# Agent-Based Model of Tax Control Under Information Spreading

**Suriya Kumacheva, Elena Gubar, Ekaterina Zhitkova, Galina Tomilina, and Edgar J. Sánchez Carrera**

**Abstract** We affirm that information about tax audit probability may be encapsulated and analyzed within the formations of taxpayer networks. Hence, our aim is to analyze how information and its dissemination as an element of tax control affects the evolutionary process of paying taxes driven by networks of economic agents. We show the process of dissemination of audit information using evolutionary game theory, taking into account the network structure of interactions between economic agents. We then show that such agents from a population of contributors are more willing to exchange information with their neighbors through the various topological network formations. To support our model, a series of numerical experiments are carried out, for different network topologies, bimatrix instant game structures, and evolutionary dynamics imitation rules. Concluding that tax evasion (whether it is little or a lot) remains present in each taxpayer network regardless of the spread of information and the topology of such networks, but that the networks once identified can be controlled in order to eliminate tax evasion.

S. Kumacheva (✉)
Faculty of Economics, St. Petersburg State University, St. Petersburg, Russia
e-mail: s.kumacheva@spbu.ru

E. Gubar · E. Zhitkova
Faculty of Applied Mathematics and Control Processes, St. Petersburg State University, St. Petersburg, Russia
e-mail: e.gubar@spbu.ru; zhitkovakaterina@mail.ru

G. Tomilina
Systems, Spain S.L., Malaga, Andalucia, Spain
e-mail: g.tomilina@yandex.ru

E. J. Sánchez Carrera
Department of Economics and Management, University of Florence, Florence, Italy

Research Fellow at the Applied Mathematics Research Center CIMA UAdeC, Saltillo, Mexico
e-mail: edgar.sanchezcarrera@uniurb.it



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## 1 Introduction

Individual and small group socioeconomic behavior has become one of the critical problems in various domains of social and applied sciences. A great deal of research work with mathematical and dynamical systems tools and simulations has been applied in psychology, anthropology, and sociology (see, e.g., [1–3, 18, 19, 23, 29, 33]). The research concludes that the choice of a particular behavioral strategy by an individual depends on the quality of the information they have, which can be complete, bounded or partial (incomplete), or completely absent. Then, a series of existing models may be improved by studying in detail how information of different quality circulates in a network/group of individuals.[1]

Current research on the behavior of tax morale/compliance versus tax evasion shows that this is a function of income levels [4, 11]; particularly, it is shown that for high fines, only people with high incomes evade, while for moderate sanctions, people with medium incomes evade more frequently. In addition, tax authorities audit low returns more intensively than average returns when the common shock is favorable and sometimes do not audit at all when the common shock is unfavorable [32, 41]. But, what about the dissemination of information on the performance of tax audits focused on improving the tax collection system? The availability of information helps strengthen people's preconceived assumptions, which results in obtaining different benefits [22, 45]. In this sense, our objective is to study the flow of information and its quality in an agent-based problem model configured in a network where acts of corruption are perceived in an environment of taxpayers and public auditors.

For example, the intensity of information about a tax audit has effects on tax evasion activities. These results are supported by Kleven et al. [25] who state that audit activities have an insignificant effect on tax evasion.[2] Therefore, increased audit activities do not necessarily improve tax collection. It is known that taxpayers tend to be less receptive to information about audit activities carried out by tax authorities. Rational taxpayers tend to realize that the intensity of the inspection by the tax authorities has a small probability of reaching 100%. This is due to the low frequency of meetings between tax authorities and taxpayers and the problem of the cliché, but in order to model or analyze the above well, the networks made up

[1] Dissemination of information largely depends on the risk propensity of agents, as well as on their social networked environment (social contacts) [5, 9, 12, 31, 37], because in many cases economic agents do not have accurate and comprehensive information about planned events (for instance, an information on a tax auditing), or they are unable to adequately assess the behavior of their neighbors. As it was assumed in [8, 27, 38], the agents correct their actions only through evaluation of other agents' potential benefits. This brings us to looking at information component as an important control parameter in studying the evolution of different behaviors in population.

[2] Kleven et al. [25] distinguish between third-party and self-reported income in a tax audit study applied for Denmark. They found that the compliance rate for self-reported income was about 83% for total positive income, 95% for capital income, 86% for stock income, and 82% for self-employment income.

of taxpayers and auditors must be considered, and this is the main contribution of the present investigation. Furthermore, it is presumed that the resources of the tax authorities are not sufficient to carry out tax audits that include all taxpayers.

On the one hand, research on the psychological tax contract postulates that tax compliance is influenced by government policies, the behavior of tax authorities, and state institutions [14, 42]. However, they do not take into account how government information flows about possible government audits, and in the same way, they do not take into account the networks formed by the economic agents involved. Recently, [40] applying a field experiment approach to an African context finds that appealing to tax morale promotes compliance but slightly less than the threat of an audit. However, once again, the role of information and its quality and fluidity on the networks of economic agents and possible tax audits is something missing in their study.

In this vein, the current research examines the question of how information of possible tax auditing impacts on the individual's choice between paying taxes or evading them [2, 3]. In earlier works [16–18, 20, 28, 30] based on the static model [7], we have formulated the combined model of tax control with information component and developed a dynamic system which describes the process of information dissemination over the network of taxpayers.

Previously, it has been studied a similar modeling [20, 28, 30] in the assumption that the agents are risk neutral. In these initial works, the information spreadings have been considered as the only factor which influences on the taxpayers' strategy. But taking into account the agents' bounded rationality, we need to consider their propensity to risk as another decision-making trigger. The first attempt to solve this problem was made in [29]. The current study is devoted to the developing of this approach.

The mathematical modeling of tax control includes a series of methods such as application of decision theory, contract theory, game theory, and probabilistic modeling. For example, game-theoretical models designed in [10, 43], and [7] have hierarchical structure based on the "principal-agent" scheme which includes tax authority and risk-neutral taxpayers as players. The foregoing models describe the optimal tax control strategy using different modifications of a "threshold rule." Therefore, the fiscal administration needs to design a sample selection of the audited agents, which is smaller than the proportion of the population, corresponding to the value $P^*$. In practice, while creating such sample selection, tax authority follows a set of criteria, which are often not obvious to taxpayers. Therefore, in this study, the actual proportion of the audited taxpayers (the probability of checking $P_L$) is assumed to be stochastic.

From a practical standpoint, the implementation of such strategies is a costly procedure which can be unattainable due to the limited state budget. The tax authority needs to develop new tools to encourage tax payments. Thus, the dissemination of information on possible tax auditing serves as an efficient tool for accomplishing this goal. However, it must be taken into consideration that, according to the "threshold rule," the real probability of information in the message disseminated among the taxpayers, further mentioned as economic agents, shall be at least equal to the optimal probability value.

Over the last decades, various models of virus and information propagation across networks have been developed. For example, [15] is one of the first papers which apply the epidemic process modeling to the spreading of rumors, ideas, and information. The majority of sources [23, 33] describe the dissemination of information as “the infection of the mind” and study the phenomenon in terms of epidemic or evolutionary processes.

The paper contains the analysis of how dissemination of information impacts the agents’ decision on whether to evade taxation or not, based on evolutionary network game model. In many countries, an information about the importance of paying taxes and their social effects are propagated in media, social network, and on the billboards. Together with this viral technology, there exist special websites, where general criteria of tax auditing are published. Based on these criteria, the taxpayer can estimate the possibility of their own audit. It is assumed that any changes in the structure of these criteria or the reinforcement of the audit strategy in the media provide a stimulus for a change in the agent’s behavior strategy. In addition, the decision also depends on a variety of other factors, such as topology of the contact network, relations between agents, etc.

As opposed to the previous studies [36, 44], where agents were chosen at random, the current research applies specific algorithms to determine social contacts of an agent for obtaining more precise evolutionary dynamics of imitation. The analysis of the information obtained from advertising, media, special websites, and online tax-related applications such as helps the taxpayers estimate the possible share of audited agents. Based on such indirect information, agents can adapt their behavior to the economic environment. In turn, these changes influence the behavior of their neighbors (agents who have social connections with them), and thereby, the information is disseminated among the entire population represented by the network. Bimatrix games of different structures are used to define the interrelation between agents and then specify the dynamics.

Along with models developed for risk-neutral agents [16–18, 20, 26, 30], the current study discusses population of agents with different propensity to risk who may change their behavior and attitudes in accordance with external circumstances, as it was done in [20, 27]. Due to this fact, various scenarios of taxpayers’ behavior are designed.

