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## Integrated task scheduling and personnel rostering of airports ground staff: A case study

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## **1. Introduction**

The staff scheduling problem, also known in the literature as *Personnel Rostering* problem, consists in planning suitable work shifts for employees in (usually large) companies, e.g., hospitals, call centers, transportation companies, or hotels, so that the demand of required goods or services can be satisfied ([Brucker et al.,](#page-10-0) [2011;](#page-10-0) [Cappanera et al.](#page-10-1), [2022;](#page-10-1) [Özder et al.](#page-11-0), [2020](#page-11-0); [Van den Bergh et al.](#page-11-1), [2013](#page-11-1)). The optimal planning of work shifts may be highly beneficial both to employers, who can minimize costs, and to employees, whose preferences concerning daily timetables or days off may be satisfied.

The basic version of the staff scheduling problem requires to cover pre-defined shifts employing workers from a roster. The working hours of employees are typically bound by their contracts, which induce the main constraints in the optimization models. To complicate matters further, the staff scheduling problem may span a multi-day planning horizon and employees may be characterized by skills required to cover specific shifts ([De Bruecker et al.,](#page-10-2) [2015\)](#page-10-2).

Recently, with the specialization and diversification of occupations in the work environment, staff scheduling problems where a worker has to perform several activities in the same shift are increasingly common,

leading to even more complex problems. A second issue workforce planners have to deal with is thus that of *Task Scheduling*, i.e., the assignment of each activity to be carried out by a sufficient number of active, skilled enough workers to cover the demand [\(Kolen et al.](#page-10-3), [2007\)](#page-10-3).

Historically, the two aforementioned problems have been addressed separately, in a sequential fashion, from the operations research community: first, the working shifts are constructed for each employee and then the tasks are assigned within the shifts. Clearly, not assigning tasks and shifts simultaneously eventually leads to lower-quality solutions ([Ernst et al.,](#page-10-4) [2004](#page-10-4)).

Of course, this kind of approach has been motivated by computational sustainability reasons. However, exploiting the increasing computational power provided by modern hardware and the algorithmic, software advances, the *integrated task scheduling and personnel rostering* (TSPR) problem was finally formalized and rigorously defined as a mixed-integer linear programming model by [Smet et al.](#page-11-2) [\(2016](#page-11-2)).

Since then, more complex versions of TSPR have also started to be analyzed to meet specific application needs; for example, continuous training of employees has been considered in the context of inclusion

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of people with disabilities [\(Maheut et al.,](#page-11-3) [2023\)](#page-11-3); ordering and grouping constraints to satisfy customers' preferences were taken into account when dealing with employees of a wellness center ([Mansini et al.](#page-11-4), [2023\)](#page-11-4); grouping was also something to be handled in the case of residential care under pandemic conditions to mitigate the spread of infections ([Moosavi et al.,](#page-11-5) [2022](#page-11-5)).

We can thus observe that staff scheduling is a very context-specific problem: each case study has its peculiarities and complexities. This is of course also and particularly true for the case of ground staff in airports, which is the focus of the present work.

The examined instance, coming from a large airport in Italy, is indeed particularly hard to handle for several reasons. First, airports are a working environment operating 24 h a day, 7 days a week, where tasks repeat continuously and may even span two consecutive days. Second, employees (i) own skills making them almost unique in the team of workers (heterogeneous workforce), (ii) have legality and collective agreements to be satisfied, and (iii) typically, there is a mix of full and part-time contracts. Third, the planning horizon is made up of multiple days (typically a week) with a (highly) time-varying demand. Fourth, each employee can perform several different activities – of widely varying lengths, some of them even a few minutes long – during the same day-shift, and high flexibility is required in schedule definition.

The main contribution of the present paper is twofold: on the one hand, we show that Mixed-Integer Programming can be used with large-scale real-world instances of the TSPR problem; on the other hand, we present novel mathematical elements within the model that allow to make computation sustainable. Findings can be of general interest for other personnel scheduling problems involving task assignment.

The rest of the manuscript is organized as follows: in Section [2](#page-1-0), the specifics of our case study are outlined; in Section [3](#page-1-1), approaches from the literature for analogous problems are reviewed and discussed; in Section [4](#page-2-0), our exact optimization model is defined in detail; in Section [5,](#page-4-0) we address a set of valid inequalities that allows to greatly improve the efficiency of the solution process; in Section [6,](#page-5-0) we report the results of computational experiments carried out on the real-world data at our disposal. Finally, in Section [7,](#page-7-0) we provide some concluding remarks.

#### **2. Problem statement**

<span id="page-1-0"></span>The problem considered in this paper originates from the research project AIMS (Artificial Intelligence for the Management of Shifts) financed by the Tuscany Region. The project, whose participants came both from the academia and from industrial companies, aimed at the development of optimization tools for personnel shift scheduling based on activities. In particular, the partners of the project had the possibility of working with real data from a medium-sized airport in Italy; the objective was to build a set of optimization tools capable of taking into account all of the complexities of real-world scenarios. Thus, in the following, the definition of the problem, although being closely related to other similar problems found in the scientific literature, has been strongly influenced by the real case study on which the whole Research and Development project was based. The project was successfully concluded at the beginning of 2023 with a formal approval by the sponsoring partner.

Given a weekly planning horizon, a set of skilled employees, and a service demand to cover, the project described in this paper consists of devising an optimization model that defines shifts and assigns them to employees to satisfy demand. In the following, we will detail the key elements of the problem, i.e., service demand and employees.

In airports, which are open 24/7, some *activities* like "*check-in*" or ''*vehicle cleaning* '' must be done repeatedly depending on the arrival and departure times of flights. For each activity, and each day in the planning horizon, the demand for service is expressed in terms of the



<span id="page-1-2"></span>



<span id="page-1-3"></span>

exact number of (skilled) employees required on a given time interval in a given *working area/section*. As an example, on Friday, for the checkin activity at the stopover employee area/gate B (working area/section) two employees, with proper skill, must be on duty from 11.25 to 11.45. As widely used in the literature, we use the term *task* to refer to these pieces of work that must be covered. So, there is a one-to-many relationship between activities and tasks: each activity corresponds to a set of tasks, and each task is associated with only one activity. Specifically, in our case study, service demand is described as a set of tasks to be covered, and each task is given by a tuple of information describing the time and location in which the corresponding activity must be performed, i.e., the day in the planning horizon, the time interval in the day – starting and ending time – and the working area; moreover, the set of skills required to perform that activity is specified. Tasks must be grouped to form *shifts*. A *shift* is defined as a time interval, characterized by a start time and a length. In the following, tasks are also referred to as *service requests* or simply *requests*.

For each employee, data contains information on their skills (qualifications), the working area where they are qualified to work, and the contract. Contractual data defines the exact number of hours an employee must be available when on duty (*shift duration*), the maximum number of hours per week, and the maximum number of working days per week.

