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## **INFLUENCE OF “LONG-TERM TIME” EFFECTS ON SOIL STIFFNESS IN LOCAL SEISMIC RESPONSE EVALUATION**

**Claudia MADI<sup>1</sup>, Giacomo SIMON<sup>2</sup> and Giovanni VANNUCCHI<sup>3</sup>**

### **SUMMARY**

Dynamic shear modulus determined at low shearing strain amplitude by field testing methods,  $G_{0,field}$ , is generally higher than the value determined by laboratory testing methods on “undisturbed” soil specimens,  $G_{0,lab}$ . Moreover the shape of the modulus ratio,  $G/G_0$ , versus shearing strain curve,  $\gamma$ , in field differs from that determined by laboratory testing. The differences derive from many causes, including specimen disturbance, incorrect laboratory representation of grain-size distribution and field confinement, also for the same average confining pressure, and long-term time effects. Long-term time effects, both on the  $G_0$  value and on the shape of the  $G/G_0$  vs.  $\gamma$  curve can be evaluated by the Anderson and Stokoe II (1978) method. The method can be applied to evaluate the local seismic response of sites for which the results of dynamic laboratory tests from “undisturbed” soil specimens and a reliable geologic dating of the deposit are available, but  $V_s$  profiles have not been determined in situ.

The aim of the research is to check to what extent the difference between the values of the initial stiffness from laboratory and in situ testing can be predicted by applying the Anderson and Stokoe II method and analyse the influence of correction of the shear modulus curve on the results of local seismic response numerical analysis, depending on the shear strain and therefore on the design input motion. The study was performed on a site of Northern Italy where dynamic laboratory and in situ tests were performed to characterise soils and geologic age was known. The adopted methodology and the results are presented and discussed in this paper.

### **1. INTRODUCTION**

Generally, in local seismic response analyses the value of the shear modulus at shearing strains less than or equal to 0.001 percent (low-amplitude shear modulus) estimated from in situ tests (Down-Hole, Cross-Hole or SASW tests), and the reduction curve of the ratio ( $G/G_0$ ) determined from laboratory tests (resonant column or cyclic torsional shear tests) are assumed as design data. The difference between the low-amplitude shear moduli estimated from in situ and laboratory tests is due to several factors, the most important of which is the secondary consolidation (i.e. the “long-term time” effect).

The dynamic shear modulus and, to a lesser extent, the damping ratio are time-dependent soil properties. The shear modulus increases and the damping ratio decreases with time at constant effective stress during the secondary consolidation. These effects could play an important role on soil dynamic behaviour and they should be properly accounted when interpreting laboratory test results to determine in situ soil stiffness for practical engineering applications.

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Actually, as shown by Anderson and Stokoe II (1978), time also affects the reduction curve of the ratio ( $G/G_0$ ) vs. the shear strain amplitude. Neglecting this effect, an error depending on the shear strain amplitude level and therefore on the design seismic input is introduced.

The aim of this research is to analyse the correction ability of the Anderson and Stokoe II method on the value of  $G_0$  measured by laboratory testing, and the influence on the local seismic response of the corrected reduction curve of the ratio ( $G/G_0$ ) vs. the shear strain amplitude to take into account the “long-term time” effect.

The study was developed in an area of Northern Italy where eluvial-colluvium deposit of known age is present. An experimental testing program including Down-Hole tests and several laboratory tests on undisturbed samples was conducted to assess the static and dynamic properties of the soils.

In the paper the results of field and laboratory testing performed to characterise the site are described. 1-D local seismic response analyses were also presented by comparing the results obtained when the “long-term time” effects were taken into account and those estimated without considering them.

## 2. ASSESSMENT OF THE SHEAR MODULUS AND OF THE DAMPING RATIO FOR THE LOCAL SEISMIC RESPONSE ANALYSIS BY LABORATORY TESTING

### 2.1 Shear Modulus

The effective stress dependent response of low-amplitude shear modulus at the end of primary consolidation measured by resonant-column tests,  $G_{0lab}$ , can be expressed mathematically as:

$$\frac{G_{0lab}}{p_a} = K \cdot \left( \frac{\sigma'_0}{p_a} \right)^n \quad (1)$$

where  $p_a$  is the atmospheric pressure,  $\sigma'_0$  is the mean principal effective stress, and  $K$  and  $n$  are coefficients.

The time-dependent response of low amplitude shear modulus at constant confining stress is characterized by two phases: an initial phase which is due mainly to primary consolidation, and a second phase in which modulus increases almost linearly with the logarithm of time. The second phase is referred to as the “long-term time” effect, expressed in an absolute sense as a coefficient of shear modulus increasing with time,  $I_G$ :

$$I_G = \frac{\Delta G}{\log \left( \frac{t_2}{t_1} \right)} \quad (2)$$

where  $t_2$ ,  $t_1$  are times after primary consolidation, and  $\Delta G$  is change in low-amplitude shear modulus from  $t_1$  to  $t_2$ .