The previous study [17, 18, 28–30] shows that a population of taxpayers can be represented as a connected network as people typically share information with their family members, neighbors, colleagues, and friends. Having taken that into account, we run the series of numerical experiments over the networks with different topology. To verify the statistical stability of the received stochastic results, a specific script was run $10^2$ and $3 \cdot 10^2$ times for analyzing the tendencies in taxpayer behavior.

The remainder of this paper is organized as follows. Section 2 develops the baseline static and dynamic models, considering behavioral models of risk-neutral agents (Sect. 2.1) and the operation of a system with different risk propensity agents (Sect. 2.2). Section 3 discusses the network model of annual process of tax auditing. Section 4.1 contains numerical simulation and the results. Final remarks and conclusions are in Sect. 5.

## 2 Aggregate System Costs and Total Tax Revenue

The efficiency of the fiscal system is defined through the analysis of its total revenue. In this paragraph, we define the aggregated system costs for two auxiliary cases using the basic information spreading model. The first of the cases describes the model of risk-neutral agents only. The second case represents the model of agents with different risk propensity. Modification of the original framework enables to differentiate definitions of total tax revenue.

### *2.1 Model of Risk-Neutral Agents*

The paragraph which is based on the results of [7] discusses a large but finite population of $n$ taxpayers, where $\xi$ is considered to be true income, $\eta$ is declared income, and $\eta \leq \xi$. Here, $\xi, \eta \in \{L, H\}$, where $0 < L < H$ and such formulation is similar to [6] and [26]. After adapting the model to the existing economic practice in the considered region, the modified model can contain a larger number of subgroups. The upgrade is feasible in terms of the model and it causes significant complications for the framework. Thus, only two groups of population are examined: $n_H$ and $n_L$, where $n_L + n_H = n$.

Let $P_L$ be the probability of random audit of agents, who declared income $\eta = L$. In the case when the tax evasion is revealed, the evader should pay $(\theta + \pi)(\xi - \eta)$, where constants $\theta$ and $\pi$ are the tax and penalty rates correspondingly.

Hence, for each profile of taxpaying, we obtain three possible taxpayers' profit functions:

$$u\left(L(L)\right) = (1 - \theta) \cdot L; \tag{1}$$

$$u\left(H(H)\right) = (1 - \theta) \cdot H; \tag{2}$$

$$u\left(L(H)\right) = H - \theta L - P_L(\theta + \pi)(H - L). \tag{3}$$

According to the various modifications of the "threshold rule" obtained in [7, 10, 43], the tax evasion among the risk-neutral taxpayers ranges from $H$ to $L$ levels of income if probability $P_L$ satisfies the condition:

$$P_L \geq P^* = \frac{\theta}{\theta + \pi}. \tag{4}$$

Due to the limited budget of the tax authority, the optimal values of auditing probabilities $P^*$ are practically extremely rare reached in real life. The tax authority needs to find additional methods of stimulating taxpayers to pay their taxes. The dissemination of information on possible tax auditing in the population of taxpayers can be considered as one of the efficient tools. Particularly, introduced information can be interpreted as a message "$P_L \geq P^*$," which is received by the agents randomly selected for auditing. In other words, it means that disseminated information contains the statement that the randomly selected taxpayers will be at least equal to the value $P^*$ which is critical for the agents' choice between tax evasion and tax payment.

Before formalizing the model of information spreading, some additional statements need to be introduced:

- Firstly, the considered population is the set of size $n_H$, which consists only of the taxpayers with high level $H$ of income. Therefore, it is assumed that the absence of information causes total tax evasion. Thus, we can denote the number of evaders as $n_{ev}$ and determine it in this case as $n_{ev} = n_H$.
- Secondly, as information is injected in the population, the number of taxpayers informed about increased probability of tax auditing becomes $n^0_{inf} = n^0_{nev}$. These agents are informed directly, and therefore, they decided not to evade. In every moment of time

$$n_H = n_{ev}(t) + n_{nev}(t)$$

or

$$\nu_{nev}(t) + \nu_{ev}(t) = 1$$

where $\nu_{nev}$, $\nu_{ev}$ are the proportions of evaders and non-evaders correspondingly, $t \in [0, T]$.

Moreover, at the initial time moment, the expected income of tax authority can be defined as the total tax revenue $TTR^N_0$ in the absence of information, which includes only payments of agents with true income level $L$:

$$TTR^N_0 = n_L\theta L + n_H\,(\theta\, L + P_L\,(\theta + \pi)(H - L)) - n\,P_L\,c, \tag{5}$$

where $c$ is the unit cost of auditing.

At the final time moment $T$, the total tax revenue $TTR^N_T$ is computed in the assumption that the information was spread and the system has come to a steady state:

$$\begin{aligned} TTR^N_T = n_L\theta L + n_H\,\left(\nu^T_{nev}\theta H + \nu^T_{ev}\,(\theta L + P_L(\theta + \pi)(H - L))\right) - \\ -n(P_L\,c + \nu^0_{inf}c_{inf}), \end{aligned} \tag{6}$$

where $\nu_{nev}^T$ is the proportion of honest taxpayers, who do not evade at the moment $t = T$; $\nu_{ev}^T$ is the proportion of agents, who continue to evade taxation at the moment $t = T$; $\nu_{inf}^0$ is the value (fraction) of the informational injection at the initial time moment ($\nu_{inf}^0 = \nu_{inf}(t_0)$); and $c_{inf}$ is the unit cost of such injection. It can be interpreted as the cost of any newsletters or services of individual information disseminators. Therefore, we can assume that $c_{inf} << c$ because it is more cost-efficient than employing inspectors and also taking into account additional expenses during the auditing.

## *2.2 Model of Agents with Various Propensities to Risk*

Nevertheless, interactions between tax authority and taxpayers are more various in the real-life communications. Thus, in the examined population, there can be found individuals with different risk propensity: risk-averse, risk-neutral, and risk-loving. The previous research [13, 21] showed that an attitude toward risk influences decision-making. In terms of the current study, this means that the total population of taxpayers should be divided into three subgroups, which were supposed in [27]. These subgroups differ from each other by the various agents' behavior profiles in the similar external conditions. Therefore, their response to the same information is different. In the absence of information, only taxpayers with low-income level and risk-avoiding agents (whose proportion is a $\nu_a$) pay at the initial moment. It means that in this case the total tax revenue is

$$TTR_0^R = n_L \theta L + n_H\, (\nu_a \theta H + (1 - \nu_a) P_L(\theta + \pi)(H - L)) - n P_L\, c. \qquad (7)$$

At the final moment $T$ when the information was spread and the system has come to a steady state, the total tax revenue $TTR_T^R$ is

$$\begin{aligned} TTR_T^R = n_L \theta L + n_H\, \Big(&\nu_a \theta H \\ &+ (1 - \nu_a)(\nu_{nev}^T \theta H + \nu_{ev}^T(\theta L + P_L(\theta + \pi)(H - L)))\Big) \\ &- n(P_L\, c + \nu_{inf}^0 c_{inf}). \end{aligned} \qquad (8)$$

Thus, considering the costs of tax auditing, the total amounts of tax collection, and the preferences/tastes of individuals with respect to the risk of being or not taxpayers, let us now analyze the networks formed in this approach. Early, the dissemination of information in a taxpayer population was considered as similar to epidemic models [16], the Markov process [19], and also the simplest evolutionary processes [17, 18, 28]. Along with [29], this paper presents the most comprehensive approach to the study of evolutionary processes on the network using the example of this model. In the following section, we represent some basic assumptions of the network evolutionary model.

## 3 Network Model

Generally, in terms of evolutionary game, a population of agents with different types of behavior is divided into several subpopulations according to a number of behavioral strategies. As opposed to the original formulation of the evolutionary game, where evolution of the population is described through the set of random meetings in the well-mixed population, we set that the social connections of each taxpayer can be represented by networks of different topology. Therefore, the feasible interactions are only possible among connected taxpayers. Hence, the evolution process differs from from the ordinary evolutionary game. Following [17–19, 28, 29], and[36], we suppose that tax audit disseminates information about future inspections through the taxpayers' contact network. The process of information transfer among agents is formulated as an evolutionary game based on the imitation rules [38, 44]. In the current paper, we discuss three rules of selecting neighbors following the imitation dynamics.

