ethic Committee of the University of Florence (reference number 0152940 date 26/05/2021) and it was carried out following Ethical guidelines of the Italian Association of Psychology and of the Declaration of Helsinki.

Procedure

In September 2021, 4 schools were recruited to participate in this project. After the authorization obtained from the school administrators and the informed consents by the parents, the intervention began in October 2021 and ended in December 2021.

The teachers of the Trained Group participated in two 2-hr training sessions with the project coordinators, where they were instructed on the objectives, purposes, structure and methods of the intervention.

Both before and after the intervention, the teachers of the Trained Group engaged in an interview with researchers to assess the feasibility of the training.

To assess training's efficacy, pre- and post-training assessments with the same tasks evaluating outcome measures were conducted in the first and last 2 weeks, respectively. During the pre-training assessment the Raven's Colored Progressive Matrices (Belacchi et al., 2008) were administered as a screening of the nonverbal intelligence of children involved in the study.

The Trained Group participated in the intervention for 2 hr a week at school and for 15 min four times a week at home, while following the classical school curriculum for the remainder of the week. The training activities seamlessly integrated traditional math lessons with innovative approaches aimed at enhancing math and problem-solving skills. Throughout the intervention, teachers were available to assist parents with any technical issue. Parents were instructed to check that their children completed the home exercises. Nevertheless, non-specific protocols were

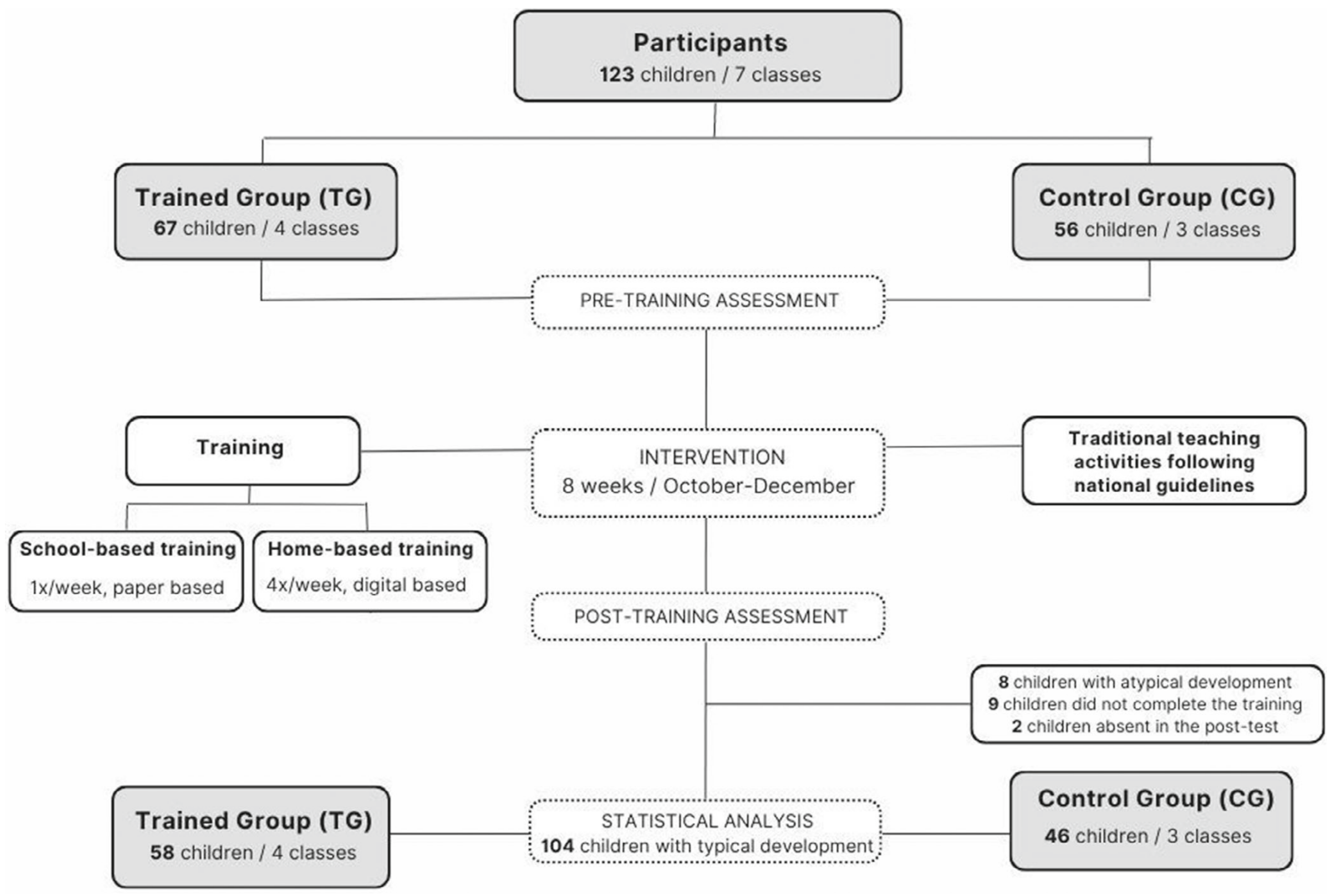


Fig. 1. Experimental procedure.

implemented to control for the variability in the conduction of the home-based training. In contrast, the Control Group engaged in the school routine as usual, without participating in any additional interventions. In this regard, a meeting was held with the teachers of the Control Group before the start of the project to ensure that, during the research period, their classes were not involved in any type of training related to EF or math.

The experimental procedure is represented in Figure 1.

Outcome Measures

Feasibility Measures. The feasibility of the training was assessed in two stages: before the initiation of the intervention and upon its conclusion.

Prior to commencing the intervention, a meeting was held with the teachers responsible for conducting the training in classrooms. The purpose of this meeting was to inquire about the feasibility of the project at school, taking into account factors such as space and time availability, the suitability of the frequency and duration of the intervention, and, notably, the practicality of the intervention at home. Questions were posed to the teachers, addressing aspects like the families' access to a computer for the child and the availability of an Internet connection at home. Teachers communicated with the families to inquire about certain aspects or address specific questions. The gathered information from both teachers and families was documented by the researcher.

Upon completion of the intervention, interviews were conducted with the teachers responsible for administering the training at school. These interviews aimed to gather information on several aspects, including the number of digital sessions completed by each child (monitored through the utilized apps), the number of paper sessions accomplished (documented by teachers after each session), and qualitative observations regarding participation levels and any difficulties encountered by the children during digital and classroom activities. Special attention was paid to those who showed low participation in the training. The researcher documented the qualitative insights provided by the teachers.

Efficacy Measures. To assess mathematical learning, the standardized AC-MT 6–11 battery (Cornoldi, Lucangeli, & Perini, 2020) was used. The tests were administered by math teachers in individual mode, in oral form. The teacher read aloud the instructions of each exercise to all children, and they completed the tasks without time-constraints. The assessment took place in a silent environment and required (Cornoldi et al., 2020; Vogel & De Smedt, 2021):

1. Mental calculation (3 additions and 3 subtractions; correct answers 0 to 6). It refers to the ability to apply calculation procedures and developed automations using different strategies, such as counting all the elements together or individually in order;
2. retrieval of arithmetic facts (correct answers 0 to 6). It refers to the ability to do simple basic operations without performing the calculation as the results of the operations are already known and stored in memory;
3. Forward enumeration (from 1 to 50). It refers to the ability to place numerical symbols in an order of magnitude, according to the ascending criteria, also known as rearrangement of sequences of numbers;
4. Backward enumeration (from 20 to 1). It refers to the ability to place numerical symbols in an order of magnitude, according to the descending criteria, also known as rearrangement of sequences of numbers.

Both accuracy and response time were recorded, with the help of a digital stopwatch. The following indexes were then computed:

1. Enumeration errors: number of errors in the forward and backward enumeration;
2. Enumeration time: sum of the response times in the forward and backward enumeration;
3. Arithmetical fact errors: number of errors in the arithmetic facts' retrieval;
4. Mental calculation errors: number of errors in mental calculations;
5. Mental calculation time: response times in mental calculations.

The math battery used shows good test–retest reliability (*r*: 0.63–0.77) according to standardization data (Cornoldi et al., 2020).

