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ORIGINAL PAPER

Groundwater management and illegality in a differential-evolutionary framework

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Abstract
It is estimated that half of all the water extracted, both in developed and developing countries, is unauthorized. This phenomenon makes the management of a groundwater even more difficult to avoid over-exploitation. To study the interaction between farmers, that could be compliant and non-compliant, and a water agency, we built a leader-follower differential game. However, we assumed that the water agency does not know neither ex-ante nor ex-post the number of compliant farmers. After illustrating the results of the dynamic game through numerical simulation using the Western La Mancha (Spain) data, we endogenize the types' choice in an evolutionary context. Finally, we perform comparative dynamics in the steady state to understand the role of the sanction to counter illegal behaviors.

Keywords Groundwater management · Unauthorized water extraction · Illegal behaviors · Leader-follower differential game · Replicator dynamics

1 Introduction

In recent years the global growth demand for food highlights the problem of water scarcity all over the world (Rosa et al. 2020). One way to mitigate the negative effects of water scarcity is an efficient management of groundwater resources to avoid an over-exploitation. However, the implementation of policy instruments to regulate the water extraction cannot be sufficient in presence of farmers' non-compliant behaviors. Indeed, in many areas of the world, both in developed and developing countries, a considerable share of water pumped is unauthorized (De Stefano and Lopez-Gunn 2012).

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According to Martínez-Santos et al. (2008), half of all agricultural firms in the Western La Mancha region may pump water without the authorization of the public authority. Dworak et al. (2010) estimates that around 30-60% of the total water extraction is illegal in Southern Europe countries. This phenomenon occurs also outside the EU, as documented by Budds (2009) in South America and by Castellano (2020) in North America. In 2010, an European Commission conference on unauthorized water usage in agriculture[1] provided a picture of the challenges of regulating water use in Europe. Empirical studies suggested that unauthorized water use can possibly be larger than authorized one in several regions of the European Union, particularly in the more arid and semi-arid southern member states. Illegal water use is a key issue to understand many of the problems related to depleting and over-exploited stocks. Unauthorized water extraction can undermine the security of access for users who have legal access to the resource, such as suppliers, farmers, industries and individuals who draw water for home use. In addition, the widespread unauthorized withdraw of groundwater can lead to significant negative environmental impacts such as the degradation of wetlands fed by groundwater and the alteration of the environment. One of the key roles of water authorities is to regulate and plan the use of water resources ensure its long-term feasibility and compatibility with a number of coexisting uses, maintaining the functionality of water-dependent ecosystems. Indeed, legal instruments may be inadequate or insufficient to achieve the stated resource objectives of long-term protection and sustainability. However, laws are one of the tools available to an administration to manage common resources such as water and the uncontrolled extraction of water can undermine the effectiveness of official regulatory and planning efforts. To counter the illegal phenomenon, a system of tax and sanction, in addition to adequate monitoring, should be implemented.

The sustainability of resources exploitation, such as groundwater, is an important issue in economic theory, encompassing a large range of analytical results with major contributions. The seminal work of Gisser and Sanchez (1980) has spurred a large literature investigating welfare gains from public intervention. Most of these analyses compare myopic farmers, under perfect competition, with a social planner's solution Feinerman and Knapp (1983), Brill and Burness (1994) while others compare strategic behaviors with the socially optimal solution Negri (1989), Provencher and Burt (1993), Rubio and Casino (2001, 2003). Both strands measure the gap between the two outcomes. A part of these studies advocates that the benefits from policy intervention are insignificant and depend on hydrological and economic parameters. The debate has been enriched including environmental externalities into the analytical framework, (Esteban and Albiac 2011; Pereau and Pryet 2018) firms heterogeneity (Biancardi and Maddalena 2018) and inter-generational competition among overlapping generations (Biancardi et al. 2020a).

However, to the best of our knowledge, only few paper deals with farmers' compliance. Biancardi et al. (2020b) study a hydro-economic model with a differential

[1] Conference on "Water and Agriculture" Events at Mercure Hotel, Boulevard de Lauzelle 61, Louvain-La-Nueve (Belgium), September 2010.

game approach to evaluate groundwater policies in presence of illegal behaviors. The effects of legal and illegal farmers' actions and the contribution of taxes and penalties imposed by public authorities capture the problem of non-compliance with resource management regimes and discuss policy options in a non-cooperative and cooperative context (only farmers are players while the water agency activity is exogenous). In Biancardi et al. (2022a) the model proposed analyzes the strategic interaction between firms that compete for a common groundwater resource in an evolutionary game, so players do not take into consideration the future consequences of their choices. In such a context, the firms' choice to be compliant or non-compliant is endogenous and the selection is given by the replicator dynamics, namely firms adopt the more rewarding strategy. Moreover, in Biancardi et al. (2022b) the authors assume a leader-follower differential game between a population of identical farmers and a water agency. Farmers can behave illegally not declaring all the water pumped. Finally, Biancardi et al. (2023) analyze the phenomenon of illegality assuming perfect foresight identical farmers that play a leader-follower game with the water agency.