A detailed description of the available data is given in [Tables](#page-1-2) [1](#page-1-2) and [2](#page-1-3).

Listed below, we summarize all the formal requirements specified for the airport considered in our study; note, however, that these specifics are common to the majority of main airports.

- 1. Every service request (task) must be satisfied with the exact number of appropriate employees.
- 2. Each employee can perform at most one shift per day.
- 3. The working hours per day of each employee must correspond to the contractual constraints.
- 4. The number of working days per week of each employee cannot be greater than a fixed number, defined in the contract.
- 5. For all employees, there must be a minimum number of hours between the end of a shift and the start of a shift on the next day.
- 6. Employees can perform only those activities they are qualified to do.
- 7. Two or more tasks cannot be performed by the same employee if they temporally overlap.

#### **3. Literature review**

<span id="page-1-1"></span>The airline industry and its activities have drawn a major focus to operations researchers since the 1950s, giving rise to many challenging

optimization problems. Airports constitute one of the key pieces of this industry; their management involves the solution of many decision problems ([Brusco et al.,](#page-10-5) [1995](#page-10-5); [Cappanera & Gallo,](#page-10-6) [2004](#page-10-6); [Dowling](#page-10-7) [et al.](#page-10-7), [1997;](#page-10-7) [Nobert & Roy,](#page-11-6) [1998;](#page-11-6) [Rong & Grunow](#page-11-7), [2009\)](#page-11-7), including personnel scheduling problems which is the focus of this work. A general description of the airport system was provided by [Clausen and](#page-10-8) [Pisinger](#page-10-8) ([2011\)](#page-10-8).

Literature on personnel scheduling problems is extremely vast; we refer the reader to the survey papers by [Brucker et al.](#page-10-0) [\(2011](#page-10-0)), [De Bruecker et al.](#page-10-2) ([2015\)](#page-10-2), [Van den Bergh et al.](#page-11-1) ([2013\)](#page-11-1) and [Özder](#page-11-0) [et al.](#page-11-0) ([2020\)](#page-11-0) for thorough reviews of models and algorithms on this class of problems. In the context of airline companies, and more generally in public transport, a distinction is made between ground staff and traveling crew workers (airline pilots, train and bus drivers, stewards, etc.); the latter case has certainly received more attention in the literature [\(Deveci & Demirel](#page-10-9), [2018;](#page-10-9) [Herbers & Hromkovic,](#page-10-10) [2005](#page-10-10); [Kasirzadeh et al.,](#page-10-11) [2017\)](#page-10-11).

The problem addressed in this paper deals with the assignment of non-preemptive tasks and shifts. As outlined by [Smet et al.](#page-11-2) [\(2016](#page-11-2)), few publications put the focus on this setting. [Dowling et al.](#page-10-7) [\(1997](#page-10-7)), who incidentally focused on airport ground staff, addressed the problem as usual in two separate, sequential phases, to minimize over/ understaffing, exploiting a simulated annealing strategy. [Meisels and](#page-11-8) [Schaerf](#page-11-8) [\(2003](#page-11-8)) addressed a timetabling problem where both tasks and possible shifts are known in advance; however, contrarily to general TSPR, only one task is assigned to each shift. [Krishnamoorthy et al.](#page-11-9) ([2012\)](#page-11-9) proposed instead an algorithm to carry out multiple task assignment to multi-skilled employees, in the case where the shifts have been assigned in advance. Alternative approaches for this same problem have been proposed by [Baatar et al.](#page-10-12) ([2015\)](#page-10-12) and [Smet et al.](#page-11-10) ([2014\)](#page-11-10).

As outlined by [Smet et al.](#page-11-2) ([2016\)](#page-11-2), the strong but yet complicated bond between task and shift assignments results in an integrated problem (TSPR) which has long been considered computationally impractical for realistically sized instances (see, e.g., [Ernst et al.](#page-10-4), [2004](#page-10-4)). However, in the same paper, [Smet et al.](#page-11-2) ([2016\)](#page-11-2) finally formalized the problem and showed it to be solvable, at least for synthetic problems, combining heuristic and decomposition strategies. Calneryte et al. ([2020\)](#page-10-13) also address the TSPR and propose a three-phase approach in which time is discretized in slots and a genetic algorithm and greedy heuristics are used in cascade to manage hard and soft constraints. More recently, [Wang et al.](#page-11-11) ([2023](#page-11-11)) proposed a clique-based formulation where, instead of assigning tasks directly to staff members as [Smet et al.](#page-11-2) ([2016\)](#page-11-2) do, they assign tasks to shifts and shifts to staff members. In addition, they proposed two heuristics, respectively based on the rolling horizon concept and the iterative shift selection to solve medium to large real instances.

Problems with a structure similar to TSPR or having TSPR as a special case have also been studied recently. [Campana et al.](#page-10-14) ([2021\)](#page-10-14) address a task and personnel scheduling problem arising in a large Italian company that provides cleaning services inside a hospital. For each task, the frequency with which it should be done inside the planning period is known as well as its duration and the skills required to perform it. A 3-phase approach is proposed which consists of the following steps: (i) for each task, determine the days on which it will be done; (ii) for each day, determine the sequence with which the tasks will be done; (iii) for each day and each operator, determine which sequence of tasks they will perform.

A general framework to address a plurality of staff scheduling problems has been recently proposed by [Kletzander and Musliu](#page-10-15) [\(2020](#page-10-15)). A simulated annealing heuristic is implemented in the framework and the integrated TSPR problem defined by [Smet et al.](#page-11-2) [\(2016](#page-11-2)) is used as a benchmark to solve instances with up to 40 employees. Compared with the results by [Smet et al.](#page-11-2) [\(2016](#page-11-2)), the general approach seems to be able to find feasible solutions for more instances, while solution quality is not competitive in most cases.

<span id="page-2-1"></span>**Table 3**

<span id="page-2-2"></span>**Table 4**







Summarizing, we can conclude that few studies have focused on the peculiar TSPR problem even after the formalization of the problem by [Smet et al.](#page-11-2) ([2016\)](#page-11-2). Most of them are heuristics. When TSPR is dealt with using frameworks defined for more general problems, the performance obtained may not be satisfactory in terms of solution quality. This shortcoming therefore motivates the interest in studying approaches that exploit the particular structure of the problem. It is precisely into this literature gap that our work fits, proposing to enrich the clique-based mathematical models provided in the literature with crucial elements to make the problem computationally tractable even for large-scale real-world instances.

#### **4. Problem formalization**

<span id="page-2-0"></span>In this section, we present the key elements of the approach we propose to address our case study.

