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Uncertainty Quantification of an Aeronautical Combustor using a 1-D Approach

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Abstract. 1-D codes are still much utilized regarding the preliminary design and the design phase of a single component. In particular, combustors is one of the most critical component in gas turbine engine and its design mainly affects all other components, starting from the turbine. During the initial phases of the design process, parameters are known approximately; during this phase, critical for the definition of a starting set of design parameters, there is little point in doing high fidelity Computational Fluid Dynamics (CFD) analyses. On the contrary, the exploration of the whole space is extremely important to better understand the behaviour of the system and to focus on the design objectives. Uncertainty quantification (UQ), mainly developed in recent years and applied in many fields, can be really helpful in the preliminary design phase and also as a support during the whole design process. This work comes with the idea to estimate the main source of geometrical uncertainties in the design phase of a combustor. The test case is based on a full annular lean burn combustor, tested at Central Institute of Aviation Motors (CIAM) during the LEMCOTEC (Low EMissions COre-engine TEChnologies) European project. Among the test points investigated in the experimental campaign, the Approach condition is analysed. The inner liner is considered to analyse the metal temperature. Therm-1D, a 1-D in-house simulation code, is used to model the combustor. After having built the baseline case of the combustor, several uncertainty analyses are investigated. In particular a classical Monte Carlo approach is compared with 4 innovative polynomial-chaos approaches for each group: Gauss quadrature, total order with Latin Hypercube sampling (LHS), probabilistic collocation and stochastic collocation. The analyses proved how the last two methods give the same results with a sensible lower amount of simulation (depending on the number of input variables). An additional sensitivity analysis to the order of the polynomial is conducted and results show how, with a 3rd order polynomial approximation, results can be very similar to those ones of the Monte Carlo simulation. Lastly, results are compared with experimental data to achieve a better understanding of the most relevant input parameters and the propagation of their uncertainty on the results.

INTRODUCTION

Historically, the majority of the works associate the concept of uncertainty only to the experimental tests, even if there are also many uncertain factors that affect the results of computational analyses. From the design point of view, the estimation and the correct propagation of the uncertainty affecting an analytical result is fundamental, especially in the field of aircraft engines. The design of the liner effusion cooling systems [1] of a combustors is critical, considering the high number of parameters that influence the wall adiabatic temperature, as highlighted in the experimental works of Andreini et al. [2]. The critical issues related to the experimental measurements on real components, as well as the high costs, and problems associated with CFD analysis have led to the development of a 1-D code that allows a preliminary thermal design with limited computational effort. In order to increase the robustness of the 1D in-house code, developed by the University of Florence ([3], [4]) an uncertainty quantification (UQ) analysis with non-intrusive methods is conducted. The approach consists to disturb the input data of the numerical code and evaluate the variation of the response of the system; subsequently the statistics on the results are calculated. The procedure described is known as Monte Carlo method [5], which will be described. A probabilistic approach based on the application of the spectral methods makes possible to evaluate the propagation of the uncertainty in numerical

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codes without increasing the computational cost too much. Spectral methods, also called stochastic expansions, allow constructing a functional linking the solution provided by the numerical code to the variables entering in it with a statistical distribution predefined. The stochastic expansion methods are based on theory developed by Wiener [6]. The first application was in mathematics on hypergeometric polynomials designed by Askey [7]. For the uncertainty quantification the most interesting methods are the polynomial chaos expansion (PCE), developed by Karniadakis-Xiu [8] based on the Askey's scheme and the stochastic collocation (SC). This paper shows a comparison among the various spectral methodologies and the various polynomial orders; moreover, these results are compared with those obtained with the Monte Carlo method. In literature, there are several works of uncertainty quantification applying at gas turbine. For film cooling systems the results shown by D'Ammaro et al. [9] showed almost identical results between polynomial chaos and Monte Carlo method, but with a 10 times reduction in the computational effort for the PC approach. A similar study was conducted by W. Shi [10] to consider the effects of manufacturing deviations on film cooling effectiveness provided by fan-shaped hole. The comparison between polynomial chaos and stochastic collocation showed by Montomoli at al. [11] for thermal loads on a gas turbine nozzle indicated a difference in results of less than one percent. A comparison of the different UQ techniques was carried out by Durocher et al. [12] in the field of NO_x formation at different flame conditions, showing a certain discrepancy between the results obtained.

