Abstract

[1] This is the second part of a series of papers called "HAG", and devoted to develop the foundations of *homotopical algebraic geometry*. We start by defining and studying generalizations of standard notions of linear algebra in an abstract monoidal model category, such as derivations, étale and smooth morphisms, flat and projective modules, etc. We then use our theory of stacks over model categories, introduced in [**HAGI**], in order to define a general notion of geometric stack over a base symmetric monoidal model category C, and prove that this notion satisfies the expected properties.

The rest of the paper consists in specializing C in order to give various examples of applications in several different contexts. First of all, when $C = k - Mod$ is the category of k-modules with the trivial model structure, we show that our notion gives back the algebraic n-stacks of C. Simpson. Then we set $C = sk - Mod$, the model category of simplicial k-modules, and obtain this way a notion of geometric D^-*-stack* which are the main geometric objects of *derived algebraic geometry*. We give several examples of derived version of classical moduli stacks, as the D^--stack of local systems on a space, the D^--stack of algebra structures over an operad, the D^--stack of flat bundles on a projective complex manifold, etc. We also present the cases where $\mathcal{C} = C(k)$ is the model category of unbounded complexes of k-modules, and $C = Sp^\Sigma$ the model category of symmetric spectra. In these two contexts we give some examples of geometric stacks such as the stack of associative dg-algebras, the stack of dg-categories, and a geometric stack constructed using topological modular forms.