The three basic EF were assessed by three tests (GoNoGo task: response inhibition; Flanker task: interference control and cognitive flexibility; NBack task: updating in working memory) and a questionnaire selected from a battery for the tele-assessments of Executive Functions in school-age children (TeleFE web platform, Anastasis Cooperativa and Hogrefe Eds. 2023). Using both direct (e.g., tasks) and indirect (e.g., questionnaires) measures to assess EF allows to detect both the cognitive and behavioral dimensions obtaining a complete view of the executive domain (Rivella, Bombonato, & Viterbori, 2022; Toplak, West, & Stanovich, 2013). TeleFE was previously developed and standardized for the Italian population and normative data and reliability indexes (internal consistency: 0.68–0.93; test–retest reliability: $p < .01$) are provided for 6 to 13 aged children (Rivella et al., 2023). Moreover, results from the normative study show absence of differences between in person and remote administration of the tests (Rivella et al., 2023).

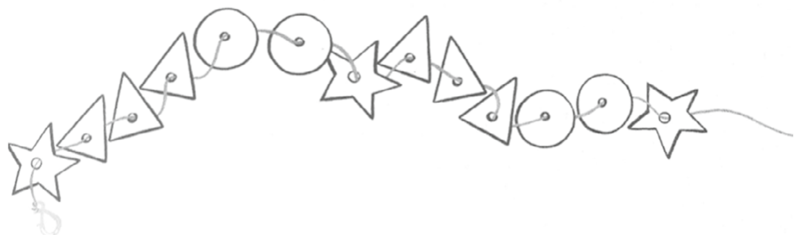
The tests were individually and remotely administered through a PC by trained psychologists in a session of about 50–60 min. Following guidelines for a correct procedure of tele-assessment in children (Ruffini, Tarchi, Morini, Giuliano, & Pecini, 2022), a psychologist was connected via Skype with a child located within a school class, under a teacher's supervision who had the role to ensure children's physical safety and to check for any technological problems. In each class, there was a maximum of 3 workstations simultaneously. The three tests were carried out without interruption and were administered by the operators following three possible orders (Latin square procedure). The TeleFE tests were selected in order to directly measure the three main EF components.

The Go/No-Go test is a response inhibitory control task which measures child's *response inhibition*. Fifty repeated stimuli (3 cm yellow or blue circles or triangles) for 4 blocks were presented individually in the center of the screen and the child had to press the spacebar as soon as possible when a given target (Go stimulus) appeared and not respond when a second given target (No-Go stimulus) appeared. In each block, Go stimulus were 35 and No-Go stimulus were 15 for a proportion of 70/30. A maximum response time was set at 1000 ms. The 4 blocks changed for the target stimuli: yellow stimuli (1st block), blue stimuli (2nd block), triangles (3rd block), circles (4th block). For each block, the number of correct responses (CR) at Go stimuli (from 0 to 35), number of CR at No-Go stimuli (from 0 to 15), and reaction time (RT) at Go CR (from 0 to 1,000 ms) were measured. The mean of the CR to the No-Go stimuli in the 4 blocks (No-Go CR) was used for the analyses.

In the Flanker test, measuring the ability to *control interference* (first and second block: single rule) and *cognitive flexibility* (third block: mixed rules), a string of five arrows (long 8.93 mm, distancing 3.84 mm) was presented in the center of the screen. The child had to focus the attention on the direction of the one central arrow (first block) or on those of the four side arrows (second block). In the third block, if the arrows were blue, the child had to answer according to the direction of the arrow in the center, while if the arrows were orange, he/she had to answer according to the direction of the arrows on the side. The center and the side arrows could be directed toward the same direction (congruent condition) or rather to the opposite direction (incongruent condition). The child was asked to press on the keyboard the letter S if the arrow (arrows) was directed to left or the letter (L) if it was directed to right. The first and second blocks were

THE NECKLACE

Below is drawn only a small part of a very long necklace. Color the stars yellow, the circles red, the triangles blue. Next, continue drawing the necklace down to another little star.



Now answer the questions and try to explain your answers.

1. Philip claims that in the thirteenth place there is a small star. Is he right?
2. Erica says that in eighteenth place there must be a circle. Is this true?
3. Elena, without stretching the necklace even further with the drawing, says that in the twentieth place there must be the second of the three triangles. Is this true?
4. In your opinion, which figure will be in the 24th place? How did you go about answering it?

Fig. 2. A problem-solving activity from Di Martino and Zan (2020).

composed of 40 trials (20 congruent stimuli; 20 incongruent stimuli); the third block was composed of 64 trials (32 congruent stimuli; 32 incongruent stimuli). For each block, the number of CR at congruent condition (from 0 to 20 for the first and second blocks, from 0 to 32 for the third block), number of CR at incongruent condition (from 0 to 20 for the first and second blocks, from 0 to 32 for the third block), RT at congruent condition (from 200 ms to 1,500 ms), RT at incongruent condition (from 200 ms to 1,500 ms) were measured. The following indexes were computed:

1. Single rule incongruent CR = mean number of correct responses in the incongruent conditions of the blocks 1–2
2. Single rule incongruent RT = mean of the reaction times in the incongruent conditions of the blocks 1–2
3. Mixed rules incongruent CR = number of correct responses in the incongruent conditions of the block 3.
4. Mixed rules incongruent RT = mean of the reaction times in the incongruent conditions of the block 3.

The N-Back test measures *updating in working memory*. Fifty stimuli (3 cm colored circles, green shapes or black letters) were serially presented in the center of the screen for 1,550 ms (ISI = 1,000 ms). The task was divided into 6 blocks, differentiated according to the characteristic of the stimuli: in the first and second blocks the child was instructed to focus on the color of the stimulus, in the third and fourth blocks on the shape, in the last two blocks on the letter (written alternatively in upper and lowercase). The child was required to respond by pressing the spacebar if the stimulus was of the same color (or shape or letter) of the previous stimulus (1-Back) or of the stimulus two back (2-Back). Each block has 16 target stimuli and 36 non target stimuli. For each block, the number of CR at target stimuli (from 0 to 16), the number of CR at non-target stimuli (from 0 to 36), the RT of the CR to the target (from 0 to 2,500 ms) were measured. The mean number of CR in the 6 blocks (N-Back CR) was computed.

A questionnaire (QUFE) was completed, by parents and teachers separately, to assess the child's executive functioning within the two main life contexts, specifically home and school. The questionnaires were filled online or in paper and pencil form, according to their preferences. They consisted of 32 items on a five Likert scale, concerning 3 areas of investigation (cognitive self-regulation, behavioral self-regulation and material management) for teachers and 5 areas of investigation (cognitive self-regulation, behavioral self-regulation, material management, flexibility of adaptation and spirit of initiative) for parents.

Training

The intervention consisted of paper and pencil activities carried out in the classroom (School-based) and digital activities carried out individually by the child at home (Home-based).

The School-based training was conducted once a week for 8 weeks involving the full class. The intervention was based on 8 problem-solving activities, chosen from the operative manual "*Problemi al centro. Matematica senza paura*." *by* Di Martino and Zan (2020) and from the *Invalsi* tests (INVALSI, 2018) for second grade children (https://invalsi-areaprove.cineca.it/docs/file/QdR_MATEMATICA.pdf). An example of a problem-solving activity is presented in Figure 2. A manual with the description and the instructions of the week's activities was delivered to the teachers. Children were provided with the written text of the problem and were instructed to write the solution process in a blank under the text. The activity was divided into four phases: teachers read aloud the problem, children reasoned individually on it for some minutes and subsequently they discussed with a pair mate the proposed solution and wrote it down on the blank sheet given to each pair, finally the discussion involved all the class. Children's reasoning behind their problems' solutions were collected through both written and graphic transcripts. The comparison and discussion activities were enhanced and enforced by the teacher as the emphasis was never directed to the problem's result.

The Home-based training was built on the basis of the tripartite model of EF (Miyake et al., 2000) and followed the principles suggested by Diamond for an effective intervention (Diamond, 2012) such as the use of fun activities gradually increasing in difficulty. The exercises were implemented on two platforms: Learning App (https://learningapps.org/) and Wordwall (https://wordwall.net/it). Thirty-six activities were developed ex novo or adapted from existing ones and at least one level of difficulty was planned (e.g., increasing speed or number of elements). 96 activities, corresponding to the different exercises and/or difficulties, were implemented: 32 for inhibition; 32 for working memory, 32 for cognitive flexibility. Children were provided by teachers on a weekly basis with a document where they could find the links to three activities a day, for 4 days a week. Children individually exercised EF components for about 15 min daily at home after school. Before proposing the intervention, the researchers made sure that all families had a technological device on which their children could carry out the digital activities.

An example of the activities proposed for each EF component follows.

In an activity requiring inhibition at the easiest level of difficulty, children must choose, in a given time interval, the wrong answer to a series of questions related to math knowledge expected for II grade (Figure 3a, "7 + 7" = 18). At the subsequent level of difficulty (Figure 3b), the indications are the same, but the questions are more complex (e.g., "after the 100 there is" = 99) and the time available to answer is less than the previous level. In this activity, even if it may appear counterintuitive to ask children to choose the wrong answer, it may help in increasing the cognitive control of their own answers, that after being automatically selected must be monitored.