The first rule describes a random choice: a randomly selected taxpayer receives information from the active agent with which he/she is connected. The second rule is based on the principle of obtaining information from the most influential neighbor: information is received by the agent with the largest number of direct connections with other agents. If there are several agents with an equally large number of contacts, the choice between them is made at random. The third method is based on the neighbor with the highest income: information is propagated by the taxpayer with the highest income and in accordance with the results of the first iteration step.

### *3.1 Instant Games*

Following the general assumptions of evolutionary games, any changes in a population happen according to interactions between agents in the series of instant games, which occur during the long-run period. This subsection represents structures of an instant game between a couple of taxpayers. As in [38, 44], the instant communications between taxpayers are defined through two-player symmetric bimatrix game $\Gamma(A, B)$ presented below, where a payoff matrix of the first player is $A$ and a payoff matrix of the second player is symmetric $B = A^T$. Every taxpayer can choose between two behaviors $X = \{ev, nev\}$, where $nev$ is the strategy "not evade" and $ev$ corresponds to strategy "to evade." To define the payoff matrix of the instant game between connected agents, we base it on the following classical games: the Prisoner's Dilemma, the Stag Hunt game, and the Hawk-Dove game. We choose these well-known bimatrix games because they are appropriate to estimate the impact of network structure and imitation rules. However, we adopt the classical structure of these bimatrix games to define the interactions between taxpayers in the socioeconomic population [30].

For example, based on classical formulation of Prisoner's Dilemma game [35], we obtain the following modified payoff matrix:

| | $C$ | $D$ |
|---|---|---|
| $C$ | $\overline{u} + SW, (\overline{u} + SW)$ | $(\overline{u} - SW, u(L(H)))$ |
| $D$ | $(u(L(H)), \overline{u} - SW)$ | $(\overline{u}, \overline{u})$ |

corresponds to behavior "to defect" which is equal to "to evade" in the original PD model. Here, we define $SW$ as social welfare, obtained for the participation in social consolidation. Value $\overline{u} = 1/2u(L(L)) + 1/2u(H(H))$ is the average profit of the "mean" agent. Thus, the current payoff matrix describes the results of possible meetings between taxpayers. These four strategy profiles can be realized: a meeting between two honest taxpayers with strategies $C$, a meeting between two evaders, who use strategies $D$, and two asymmetric situation profiles that define interactions between honest taxpayer and evader. Hereafter in the next two games, we follow the same technique.

Following the Stag Hunt game [39], payoff matrices $A$ and $B = A^T$ for the current study are formed in the following way:

| | $S$ | $I$ |
|---|---|---|
| $S$ | $(\overline{u} + SW, \overline{u} + SW)$ | $(0, \overline{u} - SW)$ |
| $I$ | $(\overline{u} - SW, 0)$ | $(\overline{u} - SW, \overline{u} - SW)$ |

Here, strategy $S$ corresponds to social strategy "to hunt a stag" in classical form of the game. In our case, this strategy dictates the behavior "to pay taxes/not evade." Strategy $I$ corresponds to individual strategy "to hunt a hare" in original game, and in our interpretation, this strategy recommends taxpayer "to evade." As in the previous case, we consider various behavioral options, which focus on increasing the mutual social benefits or satisfying personal interests.

The original Hawk-Dove game is transformed to modified game:

| | $F$ | $D$ |
|---|---|---|
| $F$ | $\left(\frac{u(L(H)) - (\theta + \pi)(H - L)}{2}, \frac{u(L(H)) - (\theta + \pi)(H - L)}{2}\right)$ | $(\overline{u} + SW, 0)$ |
| $D$ | $(0, \overline{u} + SW)$ | $(\frac{\overline{u} + SW}{2}, \frac{\overline{u} + SW}{2})$ |

where strategy $F$ corresponds "to be a Hawk" in the classical case, and this means that the agent demonstrates aggressive behavior. Strategy $D$ is "to be a Dove," and forces remain passive. We suppose that aggressive behavior leads taxpayers "to evade" and follow their interests. Passive strategy corresponds to behavior "to pay taxes/not evade," which means that taxpayers reflect not only personal interests but also the interests of the population in total. Additionally, we assume that the condition $u(L(H)) << (\theta + \pi)(H - L)$ should be satisfied, and it works for the large values of the parameters $\theta$ and $\pi$ or if the difference $(H - L)$ is large.

### 3.1.1 Instant Games: Examples of Network Formations

To exemplify the above games, let us assume the same initial distribution of evaders and honest taxpayers in all experiments: $n_{ev}^0 = 13$, $n_{nev}^0 = 12$, and the size of total population is $N = 25$. This size of population has been chosen to simplify visualization of the figures. The designed software allows carrying out similar experiments for a population of larger sizes. However, the conclusions obtained from these experiments are hampered by a sharply increasing number of cases that need to be analyzed.

Figures 1, 2, 3, 4, and 5, 6 demonstrate the evolution of the proportions of taxable population during the iteration process. We use the following notation: honest agents with strategy "to pay taxes" are drown by squares, and agents with strategy "to evade" are drown by circles, respectively. The risk status of agents is displayed by using the colors in the figures: risk-averse taxpayers are green nodes, risk-neutrals are red, and risk-loving are blue.

To define the dynamic process in the population, we use following modification of the network: strongly connected network (the probability of link formation is $1/10$) and weakly connected network (the probability of link formation is $1/3$).

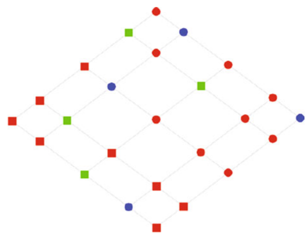

**Fig. 1** Example 1. Initial state is $n_{ev}^0 = 13$, $n_{nev}^0 = 12$, initial total tax revenue is $TTR_0^R = 52082.86$

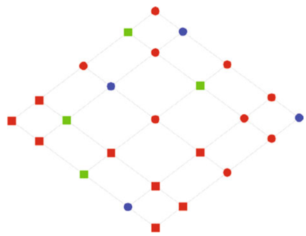

**Fig. 2** Example 1. Final state is $n_{ev}^T = 15$, $n_{nev}^T = 10$, final TTR $TTR_T^R = 65210.50$

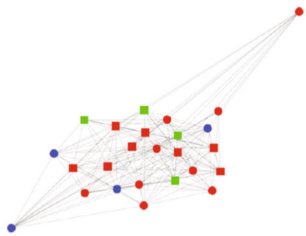

**Fig. 3** Example 2. Initial state is $n_{ev}^0 = 13$, $n_{nev}^0 = 12$, initial TTR is $TTR_0^R = 52082.86$

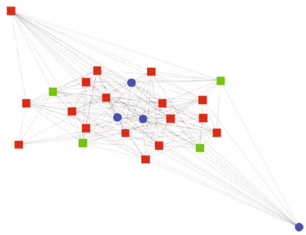

**Fig. 4** Example 2. Final state is $n_{ev}^T = 4$, $n_{nev}^T = 21$, final TTR $TTR_T^R = 82657.93$

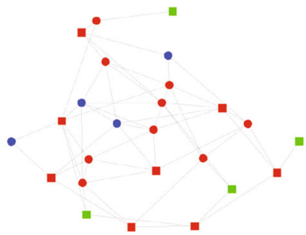

**Fig. 5** Example 3. Initial state is $n_{ev}^0 = 13$, $n_{nev}^0 = 12$, initial TTR is $TTR_0^R = 52082.86$

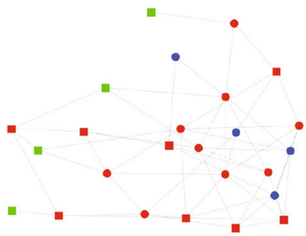

**Fig. 6** Example 3. Final state of the population is $n_{ev}^T = 7$, $n_{nev}^T = 18$, final TTR $TTR_T^R = 77899.54$

**Example 1** Here, the following combination of parameters has used: graph is grid, instant game is Prisoner's Dilemma, and imitation rule is the most influenceable neighbor. The pictures 1 and 2 present initial and final states of the system subject to cumulative method of computing of agent's profit.

**Example 2** In this example, we present the results of simulations for strong connected network with "Stag Hunt" game as an instant game, and imitation rule is "the neighbor with highest payoff"; cumulative method of computing of agent's profit is used; see Figs. 3 and 4.

**Example 3** The next example represents outcomes of simulations for weakly connected network with "Hawk-Dove" game as an instant game and imitation rule "random neighbor." Cumulative method is used as a method of computing of agent's profit; see Figs. 5 and 6.