## *4.1. Notation and basic concepts*

As a first modeling element, we need to define a *shift*. A shift is a time interval, characterized by a start time  $t_s$  and a duration  $L_s$ . We define a finite set  $S$  of possible shifts that employees can do. We discuss the choice of the possible starting times and shift durations, which strongly depend on the problem at hand, in Section [4.3](#page-3-0). In the meantime, we assume that the set  $S$  is given.

In [Tables](#page-2-1) [3](#page-2-1) and [4](#page-2-2) the notation for the considered problem is described, assuming the time horizon is one workweek.

In what follows, we describe the mathematical program, inspired by the TSPR model by [Smet et al.](#page-11-2) [\(2016](#page-11-2)), used in this work to model our problem.

For ease of notation, we introduce two matrices:  $A \in \mathbb{R}^{|R|\times|S|}$  and  $B \in \mathbb{R}^{|R| \times |R|}$ .

Matrix  $A$  is an incidence matrix, encoding the information about compatibility between request  $r \in R$  and shift  $s \in S$ ; each element  $a_{r,s}$ is defined as follows

 $a_{r,s} =$  $\int 1$  if request *r* is time-wise contained in shift *s*, 0 otherwise.

Matrix  $B$  is an incompatibility matrix, encoding the information about the incompatibility between two requests  $r_i \in R$ ,  $r_j \in R$ ; each element  $b_{i,i}$  is defined as follows

$$
b_{i,j} = \begin{cases} 1 & \text{if request } r_i \text{ is time-wise overlapped with request } r_j, \\ 0 & \text{otherwise.} \end{cases}
$$

Matrix  $B$  can also be seen as an adjacency matrix of a graph where nodes represent requests and edges indicate incompatibility among



**Fig. 1.** Example of (part of) input data.

<span id="page-3-1"></span>them. Cliques within this graph constitute sets of requests such that at most one of them can be assigned to a single employee. In the following, we will denote the set of maximal cliques in  $B$  as  $C(B)$ . An efficient algorithm to find all maximal cliques in  $B$  was proposed by [Krishnamoorthy](#page-11-9) [et al.](#page-11-9) [\(2012](#page-11-9)); we provide details about this procedure in [Appendix.](#page-10-16)

The introduction of the above two matrices allows us to encapsulate the temporal element of the problem, reducing it to a static assignment task. Of course, the concept of incompatibility between two requests can be extended to further modeling requirements, such as spacial incompatibility (two requests cannot be performed by the same employee on the same day if their location is one far away from the other).

**Example 4.1.** To make it easier to understand the notation defined above and all the elements of the problem, we provide a toy example. In [Fig.](#page-3-1) [1,](#page-3-1) a pictorial representation of a small set of requests to be satisfied and the set of possible shifts for a given day of the planning period are shown. Specifically, we are given 5 requests, i.e.,  $R =$  ${r_8, r_{12}, r_{37}, r_{44}, r_{56}}$  and each request is represented as a horizontal rectangular box of length equal to its duration. For instance, request  $r_8$  begins at 8.00 and has a duration of 2 h and a half. For that specific day, there are 3 possible shifts, i.e.,  $S = \{s_7, s_{13}, s_{16}\}$ , and each shift is represented as a horizontal rectangular box of length equal to its duration. Shift  $s_7$  begins, e.g., at 8.00 and has a duration of 4 h.

[Fig.](#page-4-1) [2](#page-4-1) shows respectively the compatibility matrix between requests and shifts (matrix  $A$ ) and the incompatibility matrix between requests (matrix *B*). For example, request  $r_8$  is time-wise contained in shift  $s_7$ , whereas it is not fully contained in shifts  $s_{13}$  and  $s_{16}$ , thus motivating the first row in matrix A [\(Fig.](#page-4-2) [2\(a\)](#page-4-2)). In addition, request  $r_8$  is timeoverlapped to all the other requests but request  $r_{56}$ , thus motivating the first row in matrix  $B$  [\(Fig.](#page-4-3) [2\(b\)](#page-4-3)).

We can now define the binary decision variables of the model:

$$
X_{j} = \begin{cases} 1 & \text{if employee } j \text{ takes at least one shift,} \\ 0 & \text{otherwise,} \end{cases}
$$
  
\n
$$
Y_{jsd} = \begin{cases} 1 & \text{if employee } j \text{ takes shift } s \text{ in day } d, \\ 0 & \text{otherwise,} \end{cases}
$$
  
\n
$$
Z_{jr} = \begin{cases} 1 & \text{if employee } j \text{ performs request } r, \\ 0 & \text{otherwise.} \end{cases}
$$
  
\n
$$
Z_{jr} = \begin{cases} 1 & \text{if employee } j \text{ performs request } r, \\ 0 & \text{otherwise.} \end{cases}
$$
  
\n
$$
Y_{jr} = \begin{cases} 1 & \text{if employee } j \text{ performs request } r, \\ 0 & \text{otherwise.} \end{cases}
$$

Before turning to the optimization model, we finally have to introduce for each shift  $s$ , the set  $I_s$  of shifts that are incompatible on the following day:

$$
I_s = \{ s_2 \in S \mid t_{s_2} + 24h - (t_s + L_s) \le \delta \},\
$$

 $\epsilon$ 

where  $\delta$  is an appropriate value, usually defined in the industry collective agreements. This set is needed to take into account that, usually, a minimum number  $\delta$  of resting hours has to pass between two shifts taken consecutively by an employee.

### *4.2. The integrated TSPR model*

We formulate our integrated TSPR as the following integer linear programming problem:

$$
\underset{X,Y,Z}{\text{arg min}} \sum_{j \in J} \sum_{d \in D} \sum_{s \in S_j} L_s Y_{jsd} + \gamma \sum_{j \in J} X_j \tag{1a}
$$

<span id="page-3-13"></span>
$$
\text{s.t. } \sum_{j \in J_r} Z_{jr} \ge d_r \qquad \forall r \in R,
$$
\n<sup>(1b)</sup>

<span id="page-3-4"></span><span id="page-3-3"></span><span id="page-3-2"></span>
$$
\sum_{s \in S_j} Y_{jsd} \le 1 \qquad \forall j \in J, \forall d \in D,
$$
 (1c)

$$
Z_{jr} \le \sum_{s \in S_j} a_{r,s} Y_{jsd} \qquad \forall j \in J, \forall r \in R, \forall d \in D,
$$
 (1d)

$$
\sum_{r \in c:j \in J_r} Z_{jr} \le 1 \qquad \forall j \in J, \ \forall c \in C(B), \tag{1e}
$$

<span id="page-3-6"></span><span id="page-3-5"></span>
$$
\sum_{d \in D} \sum_{s \in S_j} Y_{jsd} \le G_j X_j \qquad \forall j \in J,
$$
\n(1f)

$$
\sum_{d \in D} \sum_{s \in S_j} L_s Y_{jsd} \le H_j X_j \quad \forall j \in J,
$$
\n(1g)

$$
Y_{j,s_1,d} + Y_{j,s_2,d+1} \le 1 \qquad \forall j \in J, \forall s_1, s_2 \in S_j
$$