UNCERTAINTY QUANTIFICATION METHODS

Monte Carlo Method

This method has the advantage to be simple and assumes that the expected value, $\bar{E}(x)$ consists in the average calculated on a discrete sample of the population.

$$\bar{E}(x) \approx \bar{R}(x) = \frac{1}{N}\sum_{i=1}^{N} R_i \quad , \quad for\ N = \infty \rightarrow \bar{E}(x) \equiv \bar{R}(x) \tag{1}$$

The method reaches a statistical convergence when the exact stochastic solution is reached, i.e. when the number of samples N goes to infinity. Another advantage of this method is that the convergence does not depend on the number of input variables of the problem. When MC method is not at convergence yet, the error order committed is a function of N

$$o(err) = \frac{1}{\sqrt{N}} \tag{2}$$

The disadvantage of this method is that it is slow in reaching the convergence and requires a high number of evaluations N. To accelerate this method, it is common to act on the selection of samples to be evaluated. The most used technique is the Latin Hypercube sampling (LHS) [13], in which the range of each input random variable x is divided in N intervals with equal probability. With this sampling method the error order is

$$o(err) = \frac{1}{N} \tag{3}$$

In this work all Monte Carlo analyses were performed with the Latin Hypercube sampling.

Spectral Method

A stochastic function, $R(x,\xi)$, can be expressed by a series expansion:

$$R(x,\xi) = \sum_{i=1}^{\infty} \alpha_i(x)\psi_i(\xi) \tag{4}$$

Where $\alpha_i(x)$ are deterministic coefficient of the polynomials and $\psi_i(\xi)$ are the orthogonal basis, which are functions of the probabilistic distributions of the input variable. When the coefficients α_i are defined and the basis are unknown, the technique is called "*Stochastic expansion*"; otherwise, when the known coefficients and basis are unknown, the methodology takes the name of "*Polynomial Chaos Expansion*" [5]. The equation (4), in practice, is always truncated at a finite expansion order [14]:

$$R(x,\xi) \cong \sum_{i=1}^{P} \alpha_i(x)\psi_i(\xi) \qquad (5)$$

Based on how the polynomial expansion is truncated, there are different approaches. When the total order of the polynomial expansion is fixed, the approach is called "*total-order expansion*". The number of evaluations N_t depends on the order "p" of polynomial and the number of variables "n":

$$N_t = 1 + P = \frac{(n+p)!}{n!\,p!} \qquad (6)$$

The evaluation points can be select with different sampling methods. This method allows to set random sampling, LHS or to import a calculation grid defined by the user; in this work, LHS is used for the total-order expansion. Furthermore, in order to increase the robustness of the method, a double sampling is used throughout this work with respect to the value obtained from the equation (6), as recommended by Eldred [15].

When the order "p_i" of each polynomial basis ψ_i are fixed, the approach is called "*tensor-product expansion*". The total number of evaluations is given as follows

$$N_t = 1 + P = \prod_{i=1}^{n}(p_i + 1) \qquad (7)$$

The input points are fixed and they are defined by the Gauss grid. Tensor-product approach is excellent for reduced input stochastic variable numbers, while for numbers of small to medium variables the total order approach is recommended.

In this work, the results obtained with the two different approaches were investigated. For completeness, there are other methodologies: one example is the one based on Smoliak grid [16], as a reduction of the tensor-product approach, but since it is convenient only for high variable numbers, it has not been considered in this work.

THERM1D

One-dimensional codes are simulation tools that rely on simplified approaches to perform design evaluations. Low-order equations and correlations are here exploited to represent physical phenomena involved in the problem, allowing quick results. For this reason, approaches like these are ideal in the early stage of engineering design. The code used in this work is called *Therm-1D* and is intended for the preliminary design of combustor cooling systems, meant both for defining a preliminary arrangement of the cooling and for evaluating performance in different operating conditions once a configuration is defined. The procedure is able to predict wall temperature and heat loads on liners, coupling an industrial, well-proven and in-house developed flow network solver with a standard 1D heat transfer model based on Lefebvre's methodology [17]. For a fully detailed description, please refer to [18] and [4].

In Figure 1 is reported a schematic representation of the operations performed by the different codes in the procedure, which are iteratively called to solve the fluid-wall conjugate calculation. The very first input is represented by main geometrical features and boundary conditions at inlet and outlet ports of the flow network, consisting of mass-flow rate/total pressure, temperature and static pressure respectively. Starting from that information, the prediction of the air-split is performed by the solver, which makes use of dedicated models to calculate pressure drops trough the different sections. The internal heat transfer is evaluated as well, having availability of a wide library of suitable thermal correlations. Once all the thermal loads are defined, the 1-D equation for conduction is resolved to determine the equilibrium temperature into the liner thickness.