In the present paper we consider a population of heterogeneous farmers that can be compliant and not-compliant. Since in the real world it is very difficult for the water agency to control the compliant access to the resource since non-compliant farmers seek to maintain hidden their behavior, differently from previous cited works, we assume that the water agency does not know, neither ex-ante nor ex-post, the number of illegal farmers. The policy instrument to manage the groundwater is a tax chosen by the water agency in order to maximize social welfare composed of only compliant net benefits and the environmental damage. After the analysis of the model and a numerical illustrations using the data of the Western La Mancha aquifer, we endogenize the choice of being compliant and we study how to counter illegal behaviors through numerical comparative dynamics.

The paper is organized in the following way. Section 2 presents the model and solves the firms' maximization problem. Section 3 introduces the dynamics of public authority and groundwater level, determining the optimal water tax with a leader-follower game and proposes numerical simulations about policy implications on evolution of market composition and illegal pumping. Section 4 shows the types' selection through an evolutionary approach and preforms numerical simulations about the effects of sanction on compliance. Finally, Section 5 concludes.

2 The model

Let us consider a population of N farmers using water pumped w_i from a common aquifer as the only input to irrigate their crops. The linear inverse demand of water of the i-th firm, as in Kim et al. (1989), is the following:

$$P = \alpha - \beta w_i$$

where $\alpha > 0$ and $\beta > 0$ represent the intercept and the slope, respectively. Integrating the water price, we obtain:

$$\int P(w_i)\,\mathrm{d}w_i = \alpha w_i - \frac{\beta}{2}w_i^2$$

Therefore, the revenues of the *i*-th farmer are:

$$y_i(w_i) = \left(\alpha w_i - \frac{\beta}{2}w_i^2\right)p$$

where $p > 0$ represents the price of the crops cultivated. According to Gisser and Sanchez (1980) the extraction cost of the *i*-th firm is a function of both water table elevation H and water withdrawn:

$$C(w_i, H) = (c_0 - c_1 H)w_i$$

where, with respect to H, $c_0 > 0$ is the fixed cost due to the hydrologic cone and $c_1 > 0$ is the marginal pumping cost. The ratio $H = \overline{H} := \frac{c_0}{c_1}$ represents the maximum level of the aquifer (Rubio and Casino 2001).

The right of pumping water from the common resource is given by the payment of a tax $\tau \geq 0$ on individual withdrawals set by a water agency (as, among others, in Roseta-Palma 2003; Erdlenbruch et al. 2014; Biancardi et al. 2022b). However, we assume that farmers could decide to not pay the tax and, consequently, to face the risk ($\phi \in [0, 1]$) of being sanctioned by the water agency. The sanction is composed of the unpaid tax plus a fixed amount ($\sigma \geq 0$).[2] We refer with the subscript c to compliant farmers, while with the subscript nc to non-compliant farmers. The profit maximization problem of the representative compliant farmer is:

$$\max_{w_c(t)} \Pi_c = \int_0^{+\infty} \pi_c(w_c(t))e^{-rt}\,\mathrm{d}t \tag{1}$$

where

$$\pi_c = \left(\alpha w_c(t) - \frac{\beta}{2}w_c^2(t)\right)p - (c_0 - c_1 H(t))w_c(t) - \tau(t)w_c(t)$$

Analogously, the profit maximization problem of the representative non-compliant farmer is:

$$\max_{w_{nc}(t)} \Pi_{nc} = \int_0^{+\infty} \pi_{nc}(w_{nc}(t))e^{-rt}\,\mathrm{d}t \tag{2}$$

where

[2] A real world example of a such sanction is given the European Union Emission Trading System, where σ =€100 for each tonne of CO_2 emitted for which no allowance has been surrendered, in addition to buying and surrendering the equivalent amount of allowances (see https://icapcarbonaction.com/en/?option=com_etsmap &task=export &format=pdf &layout=list &systems%5B%5D=43).

$$\pi_{nc} = \left(\alpha w_{nc}(t) - \frac{\beta}{2} w_{nc}^2(t)\right)p - (c_0 - c_1 H(t))w_{nc}(t) - (\sigma + \tau(t))\phi w_{nc}(t)$$

Denoting $x \in [0, 1]$ as the share of compliant farmers and $1 - x$ as the share of non-compliant farmers, we suppose the probability of being discovered to be as follows (see, for further details, Petrohilos-Andrianos and Xepapadeas 2017 and Biancardi et al. 2022a):

$$\phi = (1 - x^{\theta})^{\eta}\psi \tag{3}$$

where $\theta, \eta > 0$ and $\psi \in [0, 1]$ is a parameter that captures the monitoring effort of the water agency. Probability (3) assumes that if the share of compliant firms tends to zero, then ϕ tends to 1. The opposite occurs if $x \to 1$.