<span id="page-3-12"></span><span id="page-3-11"></span><span id="page-3-10"></span><span id="page-3-9"></span><span id="page-3-8"></span><span id="page-3-7"></span>
$$
\text{s.t. } s_2 \in I_{s_1}, \forall d \in D,\tag{1h}
$$

$$
Y_{jsd} \le X_j \qquad \forall j \in J, \forall s \in S_j, \forall d \in D,
$$
 (1i)

$$
X_j \in \{0, 1\} \qquad \forall j \in J, \tag{1j}
$$

$$
Y_{jsd} \in \{0, 1\} \qquad \forall j \in J, \forall s \in S_j, \forall d \in D, \qquad (1k)
$$

$$
Z_{jr} \in \{0, 1\} \qquad \forall r \in R, \forall j \in J_r,
$$
\n<sup>(11)</sup>

where  $\gamma > 0$  is a suitably defined constant.

Constraint  $(1b)$  $(1b)$  forces requests covering: each request  $r$  has to be assigned to  $d_r$ , workers. Constraint  $(1c)$  $(1c)$  prevents employees from taking more than one shift per day. Moreover, by ([1d\)](#page-3-4) an employee is allowed to perform a request only if they are working at that moment. The assignment of incompatible requests to the same employee is avoided thanks to ([1e](#page-3-5)). The contractual constraints of employees are enforced by ([1f\)](#page-3-6) and [\(1g](#page-3-7)). Constraint [\(1h\)](#page-3-8) imposes the incompatibility condition between close shifts on consecutive days. By ([1i\)](#page-3-9), we model the fact that an employee can take a shift only if it is activated for the week. Finally, the (binary) domain of the variables is specified in ([1j\)](#page-3-10), ([1k\)](#page-3-11) and [\(1l](#page-3-12)).

As for the objective function, we minimize the idle time of the employees, i.e., the difference between the length of their shifts and the hours actually worked. To avoid unrealistic solutions where some employees work just one or two days throughout the week, we also add a penalty on the total number of active employees. The value of  $\gamma$ defines the trade-off between the two partly contrasting goals.

#### *4.3. Identifying the shift set*

<span id="page-3-0"></span>The choice of the shift set  $S$  is crucial for the proposed solution. Indeed, a wide set of possible choices for the shifts may result in a too large number of binary variables to be handled with reasonable computational resources by software solvers. On the other hand, an excessively limited set of shifts may lead to the problem described in Section [4.3](#page-3-0) having poor quality solutions and even being infeasible.

The possible length(s) of shifts usually depends on employees' contracts. Thus, the margin for choice mostly lies in the moments when shifts can be started. A reasonable option is to allow shifts to start every hour. Of course, with problems of moderate size, a denser set of possible shifts could be considered, to obtain solutions of better quality. On the other side, with particularly large problems a coarser grid might be necessary to reduce the computational effort up to sustainable levels.

<span id="page-4-2"></span>



(a) Request-shift compatibility matrix.

(b) Request-request incompatibility matrix.

[Fig.](#page-3-1) 2. Matrices associated with data from Fig. [1](#page-3-1).

<span id="page-4-1"></span>

Fig. 3. Example of "unfeasible" fractional solution.

<span id="page-4-4"></span>

In general, starting times for shifts should be added so that all ''long'' requests can be covered.

#### **5. Valid inequalities**

<span id="page-4-0"></span>To decrease computational solution times, valid inequalities can be introduced into the optimization model to improve the quality of the solution of the continuous relaxations.

Indeed, in the fractional solution situations occur that are impossible in feasible, binary, ones. The idea of valid inequalities is to try to add constraints that are satisfied by every feasible integer solution, and thus, redundant, but that are not satisfied at some vertices of the linear relaxation.

We begin this discussion by showing a situation that might be observed (and indeed it has been observed in our experiments) in the continuous relaxation of model ([1\)](#page-3-13). Consider the toy example shown in [Fig.](#page-3-1) [1](#page-3-1) and observe the situation depicted in [Fig.](#page-4-4) [3.](#page-4-4)

In this example, worker  $j = 23$  has been "partially" assigned to shift  $s = 16$  in the fractional relaxation with a value of  $\frac{1}{3}$ . During this shift, this worker is also partially performing three tasks,  $r = 12,37,44$ , that are partly overlapping. It is evident that this is not possible and, indeed, in the optimal, binary-valued, solution this situation cannot happen. But in the relaxed problem, this is a feasible solution, as constraints ([1d\)](#page-3-4) and ([1e](#page-3-5)) are satisfied.

<span id="page-4-3"></span>

Fig. 4. Example of "unfeasible" fractional solution.

<span id="page-4-6"></span>In fact, constraint ([1e](#page-3-5)) forbids assigning incompatible tasks to the same worker. But the constraint just asks that the sum of binary variables is no larger than 1 and this, as we can see, is satisfied in this case. What is lacking in the model is a constraint that links  $Y$  and  $Z$ variables for incompatible requests. In this example, we would like to add a constraint like

$$
Z_{23,12}+Z_{23,37}+Z_{23,44}\leq \sum_{s:\, a_{12,s}+a_{37,s}+a_{44,s}\geq 1} Y_{23,s,1},
$$

where the sum concerns all shifts that cover at least one of the requests 12, 37, 44. Generalizing:

<span id="page-4-5"></span>
$$
\sum_{r \in c \, : \, j \in J_r} Z_{jr} \le \sum_{s \in S(j,c)} Y_{jsd} \tag{2}
$$

where  $c$  is a (maximal) clique of incompatible requests.

Given a clique  $c$ , the left-hand side sums, similarly as in constraint ([1e\)](#page-3-5), all of the assignments of those tasks to the same employee  $j$ . In the example, this sum is  $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1$ . The right-hand side considers, for the day in which the tasks have to be performed, all of the possible shift assignments of employee  $i$  to a shift which covers any of the tasks in the clique; the set of shifts covering at least one of the tasks in  $c$  is denoted by  $S(i, c)$ , i.e.,

$$
S(j, c) = \left\{ s \in S_j \mid \sum_{r \in c} a_{r,s} \ge 1 \right\}.
$$

In our case, this sum is  $\frac{1}{3}$ . Changing the constraint ([1e](#page-3-5)) into ([2](#page-4-5)), the above solution is immediately cut off.