For the execution of the procedure, two iterative loops are provided. In the internal one, heat exchange on walls is resolved until the convergence with the 1D-conduction calculation. Radiation and convection for the "gas side" and radiation for the "coolant side" are here evaluated iteratively, in order to obtain liner's metal temperature. This last information is provided as an input to the overall loop in which convection on the cold side, heat-sink effect and main flow network parameters are evaluated using the updated values. The calculation continues until the convergence or arresting criteria are achieved, providing not only an output in terms of liners temperature but also main characteristics of a cooling system such as adiabatic effectiveness distribution and coolant characteristics.

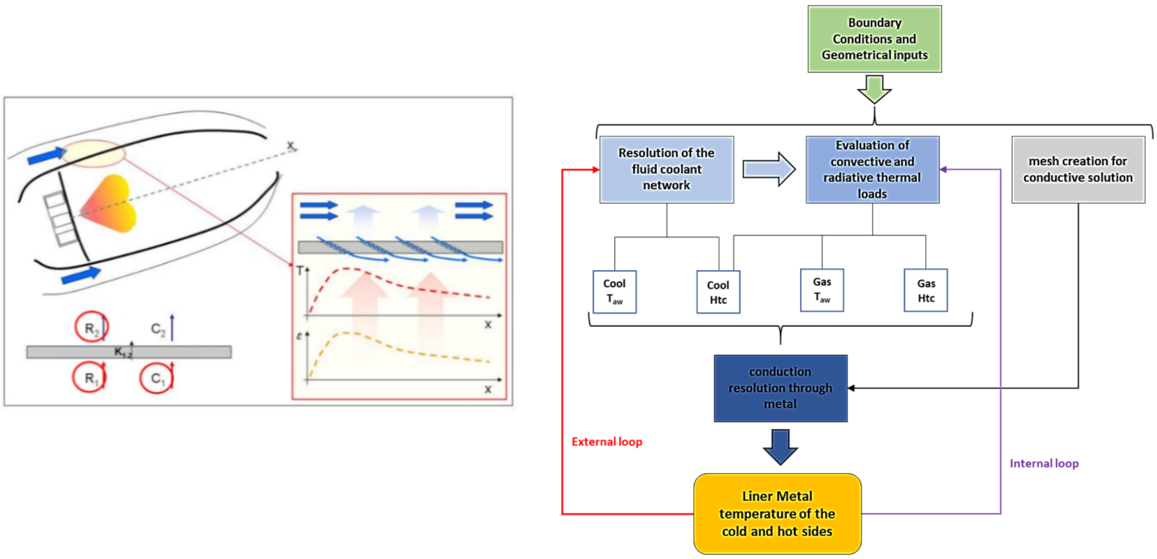


FIGURE 1 - Therm-1D procedure

In the present study, a network representative of a cross-section passing through the injector axis is modeled. The geometrical information is provided in terms of sections for air-passages and liners' thickness, as well as type and arrangement of the cooling features. Boundary conditions are set according to the characteristics of the mission point in which the real hardware is tested, meshing parameters are so defined for the finite element calculation in order to assure a proper discretization of the solid walls.

TEST CASE AND METHODOLOGY

The present work investigates a single annular combustor designed in the EU-funded research program LEMCOTEC, additional information about the test case can be found in previous works ([19], [20]). The prototype can be considered an improvement of the combustor designed, manufactured and tested in a previous research program (NEWAC) and depicted in FIGURE 2.The fluid dynamics behaviour is characterized by air passing through a dump diffuser and diverted within the cowl and to the inner/outer annuli, where it cools the liner and is partly bled outside. Once inside the cowl, it flows through the swirler and the dome cooling system. An impingement-cooled heat shield protects the dome, which provides also slot cooling in the first part of the liner. Such liners are cooled also by staggered effusion holes.