We suppose that the farmers behave myopically because they consider as negligible the impact of their decisions on the aquifer level (as in Erdlenbruch et al. 2014; Pereau et al. 2018; Biancardi et al. 2022a, b). Therefore, they maximize profits without the dynamic constraint of the water table H, that is given by natural recharge ($R > 0$), compliant and non-compliant water pumping, and natural discharge:

$$\dot{H} = \left\{R - (1 - \gamma)[xNw_c(t) + (1 - x)Nw_{nc}(t)] - (H(t) - \widehat{H})\delta\right\}\frac{1}{S_a} \tag{4}$$

where $\gamma \in (0, 1)$ is the return flow coefficient, $xNw_c(t)$ and $(1 - x)Nw_{nc}(t)$ represent the compliant and non-compliant total water pumping, $S_a > 0$ is the aquifer area times storativity. According to Pereau (2020), the natural discharge $(H(t) - \widehat{H})\delta$, with $\delta > 0$, can be a river or a groundwater-dependent ecosystem adjacent to the aquifer. The parameter $\widehat{H} > 0$ represents the minimum level of the water table for which the natural discharge is nil. Notice that if $H(t) < \widehat{H}$ then a disastrous ecosystem damage occurs. Therefore, we add the constraint $H(t) \geq \widehat{H}$ to guarantee a water table steady state value higher than this ecological threshold.

The objective function of the water agency is the Social Welfare (SW), composed of Net Benefits (NB) minus Environmental Damage (ED):

$$SW = \int_0^{+\infty} [NB(t) - ED(t)]e^{-rt}\, \mathrm{d}t \tag{5}$$

where

$$NB(t) = \left[\left(\alpha w_c(t) - \frac{\beta}{2} w_c^2(t)\right)p - (c_0 - c_1 H(t))w_c(t)\right]xN \tag{6}$$

and

$$ED(t) = [R - (H(t) - \widehat{H})\delta]\lambda + (\overline{H} - H(t))\mu$$

where $\lambda > 0$ and $\mu > 0$. According to Biancardi et al. (2022b, 2023), the water agency knows the existence of non-compliant farmers (but not their number) and so only compliant profits are taken into account. Differently, the ED is composed of the ecosystem damages costs associated with consumptive uses, namely $[R-(H(t)-\widehat{H})\delta]\lambda$ (Pereau and Pryet 2018), and non-consumptive uses, namely $(\overline{H}-H(t))\mu$ (Esteban and Dinar 2013). Following Pereau et al. (2019), we rewrite the environmental damage as:

$$ED(t)=d_0-d_1H(t) \tag{7}$$

where $d_0=(R+\Omega)\lambda+\mu\overline{H}$, $d_1=\delta\lambda+\mu$, and $\Omega=\delta\widehat{H}$. Therefore, the maximization problem of the water agency is the following:

$$\max_{\tau(t)} SW=\int_0^{+\infty}\left\{\left[\left(\alpha w_c(t)-\frac{\beta}{2}w_c^2(t)\right)p-(c_0-c_1H(t))w_c(t)\right]xN-(d_0-d_1H(t))\right\}e^{-rt}\,\mathrm{d}t$$
$$\text{s.t. } \dot{H}=\left\{R-(1-\gamma)[xNw_c(t)+(1-x)Nw_{nc}(t)]-(H(t)-\widehat{H})\delta\right\}\frac{1}{S_a} \quad \text{and} \quad H(t)\geq\widehat{H}$$

We assume that the water agency takes as given the share x. The idea behind this assumption is that non-compliant farmers seek to maintain hidden from the agency their illegal behaviors. Therefore, the water agency ignores both ex-ante and ex-post the value of the share of compliant farmers.

3 Differential game

In this section we analyze a leader-follower dynamic game in which the water agency is the leader and the farmers are the follower. The structure of the game is the following:

(1) The water agency announces the tax τ;
(2) The farmers maximize their profits choosing w_i and taking as given x, τ, and H;
(3) The water agency maximizes the SW choosing the optimal τ under the constraint of the water table dynamics and $H(t)\geq\widehat{H}$;
(4) Adopting a feedback strategy, the water agency derives the steady-state value of $H(t)$.

The following proposition and corollary hold.

Proposition 1 *Let*

$$\widetilde{H}_c=\max\left\{0,\frac{c_0-p\alpha}{c_1}\right\},\qquad \widetilde{H}_{nc}=\max\left\{0,\frac{c_0-p\alpha+\sigma\phi}{c_1}\right\} \tag{8}$$

and

$$\widetilde{\tau}_c=\max\left\{0,p\alpha-c_0+c_1H(t)\right\},\qquad \widetilde{\tau}_{nc}=\max\left\{0,\frac{p\alpha-c_0+c_1H(t)}{\phi}-\sigma\right\} \tag{9}$$

If $H(t) \in (\widetilde{H}_c, \overline{H}]$ *and* $\tau(t) \in [0, \widetilde{\tau}_c)$, *then the optimal value of the compliant water pumped is*:

$$\widetilde{w}_c = \frac{p\alpha - c_0 + c_1 H(t) - \tau(t)}{p\beta} \tag{10}$$

Otherwise, namely if $H(t) \in [0, \widetilde{H}_c]$ *and* $\tau(t) \in [\widetilde{\tau}_c, +\infty)$, *then* $\widetilde{w}_c = 0$.