## 4 Imitation Games on the Networks

In this section, we are considering that an evolutionary process occurs on the indirect network $G = (N, K)$, where $N = \{1, \ldots, n_H\}$ is a set of economic agents with high level of income and $K \subset N \times N$ is an edge set (each edge in $K$ represents two-player symmetric game between connected taxpayers) [17, 28, 29, 36]. It is assumed that the taxpayers choose strategies from a binary set $X = \{ev, nev\}$ and receive payoffs according to the payoffs matrices, defined in Sect. 3.1. Each instant time moment, agents use a single strategy against all opponents, and thus, the games occur simultaneously. The strategy state: $x(t) = (x_1(t), \ldots, x_{n_H}(t))^T$, where $x_i(t) \in X$ is a strategy of taxpayer $i$, $i = \overline{1, n_H}$, at time moment $t$. Aggregated payoff of agent $i$ will be defined as in [36], that is,

$$u_i = \omega_i \sum_{j \in M_i} a_{x_i(t), x_j(t)}, \tag{9}$$

where $a_{x_i(t), x_j(t)}$ is a component of payoff matrix, $W_i := \{j \in L : \{i, j\} \in K\}$ is a set of neighbors for taxpayer $i$, weighted coefficient $\omega_i = 1$ for cumulative payoffs, and $\omega_i = \frac{1}{|W_i|}$ for average payoffs. Vector of payoffs of the entire population is $u(t) = (u_1(t), \ldots, u_{n_H}(t))^T$. Here, the payoff matrix has one of the structures, presented in Sect. 3.1.

The state of population will be changed according to the rule, which is a function of the strategies and payoffs of neighboring agents:

$$x_i(t + i) = f(\{x_j(t), u_j(t) : j \in N_i \cup \{i\}\}). \tag{10}$$

This imitation rule represents a method of adjustment agents behavior to the changes in his/her environment, which means that taxpayer can choose another strategy if at least one neighbor has the better payoff. As an example of such dynamics, the proportional imitation rule [38, 44] can be used, in which each agent chooses a neighbor randomly, and if this neighbor receives a higher payoff by using a different strategy, then the agent will switch with a probability proportional to the payoff difference. The proportional imitation rule can be presented as

$$p\left(x_i(t + 1) = x_j(t)\right) := \left[\frac{\lambda}{|W_i|}(u_j(t) - u_i(t))\right]_0^1 \tag{11}$$

for each agent $i \in K$, where $j \in W_i$ is a uniformly randomly chosen neighbor, $\lambda > 0$ is an arbitrary rate constant, and the notation $[z]_0^1$ indicates $\max(0, \min(1, z))$.

We define three alternative imitation rules, each of them is based on either a choice made by an exampled agent, or taxpayers' payoffs, or distribution of risk statuses of taxpayers over the entire population. So, one important contribution is the application of these imitation rules only to the subgroup of risk-neutral taxpayers. According to our assumption, this group of taxpayers is the most influential among all other agents. And it has the ultimate impact on the change in tax collection. Proportions of risk-loving and risk-averse taxpayers are fixed at the initial time moment.

- **Rule 1.** *A random neighbor.* When a taxpayer $i$ receives an opportunity to revise their strategy, then she chooses an exampled agent at random with equal probability to all connected neighbors.
- **Rule 2.** *Neighbor with the highest payoff.* When agent $i$ receives an opportunity to revise her strategy, then she considers current payoffs of all taxpayers and chooses an agent (or a set of agents) with maximum payoff. If there are several agents with the maximum payoff, then an exampled agent is chosen at random among this subset.
- **Rule 3.** *The most influenceable neighbor.* Firstly, taxpayer $i$ estimates a number of connections of all the nearest neighbors and selects an exampled agent from the set of agents with the maximum number of links. If there are several agents with the maximum links, then an opponent is selected at each iteration of the dynamic process at random among the subset of influenceable agents.

### *4.1 Numerical Simulations and Experimental Results*

Let us conduct numerical simulations aimed to corroborate the theoretical approach from Sects. 2.1 to 3–4. We consider different initial distributions of risk status and the imitative behavior of economic agents. The first method is based on the results of the psychological research on risk addiction [34]: the percentage of risk-loving agents in the population of taxpayers is 18%, the proportion of risk-neutral agents is 65%, and the percentage of risk-avoiding agents is 17% correspondingly. The second approach is based on the results of [13, 21] where Gaussian curves are taken as the initial distribution of risk status among total population. The assumption that risk propensity is normally distributed over the population substantiates the usage of first quartile to represent initial proportions of risk-loving and risk-avoiding agents.

Numerical experiments are based on data on the income distribution of the population of the Russian Federation in 2019. The model uses two levels of income accessible for a taxpayer: low and high ($L$ and $H$). The groups with different income levels are determined according to economic rates, while the average-income levels of $L$ and $H$ groups are computed as mean values of the uniform and Pareto distributions [24]. Thus, the proportions of the entire taxpayer population (see Table 1) correspond with the results in [28].

**Table 1** Two modeled groups and average income

| Group | Income interval (rub./month) | Average income (rub.) | Proportion of population (%) |
|---|---|---|---|
| $L$ | less 25000 | $L = 12500$ | 51 |
| $H$ | more 25000 | $H = 50000$ | 49 |

For all experiments, the next values of parameters are fixed: a tax rate is $\theta = 13\%$, penalty rate is $\pi = 13\%$, optimal value of the probability of audit is $P^* = 0.5$, actual value of the probability of audit for agents who declared $L$ is $P_L = 0.1$, and unit cost of auditing is $c = 7455$ (rub.); as a unit cost of information injection, we consider $c_{inf} = 10\%c = 745.5$ (in ruble money).

If we take for $x_i(t)$ the $i$-th agent' profit at iteration $t$, then the $(t+1)$-th iteration can be considered as final if

$$\sqrt{\sum_{i=1}^{n}(x_i(t) - x_i(t+1))} \leq 10^{-2}$$

Series of experiments were conducted for the population of 25, 50, and 100 agents with high level of income $H$, with different initial distribution of risk status:

- 17% of risk-avoiding, 65% of risk-neutral, and 18% of risk-loving (in accordance with the results presented in [34])
- 25% of risk-avoiding, 65% of risk-neutral, and 10% of risk-loving (based on the assumption about Normal distribution of risk status)
- 10% of risk-avoiding, 65% of risk-neutral, and 25% of risk-loving (in accordance with the same reasoning)

Here, we suppose that the information about future auditing can be presented on billboards, advertising letters, etc. Following the framework of the model of information spreading, we run experiments with the different initial distribution of risk status in the population of taxpayers. All experiments include various configurations of the networks such as grid, strongly, and weakly connected random graphs. The evolutionary process of spreading information in the population of taxpayers occurs according to the rules of the bimatrix games Sect. 3.1. For each bimatrix game, we apply three types of imitation rules described above: a random neighbor, the neighbor with the highest payoff, and the most influenceable neighbor. Each agent evaluates their profit using these matrices and information about the neighboring agents' choices according to these matrices and the rules stated above. This rating is calculated by two algorithms:

- Cumulative: the computed sum of the profits from each interaction
- Average: the sum of the profits from interactions divided by their number

In order to investigate the information spreading behavior and analyze the structure of possible scenarios, each experiment was run $3 \cdot 10^2$ times for each initial distribution, and a detailed report was generated. Thereafter, the impact of information spreading and significant trends were estimated. Remember that the three examples presented above (in Sect. 3.1) represent different combinations of instant games, protocols, and payout calculation methods. For each example, we analyze the mean values of the final distribution.

Each experiment includes an estimation of the income level $TTR^R$ for each of the final distributions of honest taxpayers and evaders. The dynamics of $TTR^R$ is provided depending on the initial corresponding distributions.

We collect all results of experiments in the next figures. Figures 7, 8, 9, 10, and 11 show the dynamics of $TTR^R$ depending on the initial distribution of agents paying their taxes. The ordinate axis represents the values of $TTR^R$, and the abscissa axis shows the number of agents paying their taxes: **a**—for the average profit, **b**—for the cumulative profit.