In an activity requiring working memory, children must order a series of numbers presented on the left of the screen from the smallest to the biggest (Figure 4: 5–12–20) and subsequently choose between 2 alternatives the missing number of the sequence (12). In the subsequent levels of difficulty, the sequences get longer and numbers to choose higher.

An example of a cognitive flexibility activity requires children to select "true" ("vero") for the additions and "false" ("falso") for the subtractions regardless of the correctness of the result. Subsequently he/she must select "false" when the result is right and "true" when it is wrong (Figure 5).

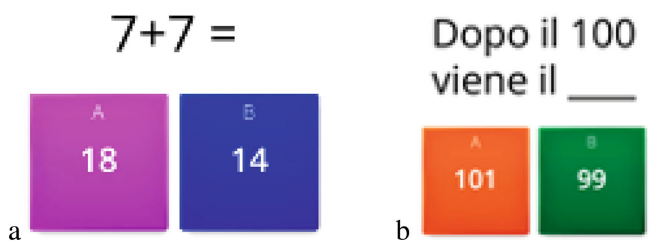


Fig. 3. Inhibition activity: "chose the wrong answer"; a. easiest level of difficulty; b. highest level of difficulty.

Fig. 4. Example of the working memory activity: "order and memorize the number sequence".

Fig. 5. Cognitive flexibility activity, level 1. Correct response: "false".

Statistical Analysis

Statistical analyses were conducted by Statistical Package for Social Science 2022, version 28.0.1.0 (142) (SPSS, IBM Corporation) and Jamovi (2021), version 1.6.

Descriptive statistics and analysis of the normality of the distribution (skewness cut-off = 2; kurtosis cut-off = 3) were carried out on all indexes.

To identify the presence of differences between the TG and the CG before the training in the performances at the math and EF tests, multivariate analysis of variance was used for the normal distributed measures, and Robust Analyses of variance were used for the non-normal distributed measures.

To measure the feasibility of the intervention, a qualitative description of data collected was conducted.

To measure the efficacy of the intervention, the following analyses were conducted:

1. For normally distributed measures, mixed Analyses of variance with group (Trained vs. Control) as between factor, time (pre vs. post) as within factor and outcome measures as dependent variable were conducted;
2. For not normally distributed measures, robust Analyses of variance tests with group (Trained vs. Control) as between factor and the differences between post training and pre-training performances (delta) as dependent variables were conducted.

To measure the changes in problem solving skill associated with the training, the scores of the TG in the problem-solving skills from the first and last sessions of the training were analyzed using planned paired t tests. Given that it was not possible to gather this measure in 15 children and that children worked in pairs, 86 productions (43 first session and 43 last session) were used for the analysis.

To investigate if improvements were related to individual differences, linear regression analyses were conducted in the TG with pre-training EF measures as predictors and pre-post training differences (delta) of the performances in the math and problem-solving tasks as dependent variables.

RESULTS

Feasibility

Four teachers participated in the interview before the start of the intervention, all demonstrating high enthusiasm and interest in the project. They did not express any initial concerns about the feasibility of implementing it in their classroom context. Given that the training's objectives aligned with the national educational goals they were pursuing, the teachers were willing to integrate this activity into their curricular math sessions. Additionally, all the teachers found the proposed duration and frequency of the training to be appropriate, with no doubts or critical issues raised during the discussion. During this session, teachers were encouraged to ask questions about the training; they focused mainly on the materials and aspects related to the software. In discussions with parents, where teachers explained the training and sought consent, they investigated any potential obstacles to carrying out the training at home. Around 5% of families mentioned the presence of only one computer at home, sometimes unavailable to the child, or its absence. However, all parents had internet connections at home or could use a Hotspot. Parents appreciated the flexibility of conducting exercises at home, allowing free choice of times and days. Teachers reported that all parents, considering the possibility of using mobile devices, were very willing to implement the training. An aspect highlighted by parents as a strength of the training was the use of digital devices for educational purposes.

After the training, a second interview was conducted with the four teachers. Out of 67 children eligible for the Home-based training, one child did not perform the post-training evaluation, and 58 children completed the activities. Teachers reported that eight children did not engage in home-based activities due to inaccessibility to digital devices ($n = 6$) or family commitments ($n = 2$). Children completed, on average, 70.71% of the proposed digital exercises. Regarding the School-based training, all children who participated completed all paper-and-pencil sessions at school. Qualitative observations by teachers documented high compliance among all children, expressing enthusiasm and interest in completing the training activities in class. They reported that children appreciated and enjoyed the classroom activities, often expressing a desire to continue the training. Teachers also noted that paper-and-pencil activities were new and unusual for the children, who were used to carrying out mathematical problems in a very linear way and with a fixed procedure. Already after the second session, the children got used to this alternative way of posing mathematical problems and demonstrated their ability to find innovative and alternative solutions based on their creativity and breaking away from familiar patterns. Group work, alternating with individual work, was well-received as it allowed children to train individually while actively comparing ideas with their classmates. Lastly, teachers expressed great appreciation for the meticulousness and excellent organization that preceded and underpinned the project on the part of the researchers. They perceived this as a notable strength of the intervention. Some of the transcript of teachers' qualitative observations on the intervention are shown in Appendix 2.

Efficacy

EF and math performances of the TG and CG are reported in Tables 1 and 2.

Analysis of the normality of the distributions showed that all EF indexes were normally distributed (skewness [−1.77; 0.09]; kurtosis [−.7; 2.52]) whereas only 4 out of 10 math measures (i.e., mental calculation errors pre and post, mental calculation time pre, arithmetic fact pre) were normally distributed (skewness [1.24; 1.66]; kurtosis [1.74; 2.44]).

QUFE questionnaires completed by the parents were not used for the analysis of the training efficacy, due to the low number of questionnaires returned to the examiners.

Table 1
Descriptive Statistics and Mixed Analysis of Variance Results at the Executive Functions Tests

		Pre-training		*Post-training*				
		TG Mean (SD)	*CG Mean (SD)*	*TG Mean (SD)*	*CG Mean (SD)*	*Group effect* F*(gdl)*, p, η_p^2	*Time effect* F*(gdl)*, p, η_p^2	*Interaction Time*Group* F*(gdl)*, p, η_p^2
No-Go CR		10.46 (2.18)	10.09 (2.33)	10.95 (1.97)	10.33 (1.91)	$F(1,192) = 2.01$, ns, 0.02	$F(1,102) = 2.77$, .099, 0.03	$F(1,102) = 0.32$, ns, 0.00
Flanker	Single rule incongruent CR	8.59 (5.09)	10.13 (4.72)	12.44 (5.68)	12.51 (5.37)	$F(1,98) = 1.03$, ns, 0.01	$F(1,98) = 39.22$, <.001, 0.29	$F(1,98) = 1.98$, ns, 0.02
	Single rule incongruent RT	982.32 (192.95)	912.96 (207.95)	959.55 (136.69)	903.98 (138.73)	$F(1,93) = 3.38$, .07, 0.04	$F(1,93) = 0.29$, ns, 0.00	$F(1,93) = 0.00$, ns, 0.00
	Mixed rules incongruent CR	11.89 (5.61)	12.79 (6.01)	15.39 (6.19)	16.84 (5.68)	$F(1,91) = 1.29$, ns, 0.01	$F(1,91) = 29.99$, <.001, 0.25	$F(1,91) = 0.05$, ns, 0.00
	Mixed rules incongruent RT	1,212.75 (269.73)	1,198.85 (238.47)	1,213.38 (240.94)	1,213.59 (223.16)	$F(1,86) = 0.01$, ns, 0.00	$F(1,86) = 0.52$, ns, 0.01	$F(1,86) = 0.1$, ns, 0.00
N-Back CR		37.72 (8)	37.29 (8.24)	41.03 (6.33)	39.67 (7.55)	$F(1,99) = 0.4$, ns, 0.00	$F(1,99) = 25.47$, <.001, 0.21	$F(1,99) = 0.65$, ns, 0.01
QUFE		121.24 (28.53)	131 (24.4)	127.76 (26.12)	121.57 (26.64)	$F(1,83) = 0.13$, ns, 0.00	$F(1,83) = 0.8$, ns, 0.01	$F(1,83) = 14.63$, <.001, 0.15

CG = control group; CR = correct responses; RT = reaction time; TG = trained group.