Analogously, if $H(t) \in (\widetilde{H}_{nc}, \overline{H}]$ *and* $\tau(t) \in [0, \widetilde{\tau}_{nc})$, *then the optimal value of the non-compliant water pumped is*:

$$\widetilde{w}_{nc} = \frac{p\alpha - c_0 + c_1 H(t) - (\sigma + \tau(t))\phi}{p\beta} \tag{11}$$

Otherwise, namely if $H(t) \in [0, \widetilde{H}_{nc}]$ *and* $\tau(t) \in [\widetilde{\tau}_{nc}, +\infty)$, *then* $\widetilde{w}_{nc} = 0$.

Proof See Mathematical appendix. □

Based on Eqs. (10) and (11), we underline that the water agency determines an optimal τ in order to reduce the pumping of legal farmers and illegal ones only if they are discover and increases the water table.

Corollary 2 *The non-compliant farmers pump more than the compliant farmers (namely,* $\widetilde{w}_c < \widetilde{w}_{nc}$*) if*

$$\sigma < \frac{(1-\phi)}{\phi}\tau(t) \tag{12}$$

That is if the sanction is relatively low.

Substituting the values of $\widetilde{w}_c$ and $\widetilde{w}_{nc}$ given by (10) and (11), respectively, in (5) subject to dynamics in (4) and $H(t) > \widehat{H}$, the Hamilton-Jacobi-Bellman (HJB) equation for the water agency follows:

$$\begin{aligned} rV(H,t) = \max_{\tau(t)} \Bigg\{ & \left[\left(\alpha\widetilde{w}_c - \frac{\beta}{2}\widetilde{w}_c^2 \right) p - (c_0 - c_1 H(t))\widetilde{w}_c \right] xN - d_0 + d_1 H(t) \\ & + \frac{V'(H,t)}{S_a}[R - (1-\gamma)(xN\widetilde{w}_c + (1-x)N\widetilde{w}_{nc}) + \Omega - \delta H(t)] \Bigg\} \end{aligned} \tag{13}$$

where $V(H, t)$ and $V'(H,t)$ are the optimal control value function and its derivative with respect to the state variable H, respectively. The analysis of the HJB equation can be found in Mathematical appendix. The following proposition holds.

Proposition 3 *A unique steady-state for the feedback equilibrium of the game exists*:

$$H^* = -\frac{Y}{\widehat{Y}} \tag{14}$$

where

$$\widehat{Y} = \left\{ \frac{(1-\gamma)[2(1-\gamma)((1-x)\phi + x)^2 A_2 - c_1 S_a x]N}{pS_a x\beta} - \delta \right\} \frac{1}{S_a}$$

$$Y = \left\{ R - \frac{[xS_a(\alpha p - c_0 - \phi\sigma(1-x)) - B_2(1-\gamma)((1-x)\phi + x)^2]N(1-\gamma)}{pS_a x\beta} + \Omega \right\} \frac{1}{S_a}$$

The feedback equilibrium water table trajectory is given by:

$$H(t) = H^* + (H_0 - H^*)e^{\widehat{Y}t} \tag{15}$$

where H_0 is the initial value of the water table.

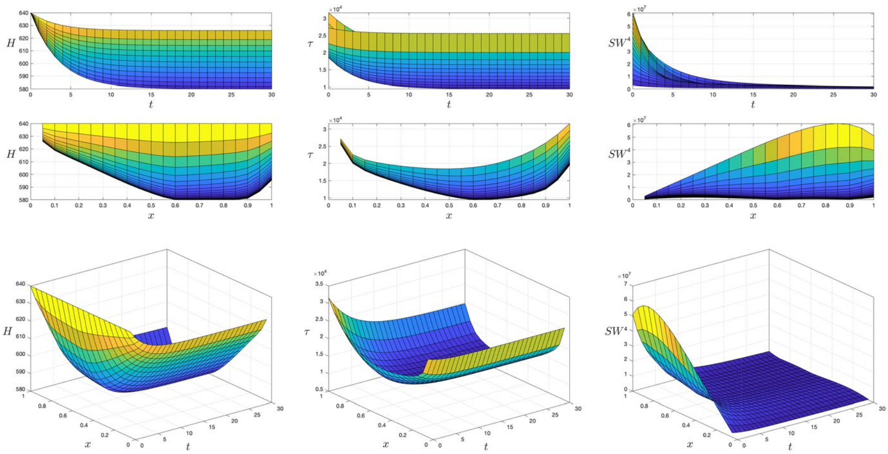
Proof See Mathematical appendix. □

To better understand the implications of (15), we perform now numerical simulations using the Western La Mancha aquifer data, widely used in the literature (see, among others, Esteban and Albiac 2011; de Frutos Cachorro et al. 2014; Pereau et al. 2019; Biancardi et al. 2022b, 2023). Due to the heterogeneity of the farmers, we follow the setting provided by Pereau et al. (2018), see Table 1 for the parameter values. For graphical reasons we split the simulations in two figures, Fig. 1 (water table H, water tax τ, social welfare SW) and Fig. 2 (total pumping W, compliant pumping $W_c = xNw_c$, non-compliant pumping $W_{nc} = (1-x)Nw_{nc}$).