Shear modulus at low shear strain from field tests,  $G_{0field}$ , is generally higher than the value obtained from laboratory tests on “undisturbed” and representative soil specimens. By supposing that “long-term time” effect are the first cause, but not the only one, of the difference between the  $G_0$  values obtained from in situ and laboratory tests, the relationship between the in situ low-amplitude shear modulus,  $G_{0field}$ , and the low-amplitude shear modulus at the end of primary consolidation measured from resonant-column tests,  $G_{0lab}$ , can be expressed mathematically as:

$$G_{0field} = G_{0lab} + F_A \cdot I_G \quad (3)$$

where  $F_A$  is an age factor for site [Anderson and Stokoe II, 1978].

The age factor  $F_A$  can be estimated using the following relationship:

$$F_A = \log \left( \frac{t_c}{t_p} \right) \quad (4)$$

where

$t_c$  = time since start of most recent significant change in stress history at the site, and

$t_p$  = time to complete primary consolidation at site as a result of stress change.

Numerically,  $I_G$  equals the value of  $\Delta G$  for one logarithmic cycle of time. The long-term time effect is also expressed in relative terms by the normalized shear modulus increase with time,  $N_G$ , to remove some of influences of confining pressure.

$$N_G(\%) = \frac{I_G}{G_0} \cdot 100 \quad (5)$$

At higher strain levels, the effects of nonlinearity and inelasticity produce a reduction of the shear modulus which can be expressed mathematically as [Yokota et al., 1981]:

$$\frac{G_{lab}}{G_{0lab}} = \frac{1}{1 + \alpha \cdot \gamma^\beta} \quad (6)$$

where  $\alpha$  and  $\beta$  are coefficients.

This relationship does not take into account modulus variations due to effects such as specimen disturbance and incorrect laboratory representation of field confinement pressure.

Increase in shear modulus with duration of confinement also occurs at shearing strains from 0.001 to 0.1 percent. This increase in high-amplitude modulus is equal to or slightly less than that which occurs at low-amplitude shearing strains. Therefore the expected field modulus-strain curve would be represented mathematically by:

$$G_{field} = G_{lab} + A_r \quad (7)$$

in which

$$A_r = G_{0field} - G_{0lab} = F_A \cdot I_G \quad (8)$$

The concept of an arithmetic increase implies that the shape of the modulus ratio,  $G/G_0$ , versus the shearing strain curve is not unique but changes with time.

## 2.2 Damping ratio

The relationship between the shear modulus and the damping ratio is well fitted by the following equation [Yokota et al., 1981]:

$$D = D_{max} \cdot \exp\left(-\lambda \cdot \frac{G}{G_0}\right) \quad (9)$$

where  $D_{max}$  and  $\lambda$  are coefficients.

The damping ratio decreases almost linearly with increasing logarithm of time at constant effective stress during secondary compression. Marcuson e Whals (1978) showed that the damping ratio decreases approximately 12 percent for kaolinite ( $w_L = 66\%$  and  $I_p = 35\%$ ) and 25 percent for bentonite ( $w_L = 120\%$  and  $I_p = 60\%$ ) per logarithmic cycle of dimensionless time ratio,  $T_r$ , during secondary compression. The time ratio is defined as any time,  $t$ , divided by the time to 100 percent primary consolidation. The damping ratio decreases with increasing stiffness and increasing pressure.

As the low-amplitude damping ratio values measured from resonant column tests,  $D_{0lab}$ , are very uncertain because of several factors, and therefore the  $D_0$  decreasing rate with time from a linear regression  $D_0$  versus  $\log t$  is not very reliable, in analyses of the local seismic response we assume the relationship between shear modulus and damping ratio of Eq. (9). The coefficients  $D_{max}$  and  $\lambda$  were obtained from a regression between the  $D(\gamma)$

values measured from resonant column and cyclic torsional tests and the corresponding values of the in situ  $G/G_0(\gamma)$  ratio, estimated considering the long-term time effects.

### 3. SITE CHARACTERIZATION

#### 3.1 Localization and stratigraphy

The site investigated is located in North Italy, on the edge of the Brescia pre-Alps near Lake Iseo, which is in the bottom of a deep valley with glacial origins. The site, to the South of Lake Iseo, is deemed geologically representative of a vast area of Lombardy and its stratigraphic and geotechnical characteristics mean it is particularly exposed to local seismic amplification phenomena. In order to draw up regional guidelines for assessment of seismic amplification at the site, the Lombardy Regional Government promoted a research project which was coordinated by the Department of Structural Engineering of the Technical University of Milan. The site analysed in the present paper was included in the experimental program which comprised a detailed geological and geotechnical survey. The geotechnical investigations carried out on the site included a large amount of in situ and laboratory test both in static and dynamic conditions.