Table 2
Descriptive Statistics and Mixed Analysis of Variance Results at the Math Measures

	Pre-training		*Post-training*		
	TG Mean (SD)	*CG Mean (SD)*	*TG Mean (SD)*	*CG Mean (SD)*	F, p
Mental calculations (errors)	1.28 (1.53)	1.17 (1.29)	0.45 (0.86)	0.89 (1.02)	Interaction Time*Group $F = 5.49$, $p < .05$, 0.05 Time effect $F = 22.78$, <.001, 0.18 Group effect $F = 0.67$, $p = .41$
Mental calculations (time)	62.66 (34.26)	49.29 (27.8)	39.41 (21.93)	35.06 (17.29)	$F = 2.92$, $p = .09$
Arithmetic facts (errors)	1.07 (1.35)	0.76 (1.08)	0.41 (0.75)	0.67 (0.87)	$F = 2.98$, $p = .09$
Enumeration (errors)	0.72 (1.52)	0.63 (1.34)	0.31 (0.63)	0.13 (0.4)	$F = 0.17$, ns
Enumeration (time)	68.02 (37.72)	53.67 (19.32)	54.53 (25.84)	46.03 (13.86)	$F = 4.42$, $p < .05$

CG = control group; TG = trained group.

Multivariate analysis of variance showed the absence of differences in the pre-training performances between the two groups except for the single incongruent rule RC ($F(1,85) = 4.38$, $p < .05$, $\eta_p^2 = 0.05$), the QUFE ($F(1,85) = 4.03$, $p < .05$, $\eta_p^2 = 0.05$), the time of mental calculation ($F(1,85) = 0.64$, $p < .05$, $\eta_p^2 = 0.01$) where the TG was worse than the CG. The robust analysis of variance showed the absence of differences for all not normally distributed pre-training measures except for enumeration time ($F = 5.36$, $p < .05$).

Mixed analyses of variance on the EF measures (Table 1) showed an effect of Time that was significant in Flanker single and mixed rules incongruent CR and in N-Back CR and tended to significance in No-Go CR. The Group approached significance in the Flanker single rule RT. No significant Time x Group interactions were found except for the teacher QUFE as the TG had larger pre-post training improvements than the CG.

Mixed analyses of variance on the mental calculation errors (Table 2) showed a significant effect of Time and of the interaction Time x Group as the TG had larger pre-post training improvements than the CG. Robust analysis of variance on the pre-post training delta showed significant differences between groups in enumeration time and a tendency to significant differences in arithmetic facts accuracy and mental calculation time as the TG had larger pre-post training differences than the CG.

The scores attributed to the strategies used in the problem-solving tasks by TG (Table 3) in the first and last session of the training were analyzed by paired t-tests (Bonferroni post-hoc corrections for 10 comparisons, $p < .01$). Results showed significant improvement on all

TABLE 3
Descriptive and Inferential Statistics of the Scores Obtained by the Trained Group in the Problem-Solving Tasks

		Pre-training *Mean* (SD)	*Post-training* *Mean* (SD)	t *(df)*, p
Problem text comprehension	Adherence of the text produced to the request of the problem question	2.84 (.84)	3.77 (1.11)	−4.77 (42), $p<.001$
Answer argumentation	Use of connectives	2.14 (1.08)	2.89 (1.2)	−3.23 (42), $p<.01$
	Correctness and order in the exposition	2.14 (.68)	2.65 (.78)	−3.8 (42), $p<.001$
	Relevance to the solution to be achieved	2.74 (.76)	2.98 (.96)	−1.66 (42), $p=.053$
Listening and communication	Integration of oral and written explanation	2.86 (.68)	3.91 (.72)	−14.13 (42), $p<.001$
Control cognitive processes	Description of the solution processes	2.12 (.88)	3.23 (.97)	−5.74 (42), $p<.001$
	Description of the solution reasoning	1.74 (.85)	3.16 (1.15)	−6.14 (42), $p<.001$
Decision strategies	Clarification in the text of the information about the solution of the problem	2.65 (.53)	3.28 (1.6)	−3.85 (42), $p<.001$
Graphic representation of the problem	How much the graphic solution produced is functional to the solution process adopted	2.12 (.85)	2.28 (.98)	ns
	Integration of the graphic schemes produced with the argumentative answer	2.16 (.87)	2.23 (1)	ns
Problem-solving Total score		23.51 (4.73)	30.37 (6.38)	−7.08 (42), $p<.001$

items except for the graphic representations of the problem index.

The linear regression analyses showed that EF (all measures included) at the pre-training significantly predicted the pre–post training delta in the mental calculations errors ($R^2=0.32$, $F(7,48)=2.74$, $p<.05$) and time ($R^2=0.27$, $F(7,48)=2.18$, $p=.056$) and in the enumeration time ($R^2=0.21$, $F(7,48)=4.2$, $p=.001$) with N-Back CR as the unique significant predictor in all cases ($\beta=0.35$, $p<.05$; $\beta=0.34$, $p<.05$; $\beta=0.54$, $p<.001$).

Pre-training EF measures did not significantly predict pre-post training delta in the arithmetic facts ($R^2=0.11$, $F(7,48)=0.72$, ns), in enumeration errors ($R^2=0.21$, $F(7,48)=1.54$, ns) nor in problem-solving ($R^2=0.12$, $F(7,33)=0.50$, ns).

DISCUSSION

The aim of this pilot study was to verify the feasibility and the efficacy of a training program designed to enhance math and problem-solving skills in second-grade children. Recognizing the significant role of EF in mathematical abilities and considering the limited generalizability of EF training to math learning, this study employed an integrated training approach. The training comprised school-based problem-solving group activities and home-based individual digital exercises targeting EF within the math domain. Furthermore, adhering to the fractionated model and the principles proposed by Diamond and Lee (2011) for effective EF interventions, the training was developed to address all EF components. It was designed to be intensive, challenging, and user-friendly.

Aligned with the development trajectories of EF and mathematical skills, this study focused on second-grade children. During this phase, fundamental math skills become automatized, and problem-solving abilities undergo significant development. Thus, second grade represents a critical age, as it precedes the potential emergence of specific learning disorders that can be diagnosed.

Based on the qualitative analysis of the teachers' reports on the children's behavior, the training proposed in the present study appeared to be highly feasible. Teachers verbally expressed that all children participated with a positive and thoughtful attitude, displaying great commitment throughout the entire training period. This enthusiasm was evident both in the computerized activities conducted at home and in the classroom paper-and-pencil activities. Teachers highlighted their perception of children's eagerness and appreciation for the problem-solving activities and considered them an enrichment of the traditional didactic approach. Consequently, all teachers perceived the intervention as fully integrable into daily routines without imposing an extra burden. Referring to the home-based training, 87.88% of eligible children completed an average of about 71% of the 96 exercises. These findings affirm the feasibility of interventions aiming to address individual differences in learning by integrating classroom activities aligned with the curriculum (in the present study math) with personalized and digitalized home-based exercises focusing on the cognitive processes underpinning learning, such as EF (Carretti et al., 2017; García-Madruga et al., 2013;

Sánchez-Pérez et al., 2018). Nevertheless, despite utilizing free online platforms, eight children did not complete the training due to difficulties arising from the inaccessibility of digital devices or family commitments. This outcome aligns with previous literature emphasizing the need to control for socio-economic disparities when proposing digital home-based training, ensuring accessibility to digital devices and the availability of time by the families (Van Dijk, 2020).

Regarding efficacy results, the Trained Group exhibited significantly greater improvements in behavioral self-regulation and several math skills compared to the Control Group.

The pre–post training assessment conducted by teachers, using structured observational questionnaires (QUFE), indicated a significantly higher improvement in the overall score of behavioral self-regulation in the Trained Group in comparison to the Control Group. This result suggests that the impact of the training in this study, though specifically focused on EF and math, extends to the behavior of children in other classroom and school activities. Indeed, teachers noted that, by the end of the study, the trained children were more inclined to assist each other, listen to others' ideas, and collaborate more effectively with their peers. This observation indicates higher levels of self-regulation and organizational strategies during group activities. This result contributes to the existing literature as it represents the first study to measure the effect of an EF-math training on behavioral self-regulation in children. It underscores the importance of incorporating tools for the indirect assessment of child self-regulation behaviors in protocols designed to detect the effects of EF training (Bombonato et al., 2020). However, it is important to note that this result requires further confirmation, as it may have been influenced by teachers' expectations and the presence of lower scores on the questionnaire in the Trained Group compared to the Control Group.