Figure 1a shows the water table in a three-dimensional box (t, x, H). Notice that $x \in (0, 1]$, because x is in the denominator of both Y and $\widehat{Y}$ (in the simulations $x \in [0.05, 1]$ for graphical reasons). This leaves out the possibility of a population composed of only non-compliant farmers. Starting from a situation without pumping ($H_0 = \overline{H} = 640$ m), as one might expect, H decreases as time increases. The steady-state value H^* is a little bit greater than the minimum level $\widehat{H}$. This result is in line with the model that establishes that the agency determines the water tax τ such that the equilibrium H^* is always above $\widehat{H}$. Interesting, H is a non-monotonic function of the share of compliant farmers x, graphically a convex parabola. The aquifer reaches its minimum level when, approximately, $x \in (0.6, 0.9)$ that coincides with the interval in which compliant pumping (higher than non-compliant one) achieves its maximum value (see Fig. 2b). Since the water tax is an increasing function of H (for $A_2 > 0$ as in this parameter set, see (26) in Mathematical appendix), then analogous trend occurs also for the water tax, namely decreasing in time and a convex parabola shape in x (see Fig. 1b). Its minimum interval coincides with the maximum interval of the total pumping, $x \in (0.6, 0.7)$. The increase of the water tax by the agency, even when the height of the water table increases, can be explained by the reduction of pumping costs and this leads to an increase in pumping. So, to avoid this negative effect on the height of the water table, the agency uses the tax as a deterrent to discourage high pumping. Conversely, the Social Welfare is a concave parabola shape

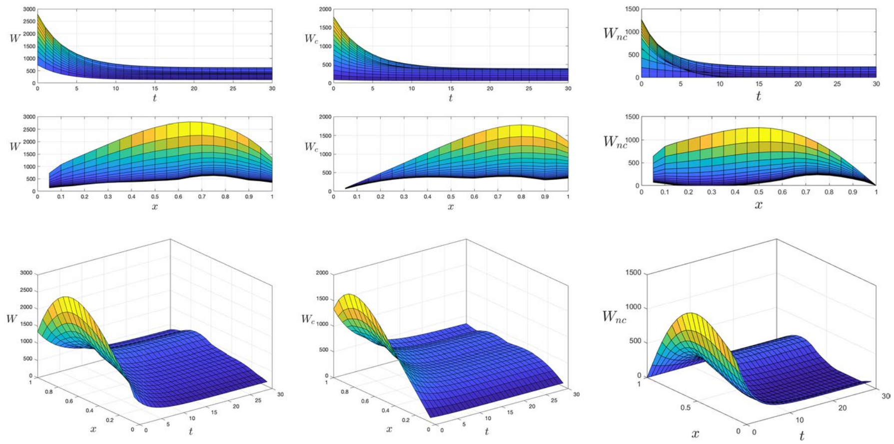
Table 1 Parameter values

Parameters	Description	Units	Value
α	Intercept of the inverse water demand	€/Mm^3	30245.57
β	Slop of the inverse water demand	€/Mm^3	687.28
c_0	Fixed pumping cost	€/Mm^3	320000
c_1	Marginal pumping cost	€/Mm^3m	500
γ	Return flow coefficient	–	0.2
S_a	Aquifer area	Mm^2	126.5
R	Natural recharge	Mm^3	360
$\overline{H}$	Maximum water level and initial condition	m	640
$\widehat{H}$	Minimum water level for nil natural discharge	m	580
λ	Natural discharge consumptive uses cost	€	7500
μ	Natural discharge non-consumptive uses cost	€	12500
r	Discount rate	–	0.03
δ	Slope of the natural drainage	€/Mm^3	5.53
η	Monitoring effort	–	0.8
N	Number of farmers	–	100
p	Output price	€	1.5
θ	Monitoring probability parameter	–	2
η	Monitoring probability parameter	–	2
ψ	Water agency efficiency parameter	–	0.5
σ	Administrative sanction	€/Mm^3	50000



(a) Water table. (b) Water tax. (c) Social welfare.

Fig. 1 Surfaces of water table, water tax and social welfare



(a) Total pumping. (b) Compliant pumping. (c) Non-compliant pumping.

Fig. 2 Surfaces of total, compliant and non-compliant pumping

in x (see Fig. 1c). This reflects the trend of compliant pumping (Fig. 2b) since SW is a function of only compliant net benefits (see (6)). Obviously, SW is a decreasing function of time since pumping reduces the aquifer level and so the environmental damage increases.