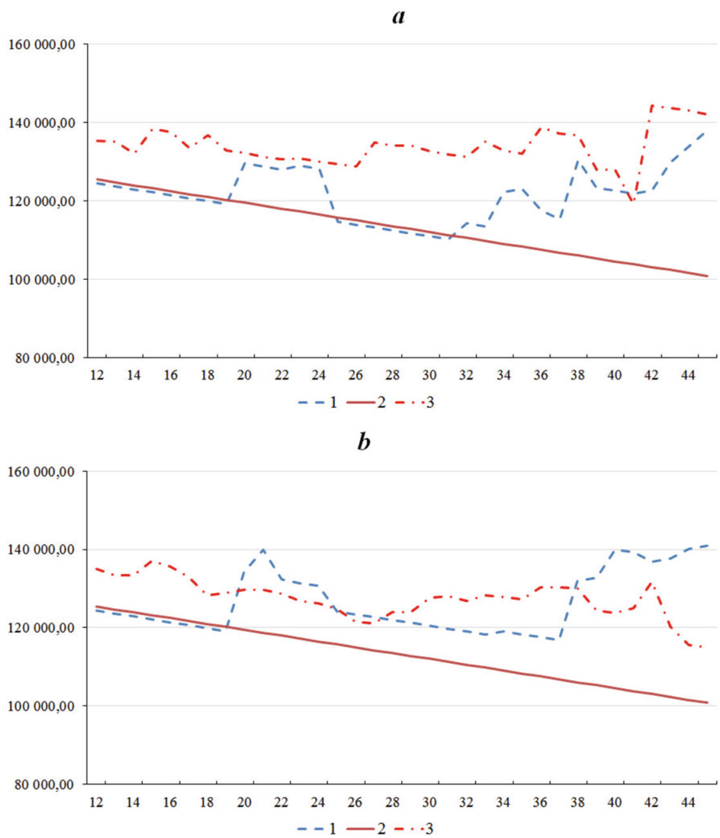


**Fig. 7** Experiment 1. The type of game: the Prisoner's Dilemma. The rule: the neighbor with the highest payoff. 1—$TTR^R$ for the type of network grid, 2—$TTR^R$ for the strongly connected graph, 3—$TTR^R$ for the weakly connected graph

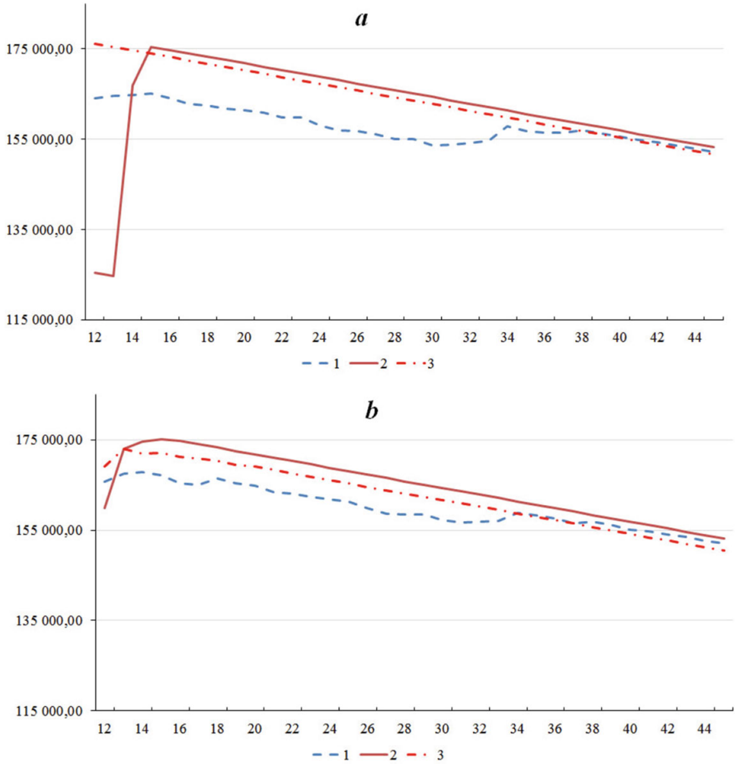


**Fig. 8** Experiment 2. The type of game: The Stag Hunt game. The rule: A random neighbor. 1—$TTR^R$ for the type of network grid, 2—$TTR^R$ for the strongly connected graph, 3—$TTR^R$ for the weakly connected graph

The first step includes a series of experiments with three types of graphs (grid, random strongly connected graph, random weakly connected graph) and the initial distribution of risk status where 25% are risk-avoiding agents, 65% are risk-neutral agents, and 10% are risk-loving. The total number of high-income agents is 50. The value of total tax revenue in the absence of information is $TTR_0^R = 102123.25$.

**Experiment 1** The Prisoner's Dilemma is considered as an instant game. The neighbor with the highest payoff is chosen as the imitation rule. The average type of payoff is used. The minimum value of $TTR^R$ is reached in the case of a strongly connected graph: $TTR_T^R = 100\ 827.06$ ($TTR_T^R < TTR_R^0$), the initial state of the population is $n_{ev}^0 = 5$, $n_{nev}^T = 45$; the final state of population is $n_{ev}^0 = 38$, $n_{nev}^T = 12$. The maximum value of $TTR^R$ is reached in the case of a grid: $TTR_T^R = 144\ 302.94$, initial state of the population is $n_{ev}^0 = 8$, $n_{nev}^T = 42$; final state of population is $n_{ev}^0 = 12$, $n_{nev}^T = 38$. Comparing the results presented in Fig.7a and b, we obtain that the level of $TTR^R$ is decreasing and is inverse to the number of agents paying their taxes. In grid and strongly connected graph, there is

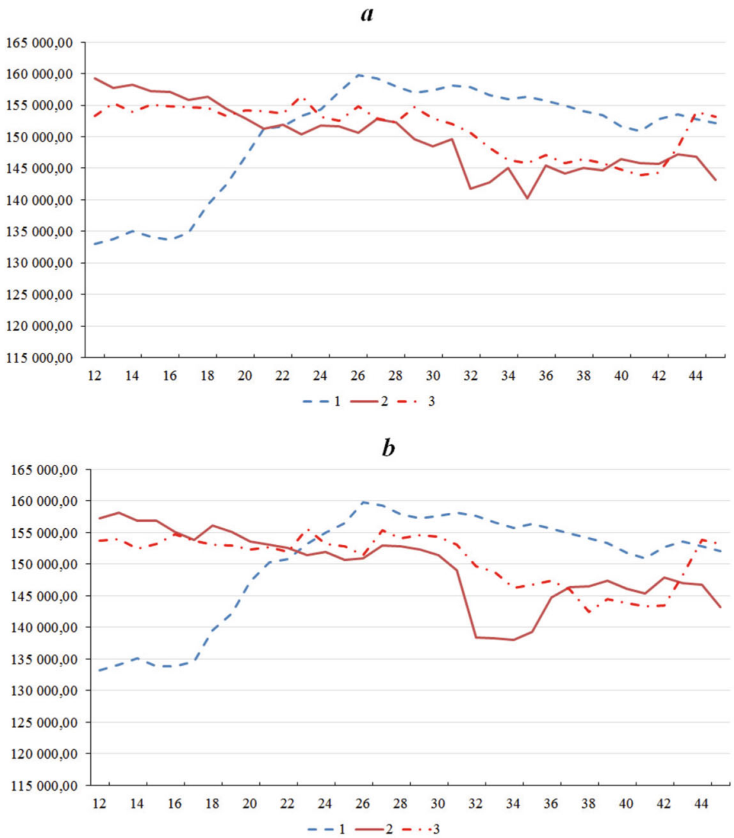


**Fig. 9** Experiment 3. The type of game: Hawk-Dove. The rule: the most influenceable neighbor. 1—$TTR^R$ for the type of network grid, 2—$TTR^R$ for the strongly connected graph, 3—$TTR^R$ for the weakly connected graph

an increase in the total tax revenue in terms of an average type of profit. At the same time, the leaps of the income level can be explained by the fact that the majority of agents accept the strategy "to evade" (from 50% to 85% of agents), and in a strongly connected graph, total 100% of agents accept the strategy of "to evade."

**Experiment 2** As an instant game, the Stag Hunt game is considered and the imitation rule is a random neighbor. The type of payoff is average.

In the case of a weakly connected graph, the minimum value of $TTR^R$ is reached: $TTR_T^R = 124\ 762.37$, initial state of the population is $n_{ev}^0 = 37$, $n_{nev}^T = 13$; final state of population is $n_{ev}^0 = 38$, $n_{nev}^T = 12$. In a strongly connected graph, the maximum value of $TTR^R$ is reached: $TTR_T^R = 176\ 184.72$, initial state of the population for this case is $n_{ev}^0 = 38$, $n_{nev}^T = 12$; final state of population is $n_{ev}^0 = 6$, $n_{nev}^T = 44$.