Another case of ''unfeasible'' fractional solution can be observed in the example in [Fig.](#page-4-6) [4,](#page-4-6) again referred to the toy example instance in [Fig.](#page-3-1) [1.](#page-3-1)

Here, the assignment of worker  $j = 14$  on day  $d = 1$  has been split among two shifts,  $s = 7$  and  $s = 13$ . Then, requests 8 and 56 can both be (partly) assigned to worker  $j$  without breaking constraint  $(1d)$  $(1d)$  nor ([1e\)](#page-3-5). Note that the addition of constraint ([2](#page-4-5)) does not cut this solution



<span id="page-5-1"></span>[Fig.](#page-3-1) 5. An optimal solution for the toy problem from Fig. [1](#page-3-1). The assignment of tasks to shifts and the staff rostering is color-coded. Worker  $w_{23}$  has been assigned to shift  $s_{13}$  and covers requests  $r_{12}$  and  $r_{56}$  (color-coded in green). The other three workers, namely  $w_{14}$  (blue),  $w_{30}$  (red), and  $w_{35}$  (brown) cover a request each and they have been assigned respectively to shifts  $s_7$ ,  $s_{16}$ , and  $s_7$ .

off, as  $1 = Z_{14,8} + Z_{14,56} = Y_{14,7,1} + Y_{14,13,1} = 1$ . However, the relaxation bound can be strengthened. Since the overlapping requests 8 and 56 cannot be performed by the same worker, we notice that at least two fully active shifts (from different employees) need to be active to cover both. Hence, we should include in the model a constraint like

$$
\sum_{j \in J_8 \cup J_{56}} \sum_{\substack{s \in S_j \\ a_{8,s} + a_{56,s} \ge 1}} Y_{j,s,1} \ge 2.
$$

Generalizing, we can insert into model  $(1)$  $(1)$  $(1)$ , for any (maximal) clique  $c \in C(B)$  of incompatible requests, the constraint

$$
\sum_{j \in \bigcup_{r \in c} J_r} \sum_{\sum_{s \in S_j} s \in S_j} Y_{jsd} \ge \sum_{r \in C} d_r,\tag{3}
$$

where we are also taking into account the number of operators required to satisfy each request.

We conclude the section reporting, for the toy problem shown in [Fig.](#page-3-1) [1](#page-3-1), an optimal solution of the corresponding mathematical model. [Fig.](#page-5-1) [5](#page-5-1) shows that 4 workers are needed to cover the 5 requirements in  $R$ .

#### **6. Experimental results**

<span id="page-5-0"></span>In this section, we describe the insights gathered dealing with our real-world case study, presenting and discussing the obtained results.

## *6.1. Case study*

Our case study concerns a major international airport in Northern Italy for which we collected 10 instances, corresponding to different weeks spanning from 2022 to 2023.

The relevant information available for each request and each employee corresponds to that reported in [Tables](#page-1-2) [1](#page-1-2) and [2](#page-1-3) respectively. Each employee is associated with a set of skills, and each request specifies one or more skills for any employee to be assigned to the corresponding task. Three instances include requests associated with a specific section within the department; in these cases, employees are also associated with (possibly multiple) sections; to handle this kind of specification, the combinations of skill and section can be considered as a whole specific skill. Overall, 58 possible skills can be associated with the employees.

We report some statistics for the 10 instances in our benchmark in [Table](#page-6-0) [5.](#page-6-0) Specifically, for each instance we can find: the number of employees, the average number of skills for each employee, the number of requests, descriptive statistics (average, minimum, and maximum values) about the number of employees skilled for a request, same descriptive statistics about number of requests that employees own the skill to perform, and minimum and maximum number of daily requests.

For all employees in each test instance, the maximum number of working days per week is 5. Employees are associated, by their contracts, with shifts of length 4, 6, or 8 h; aggregate information about this data is reported in [Table](#page-6-1) [6](#page-6-1).

#### *6.2. Experimental setup and evaluation metrics*

Each problem instance is modeled and implemented according to formulation ([1](#page-3-13)). The values of  $G_j$  and  $H_j$  for the *j*th employee,  $L_s$  and  $t_s$  for shift s, and  $d_r$  and the set  $J_r$  for request r are directly determined by instance data. The value of the trade-off parameter is set to  $\gamma = 50$ .

As for the set  $S_i$  of feasible shifts for employee j, we followed the methodology described in Section [4.3](#page-3-0): first, we define shifts starting every hour o'clock. Then, it can occasionally happen that a request cannot be covered by any of the already available shifts; in those cases, a new shift option, starting exactly at the request start time, is generated.

For each instance, we solve 4 variants of model ([1\)](#page-3-13) (hereafter called configurations). Specifically, model ([1](#page-3-13)) is run:

- (i) without the addition of any valid inequality,
- (ii) adding to the model only the set of inequalities ([2](#page-4-5)),
- (iii) adding to the model only the set of inequalities ([3](#page-5-2)),
- (iv) adding both sets of valid inequalities.

The solver is enabled to carry out its presolve phase. For each model variant, however, we also solve the pure continuous relaxation, to assess the effect of the presolve in terms of performance.

<span id="page-5-2"></span>To provide a condensed view of the results, in the following we are making use of performance profiles ([Dolan & Moré,](#page-10-17) [2002\)](#page-10-17). Performance profiles provide a unified view of the relative performance of the solvers on a suite of test problems. Formally, consider a benchmark of P problem instances and a set of solvers S. For each solver  $\sigma \in S$  and problem  $\pi \in \mathcal{P}$ , we define

 $c_{\pi,\sigma}$  = the cost for solver  $\sigma$  to solve problem  $\pi$ ,

where cost is the performance metric we are interested in. In particular, we will be interested in CPU time. We then consider the ratio

$$
\eta_{\pi,\sigma} = \frac{c_{\pi,\sigma}}{\min_{\sigma \in S} \{c_{\pi,\sigma}\}},
$$

which expresses a relative measure of the performance on problem  $\pi$ of solver  $\sigma$  against the performance of the best solver for this problem. If a solver fails to solve a problem, we shall put  $\eta_{\pi,\sigma} = \eta_M$ , with  $\eta_M \ge \max\{\eta_{\pi,\sigma} \mid \pi \in \mathcal{P}, \sigma \in \mathcal{S}\}.$ 

Finally, the performance profile for a solver  $\sigma$  is given by the function

$$
\rho_{\sigma}(\tau) = \frac{1}{|\mathcal{P}|} \cdot \left| \left\{ \pi \in \mathcal{P} \mid \eta_{\pi,\sigma} \leq \tau \right\} \right|,
$$

which represents the estimated probability for solver  $\sigma$  that the performance ratio  $\eta_{\pi,\sigma}$  on an arbitrary instance  $\pi$  is at most  $\tau \in \mathbb{R}$ . The function  $\rho_{\sigma}(\tau) : [1, +\infty] \to [0, 1]$  is, in fact, the cumulative distribution of the performance ratio.