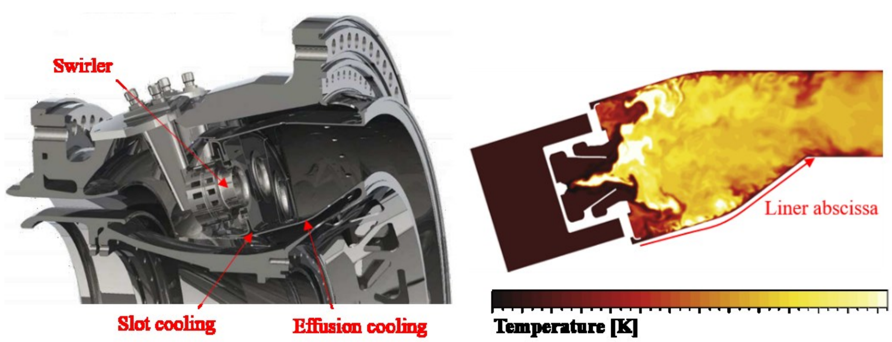
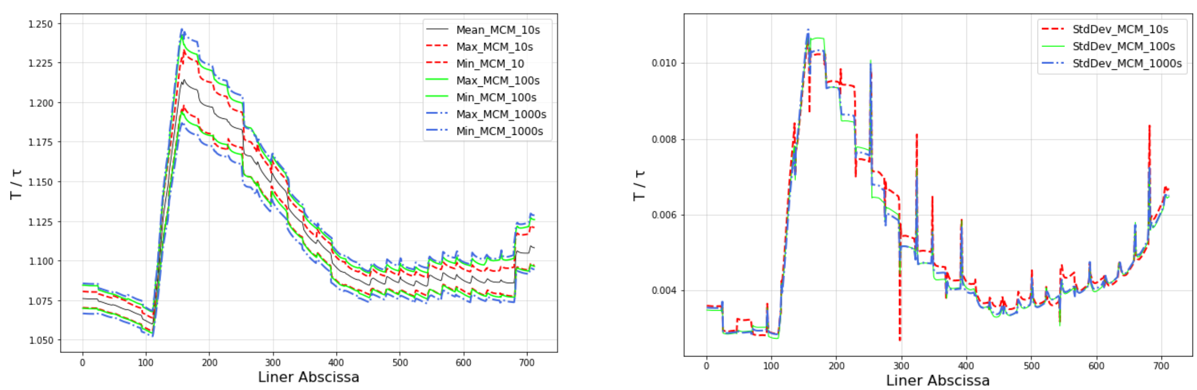


FIGURE 2 - NEWAC combustor (left) and expected temperature field at Approach condition for the LEMCOTEC combustor (right). Pictures adapted from [19]

The goal for this work is to investigate different UQ methodologies in order to determine the performance of the different approaches as well as to determine some guidelines for future activities. Concerning the analysis, 2 inputs are chosen with truncated Gaussian distribution: streamwise angle of the effusion cooling holes and the C_d: mean value and standard deviation are respectively 30°, 2.5° and 0.7, 0.05. Truncation is chosen at ± 2σ. The inner liner only is considered (FIGURE 2). Initially, a Monte Carlo sensitivity analysis is conducted in order to show the sensitivity of this method to the number of simulations, generating and testing 10/100/1000 samples. Relating to the Uncertainty Quantification 3 method are investigated: tensor-product expansion polynomial, total-order expansion polynomial, and stochastic collocation. For all these methods several orders are investigated and results are provided in the following chapter. Tensor-product expansion requires a collocation grid defined by Gauss. Total-order expansion results are all generated from an LHS with double oversampling with respect to equation (6). Stochastic collocation requires the same grid as the tensor-product expansion. Moreover, the different methods are then compared in order to benchmark the approaches in terms of accuracy and computational effort.