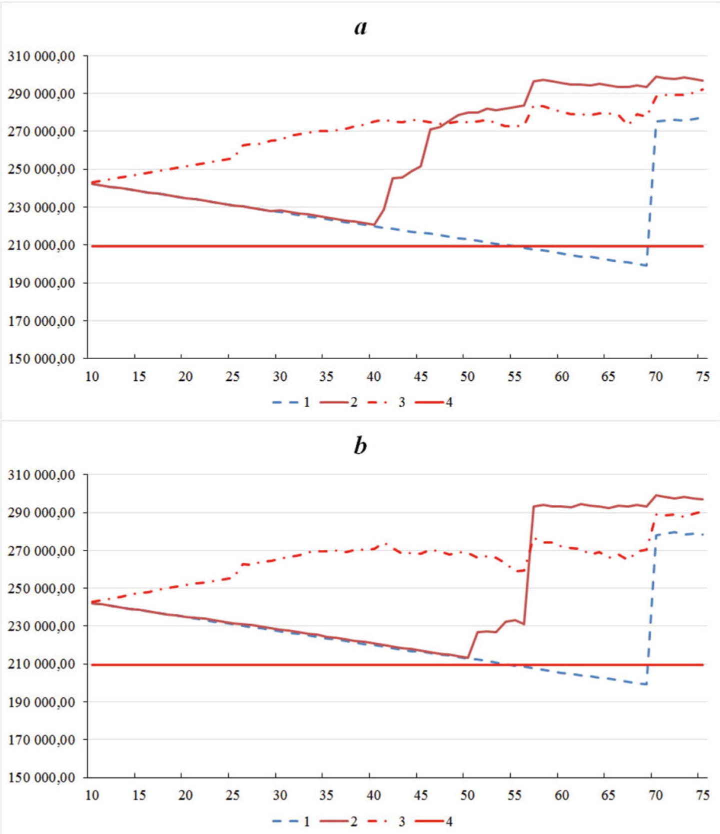


**Fig. 10** Experiment 4. The network topology: random weakly connected graph. The imitation rule: the most influenceable neighbor. 1—$TTR_T^R$ for the Prisoner's Dilemma, 2—$TTR_T^R$ for the Stag Hunt game, 3—$TTR_T^R$ for the Hawk-Dove game, 4—$TTR_0^R$

The received dynamics of $TTR^R$ on the number of agents paying their taxes $n_{nev}$ shows smooth decrease in the total tax revenue $TTR^R$. In almost 95% of cases, agents choose the strategy "to pay" accordingly, there are no penalties, and the value of $TTR^R$ decreases. Nevertheless, the total revenue $TTR_T^R$ is higher than $TTR_0^R$. This occurs due to the fact that the percentage of risk-loving agents is only 10%, while the percentage of risk-avoiding agents is higher.

**Experiment 3** The Hawk-Dove game is considered as the instant game, and the most influenceable neighbor is chosen as the imitation rule. In this experiment, the type of payoff is cumulative.

In a grid, the minimum value of $TTR^R$ is reached: $TTR_T^R = 132\,978.54$, initial state of the population is $n_{ev}^0 = 37$, $n_{nev}^T = 12$; final state of population is $n_{ev}^0 = 32$, $n_{nev}^T = 17$. In a grid, the maximum value of $TTR^R$: $TTR_T^R = 159\,768.01$, initial

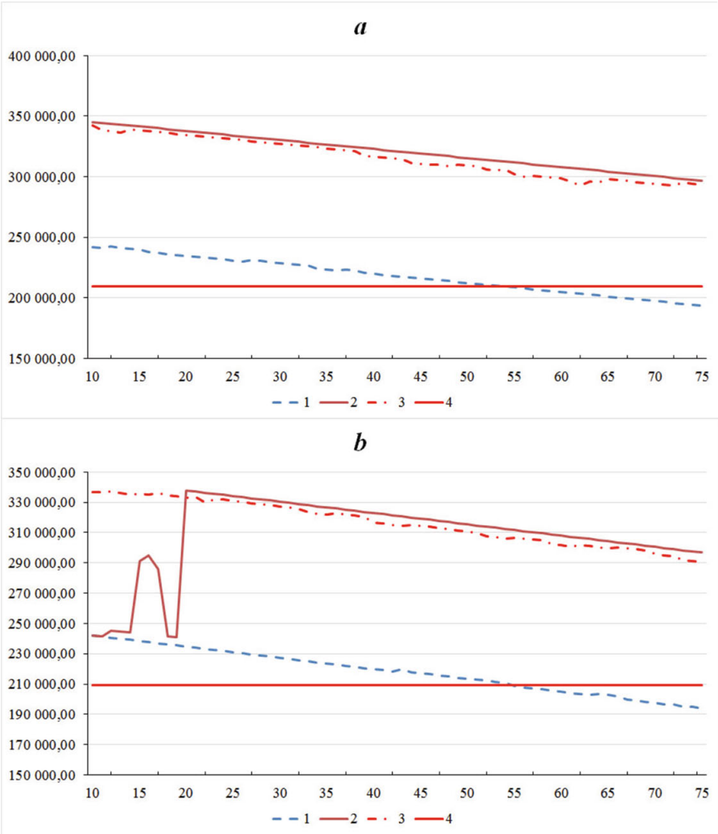


**Fig. 11** Experiment 5. The network topology: random weakly connected graph. The imitation rule: neighbor with the highest payoff. 1—$TTR_T^R$ for the Prisoner's Dilemma, 2—$TTR_T^R$ for the Stag Hunt game, 3—$TTR_T^R$ for the Hawk-Dove game, 4—$TTR_0^R$

state of the population is $n_{ev}^0 = 23$, $n_{nev}^T = 26$; final state of population is $n_{ev}^0 = 8$, $n_{nev}^T = 42$.

With the considered initial distribution of risk status, 100% of the agents in random strongly and weakly connected graphs adopt the strategy "to pay," and for the network grid, 85% of agents act similarly. Thus, the total revenue $TTR^R$ depends on the proportion of the initial and final number of agents with different strategies. This also explains the leaps in the curves.

The following is a series of experiments run with different types of considered instant games on a specific network topology. In these examples, we consider the initial distribution of risk status: 10% are risk-avoiding, 65% are risk-neutral, and 25% are risk-loving. The total number of high-income agents is 100. The value of $TTR_0^R = 209\ 352.66$.

**Experiment 4** A connecting network has a structure of random weakly connected graph; the most influenceable neighbor is the imitation rule. In Fig.10, the curves corresponding to $TTR^R$ of different instant games are denoted as 1, $TTR_T^R$ for the Prisoner's Dilemma; 2, $TTR_T^R$ for the Stag Hunt game; and 3 $TTR_T^R$ for the Hawk-Dove game, 4—$TTR_0^R$. The simulations show that from 59% to 90% agents choose the strategy "to evade." This indicator impacts on the total tax revenue in case of the Prisoner's Dilemma and the minimum of $TTR_T^R < TTR_0^R$. In two other cases, the value of $TTR_T^R$ increases dramatically in the end of the taxation period due to the existing penalties. In this case, if the final proportion of tax-evading agents is more than two times lower than the proportion of tax-paying agents, then $TTR_T^R > TTR_0^R$.

**Experiment 5** A connecting network has the structure of random weakly connected graph; the most influenceable neighbor is the imitation rule. In Fig.11, the curves corresponding to $TTR_T^R$ of different instant games are denoted as 1, $TTR_T^R$ for the Prisoner's Dilemma; 2, $TTR_T^R$ for the Stag Hunt game; 3, $TTR_T^R$ for the Hawk-Dove game; and 4, $TTR_0^R$.

In all of the three cases, the total tax revenue is decreasing, but for the instant game structured according to the Prisoner's Dilemma, the result is contradictory.

On the one hand, in 100% experiments, agents choose the strategy "to evade" and the final distribution between tax-paying and tax-evading agents is 9:1. In this case, penalties do not cover the costs of the information propagation and $TTR_T^R < TTR_0^R$. This tendency proves that individual preferences prevail over social interests. In case of two other games, the strategy "to evade" is chosen by the 95–100% of agents, and the costs of information propagation are covered by the total tax revenue; they are higher than $TTR_0^R$.

A vital part of the experiment includes comparison of the results obtained for simulations with and without specified risk status of the agents.