Furtherly, the improvement highlighted by teachers was not corroborated by direct measures of EF. Although, on average, the Trained Group displayed larger pre-post training differences than the Control Group, no significant distinctions emerged between the two groups in the direct measures of inhibition, updating or shifting. This outcome was unexpected and deviates from prior studies that identified substantial improvement in children's executive functioning after EF training (for meta-analysis see Kassai et al., 2019; Takacs & Kassai, 2019). However, it is crucial to acknowledge that results from the literature are not uniform, as the efficacy of EF training may hinge on the nature of the training and the characteristics of the population involved (Takacs & Kassai, 2019). Given that the present study exclusively included typical children with consistent functional profiles and employed a short-term training regimen, more pronounced effects on direct measures of EF might be observed with larger and more diverse samples, extended training durations, incorporation of more self-adaptive and challenging exercises, and heightened remote monitoring of the child's activity and compliance (Diamond & Lee, 2011; Pecini, Spoglianti, Bonetti, & Di Lieto, 2019). Additionally, the significant effect of Time observed in several EF measures, irrespective of the group, could be attributed to a learning effect, a common occurrence when using EF tasks (Quattropani, 2008). Since EF tasks typically entail unfamiliar instructions and necessitate the development of new strategies for resolution, subjects are likely to improve upon re-evaluation, thus potentially diminishing the power of the comparison between the trained and the non-trained subjects. Finally, the limited correlation between direct (i.e., tasks) and indirect (i.e., questionnaire) measures of EF, as observed in the literature (Krivitzky, Bosenbark, Ichord, Jastrzab, & Billinghurst, 2019; Silver, 2014), underscores the necessity of employing both assessment modalities.

A more pronounced effect of the training was observed in the math domain, particularly in the areas of number skills, calculation and problem-solving. The Trained Group exhibited greater improvement than the Control Group in accuracy and speed of mental calculations, as well as speed of enumeration and access to arithmetic facts. Notably, there was a discernible trend toward a statistical significance in mental calculation time and arithmetic facts accuracy. The lack of significant training effects on enumeration accuracy may be indicative of a ceiling effect, given the high performances achieved by the majority of the children. These results suggest that the training implemented in the present study had a positive impact on mentally manipulating numbers and counts, automating the number line, and accessing stored arithmetic facts in long-term memory. This aligns with the findings of Sánchez-Pérez et al. (2018), who employed a computer-based training program that integrated working memory and math activities into school routines. In contrast to other studies that solely focused on EF training and did not observed significant far-reaching effects on math skills (Alloway, Bibile, & Lau, 2013; Ang, Lee, Cheam, Poon, & Koh, 2015; Dunning, Holmes, & Gathercole, 2013; Elliott, Gathercole, Alloway, Holmes, & Kirkwood, 2010; Rode, Robson, Purviance, Geary, & Mayr, 2014), our study, along with that of Sánchez-Pérez et al. (2018), supports the effectiveness of interventions that combine activities in both the EF and math domains.

Concurrently, children who underwent training exhibited multiple improvements in problem-solving tasks, particularly in problem text comprehension, answer argumentation, integration of oral and written responses, as well as control cognitive processes and decision strategies. This outcome was anticipated and can be deemed a proximate effect of the training, given that the exercises for problem-solving tasks conducted in class were specifically designed to enhance reasoning strategies and math metacognition (Di Martino & Zan, 2020).

Collectively, the results of the present study suggest that while the proposed training integrates EF and math exercises, the effect on the two domains may be partially dissociated, particularly when EF are measured by direct tasks. Nevertheless, as previously mentioned, the absence of training effects on EF measures can be attributed to several factors, including the sensitivity of EF tasks to the test–retest procedure. Supporting this interpretation, a regression analysis revealed a significant model when using EF individual profiles as predictors of improvements in the math domain. Children with higher EF demonstrated greater gains from the training than those with lower EF, particularly in measures of mental calculation accuracy and speed, as well as enumeration speed. This result aligns with previous literature (Bull, Espy, & Wiebe, 2008; Simanowski & Krajewski, 2019; Viterbori et al., 2015), reinforcing the relationship between EF and math and emphasizing the importance of considering individual EF profiles to personalize and optimize math learning interventions. Notably, the significant EF predictor was updating in working memory, measured by the N-back task. This finding supports prior literature suggesting that possessing strong working memory skills serves as a protective factor in learning math, while children with lower working memory may be at risk of math difficulties (David, 2012; Passolunghi & Mammarella, 2012; Schuchardt, Maehler, & Hasselhorn, 2008). An educational implication of this finding is that enhancing working memory may be a prerequisite for math training, especially in children with special educational needs or neurodevelopmental disorders who typically exhibit lower EF (Bandettini, Salterini, Panesi, & Ferlino, 2020; Wiley, Ghanim, Taylor, & Murias, 2021).

Limitations and Future Directions

The present study was designed as a pilot study and has some limitations that need to be acknowledged. Firstly, the small number of children involved in the training and the specific school grade selected necessitate further research on larger samples and across various primary school grades. Consequently, the results of the study cannot be generalized to all primary school populations, and the proposed activity should be adapted based on the developmental trajectories of EF and math during school age.

A second limitation of the study is that the training integrated EF and math exercises using a combined digital-paper-and-pencil approach. While this characteristic represents a strength of the training, probably enhancing the ecological nature of the exercise and facilitating efficacy transferability, it precludes the separate measurement of the specific contribution of EF and problem-solving exercises, as well as the impact of digital and paper and pencil activities, on the observed improvements. Further studies should compare the effects of integrated interventions, like the one used in the present study, with those achieved by addressing EF and math separately. Moreover, further investigations are needed to untangle the roles of different factors in training effectiveness, including EF versus math components, digital versus paper, and home versus school settings.

Additionally, as the training in this study adopts an inclusive approach by adapting activities to children with atypical development, future studies should assess the feasibility and effectiveness of adapted training in children with special needs. Another limitation of the present study was the inability to control possible interfering factors in the home context during the training, such as the exercises' frequency or the environment where children performed activities.

A further limitation involves the partial return of QUFE by families, which was not utilized for data analysis. In the future studies, experimenters should strive to enhance awareness among families from the outset about completing the questionnaires, underscoring the importance of active collaboration in collecting this type of data. Consistently, such data are crucial for obtaining a comprehensive understanding of children's EF in a home environment.

CONCLUSIONS

The intervention demonstrated feasibility in terms of time and resources, as perceived by both teachers and children. It presented a novel model for integrating traditional paper-and-pencil school tasks with digital home-based activities, aimed at enhancing math skills and the underlying cognitive processes in second-grade children. While the effectiveness of the training and the relationship between EF and math improvements require further studies and verifications, the intervention employed in the present study has proven to be effective in improving several math and problem-solving skills, along with behavioral self-regulation.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

The data presented in this study are available on request from the corresponding author.

REFERENCES

Alloway, T. P., Bibile, V., & Lau, G. (2013). Computerized working memory training: Can it lead to gains in cognitive skills in students? *Computers in Human Behavior*, *29*(3), 632–638. https://doi.org/10.1016/j.chb.2012.10.023

Ang, S. Y., Lee, K., Cheam, F., Poon, K., & Koh, J. (2015). Updating and working memory training: Immediate improvement, long-term maintenance, and generalisability to non-trained tasks. *Journal of Applied Research in Memory and Cognition*, *4*(2), 121–128. https://doi.org/10.1016/j.jarmac.2015.03.001

Baker, D., Street, B., & Tomlin, A. (2003). Mathematics as social: Understanding relationships between home and school numeracy practices. *For the Learning of Mathematics*, *23*(3), 11–15.

Bandettini, A., Salterini, F., Panesi, S., & Ferlino, L. (2020). Preschoolers with typical development or special education needs in a low SES context: The relationship between early achievement and their cognitive underpinnings. *EDULEARN20 Proceedings* (pp. 1479–1487). IATED.

Belacchi, C., Scalisi, T. G., Cannoni, E., & Cornoldi, C. (2008) *CPM – Coloured progressive matrices standardizzazione italiana*. Firenze: Giunti O.S.

Bergman Nutley, S., Söderqvist, S., Bryde, S., Thoreil, L. B., Humphreys, K., & Klingberg, T. (2011). Gains in fluid intelligence after training non-verbal reasoning in 4-year-old children: A controlled, randomized study. *Developmental Science*, *14*, 591–601. https://doi.org/10.1111/j.1467-7687.2010.01022.x

Bergman-Nutley, S., & Klingberg, T. (2014). Effect of working memory training on working memory, arithmetic and following instructions. *Psychological Research*, *78*(6), 869–877. https://doi.org/10.1007/s00426-014-0614-0

Blair, C. (2017). Educating executive function. *Wiley Interdisciplinary Reviews: Cognitive Science*, *8*(1–2), e1403. https://doi.org/10.1002/wcs.1403

Blair, C., & Raver, C. C. (2015). School readiness and self-regulation: A developmental psychobiological approach. *Annual Review of Psychology*, *66*, 711–731. https://doi.org/10.1146/annurev-psych-010814-015221

Bombonato, C., Del Lucchese, B., Ruffini, C., Di Lieto, M. C., Brovedani, P., Sgandurra, G., Cioni, G., & Pecini, C. (2023). Far transfer effects of trainings on executive functions in neurodevelopmental disorders: A systematic review and metanalysis. *Neuropsychology Review*, *1-36*. https://doi.org/10.1007/s11065-022-09574-z

Bombonato, C., Di Lieto, M. C., Cristofani, P., Spoglianti, S., Chilosi, A., & Pecini, C. (2020). Funzionamento esecutivo dei ragazzi con Disturbo Specifico di Apprendimento secondo la percezione dei genitori. Indagine con il questionario BRIEF-2. *Psicologia Clinica dello Sviluppo*, *24*(2), 285–301.