RESULTS

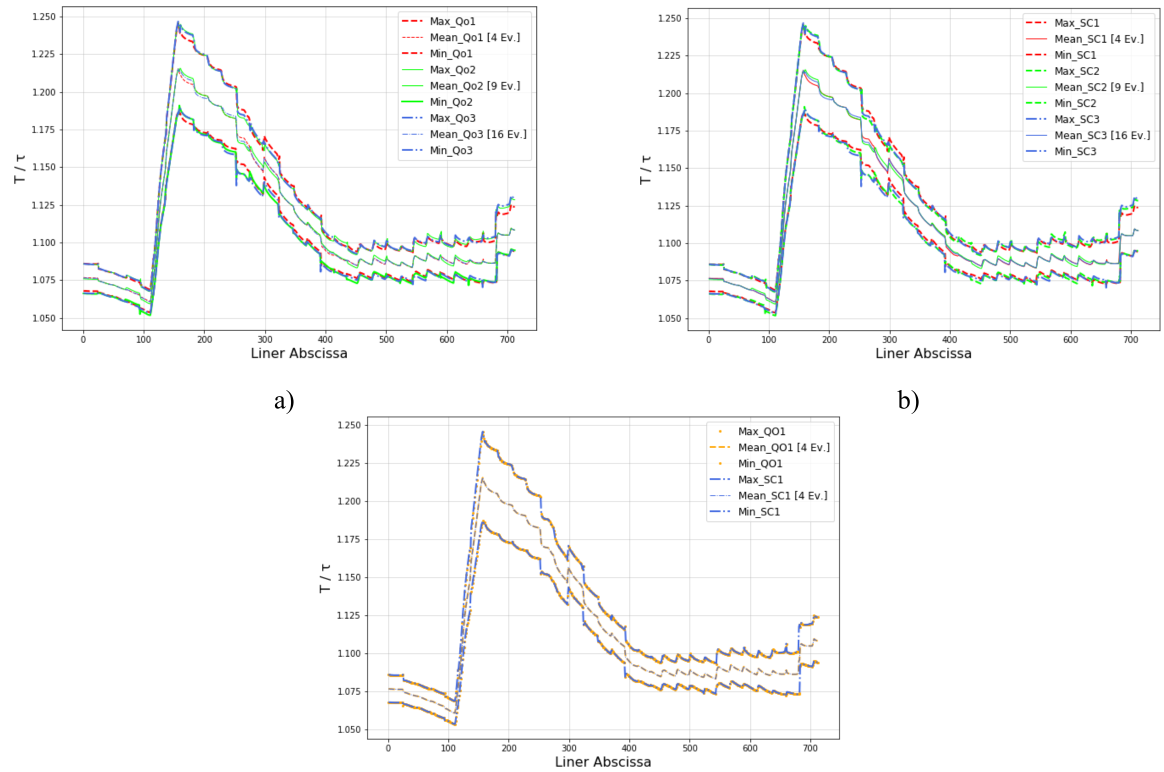
As far as the results are concerned, the first outcome to show is the sensitivity to the Monte Carlo analysis. It is known that Monte Carlo has a low rate of convergence, especially when random sampling is adopted. Regarding this analysis, LHS instead of random sampling is used in order to increase the rate of convergence [14]. Throughout the work, results are provided in a non-dimensional way: the wall liner temperature is scaled based on a reference value in accordance with the industrial partner. Results of Monte Carlo sensitivity are provided in FIGURE 3. As already stated in the previous chapter, 3 different analyses are carried out with a different number of samples: 10, 100, and 1000 samples are selected. FIGURE 3a displays the results obtained from the 3 Monte Carlo analyses. It is proved that the mean value is exactly the same for all the three analyses and this is due to the sampling method chosen: LHS equally spaces the domain in which to select the random sample. Therefore, a more organized choice of samples is done: LHS performs better than pure random sampling. For this reason, only a single line for the mean is plotted. FIGURE 3b shows the standard deviation and the min/max values for all the analyses. Standard deviation, as well as liner temperature, displays some peak and valley trend due to the combustor configuration: each peak or valley corresponds to an effusion row in the combustor. It is clear also, how increasing the number of sample, a wider range is investigated, thus increasing the possibility to consider more extreme values.



a) b)

FIGURE 3 - Monte Carlo analysis results: a) range b) standard deviation

Going to polynomial chaos, the first method taken into consideration is the quadrature order approximation. A sensitivity analysis of the order of this method is conducted and results are presented in FIGURE 4a. 1^{st}, 2^{nd}, and 3^{rd} order are taken into consideration and the graph shows how the 1^{st} order approximation is sufficient enough in order to characterize the response function. The number of evaluations required is reported in the legend. FIGURE 4b shows the order sensitivity to the stochastic collocation method. The stochastic collocation, as previously highlighted, is very similar to the quadrature order method: number of evaluations required is the same, however, stochastic collocation compute the basis of the polynomial for known coefficients whereas the quadrature order computes coefficients for known basis functions. The results, as already investigated by Montomoli et al. [11] are the same and are shown in FIGURE 4c for the 1^{st} order only.



a) b)

c)

FIGURE 4 – Polynomial order sensitivity for a) quadrature method, b) stochastic collocation method and c) comparison.

The second method group taken into consideration is the total-order approximation. This method requires less number of evaluations compared to the quadrature order approximation, especially when the number of variables is high (approximately more than 5). Number of evaluations is calculated from Equation (3), and an oversampling of 2 is adopted throughout the work. As for the previous method, FIGURE 5a shows a sensitivity to the approximation order: 2nd, 3rs, and 4th order approximation are considered. The maximum and minimum values for each linear abscissa are evaluated, as well as the mean value. Mean value is the same for all the order considered (FIGURE 5b), whereas standard deviation (FIGURE 5c) varies depending of the order. Generally, it can be stated that the higher the order does not entail necessary a better approximation: from FIGURE 5a it is clear how, increasing the order, can lead to a more scattered plot with peak and valley, with a non-smooth trend. An important consideration requires attention: FIGURE 5d displays the 2nd order approximation with collocation ratio equal to 1: clearly, results are non-sense; even if the mean value is obtained correctly, maximum and minimum, as well as plenty of values in the middle, are wrongly approximated. this last figure is included in order to prove the effectiveness of this method only when collocation ratio is equal to 2.