**Experiment 6** The Hawk-Dove game is considered, the imitation rule is the most influential neighbor, and the payoff is cumulative. The number of agents with high income is 25. The initial distribution of risk status: 17% are risk-avoiding, 65% are risk-neutral, and 18% are risk-loving. The initial total tax revenue is $TTR_0^R = 52082.86$. Figure 12 shows the dynamics of total tax revenue for different network structures: 1–3 is grid, random strongly connected graph, and random weakly connected graph with different risk statuses, respectively, and 4–6 is grid, random strongly connected graph, and random weakly connected graph without risk statuses (all high-income agents are considered risk-neutral).

The series of experiments statistically demonstrates that regardless of network topology and the imitation rule, the tendency of evasion persists. It is chosen by the majority of taxpayers in the most of experiments. This result is correct for the Prisoner's Dilemma structure of the instant game.

Figure 13 shows the dynamics of total income for different types of games: $1-3$, the Prisoner's dilemma, the Stag Hunt game, and the Hawk-Dove game with different risk statuses, respectively, and 4–6, the same games, but without risk statuses.

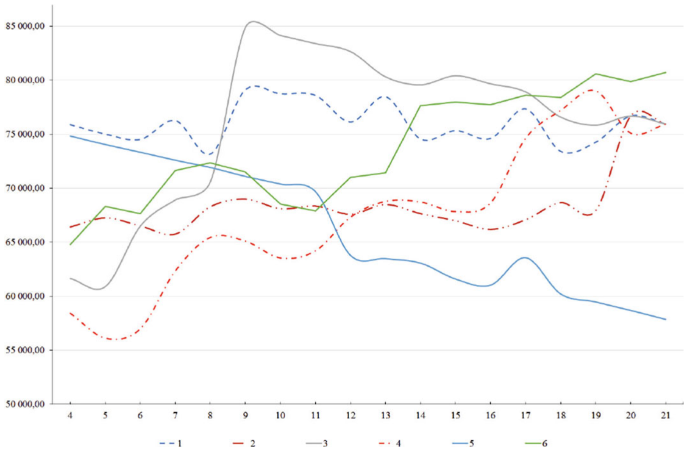


**Fig. 12** Experiment 6. The instant game: Hawk-Dove game. Imitation rule: the most influential. Notation: 1–3 is grid, random strongly connected graph, and random weakly connected graph with different risk statuses, respectively, and 4–6 is grid, random strongly connected graph, and random weakly connected graph without risk statuses (all high-income agents are considered risk-neutral)

We have also received a significant difference in the behavior of the curves depending on the number of risk-loving agents. The larger is their proportion, the greater is the probability of the event $\{TTR_T < TTR_0\}$. Moreover, in the most cases, if the imitation rule "The most influential neighbor" is used, then $TTR_T$ grows according to increasing of the number of agents, but total tax revenue decreases for all other protocols. Furthermore, in the most experiments, $TTR_T > TTR_0$, both for the game with risk-neutral players and for the game with risk-statuses.

The series of numerical simulation discovers the following tendencies. When the Prisoner's Dilemma is used as an instant game, then in long-run period, taxpayers prefer to evade, and hence, they do not pay taxes. This behavior describes the "selfish" preferences of individuals, which means that agents choose immediate profit and do not take into account possible effects of their choices. Thereby, we can say that agents do not pay attention to the received information about possible tax audit. This selfish behavior leads to deterioration of social environment and decrease in tax collection. Two other cases where the Stag Hunt game and the Hawk Dove game are considered as instant games provide completely different results. The majority of agents choose social admissible behavior and pay taxes. This choice leads to increase in tax collection, and we can conclude that information brings positive changes in the tax audit process.

The second issue of the simulation experiments is that the consideration of risk-status and influence of information on the specific group of agents (e.g., risk-neutral subgroup) provides a significant growth of the total tax revenue. Moreover, experiments show that if the topology of the network is described by weakly connected graph, then the behavior of the entire population is close to the behavior

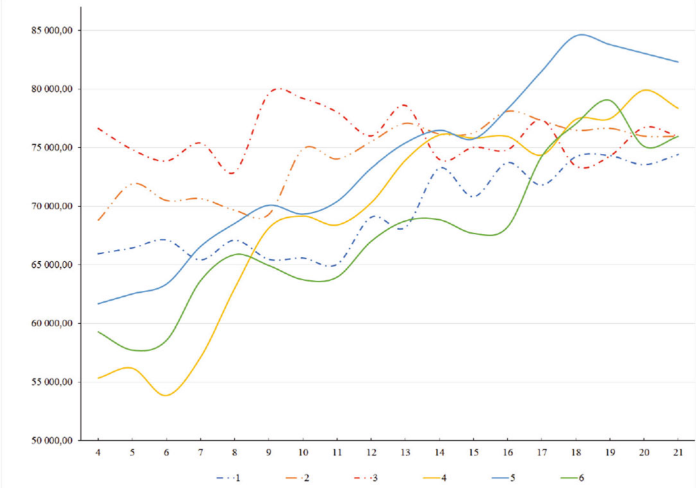


**Fig. 13** Experiment 6. Notation: 1–3, the Prisoner's dilemma, the Stag Hunt game, and the Hawk-Dove game with different risk statuses, respectively, and 4–6, the same games, without risk statuses

of an unstructured population. Other types of topology also impact the behavior of population immensely. At the same time, imitation protocol does not influence the behavior of the entire population. However, in few cases, random neighbor protocol has a weak impact. If the Prisoner's Dilemma is taken as an instant game, this protocol leads to an increase in number of unsocial strategies.

## 5 Concluding Remarks

This research article analyzed the impact of the dissemination of information from future audits in a population subject to taxes to stimulate the fair payment of taxes, when the agents differ in risks and conform in networks. In the theoretical part, the problem of tax control was represented as the joint model, which includes the application of the "threshold rule" of taxation and the evolutionary dynamics to imitate the information diffusion process. Unlike previous research, in this article, we consider an evolutionary game for the structured population rather than a population of random agents. Numerical simulations have shown the main trends that emerge with different combinations of simulation rules, instant games, and network topologies.

Particularly, the experiments/simulations statistically show us that, regardless of the network topology and the imitation rule, the evasion tendency persists. This result is correct for the instant game Prisoner's Dilemma structure.

Another result showing in the simulation experiments is that the consideration of the risk status and the influence of the information on the specific group of agents (e.g., risk-neutral subgroup) provides a significant growth of total tax revenue. Furthermore, experiments show that if the topology of the network is described by a loosely connected graph, then the behavior of the entire population approaches the behavior of an unstructured population. Other types of topology also have a huge impact on the behavior of the population. At the same time, the imitation protocol does not influence the behavior of the entire population. However, in some cases, the random neighbor protocol has a weak impact. If the Prisoner's Dilemma is taken as an instant game, this protocol leads to an increase in the number of antisocial strategies.

To conclude our investigation, we propose a possible direction for future research. The economic agents integrated in networks where there is dissemination of information must be fundamentally considered by the tax administrations in order to offer taxpayers tax incentives but also sanctions or fines in case of complying or not with tax payments. It is therefore crucial to address the fundamental issues of tax enforcement and information sharing in order to optimize the tax system's revenue collection, particularly within the agent networks. We also propose that the combination of network methods with artificial intelligence concepts such as intelligent agents could be the essential components of a new framework for risk analysis and for the provision of advanced tax agency services.