Bowen, D. J., Kreuter, M., Spring, B., Cofta-Woerpel, L., Linnan, L., Weiner, D., … Fernandez, M. (2009). How we design feasibility studies. *American Journal of Preventive Medicine*, *36*(5), 452–457. https://doi.org/10.1016/j.amepre.2009.02.002

Brooks, M. E., & Pui, S. Y. (2010). Are individual differences in numeracy unique from general mental ability? A closer look at a common measure of numeracy. *Individual Differences Research*, *8*(4), 257–265.

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at age 7 years. *Developmental Neuropsychology*, *33*(3), 205–228. https://doi.org/10.1080/87565640801982312

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives*, *8*, 36–41. https://doi.org/10.1111/cdep.12059

Burns, M. K., Kanive, R., & DeGrande, M. (2012). Effect of a computer-delivered math fact intervention as a supplemental intervention for math in third and fourth grades. *Remedial and Special Education*, *33*(3), 184–191. https://doi.org/10.1177/0741932510381652

Buttelmann, F., & Karbach, J. (2017). Development and plasticity of cognitive flexibility in early and middle childhood. *Frontiers in Psychology*, *8*, 1040. https://doi.org/10.3389/fpsyg.2017.01040

Carretti, B., Borella, E., Elosúa, M. R., Gómez-Veiga, I., & García-Madruga, J. A. (2017). Improvements in reading comprehension performance after a training program focusing on executive processes of working memory. *Journal of Cognitive Enhancement*, *1*(3), 268–279. https://doi.org/10.1007/s41465-017-0012-9

Clements, D. H., Sarama, J., & Germeroth, C. (2016). Learning executive function and early mathematics: Directions of causal relations. *Early Childhood Research Quarterly*, *36*, 79–90. https://doi.org/10.1016/j.ecresq.2015.12.009

Cornoldi, C., Lucangeli, D., & Perini, N. (2020) *AC-MT 6–11 anni. Prove per la classe. Valutazione standardizzata delle abilità di calcolo e di soluzione di problemi*. Trento: Erickson.

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, *3*(2), 63–68. https://doi.org/10.1016/j.tine.2013.12.001

David, C. V. (2012). Working memory deficits in math learning difficulties: A meta-analysis. *International Journal of Developmental Disabilities*, *58*(2), 67–84. https://doi.org/10.1179/2047387711Y.0000000007

De Franchis, V., Usai, M. C., Viterbori, P., & Traverso, L. (2017). Preschool executive functioning and literacy achievement in grades 1 and 3 of primary school: A longitudinal study. *Learning and Individual Differences*, *54*, 184–195. https://doi.org/10.1016/j.lindif.2017.01.026

de Santana, A. N., Roazzi, A., & Nobre, A. P. M. C. (2022). The relationship between cognitive flexibility and mathematical performance in children: A meta-analysis. *Trends in Neuroscience and Education*, *28*, 100179. https://doi.org/10.1016/j.tine.2022.100179

De Witte, K., Haelermans, C., & Rogge, N. (2015). The effectiveness of a computer-assisted math learning program. *Journal of Computer Assisted Learning*, *31*(4), 314–329. https://doi.org/10.1111/jcal.12090

Dehaene, S. (2011) *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Di Martino, P., & Zan, R. (Cur.) (2020) *Problemi per crescere. Matematica senza paura*. Firenze: Giunti Scuola.

Diamond, A. (2012). Activities and programs that improve children's executive functions. *Current Directions in Psychological Science*, *21*(5), 335–341. https://doi.org/10.1177/0963721412453722

Diamond, A. (2013). Executive functions. *Annual Review of Psychology*, *64*, 135–168. https://doi.org/10.1146/annurev-psych-113011-143750

Diamond, A., & Lee, K. (2011). Interventions shown to aid executive function development in children 4 to 12 years old. *Science*, *333*(6045), 959–964. https://doi.org/10.1126/science.1204529

Dong, Y., Clements, D. H., Sarama, J., Dumas, D., Banse, H. W., & Day-Hess, C. (2022). Mathematics and executive function competencies in the context of interventions: A quantile regression analysis. *The Journal of Experimental Education*, *90*(2), 297–318. https://doi.org/10.1080/00220973.2020.1777070

Dowker, A. (2009). What works for children with mathematical difficulties. *DCSF*. Report 00086-2009BKT-EN.

Dunning, D. L., Holmes, J., & Gathercole, S. E. (2013). Does working memory training lead to generalized improvements in children with low working memory? A randomized controlled trial. *Developmental Science*, *16*(6), 915–925. https://doi.org/10.1111/desc.12068

Elliott, J. G., Gathercole, S. E., Alloway, T. P., Holmes, J., & Kirkwood, H. (2010). An evaluation of a classroom-based intervention to help overcome working memory difficulties and improve long-term academic achievement. *Journal of Cognitive Education and Psychology*, *9*(3), 227–250. https://doi.org/10.1891/1945-8959.9.3.227

Ellis, A., Ahmed, S. F., Zeytinoglu, S., Isbell, E., Calkins, S. D., Leerkes, E. M., … Davis-Kean, P. E. (2021). Reciprocal associations between executive function and academic achievement: A conceptual replication of Schmitt et al. *Journal of Numerical Cognition*, *7*(3), 453–472. https://doi.org/10.5964/jnc.7047

Farah, M. J., Shera, D. M., Savage, J. H., Betancourt, L., Giannetta, J. M., Brodsky, N. L., … Hurt, H. (2006). Childhood poverty: Specific associations with neurocognitive development. *Brain Research*, *1110*(1), 166–174. https://doi.org/10.1016/j.brainres.2006.06.072

Fay-Stammbach, T., Hawes, D. J., & Meredith, P. (2014). Parenting influences on executive function in early childhood: A review. *Child Development Perspectives*, *8*(4), 258–264. https://doi.org/10.1111/cdep.12095

Fischer, U., Moeller, K., Cress, U., & Nuerk, H. C. (2013). Interventions supporting children's mathematics school success. *European Psychologist*, *18*, 89–113. https://doi.org/10.1027/1016-9040/a000141

Friso-Van den Bos, I., Van der Ven, S. H., Kroesbergen, E. H., & Van Luit, J. E. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review*, *10*, 29–44. https://doi.org/10.1016/j.edurev.2013.05.003

Gandolfi, E., Viterbori, P., Traverso, L., & Usai, M. C. (2014). Inhibitory processes in toddlers: A latent-variable approach. *Frontiers in Psychology*, *5*, 381. https://doi.org/10.3389/fpsyg.2014.00381

García-Madruga, J. A., Elosúa, M. R., Gil, L., Gómez-Veiga, I., Vila, J. Ó., Orjales, I., … Duque, G. (2013). Reading comprehension and working memory's executive processes: An intervention study in primary school students. *Reading Research Quarterly*, *48*(2), 155–174. https://doi.org/10.1002/rrq.44

Gómez, D. M., Jiménez, A., Bobadilla, R., Reyes, C., & Dartnell, P. (2015). The effect of inhibitory control on general mathematics achievement and fraction comparison in middle school children. *ZDM*, *47*, 801–811. https://doi.org/10.1007/s11858-015-0685-4

Holmes, J., & Gathercole, S. E. (2014). Taking working memory training from the laboratory into schools. *Educational Psychology*, *34*(4), 440–450. https://doi.org/10.1080/01443410.2013.797338

INVALSI. (2018). *Quadro di riferimento delle prove di matematica del sistema nazionale di valutazione*. Retrieved from https://invalsi-areaprove.cineca.it/docs/file/QdR_MATEMATICA.pdf

Kadosh, R. C., Dowker, A., Heine, A., Kaufmann, L., & Kucian, K. (2013). Interventions for improving numerical abilities: Present and future. *Trends in Neuroscience and Education*, *2*(2), 85–93. https://doi.org/10.1016/j.tine.2013.04.001

Kassai, R., Futo, J., Demetrovics, Z., & Takacs, Z. K. (2019). A meta-analysis of the experimental evidence on the near-and far-transfer effects among children's executive function skills. *Psychological Bulletin*, *145*(2), 165–188. https://doi.org/10.1037/bul0000180

Kaufman, L., & von Aster, M. (2012). The diagnosis and management of dyscalculia. *Deutsches Ärzteblatt International*, *109*(45), 767–778. https://doi.org/10.3238/arztebl.2012.0767

Kilpatrick, J., Swafford, J., & Findell, B. (2002) *Adding it up: Helping children learn mathematics*. Washington, DC: The National Academies Press.