FIGURE 6 presents the comparison among the previous methods adopted. In particular, in FIGURE 6a it can be seen the difference regarding mean, minimum and maximum value. Stochastic collocation is not considered anymore in these final graphs, as it is demonstrated that results are equal to that one of quadrature order method. FIGURE 6b shows the standard deviation for each method. Overall, FIGURE 6 proves how the polynomial chaos method is a new and efficient way in order to compute the statistic of the output and uncertainty quantification analyses. For this test case, a Quadrature order method with 4 evaluations required, performs exactly the same way as a Monte Carlo Analyses with 1000 evaluations. This can be certainly helpful especially when the computational cost becomes really expensive.

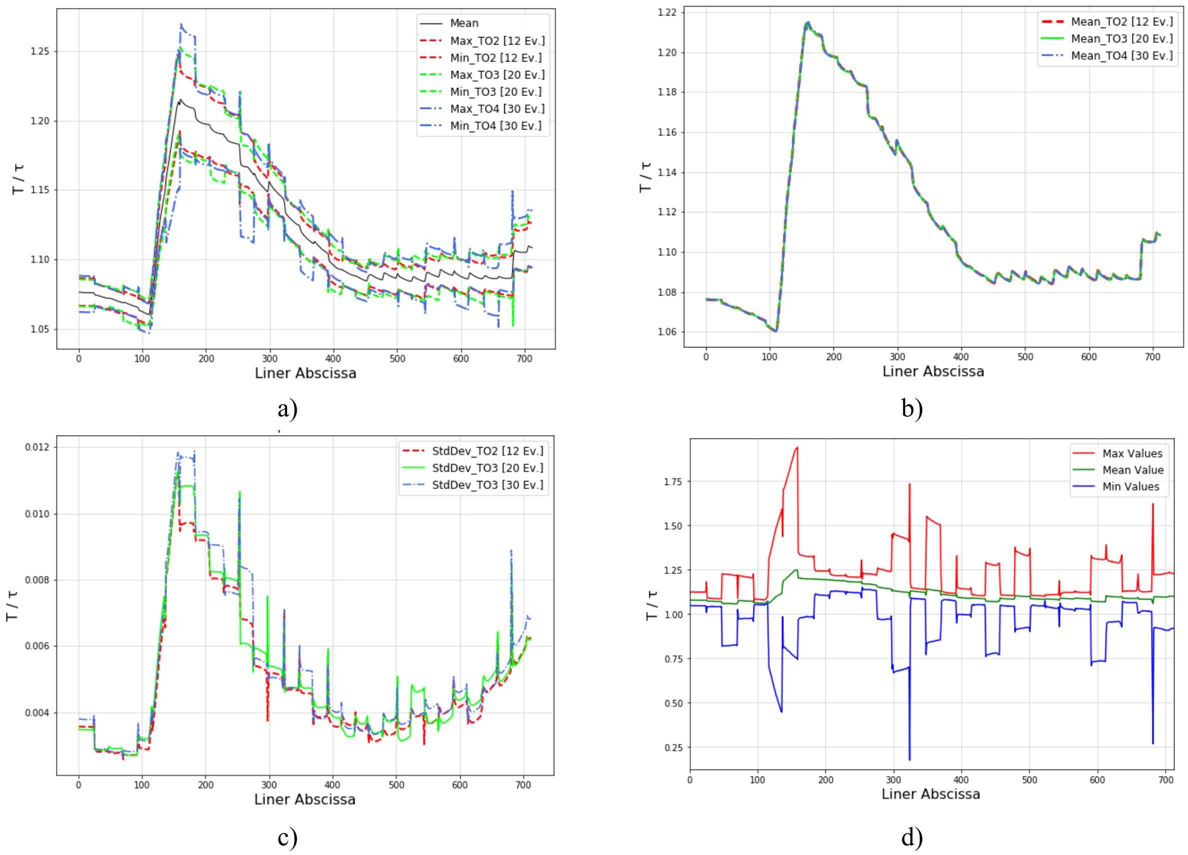


FIGURE 5 - Total order approximation: a) order sensitivity, b) mean, c) standard deviation, d) 1st order