Note that the value of  $\rho_{\sigma}(1)$  is the fraction of problems where solver  $\sigma$  attained the best performance; on the other hand,  $\lim_{\tau \to \eta_M^-} \rho_{\sigma}(\tau)$ denotes the fraction of problems solved from the given benchmark.

In addition to performance profiles, we will also make use of the cumulative distribution of absolute gaps for a given metric  $v$ ; in

**Table 5**

<span id="page-6-0"></span>Statistics for the 10 instances.



**Table 6**

<span id="page-6-1"></span>Percentage of employees associated with a given shift length for each problem instance.

Shift length							
	4 h (%)	6 h (%)	8 h (%)				
	0	0	100				
2	17	22	61				
3	14	17	69				
4	17	17	66				
5	17	17	66				
6	17	17	66				
7	17	17	66				
8	0	0	100				
9	6	0	94				
10	0	0	100				

particular, this tool has a similar concept as performance profiles and is obtainable setting

$$
\eta_{\pi,\sigma} = \frac{|v_{\pi,\sigma} - \text{opt}_{\sigma \in S} \{v_{\pi,\sigma}\}|}{\text{opt}_{\sigma \in S} \{v_{\pi,\sigma}\}},
$$

where the opt operator denotes the minimum or the maximum according to the metric  $v$  selected. Distribution of relative gap is particularly useful when evaluating results in terms of objective values. Of course, in this case, we have  $\rho_{\sigma}(\tau) : [0, +\infty] \to [0, 1].$ 

As performance metrics of our experiments, we are interested in the following values:

- the optimal value of the problem continuous relaxation, denoted by  $f_{\text{rel}}$ ;
- the optimal value at root relaxation (when the presolve step is enabled), denoted by  $f_{\text{root}}$ ;
- the objective value of the first feasible solution encountered during the optimization process, denoted by  $f_f$ ;
- the objective value of the best feasible solution encountered during the optimization process, denoted by  $f_{\text{best}}$ ;
- the runtime required to solve the relaxation, find a feasible solution, and find the best solution, denoted respectively by  $t_{rel}$ ,  $t_f$ ,  $t_{\text{best}}$ .

The notation is entirely summarized in [Table](#page-6-2) [7.](#page-6-2)

## *6.3. Results*

[Figs.](#page-8-0) [6](#page-8-0), [7](#page-8-1) and [9](#page-9-0) respectively show the cumulative distributions of the relative gaps for the metrics  $f_{\text{best}}$ ,  $f_f$ , and  $f_{\text{root}}$  and  $f_{\text{rel}}$ ; the two latter metrics are coupled in the same plot, sharing the reference optimal values, to appreciate the impact of the presolve step. [Fig.](#page-9-1) [8](#page-9-1) shows the performance profile for the  $t_f$  metric.

[Table](#page-7-1) [8](#page-7-1) provides an overview of the experimental results. For each instance, we compare the numerical results concerning the objective values  $f_{\text{rel}}$ ,  $f_{\text{root}}$ ,  $f_f$ , and  $f_{\text{best}}$ , the best lower bound value  $l_B$  and computation times  $t_{rel}$ ,  $t_f$  and  $t_{best}$  related to the four configurations of



<span id="page-6-2"></span>

the model obtained by including or not the proposed valid inequalities. Times are expressed in CPU seconds. In addition, for each instance and each configuration, [Table](#page-7-1) [8](#page-7-1) reports the percentage relative gap (%gap) between  $f_{\text{best}}$  and  $l_B$ .

It can be noticed that the introduction of valid inequalities allows to obtain relatively small gaps. Instance 9 was solved to optimality (%gap = 0) by the model that includes VI [\(3\)](#page-5-2); in this case, the running time was 30 026.05 s. In all the other cases, the model returned either a suboptimal solution with a (usually small) gap or no feasible solution within the time limit (a  $-$ ' in [Table](#page-7-1) [8](#page-7-1) indicates the absence of the value).

From the results it clearly emerges that the proposed valid inequalities are significantly effective. In fact, the model with none of them never returned a feasible solution within the time limit, while the addition of any of the proposed sets of inequalities allowed the model to return at least a feasible solution for all the instances except one case — instance 8 using VI [\(3\)](#page-5-2).

[Table](#page-7-2) [9](#page-7-2) shows for each configuration the number of instances for which a specific configuration is ranked first (best), second, third, or fourth (worst) in terms of both objective value  $f_{\text{best}}$  and best-bound value  $l_B$  yielded. It can be noticed that, if we consider just the objective value, configuration ([2](#page-4-5)) dominates the others, performing slightly better than  $(2)-(3)$  $(2)-(3)$  $(2)-(3)$ . The cumulative distribution presented in [Fig.](#page-8-0) [6](#page-8-0) shows that configuration [\(3\)](#page-5-2) is dominated by these other ones. Moreover, we can observe that each time configuration ([3](#page-5-2)) found the best solution, it was always equaled by configuration [\(2\)](#page-4-5). These considerations suggest that VI  $(2)$  is more effective than VI  $(3)$  $(3)$  to get the best objective value.

Instead, if we consider the best-bound value  $l_R$  reported in [Table](#page-7-2) [9](#page-7-2), configuration ([3](#page-5-2)) dominates all the others; we can also observe the same behavior in [Fig.](#page-9-0) [9](#page-9-0), where the cumulative distribution corresponding to configuration ([3\)](#page-5-2) dominates that referred to configuration ([2](#page-4-5)).

Considering the objective value of the first feasible solution  $f_f$ found by each configuration, the cumulative distributions plotted in [Fig.](#page-8-1) [7](#page-8-1) show that [\(2\)](#page-4-5) dominates the others; also  $(2)$ – $(3)$  $(3)$  $(3)$  has a better performance than ([3](#page-5-2)), confirming that set of inequalities [\(2\)](#page-4-5) leads the solver to reach a good quality solution even when solving the instance to the optimum is not possible.

#### **Table 8**

<span id="page-7-1"></span>Results of the computational experiments on the 10 problem instances.