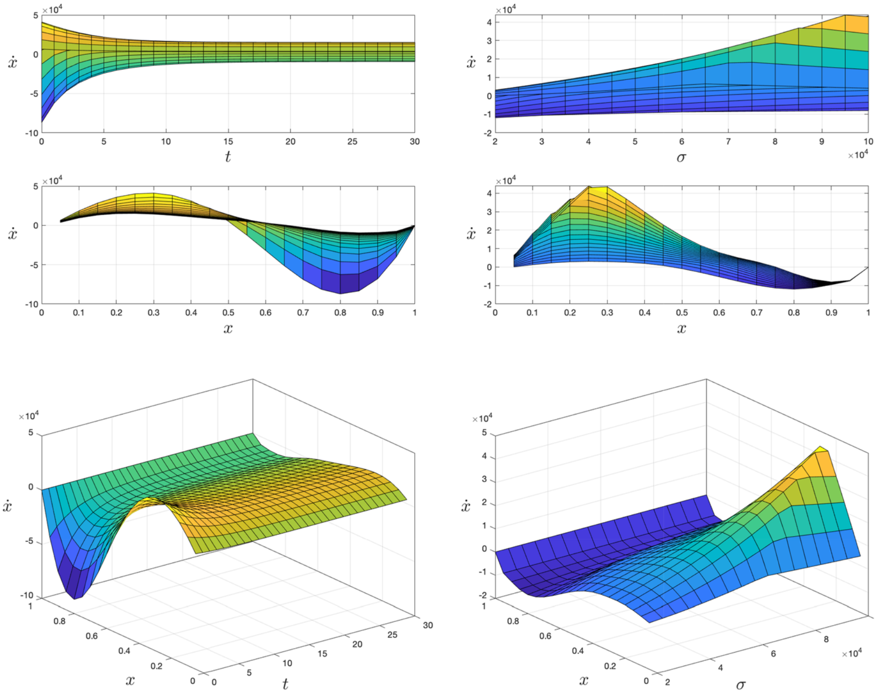
As mentioned above, there is an inverse relationship between pumping and water table. Therefore, at increasing time all types of pumping (total, compliant and non-compliant) decrease (see Fig. 2a–c). Interesting, both compliant and non-compliant pumping show an increasing and concave relation with respect to x. Moreover, the total pumping in $x = 1$ is higher than in $x = 0.05$ (specularly, H in $x = 1$ is lower than in $x = 0.05$, see Fig. 1a). This occurs because the compliant pumping is higher than the non-compliant one, since the inequality (12) is not satisfied and the sanction is relatively high.

4 Evolutionary game

We assume now that farmers can choose to be compliant or non-compliant. The selection dynamics is given by the the well-know replicator equation (see, for a general treatment, Bomze and van Damme 1992, Hofbauer 1999, and, for applications to water dynamics, Antoci et al. 2017, Biancardi et al. 2022a):

$$\dot{x} = x(1 - x)[\pi_c(t, x, H) - \pi_{nc}(t, x, H)] \tag{16}$$

If $\pi_c(t, x, H) > \pi_{nc}(t, x, H)$ then the share of compliant farmers increases, the opposite occurs if $\pi_c(t, x, H) < \pi_{nc}(t, x, H)$. Conversely, if $\pi_c(t, x, H) = \pi_{nc}(t, x, H)$ then the share of compliant farmers does not change over time. Remember that from the analysis of the model, $x \in (0, 1]$. This means that dynamics (16) admits only two types of steady states: $x = 1$ and steady states in the interval (0, 1). From simulations it emerges that (16) admits only one attractive internal steady state (see Fig. 3a).

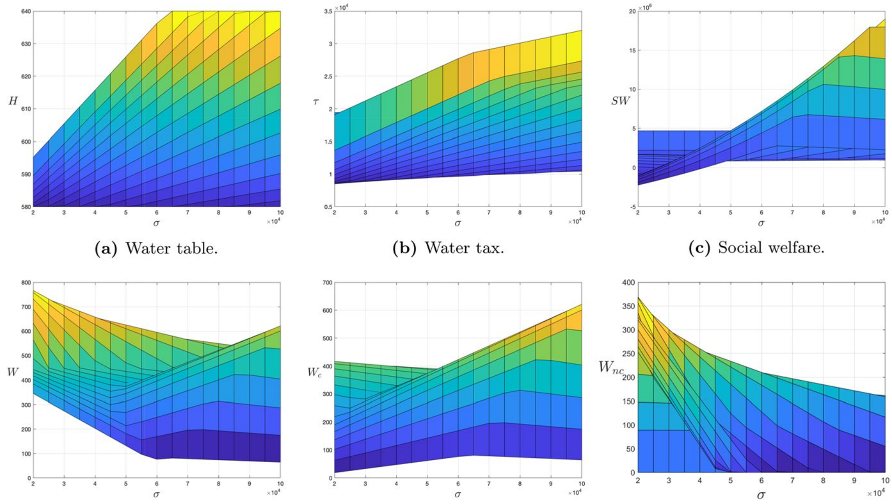


(a) Replicator dynamics in the box $(t, x, \dot{x})$. **(b)** Replicator dynamics in the box $(\sigma, x, \dot{x})$.

Fig. 3 Replicator dynamics

We endogenize the choice of being compliant to understand the farmers' behavior with respect to the sanction σ. For this reason we take the steady state value of the water table ($H = H^*$) and we do not fix σ, which becomes a variable of the model. Fig. 3b shows the replicator dynamics in the box $(\sigma, x, \dot{x})$, where we can see that the internal steady state is always attractive.