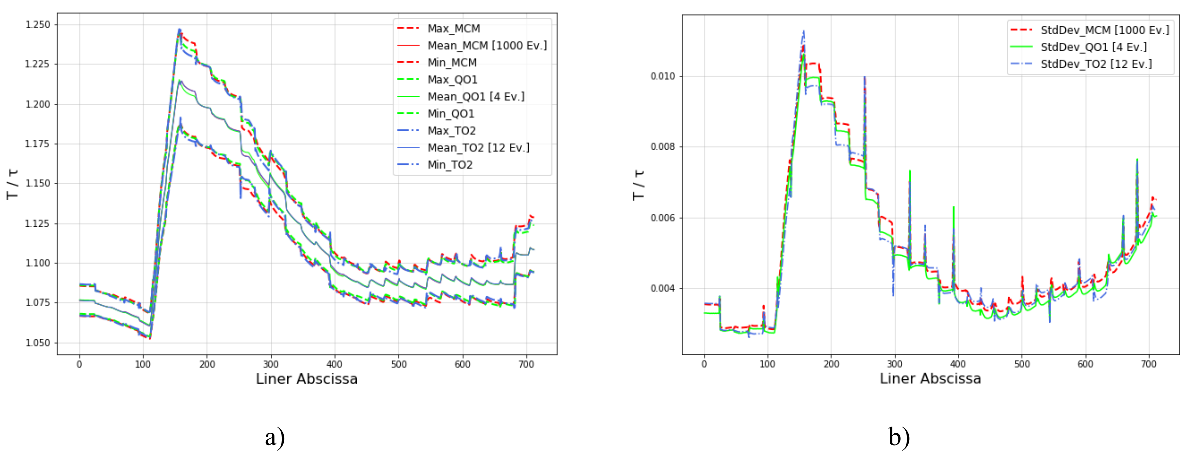


FIGURE 6 – Comparison among methods: a) Range, b) Standard deviation

In conjunction with the UQ analysis, two additional information can be printed out. The first one is the sensitivity analyses through Sobol's indices, while the second one is the output probabilistic graph that can show the different probability zones in term of PDF. Regarding the first one, results are displayed in FIGURE 7. This graph is extremely useful because can state which parameter has the most influence on the results; moreover, it shows the relative zone, as a function of the linear abscissa, in which these parameters have their effect. The combustor can be divided into 3 main zones: in the first one and in the last one the discharge coefficient C_d has a major effect on the result (in terms of liner temperature), while in the middle zone is the perforation angle that dominates the output.

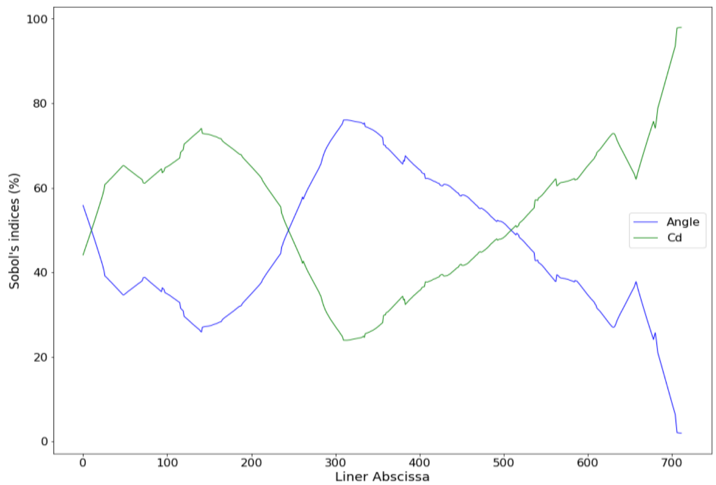
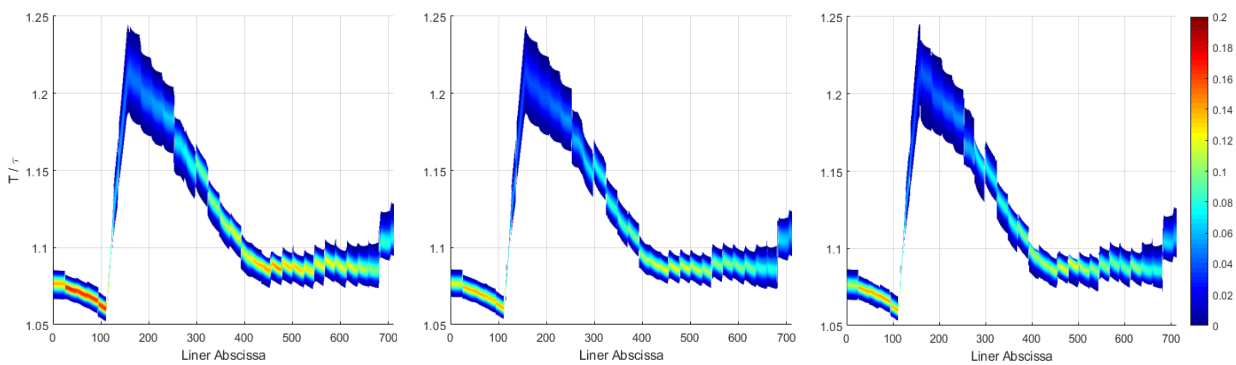