$\mathcal{P}$	VI	Objective value				$l_B$	%gap	Time (s)		
		$f_{rel}$	$f_{\text{root}}$	$f_f\,$	$f_{\text{best}}$			$t_{\rm rel}$	$t_f$	$f_{\rm best}$
	$\rm No$	1535.67	1842.8	$\equiv$	$\overline{\phantom{0}}$	2038	$\overline{\phantom{0}}$	689.1	$\equiv$	
	(2)	13122	13 122.0	13 680	13 680	13 122	4.1	996.4	80 542	80 542
$\,1\,$	(3)	13176	13 176.0	14 7 20	13 680	13 194	3.6	1 0 58.1	18731	59 860
	$(2)-(3)$	13218	13 218.0	15 210	13738	13 248	3.6	2062.8	61 660	66 942
	No	910	1 3 8 7 .0	$\overline{\phantom{0}}$	$\overline{\phantom{0}}$	3 6 20	$\overline{\phantom{a}}$	61.7	$\overline{\phantom{a}}$	$\qquad \qquad -$
$\boldsymbol{2}$	(2)	4791	4794.0	6976	4 9 0 2	4814	1.8	106.2	5687	77915
	(3)	4786	4 802.0	6992	5 0 24	4 8 0 6	4.3	116.8	2208	99 669
	(2)–(3)	4791	4794.0	6990	4 8 9 4	4814	1.6	210.1	6229	79 108
3	$\rm No$	1554	1558.4	$\equiv$	-	2 3 1 0	$\overline{\phantom{0}}$	83.5	$\equiv$	$\qquad \qquad -$
	(2)	5028	5 0 28.0	5 8 6 2	5 1 0 6	5 0 3 2	1.4	229.3	8847	79 797
	(3)	5028	5 0 28.0	7 3 6 8	5 1 2 8	5 0 6 8	1.2	176.0	4504	63 182
	$(2)-(3)$	5028	5 0 28.0	7330	5 1 0 6	5 0 3 2	1.4	344.2	9107	81 904
	No	1554	1567.1	$\overline{a}$	÷.	2 5 2 0	$\equiv$	84.6	$\overline{\phantom{a}}$	$\equiv$
4	(2)	5018	5 018.0	6016	5 1 2 8	5 0 2 2	$2.1\,$	194.8	14 183	70 914
	(3)	5018	5 018.2	7424	5 1 4 6	5 1 0 8	0.7	179.9	6267	53 227
	(2)–(3)	5018	5 0 1 8 .0	7370	5 1 3 0	5 0 20	2.1	327.8	8726	66 433
5	$\rm No$	1570	1584.0	÷.	$\overline{\phantom{0}}$	2 3 3 4	$\overline{\phantom{a}}$	86.8	$\equiv$	$\overline{\phantom{0}}$
	(2)	5154	5 1 5 4 0	6 1 1 2	5 2 1 8	5 1 7 2	0.9	195.8	14 8 5 5	83 639
	(3)	5154	5 1 5 5 . 6	7462	5 2 3 2	5 202	0.6	197.3	6512	96 570
	$(2)-(3)$	5154	5 1 5 4 .0	5914	5 2 1 8	5 1 7 0	0.9	333.8	8885	76 228
	$\rm No$	1554	1564.3	$\equiv$	$\equiv$	2 2 3 0	$\overline{\phantom{0}}$	81.7	$\bar{\phantom{a}}$	$\overline{\phantom{0}}$
6	(2)	5061.25	5 0 6 1.3	5 8 0 8	5 2 2 6	5 0 9 6	2.5	200.1	24 8 5 6	100 000
	(3)	5060	5 060.0	7 3 2 4	5 3 1 0	5 1 3 8	3.2	214.2	4936	28 7 8 5
	(2)–(3)	5061.25	5 061.3	5736	5 1 6 6	5 0 9 2	1.4	386.5	19849	58 931
	$\rm No$	1554	1566.3	$\equiv$	$\overline{\phantom{0}}$	2 2 7 0	$\overline{\phantom{a}}$	83.5	$\equiv$	$\overline{\phantom{a}}$
$\boldsymbol{7}$	(2)	5110	5 1 1 0 .0	5 8 2 6	5 2 4 6	5 1 6 2	1.6	207.8	27 25 9	96 643
	(3)	5110	5 1 1 0 .0	7458	5 2 4 6	5 2 1 4	0.6	212.8	8383	80 930
	$(2)-(3)$	5110	5 1 1 0 .0	5984	5 2 3 0	5 1 4 6	1.6	368.6	19714	97 345
8	No	1524.53	2 2 2 0.7	$\equiv$	$\overline{a}$	3 3 3 8	$\overline{\phantom{0}}$	826.5	$\equiv$	
	(2)	12852	12 852.0	15718	13 108	12852	2.0	13 820.7	63 288	77 174
	(3)	12852	12852.0	$\overline{\phantom{0}}$	$\overline{\phantom{a}}$	12 852		1 509.4		
	$(2)-(3)$	12852	12 852.0	16 374	14 8 16	12 852	13.3	2 2 4 3 .0	46 028	63 262
	$\rm No$	1610	2058.0	$\overline{a}$	÷.	3 0 6 0	$\overline{\phantom{0}}$	65.3	$\equiv$	$\overline{\phantom{0}}$
9	(2)	5748	5 7 4 8 .0	8 4 2 2	5788	5 7 4 8	$0.7\,$	231.1	2592	12 9 64
	(3)	5748	5 7 4 8 .0	8770	5788	5788	$\bf{0}$	165.8	1973	11 024
	$(2)-(3)$	5748	5 7 4 8 .0	6 3 4 4	5788	5 7 4 8	0.7	354.2	519	1802
	$\rm No$	1719.18	2 3 6 2 .5	L.	÷	4 1 8 8	$\overline{a}$	560.1	$\equiv$	
10	(2)	11862	11 862.0	12 208	12 208	11 862	2.8	870.6	34 5 5 6	34 5 5 6
	(3)	11862	11 862.0	12674	12 208	11 898	$2.5\,$	732.1	17937	25 4 29
	$(2)-(3)$	11862	11 862.0	13 122	12 208	12 070	$1.1\,$	1 846.1	14 006	17864

#### **Table 9**

<span id="page-7-2"></span>Ranking of the models based on the objective value and the best bound: for each configuration of the model the number of instances for which it placed first, second, third, and fourth is reported.

VI	$f_{\text{best}}$				$l_B$			
	1st	2nd	3rd	4th	1st	2nd	3rd	4th
No	0	0	$\Omega$	10	0	$\mathbf 0$	0	10
(2)		3	0	0	$\Omega$	6		0
(3)	3		5	0	6	2	2	0
$(2)-(3)$		$\mathcal{P}$		$\bf{0}$	3	$\mathbf{0}$		

On the contrary, it can be observed in [Fig.](#page-9-1) [8](#page-9-1) that the configuration that uses the set of inequalities [\(3\)](#page-5-2) yields feasible solutions of reasonable quality (the gap is relatively small anyway) faster than the other methods except one case where it does not find any. Moreover, we observe that adding this set of inequalities to the model together with ([2](#page-4-5)) seems to speed up the return of the first solution. In any case, using both valid inequalities proves to have a balanced performance between the quality of the solution, its best bound, and the computation time to obtain the first feasible solution.