## References


1. Accinelli, E., Sánchez Carrera, E.J.: Corruption, inequality and income taxation. In: Bourguignon, J.P., Jeltsch, R., Pinto, A., Viana, M. (eds). Dynamics, Games and Science. CIM Series in Mathematical Sciences, vol. 1. Springer, Cham (2015)
2. Antocia, A., Russua, P., Zarrib, L.: Tax Evasion in a behaviorally heterogeneous society: an evolutionary analysis. Econ. Model. **10**(42), 106–115 (2014)
3. Antunes, L., Balsa, J., Urbano, P., Moniz, L., Roseta-Palma, C.: Tax compliance in a simulated heterogeneous multi-agent society. In: Sichman J.S., Antunes, L. (eds.) Multi-Agent-Based Simulation VI. MABS 2005. LNCS, vol. 3891, pp. 147–161. Springer, Berlin, Heidelberg (2006)
4. Bag, P.K., Wang, P.: Income tax evasion and audits under common and idiosyncratic shocks. J. Econ. Behav. Organ. **184**(C), 99–116 (2021)
5. Barone, M., Coscia, M.: Birds of a feather scam together: trustworthiness homophily in a business network. Soc. Netw. **54**, 228–237 (2018)
6. Boure, V., Kumacheva, S.: A model of audit with using of statistical information about taxpayers' income. Vestnik SPbGU **10**(1–2), 140–145 (2005) (in Russian)
7. Boure, V., Kumacheva, S.: A game theory model of tax auditing using statistical information about taxpayers. Vestnik SPbGU **10**(4), 16–24 (2010) (in Russian)
8. Carrera, E.J.S., Gubar, E., Oleynik, A.F.: Network structures and poverty traps. Dyn. Games Appl. **9**, 236–253 (2019). https://doi.org/10.1007/s13235-018-0256-8
9. Centola, D.: How Behavior Spreads: The Science of Complex Contagions. Princeton University Press, Princeton (2018)
10. Chander, P., Wilde, L.L.: A General characterization of optimal income tax enforcement. Rev. Econ. Stud. **65**, 165–183 (1998)

11. Cummings, R.G., Martinez-Vazquez, J., McKee, M., Torgler, B.: Tax morale affects tax compliance: evidence from surveys and an artefactual field experiment. J. Econo. Behav. Organ. **70**(3), 447–457 (2009)
12. Farhadi, F., Tavafoghi, H., Teneketzis, Jamaloddin Golestani, D.S.: An efficient dynamic allocation mechanism for security in networks of interdependent strategic agents. Dyn. Games Appl. **9**, 914–941 (2019). https://doi.org/10.1007/s13235-018-0284-4
13. Friedman, M., Savage. L.J.: The utility analysis of choices involving risk. J. Polit. Econ. **56**(4), 279–304 (1948)
14. Frey, B.S., Torgler, B.: Tax morale and conditional cooperation. J. Comp. Econ. **35**(1), 136–59 (2007)
15. Goffman, W., Newill, V.A.: Generalization of epidemic theory: an application to the transmission of ideas. Nature **204**(4955), 225–228 (1964)
16. Gubar, E.A., Kumacheva, S.Sh., Zhitkova, E.M., Porokhnyavaya, O.Yu.: Propagation of information over the network of taxpayers in the model of tax auditing. In: 2015 International Conference on Stability and Control Processes in Memory of V.I. Zubov (SCP 2015) Proceedings, pp. 244 – 247. IEEE Conference Publications, St. Petersburg (2015)
17. Gubar, E., Kumacheva, S., Zhitkova, E., Kurnosykh, Z.: Evolutionary behavior of taxpayers in the model of information dissemination. In: Constructive Nonsmooth Analysis and Related Topics (Dedicated to the Memory of V.F. Demyanov, CNSA 2017) Proceedings, pp. 1–4. IEEE Conference Publications, St. Petersburg (2017)
18. Gubar, E., Kumacheva, S., Zhitkova, E., Kurnosykh, Z., Skovorodina, T.: Modelling of information spreading in the population of taxpayers: evolutionary approach. Contrib.Game Theory Manag. **10**, 100–128 (2017)
19. Gubar, E., Kumacheva, S., Zhitkova, E., Tomilina, G.: Modeling of the impact of information on tax audits on the risk statuses and evasions of individuals. Vestnik SPbGU **15**(2), 245–258 (2019) (in Russian)
20. Gubar, E., Kumacheva, S., Zhitkova, E., Tomilina, G., Sanchez Carrera E.J.: Games and network structures on corruption, income inequality, and tax control. Contrib. Game Theory Manag. **13**, 152–172 (2020)
21. Kahneman, D., Tversky, A.: Advances in prospect theory: cumulative representation of uncertainty. J. Risk Uncertain. **5**, 297–323 (1992)
22. Kaçamak, A.Y.: The role of information aggregators in tax compliance. Int. Tax Public Financ. **29**, 237–285 (2022). https://doi.org/10.1007/s10797-021-09666-1
23. Kandhway, K., Kuri, J.: Optimal control of information epidemics modeled as Maki Thompson rumors. In: Preprint submitted to Communications in Nonlinear Science and Numerical Simulation (2014)
24. Kendall, M.G., Stuart, A.: Distribution Theory (in Russian). Nauka, Moscow (1966)
25. Kleven, H.J., Knudsen, M.B., Kreiner, C.T., Pedersen, S., Saez, E.: Unwilling or unable to cheat? Evidence from a tax audit experiment in Denmark. Econometrica **79**(3), 651–692 (2011)
26. Kumacheva, S.Sh.: Tax auditing using statistical information about taxpayers. Contrib. Game Theory Manag. **5**, 156–167 (2012)
27. Kumacheva, S.Sh., Gubar, E.A.: Evolutionary model of tax auditing. Contrib. Game Theory Manag. **8**, 164–175 (2015)
28. Kumacheva, S., Gubar, E., Zhitkova, E., Tomilina, G.: Evolution of risk-statuses in one model of tax control. In: Petrosyan L., Mazalov V., Zenkevich N. (eds.) Frontiers of Dynamic Games. Static & Dynamic Game Theory: Foundations & Applications, pp. 121–138. Birkhauser, Cham (2018)
29. Kumacheva, S., Gubar, E., Zhitkova, E., Tomilina, G.: Modeling the behaviour of economic agents as a response to information on tax audits. In: Agarwal N., Sakalauskas L., Weber G.-W. (eds.) Modeling and Simulation of Social-Behavioral Phenomena in Creative Societies. First International EURO Mini Conference (MSBC 2019) Proceedings, pp. 96– 11. Springer, Berlin (2019)

30. Kumacheva, S., Gubar, E., Zhitkova, E., Tomilina, G.: Analysis of economic behaviour in evolutionary model of tax control under information diffusion. In: Petrosyan L., Mazalov V., Zenkevich N. (eds.) Frontiers of Dynamic Games. Static & Dynamic Game Theory: Foundations & Applications, pp. 121–140. Birkhauser, Basel (2020)
31. Mazalov, V.V., Chirkova, J. V.: Networking Games: Network Forming Games and Games on Networks. Elsevier Publishing Company, Academic Press (2019)
32. Mazzolini, G., Pagani, L., Santoro, A.: The deterrence effect of real-world operational tax audits on self-employed taxpayers: evidence from Italy. Int. Tax Public Financ. (2021). https://doi.org/10.1007/s10797-021-09707-9
33. Nekovee, A.M., Moreno, Y., Bianconi, G., Marsili, M.: Theory of rumor spreading in complex social networks. Physica A **374**, 457–470 (2007)
34. Niazashvili, A.: Individual Differences in Risk Propensity in Different Social Situations of Personal Development. Moscow University for the Humanities, Moscow (2007)
35. Owen, G.: Game Theory. Saunders Company, Philadelphia, London, Toronto (1968)
36. Riehl, J.R., Cao M.: Control of stochastic evolutionary games on networks. In: 5th IFAC Workshop on Distributed Estimation and Control in Networked Systems, pp. 458 – 462. Philadelphia (2015)
37. Shirata, Y. Evolution of a collusive price in a networked market. Dyn. Games Appl. **10**, 528–554 (2020). https://doi.org/10.1007/s13235-019-00322-2
38. Sandholm, W.H.: Population Games and Evolutionary Dynamics. The MIT Press, Cambridge (2010)
39. Skyrms, B.: The Stag Hunt and the Evolution of Social Structure. Cambridge University Press, Cambridge (2003)
40. Shimeles, A., Gurara, D.Z., Woldeyes, F.: Taxman's dilemma: coercion or persuasion? Evidence from a Randomized Field Experiment in Ethiopia. Am. Econ. Rev. **107**(5), 420–424 (2017)
41. Stö, S., Traxler, C.: Tax evasion and auditing in a federal economy. Int. Tax Public Financ. **12**, 515–531 (2005). https://doi.org/10.1007/s10797-005-1542-0
42. Torgler, B.: The importance of faith: tax morale and religiosity. J. Econ. Behav. Organ. **61**(1), 81–109 (2006)
43. Vasin, A., Morozov, V.: The Game Theory and Models of Mathematical Economics. MAKS Press, Moscow (2005). (in Russian)
44. Weibull, J.: Evolutionary Game Theory. The MIT Press, Cambridge (1995)
45. Westerhout, E.W.M.T.: The capital tax and welfare effects from asymmetric information on equity markets. Int. Tax Public Financ. **9**, 219–233 (2002). https://doi.org/10.1023/A:1016263125158