As one might expect, the non-compliant water pumping decreases for increasing values of σ (see Fig. 4f). The decay is not constant, but greater for relatively low values of the sanction and smaller for relatively high values. A tightening of the fine pushes up farmers to be compliant and, consequently, the compliant water pumping generically increases (see Fig. 4e). These contrasting effects are reflected on the trend of the total water pumping that seems to be decreasing and convex in σ (see Fig. 4d). A possible explanation could be the combined impact on water pumping of water table and water tax (as we can see in Fig. 4a and b both go up). Indeed, the increase of water table represents a reduction of extraction cost, counterbalanced by a raise of the water tax. Finally, Fig. 4c shows how the social welfare goes up at increasing values of the sanction level because net benefit increases (due to a rise of compliant pumping) while environmental damage decreases (due to a rise of water table).



(a) Water table. (b) Water tax. (c) Social welfare.

(d) Total pumping. (e) Compliant pumping. (f) Non-compliant pumping.

Fig. 4 Comparative dynamics

5 Conclusions

We have analyzed the interaction between heterogeneous farmers, compliant and non-compliant, and a water agency. The framework adopted is the leader-follower, in which the leader is the water agency and the follower is a population of farmers. The number of illegal farmers is unknown to the water agency that maximizes social welfare under the constraint of the aquifer level dynamics choosing a water tax. In the second stage, we endogenize the selection of being compliant or non-compliant through an evolutionary context, in which the farmers choose the more rewarding strategy. This allows us to derive policy suggestions in order to counter illegal behaviors through a comparative dynamics analysis.

As expected all the functions studied (namely, water table, taxation, social welfare, total pumping, compliant pumping, and non-compliant pumping) decrease at increasing time since the initial condition is the maximum level of the aquifer. Differently, the relationships between the functions studied and the share of compliant farmers x is non-monotonic. In particular, the relationship between the water table and x is a convex parabola, and the aquifer is higher in $x = 0.05$ than in $x = 1$ (compliant farmers pump more than non-compliant ones). This happens if the sanction is relatively high and so the compliant farmers pump more than non-compliant ones.

From numerical simulations it emerges that the replicator equation admits an unique attractive internal steady state that we can use to perform comparative dynamics on the sanction σ. An increase of σ has positive effects on water table, water tax and social welfare, ambiguous on pumping. Indeed, the non-compliant water pumping decreases for increasing values of σ but not in a constant way, while the compliant water pumping generically increases. These contrasting effects are reflected on the trend of the total water pumping that seems to be decreasing and convex in σ.

We can conclude that a combined use of taxation and sanction can help the water agency in the management of an aquifer in presence of illegal behaviors.

Mathematical appendix

Proof of Proposition 1

The first order conditions with respect to water pumped are:

$$\frac{\partial \pi}{\partial w_c} = p\left(\alpha - \beta w_c\right) - \tau(t) - c_0 + c_1 H(t) = 0$$
$$\frac{\partial \pi}{\partial w_{nc}} = p\left(\alpha - \beta w_{nc}\right) - c_0 + c_1 H(t) - (\sigma + \tau(t))\phi = 0$$

Solving, we get:

$$\widetilde{w}_c = \frac{p\alpha - c_0 + c_1 H(t) - \tau(t)}{p\beta}$$
$$\widetilde{w}_{nc} = \frac{p\alpha - c_0 + c_1 H(t) - (\sigma + \tau(t))\phi}{p\beta}$$

Notice that $\widetilde{w}_c > 0$ if and only if:

$$\tau(t) < \widetilde{\tau}_c := p\alpha - c_0 + c_1 H(t)$$

Moreover, $\widetilde{\tau}_c > 0$ if and only if:

$$H(t) > \widetilde{H}_c := \frac{c_0 - p\alpha}{c_1}$$

Analogously, $\widetilde{w}_{nc} > 0$ if and only if:

$$\tau(t) < \widetilde{\tau}_{nc} := \frac{p\alpha - c_0 + c_1 H(t)}{\phi} - \sigma$$

Notice that $\widetilde{\tau}_{nc} > 0$ if and only if:

$$H(t) > \widetilde{H}_{nc} := \frac{c_0 - p\alpha + \sigma\phi}{c_1}$$

This concludes the proof. □

HJB equation solution

Assuming an interior solution and differentiating the right-side of equation (13) with respect to τ, we lead:

$$\tau(t) = \frac{(1-\gamma)[(1-x)\phi + x]V'(H,t)}{xS_a} \tag{17}$$

Replacing τ given by (17) in $\widetilde{w}_{nc}$ and $\widetilde{w}_c$ given by (10) and (11), we obtain:

$$w_{nc} = \frac{p\alpha - c_0 + c_1 H(t) - \sigma\phi}{p\beta} - \frac{\phi(1-\gamma)[(1-x)\phi + x]V'(H,t)}{px\beta S_a} \tag{18}$$

and

$$w_c = \frac{p\alpha - c_0 + c_1 H(t)}{p\beta} - \frac{(1-\gamma)[(1-x)\phi + x]V'(H,t)}{px\beta S_a} \tag{19}$$