It is also worth analyzing the effectiveness of the presolve phase carried out by the solver. In [Fig.](#page-9-0) [9](#page-9-0) we plot the cumulative distributions of both the root relaxation produced by setting the presolve option

to the default and solving the linear relaxation of the models without a presolve phase. It can be observed that although the presolve phase increased the performance of all the models (indeed, for both configurations  $(2)$  $(2)$  and  $(2)$  $(2)$  $(2)$ – $(3)$  the number of times they reached the highest relaxation value increased of 1), it is especially effective for configuration [\(3\)](#page-5-2). The increase has such an extent that it went from being dominated by the other two to almost dominating them both. [Fig.](#page-9-0) [9](#page-9-0) also shows that without performing this presolve phase, the use of both sets of valid inequalities ([2\)](#page-4-5) and ([3](#page-5-2)) had the best performance. This evidence remarks the positive effect of the proposed inequalities, which may be even greater in those contexts where the solver cannot rely on such an effective presolve stage.

#### **7. Conclusions**

<span id="page-7-0"></span>This study faced a complex staff scheduling problem arising at a major international airport in Northern Italy and involving ground staff. Specifically, we addressed the integrated task scheduling and personnel rostering problem, jointly coping with two problems that are commonly solved in cascade. Inspired by an integrated model from the literature, we proposed two families of valid inequalities.

Computational results showed that jointly solving the task scheduling and the staff rostering problems can be particularly challenging at least for instances coming from the case-study addressed. Even



Fig. 6. Cumulative distribution of the relative gap for the best objective value  $(f_{best})$  found by each method.

<span id="page-8-0"></span>

**Fig. 7.** Cumulative distribution of the relative gap for the objective values at the first feasible solution encountered  $(f_f)$  found by each method.

<span id="page-8-1"></span>finding feasible solutions becomes critical if the model is not equipped with valid inequalities that significantly improve the performance of the solver used. Indeed, a basic model run without the addition of any valid inequalities fails to provide a feasible solution within the time limit imposed (about 28 h) for all 10 instances. On the contrary, introducing inequalities ([2](#page-4-5)) and ([3](#page-5-2)) may be crucial for finding high-quality solutions in reasonable computation times. Summing up, configuration ([2](#page-4-5)) proved to be a good choice when interested in highquality solutions; configuration ([3\)](#page-5-2) instead proved to be able to find good feasible solutions in the shortest time and a narrow gap to the

optimum value; configuration ([2](#page-4-5))–[\(3\)](#page-5-2) is a reasonable middle ground between them, able to balance the quality of the best solution and the computation time to get a feasible solution.

We conclude by briefly outlining what future lines of research may be. The proposed approaches have proven successful in solving medium to large instances of the problem, but as the number of requests and employees increases, their performance may degrade. To evaluate the robustness of our methodology, experiments on different problem benchmarks would certainly be of interest. Moreover, it will be interesting to investigate decomposition methods, as well as alternative



**Fig. 8.** Performance profiles of the time to retrieve a feasible solution  $(t_f)$  required by each method.

<span id="page-9-1"></span>

<span id="page-9-0"></span>Fig. 9. Cumulative distributions of the relative gap from best objective values; for each model we consider both the root relaxation value after presolve ( $f<sub>non</sub>$ </sub>) and the optimal value of pure continuous relaxation ( $f_{\text{rel}}$ ). Note that the reference value for each problem is the overall best, so that we can observe the impact of the presolve operations in increasing the root relaxation bound.

formulations of the problem, e.g., network-based formulations. Furthermore, in such a complex environment as airports, it is crucial to be able to deal with disruptions that may occur as a consequence of flight delays or other events that make planned solutions infeasible. Preliminary results have shown that the proposed approaches are particularly efficient if, as a result of parameter changes, it becomes necessary to reassign tasks to employees on a given day while maintaining planned assignments on other days of the time horizon. Disruption management surely identifies an interesting future development and deserves further study.

## **CRediT authorship contribution statement**

**Paola Cappanera:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing. **Leonardo Di Gangi:** Data curation, Investigation. **Matteo Lapucci:** Conceptualization, Formal

analysis, Visualization, Writing – original draft, Writing – review & editing. **Giulia Pellegrini:** Data curation, Software. **Marco Roma:** Investigation, Visualization. **Fabio Schoen:** Conceptualization, Methodology, Funding acquisition, Project administration. **Alessio Sortino:** Investigation, Software, Validation.

#### **Declaration of competing interest**

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Fabio Schoen reports financial support was provided by Tuscany Region.

#### **Data availability**

The data that has been used is confidential.

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#### **Appendix. Finding maximal cliques of incompatible activities**

<span id="page-10-16"></span>Following the ideas by [Krishnamoorthy et al.](#page-11-9) [\(2012](#page-11-9)), to efficiently find all maximal cliques of incompatible tasks, Algorithm [1](#page-10-18) can be employed, which exploits the temporal ordering information of requests.

<span id="page-10-18"></span>

```
Require: R set of requests
1: T = \emptyset2: for r \in R do
 3: Let (b_r, e_r) the starting and ending times of request r4: T = T \cup \{(b_r, r)\} \cup \{(e_r, r)\}\5: end for
6: Cast the set of pairs T into a list non-decreasingly order based on
    the first element of each item (order by times).
7: C = \emptyset8: c = \emptyset9: for i = 1, ..., 2|R|: do
10: (t, r) = T[i]11: if t == b_r then
12: c = c \cup \{r\}13: else
14: C = C \cup c15: c = c \setminus \{r\}16: end if
17: end for
18: return
```
The algorithm performs the following instructions: first, a sorted list  $T$  is created, in non-decreasing order, of all beginning and ending times of all requests. In the list, the identifier  $r$  of the corresponding task is also stored. The set of identified cliques  $C$  and a "running" clique set  $c$  are initialized as empty. Then, the list is iteratively examined; if the current value of  $t$  is a starting time, the corresponding request  $r$  is added to the running clique  $c$ . Otherwise,  $c$  is established as a clique, being added to the set of cliques  $C$ , and then the request  $r$  is removed from it.

We can immediately realize that, at any time, the running clique  $c$  indeed contains incompatible requests: all jobs in  $c$  have started at a time before  $t$  and are finishing at a moment after  $t$ . It is also easy to figure out that all maximal cliques are detected by the algorithm. In particular, a maximal clique of incompatible requests is added to  $C$ as soon as  $t$  represents the end time of the request of the clique that finishes earliest. At that moment, all the requests of the clique have already started and thus have been inserted (and not yet removed) into the running clique  $c$ .

We shall also note that, for the algorithm to be correct, if two requests  $r_1$  and  $r_2$  are such that  $b_{r_1} = e_{r_2}$ , then the pair  $(b_{r_1}, r_1)$  should appear later in the ordered list than the pair  $(e_{r_2}, r_2)$ . Otherwise,  $r_1$ and  $r_2$ , that are not incompatible, would be placed together into an incompatibility clique.

Finally, we shall note that the algorithm is computationally cheap: running time is dominated by the sorting operation, thus the computational complexity is  $\mathcal{O}(|R| \log(|R|))$ .

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