Klingberg, T., Fernell, E., Olesen, P. J., Johnson, M., Gustafsson, P., Dahlstrom, K., … Westerberg, H. (2005). Computerized training of working memory in children with ADHD-A randomized, controlled trial. *Journal of the American Academy of Child and Adolescent Psychiatry*, *44*(2), 177–186. https://doi.org/10.1097/00004583-200502000-00010

Kolkman, M. E., Hoijtink, H. J., Kroesbergen, E. H., & Leseman, P. P. (2013). The role of executive functions in numerical magnitude skills. *Learning and Individual Differences*, *24*, 145–151. https://doi.org/10.1016/j.lindif.2013.01.004

Korzeniowski, C., Ison, M. S., & Difabio de Anglat, H. (2021). A summary of the developmental trajectory of executive functions from birth to adulthood. In *Psychiatry and neuroscience update*. (pp. 459–473). Cham, Switzerland: Springer.

Krivitzky, L., Bosenbark, D. D., Ichord, R., Jastrzab, L., & Billinghurst, L. (2019). Brief report: Relationship between performance testing and parent report of attention and executive functioning profiles in children following perinatal arterial ischemic stroke. *Child Neuropsychology*, *25*(8), 1116–1124. https://doi.org/10.1080/09297049.2019.1588957

Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schönmann, C., Plangger, F., … von Aster, M. (2011). Mental number line training in children with developmental dyscalculia. *NeuroImage*, *57*(3), 782–795. https://doi.org/10.1016/j.neuroimage.2011.01.070

Lee, K., Bull, R., & Ho, R. M. (2013). Developmental changes in executive functioning. *Child Development*, *84*(6), 1933–1953. https://doi.org/10.1111/cdev.12096

Meltzer, L. (2018a) *Executive function in education. From theory to practice.* (2nd ed.). New York: The Guilford Press.

Meltzer, L. (2018b). Creating strategic classrooms and schools: Embedding executive function strategies in the curriculum. In L. Meltzer (Ed.), *Executive function in education: From theory to practice.* (pp. 263–299). New York: The Guilford Press.

Miller, M. R., Giesbrecht, G. F., Müller, U., McInerney, R. J., & Kerns, K. A. (2012). A latent variable approach to determining the structure of executive function in preschool children. *Journal of Cognition and Development*, *13*(3), 395–423. https://doi.org/10.1080/15248372.2011.585478

Miller, M. R., Müller, U., Giesbrecht, G. F., Carpendale, J. I., & Kerns, K. A. (2013). The contribution of executive function and social understanding to preschoolers' letter and math skills. *Cognitive Development*, *28*(4), 331–349. https://doi.org/10.1016/j.cogdev.2012.10.005

MIUR. (2012). Retrieved from. https://www.miur.gov.it/documents/20182/51310/DM+254_2012.pdf

Miyake, A., & Friedman, N. P. (2012). The nature and organization of individual differences in executive functions: Four general conclusions. *Current Directions in Psychological Science*, *21*(1), 8–14. https://doi.org/10.1177/0963721411429458

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology*, *41*(1), 49–100. https://doi.org/10.1006/cogp.1999.0734

Moffitt, T. E., Arseneault, L., Belsky, D., Dickson, N., Hancox, R. J., Harrington, H., … Caspi, A. (2011). A gradient of childhood self-control predicts health, wealth, and public safety. *Proceedings of the National Academy of Sciences*, *108*(7), 2693–2698. https://doi.org/10.1073/pnas.1010076108

Monette, S., Bigras, M., & Lafrenière, M. A. (2015). Structure of executive functions in typically developing kindergarteners. *Journal of Experimental Child Psychology*, *140*, 120–139. https://doi.org/10.1016/j.jecp.2015.07.005

Noble, K. G., McCandliss, B. D., & Farah, M. J. (2007). Socioeconomic gradients predict individual differences in neurocognitive abilities. *Developmental Science*, *10*(4), 464–480. https://doi.org/10.1111/j.1467-7687.2007.00600.x

Noble, K. G., Norman, M. F., & Farah, M. J. (2005). Neurocognitive correlates of socioeconomic status in kindergarten children. *Developmental Science*, *8*(1), 74–87. https://doi.org/10.1111/j.1467-7687.2005.00394.x

Passolunghi, M. C., & Mammarella, I. C. (2012). Selective spatial working memory impairment in a group of children with mathematics learning disabilities and poor problem-solving skills. *Journal of Learning Disabilities*, *45*(4), 341–350. https://doi.org/10.1177/0022219411400746

Pecini, C., Spoglianti, S., Bonetti, S., & Di Lieto, M. C. (2019). Teleriabilitazione nei disturbi specifici di apprendimento. In G. Stella, M. Zoppello & M. Scorza (Eds.), *DSA: identificazione, valutazione e trattamento. Indicazioni per gli operatori*. Milano, Italy: FrancoAngeli.

Peng, P., & Kievit, R. A. (2020). The development of academic achievement and cognitive abilities: A bidirectional perspective. *Child Development Perspectives*, *14*(1), 15–20. https://doi.org/10.1111/cdep.12352

Quattropani, M. C. (2008). La valutazione neuropsicologica. (pp. 1000–1032).

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, *20*(2), 110–122. https://doi.org/10.1016/j.lindif.2009.10.005

Rivella, C., Bombonato, C., & Viterbori, P. (2022). La valutazione delle funzioni esecutive nel periodo prescolare e scolare. In G. M. Marzocchi, C. Pecini, M. C. Usai & P. Viterbori (Eds.), *Le funzioni esecutive nei disturbi del neurosviluppo. Dalla valutazione all'intervento*. (pp. 61–73). Firenze, Italy: Hogrefe.

Rivella, C., Ruffini, C., Bombonato, C., Capodieci, A., Frascari, A., Marzocchi, G. M., … Viterbori, P. (2023). TeleFE: A new tool for the tele-assessment of executive functions in children. *Applied Sciences*, *13*(3), 1728. https://doi.org/10.3390/app13031728

Rode, C., Robson, R., Purviance, A., Geary, D. C., & Mayr, U. (2014). Is working memory training effective? A study in a school setting. *PLoS One*, *9*(8), e104796. https://doi.org/10.1371/journal.pone.0104796

Rosas, R., Espinoza, V., Porflitt, F., & Ceric, F. (2019). Executive functions can be improved in preschoolers through systematic playing in educational settings: Evidence from a longitudinal study. *Frontiers in Psychology*, *10*, 2024. https://doi.org/10.3389/fpsyg.2019.02024

Ruffini, C., Marzocchi, G. M., & Pecini, C. (2021). Preschool executive functioning and child behavior: Association with learning prerequisites? *Children*, *8*(11), 964. https://doi.org/10.3390/children8110964

Ruffini, C., Osmani, F., Martini, C., Giera, W. K., & Pecini, C. (2023). The relationship between executive functions and writing in children: A systematic review. *Child Neuropsychology*, *30*(1), 105–163. https://doi.org/10.1080/09297049.2023.2170998

Ruffini, C., Spoglianti, S., Bombonato, C., Bonetti, S., Di Lieto, M. C., & Pecini, C. (2021). Dialogic Reading to empower executive functions in preschoolers. *Children (Basel, Switzerland)*, *8*(5), 373. https://doi.org/10.3390/children8050373

Ruffini, C., Tarchi, C., Morini, M., Giuliano, G., & Pecini, C. (2022). Tele-assessment of cognitive functions in children: A systematic review. *Child Neuropsychology*, *28*(6), 709–745. https://doi.org/10.1080/09297049.2021.2005011

Sánchez-Pérez, N., Castillo, A., López-López, J. A., Pina, V., Puga, J. L., Campoy, G., … Fuentes, L. J. (2018). Computer-based training in math and working memory improves cognitive skills and academic achievement in primary school children: Behavioral results. *Frontiers in Psychology*, *8*, 2327. https://doi.org/10.3389/fpsyg.2017.02327