FIGURE 7 - Sobol's indices

Concerning the second type of results, FIGURE 8 presents the previous graphs in terms of associated probabilistic density function (PDF). The results are plotted for all the methods, as additional proof of the accuracy of these methods. Again, the stochastic collocation it is not included because results are exactly the same as the quadrature order, as already stated above. The PDF exhibits almost the same results for the 3 methods. The PDF value does not depend on the range (min, max) but it is just derived from the PDF obtained for that particular result. From linear abscissa 0 to 100 and approximately from 350 to 650 it can be noticed a strong red-trend color in the middle, representing the modal value. Within these zones, regardless of the range (min and max), the probability is highly limited in a narrow region. This effect can be attributed to the role played in the very first part of the liner by the slot cooling and, more downstream, by the increasing contribution of film superposition. On the other hand, from linear abscissa 150 to 300 the PDF is much more widely spread in the range (the color is almost blue everywhere) and this indicates that the region is affected by a high influence of the uncertainty of the inputs. This is associated to the region where the slot cooling suddenly reduces its effectiveness (due to both its natural decay and interaction with the swirling flow with the liner) and contextually a limited coverage offered by effusion cooling.



a) b) c)

FIGURE 8 – Probabilistic graphs: a) Monte Carlo; b) Quadrature order; c) Total order

To conclude, experimental results are also provided in FIGURE 9. The liner temperature was measured at different streamwise locations but also in different sectors of the full annular combustor. Despite the difference between the prediction and the experimental data, the scope of this paper was not to match the measurements but to perform an UQ analysis exploiting a simplified but representative modelling strategy. Further CFD analyses aimed at overcoming the limitations of 1-D modelling and its approximations will have the responsibility to improve the prediction of the thermal loads for the thermal calculation.

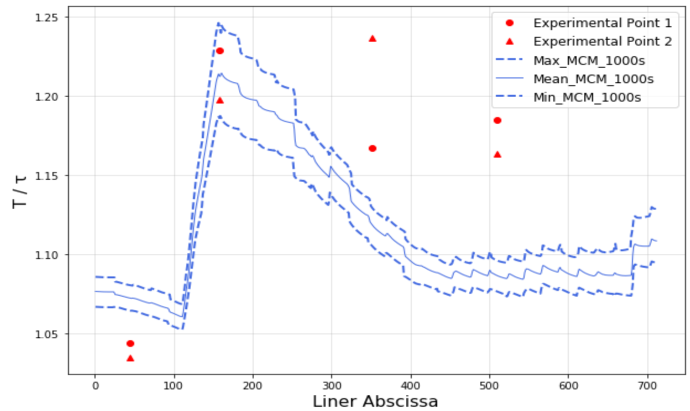


FIGURE 9 - Comparison between Monte Carlo and experimental data.

CONCLUSIONS

This work presents an uncertainty quantification approach applied to the preliminary design phase of a combustor system. An open-source code, Dakota, is used for the UQ analysis, while the in-house software Therm1D is used for the simulating the combustor. The test case is based on a single annular combustor configuration designed and tested in the EU project LEMCOTEC. In particular, only the inner liner is studied. The objective was to investigate different UQ methodologies: the Monte Carlo method and the polynomial chaos approach. Concerning the second approach, 3 sub-methods are investigated: 2 of them rely on the family of the probabilistic collocation approach (quadrature order and total order, depending on the approximation) and the other one is the Stochastic collocation. Geometric uncertainties are selected: streamwise inclination angle of the effusion cooling holes and the discharge coefficient C_d. Results proved how all of them perform very similarly. Moreover, results also proved how these methods can achieve the same results as a Monte Carlo analysis requiring thousands of evaluations, both in terms of range and standard deviation. An extremely important feature of this UQ analysis is that the sensitivity analysis is obtained by default: Sobol's indices are evaluated for each analysis. This study confirms that both inclination angle and C_d contribute to the final output. However, they have a different contribution depending on the zone selected. A probabilistic graph is also provided, proving the usefulness of the methods analyzed. Above all, the most important consideration is to bear in mind that UQ is a powerful methodology that can give a better understanding of the phenomena involved. UQ applied to a specific case is able to give an accurate and quantitative uncertainty of the outputs based on the input data provided. It was not the scope of this methodology to match the experimental data. Instead, as already explained, this is a powerful tool able to quantify the uncertainty of the output based on particular input uncertain data. It is clear how the selection of the input is of critical importance when approaching a UQ analysis. Results are based on input data; therefore, particular attention should be put in this preliminary phase. For this analysis results showed how an input variation as the one chosen can lead to an output variation of almost 10%, with equal probability. In terms of temperature, this could be a potential risk.

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