Sustituting (18) and (19) in HJB, and rearranging the terms, it follows:

$$\begin{aligned} rV(H,t) &= \frac{N(1-\gamma)^2[(1-x)\phi + x]^2}{2p\beta x S_a^2} \cdot (V'(H,t))^2 + \\ &+ \frac{(1-\gamma)N[(1-x)\sigma\phi - p\alpha + c_0 - c_1 H(t)] + \beta p[R + \Omega - \delta H(t)]}{p\beta S_a} \\ &\cdot V'(H,t) \\ &+ \frac{[\alpha p - c_0 + c_1 H(t)]^2 x N}{2p\beta} - d_0 + d_1 H(t) \end{aligned} \tag{20}$$

Being the game a linear-quadratic variety, we postulate a quadratic function of the form:

$$V(H,t) = AH^2(t) + BH(t) + C$$

with the first derivative:

$$V'(H,t) = 2AH(t) + B$$

where A, B and C are constant parameters of the unknown value function which are to be determined. Substituting the equations $V(H,t)$ and $V'(H,t)$ in the HJB, we obtain a system of three Riccati equations for the coefficients of the value function:

$$rA = \frac{2N(1-\gamma)^2[(1-x)\phi + x]^2}{px\beta S_a^2}A^2 - \frac{2[(1-\gamma)c_1 N + \delta\beta p]}{p\beta S_a}A + \frac{xNc_1^2}{2\beta p} \tag{21}$$

$$\begin{aligned} rB &= \frac{2N(1-\gamma)^2[(1-x)\phi + x]^2}{px\beta S_a^2}AB \\ &+ \frac{2\left\{(1-\gamma)[(1-x)\phi\sigma - p\alpha + c_0]N + \beta p(R+\Omega)\right\}}{p\beta S_a}A \\ &- \frac{(1-\gamma)c_1 N + \delta\beta p}{p\beta S_a}B + \frac{(\alpha p - c_0)c_1 x N}{\beta} + d_1 \end{aligned} \tag{22}$$

$$rC = \frac{N(1-\gamma)^2[\phi(1-x)+x]^2}{2p\beta x S_a^2}B^2 + \frac{(1-\gamma)[(1-x)\phi\sigma - p\alpha + c_0]N + \beta p(R+\Omega)}{p\beta S_a}B + \frac{(\alpha p - c_0)^2 xN}{p\beta S_a} - d_0 \tag{23}$$

Equation (21) admits two real and distinct solutions A_1 and A_2:

$$A_{1,2} = \frac{xS_a\left\{\beta p(rS_a + 2\delta) + 2(1-\gamma)c_1 N \pm \sqrt{D}\right\}}{4N(1-\gamma)^2[(1-x)\phi + x]^2}$$

where

$$D = 4\left\{N(1-\gamma)c_1[(1-x)\phi + 1 + x] + \frac{\beta p(rS_a + 2\delta)}{2}\right\} \cdot \left\{N(1-\gamma)(1-x)(1-\phi)c_1 + \frac{\beta p(rS_a + 2\delta)}{2}\right\}$$

is always positive. The solution has to satisfy the stability condition $\frac{d\dot{H}}{dH} < 0$. Substituting (18) and (19) in the dynamics of the water table (4), and considering that $V'(H,t) = 2AH(t) + B$, the stability condition becomes:

$$\frac{d\dot{H}}{dH} < 0 \iff \frac{1}{S_a}\left\{\frac{N(1-\gamma)[2(1-\gamma)((1-x)\phi + x)^2 A - c_1 S_a x]}{px\beta S_a} - \delta\right\} < 0$$

satisfied for $A = A_2$. Moreover, from equation (22), we determine the value of $B = B_2$ as:

$$B_2 = -\frac{xS_a\left\{[2(1-\gamma)[(1-x)\phi\sigma - p\alpha + c_0]A_2 + xS_a c_1(\alpha p - c_0)]N + p\beta[2(R+\Omega)A_2 + d_1 S_a]\right\}}{N(1-\gamma)\{2[(1-x)\phi + x]^2(1-\gamma)A_2 - xc_1 S_a\} - p\beta x S_a(rS_a + \delta)}$$

Finally,

$$w_c^* = \frac{\alpha p - c_0 + c_1 H(t)}{\beta p} - \frac{(1-\gamma)[(1-x)\phi + x](2A_2 H(t) + B_2)}{x\beta S_a p} \tag{24}$$

$$w_{nc}^* = \frac{\alpha p - c_0 + c_1 H(t) - \sigma\phi}{\beta p} - \frac{\phi(1-\gamma)[(1-x)\phi + x](2A_2 H(t) + B_2)}{x\beta S_a p} \tag{25}$$

and

$$\tau^* = \frac{(1-\gamma)[(1-x)\phi + x]\big(B_2 + 2A_2 H(t)\big)}{xS_a} \tag{26}$$

Proof of Proposition 3

Substituting the values of w_c^* and w_{nc}^*, given by (24) and (24), respectively, in the water table dynamics (4) we get:

$$\dot{H} = \widehat{Y}H + Y \tag{27}$$

Solving (27) we obtain the optimal trajectory (15). $\square$

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Declarations

Competing interests The authors declare no competing interests.

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