Scerif, G., Blakey, E., Gattas, S., Hawes, Z., Howard, S., Merkley, R., … Simms, V. (2023). Making the executive 'function' for the foundations of mathematics: The need for explicit theories of change for early interventions. *Educational Psychology Review*, *35*(4), 110. https://doi.org/10.1007/s10648-023-09824-3

Schmitt, S. A., Geldhof, G. J., Purpura, D. J., Duncan, R., & McClelland, M. M. (2017). Examining the relations between executive function, math, and literacy during the transition to kindergarten: A multi-analytic approach. *Journal of Educational Psychology*, *109*(8), 1120–1140. https://doi.org/10.1037/edu0000193

Schuchardt, K., Maehler, C., & Hasselhorn, M. (2008). Working memory deficits in children with specific learning disorders. *Journal of Learning Disabilities*, *41*(6), 514–523. https://doi.org/10.1177/0022219408317856

Scionti, N., Cavallero, M., Zogmaister, C., & Marzocchi, G. M. (2020). Is cognitive training effective for improving executive functions in preschoolers? A systematic review and meta-analysis. *Frontiers in Psychology*, *10*, 2812. https://doi.org/10.3389/fpsyg.2019.02812

Scionti, N., & Marzocchi, G. M. (2021). The dimensionality of early executive functions in young preschoolers: Comparing unidimensional versus bidimensional models and their ecological validity. *Child Neuropsychology*, *27*(4), 491–515. https://doi.org/10.1080/09297049.2020.1868419

Shaw, R., & Lewis, V. (2005). The impact of computer-mediated and traditional academic task presentation on the performance and behaviour of children with ADHD. *Journal of Research in Special Educational Needs*, *5*(2), 47–54. https://doi.org/10.1111/J.1471-3802.2005.00041.x

Silver, C. H. (2014). Sources of data about children's executive functioning: Review and commentary. *Child Neuropsychology*, *20*(1), 1–13. https://doi.org/10.1080/09297049.2012.727793

Simanowski, S., & Krajewski, K. (2019). Specific preschool executive functions predict unique aspects of mathematics development: A 3-year longitudinal study. *Child Development*, *90*(2), 544–561. https://doi.org/10.1111/cdev.12909

Söderqvist, S., & Bergman Nutley, S. (2015). Working memory training is associated with long term attainments in math and reading. *Frontiers in Psychology*, *6*, 1711. https://doi.org/10.3389/fpsyg.2015.01711

Soneson, E., Howarth, E., Ford, T., Humphrey, A., Jones, P. B., Thompson Coon, J., … Anderson, J. K. (2020). Feasibility of school-based identification of children and adolescents experiencing, or at-risk of developing, mental health difficulties: A systematic review. *Prevention Science*, *21*(5), 581–603. https://doi.org/10.1007/s11121-020-01095-6

Steinberg, J., & Roditi, B. N. (2018). The strategic math classroom: How executive function impacts math learning. In L. Meltzer (Ed.), *Executive function in education: From theory to practice*. (pp. 300–325). New York: Guilford Press.

Suan, J. S. (2014). Factors affecting underachievement in mathematics. *Proceeding of the Global Summit on Education GSE*, 5.

Takacs, Z. K., & Kassai, R. (2019). The efficacy of different interventions to foster children's executive function skills: A series of meta-analyses. *Psychological Bulletin*, *145*, 653–697. https://doi.org/10.1037/bul0000195

Toplak, M. E., West, R. F., & Stanovich, K. E. (2013). Practitioner review: Do performance-based measures and ratings of executive function assess the same construct? *Journal of Child Psychology and Psychiatry*, *54*(2), 131–143.

Traverso, L., Viterbori, P., Gandolfi, E., Zanobini, M., & Usai, M. C. (2022). The contribution of inhibitory control to early literacy skills in 4-to 5-year-old children. *Early Childhood Research Quarterly*, *59*, 265–286. https://doi.org/10.1016/j.ecresq.2021.11.010

Traverso, L., Viterbori, P., & Usai, M. C. (2019). Effectiveness of an executive function training in Italian preschool educational services and far transfer effects to pre-academic skills. *Frontiers in Psychology*, *10*, 2053. https://doi.org/10.3389/fpsyg.2019.02053

Usai, M. C., Viterbori, P., & Traverso, L. (2018). Preschool executive function profiles: Implications for math achievement in grades 1 and 3. *Journal of Research in Childhood Education*, *32*(4), 404–418. https://doi.org/10.1080/02568543.2018.1496957

Usai, M. C., Viterbori, P., Traverso, L., & De Franchis, V. (2014). Latent structure of executive function in five-and six-year-old children: A longitudinal study. *European Journal of Developmental Psychology*, *11*(4), 447–462. https://doi.org/10.1080/17405629.2013.840578

Van der Ven, S. H. G., Kroesbergen, E. H., Boom, J., & Leseman, P. P. M. (2012). The development of executive functions and early mathematics: A dynamic relationship. *British Journal of Educational Psychology*, *82*, 100–119. https://doi.org/10.1111/j.2044-8279.2011.02035.x

Van Dijk, J. (2020) *The digital divide*. Hoboken: John Wiley & Sons.

Viterbori, P., Usai, M. C., Traverso, L., & De Franchis, V. (2015). How preschool executive functioning predicts several aspects of math achievement in grades 1 and 3: A longitudinal study. *Journal of Experimental Child Psychology*, *140*, 38–55. https://doi.org/10.1016/j.jecp.2015.06.014

Vogel, S. E., & De Smedt, B. (2021). Developmental brain dynamics of numerical and arithmetic abilities. *npj Science of Learning*, *6*(22), 22. https://doi.org/10.1038/s41539-021-00099-3

Wiley, B., Ghanim, F., Taylor, K., & Murias, K. (2021). P. 124 baseline assessment of attention and executive function deficits in children with neurodevelopmental disorders: Data from a Speciality attention clinic. *Canadian Journal of Neurological Sciences*, *48*(s3), S54–S55. https://doi.org/10.1017/cjn.2021.400

Yeniad, N., Malda, M., Mesman, J., Van IJzendoorn, M. H., & Pieper, S. (2013). Shifting ability predicts math and reading performance in children: A meta-analytical study. *Learning and Individual Differences*, *23*, 1–9. https://doi.org/10.1016/j.lindif.2012.10.004

Zelazo, P. D. (2020). Executive function and psychopathology: A neurodevelopmental perspective. *Annual Review of Clinical Psychology*, *16*, 431–454. https://doi.org/10.1146/annurev-clinpsy-072319-024242

APPENDIX 1: GRID FOR THE ASSESSMENT OF PROBLEM-SOLVING SKILLS

	Indexes		*Questions*
Language skills	Problem text comprehension		Is the text produced adherent to the request of the problem question?
	Answer argumentation		1. Is the text written using logical connectors such as "why" and "therefore"? 2. Is the text correct and ordered in the exposition of the argument? 3. Is the text relevant to the solution to be achieved?
	Listening and communication		Is the written answer integrated by an oral explanation?
Problem-solving skills	Use of metacognitive strategies	Control cognitive processes	1. Does the text adequately describe the process used to arrive at the solution? 2. Does the text describe the causes of the reasoning that led to the solution?
		Decision strategies	3. Does the text clearly contain information about the solution of the problem?
		Graphic representation of the problem	4 Is the graphic solution produced functional to the solution process adopted?

APPENDIX 2: TRANSCRIPTS OF SOME OF THE TEACHERS' QUALITATIVE OBSERVATIONS (ENGLISH TRANSLATION)

"An important aspect of this project was the connection to the university world, which gave a new value to our classroom work. It is as if the project has 'elevated' the level of our teaching, creating a unique blend of academic and school experience."

"Children were highly engaged in problem-solving activities in groups. They carried out their activities with enthusiasm and motivation. They look forward to the weekly meeting in order to carry out the problem solving exercises, which is not very common."

"It was a pleasure to see children working with a new method which provided significant value to the learning experience."

"Children were initially bewildered at not using the classic problem-solving approach. After a few times, they were already able to find innovative solutions to the problems presented by actively collaborating with each other in a productive and creative discussion."

"Innovative ways, such as the use of digital technology and close collaboration between school and families, have proven to be clear strengths in this project. The positive reception by families and active collaboration were key elements that contributed to the success of this project."

"There is a great need for children to work on problem solving, especially in a collaborative mode and in small groups. The approach we used with this training provided a stimulating challenge for students, helping them to adapt and overcome the restrictions related to the pandemic."

"Rigor and careful control of the frequency were central elements in ensuring the success of this project. The careful management of this variable was challenging, but the constant monitoring contributed significantly to the overall quality of our work."

"The meticulous planning of activities is evident, showcasing the significant organizational effort and time invested. The benefits of this approach are clearly seen in the high-quality teaching and the overall positive student experience."