The soil profile consists of an eluvial-colluvium deposit, composed prevalently by layers of stiff weakly sandy clayey silt and silty clay to a depth of 24.2m, laying on a marly-calcareous bedrock. A layer of gravel in clayey-silty matrix extends from a depth of 1.5m to 3.0m and a layer of gravel with sandy silty clay and angular marly cobbles was encountered from 23.3m to 24.2m (Figure 1). The water table was not encountered. From available geological information, it is gathered that the date of the last significant event in the formation of this deposit dates back to between 27000 and 15000 years ago.

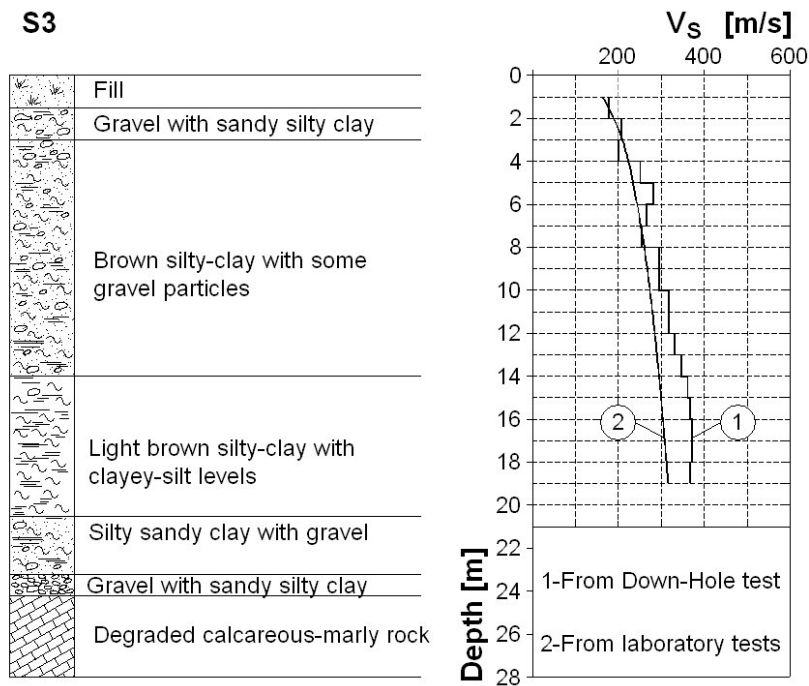


Figure 1: Soil profile from borehole S3 and shear waves velocity profile

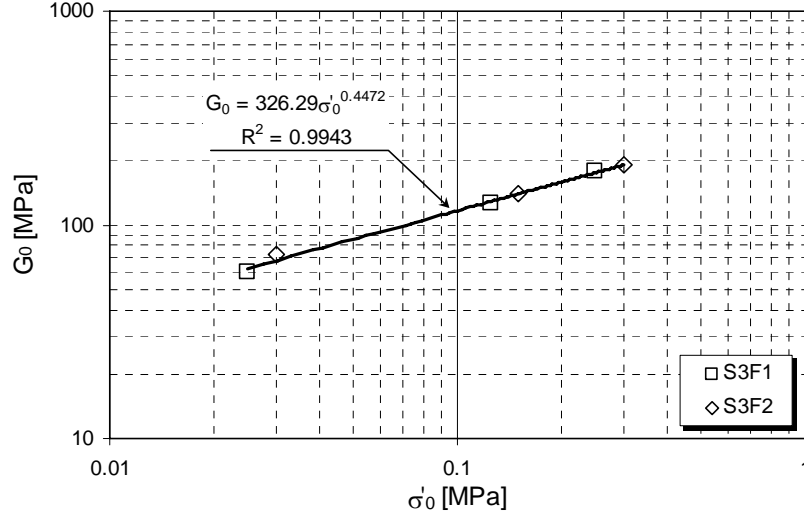
#### 3.2 Geotechnical properties for local seismic response evaluation

The geotechnical properties of the subsoil, which can be considered homogeneous in order to evaluate the local seismic response, are as follows: Gravel content = 8-17%, Fine content = 72-75%,  $\gamma = 20 \text{ kN/m}^3$ ,  $w_L = 40\%-49\%$ ,  $w_p = 16\%-19\%$ ,  $w = 20\%-22\%$ ,  $I_c = 0.8-0.9$ ,  $e_0 = 0.54-0.56$ ,  $K_0 = 0.54-0.57$ ,  $C_c = 0.147-0.153$ ,  $C_s = 0.021-0.022$ ,  $C_\alpha = 0.0008$ ,  $c_v = 9 \cdot 10^{-7} - 2 \cdot 10^{-7} \text{ m}^2/\text{sec}$ .

To assess dynamic properties (shear modulus and damping ratio versus shear strain), multistage tests with measurement of small-strain shear modulus,  $G_0$ , and damping ratio,  $D_0$ , were performed on two samples using a

Resonant Column (RC) and Cycling Torsional Shear (CTS) apparatus. The diameter of the laboratory specimens was 38.1 mm with a height of 76.2 mm, therefore they were composed of fine fraction only.

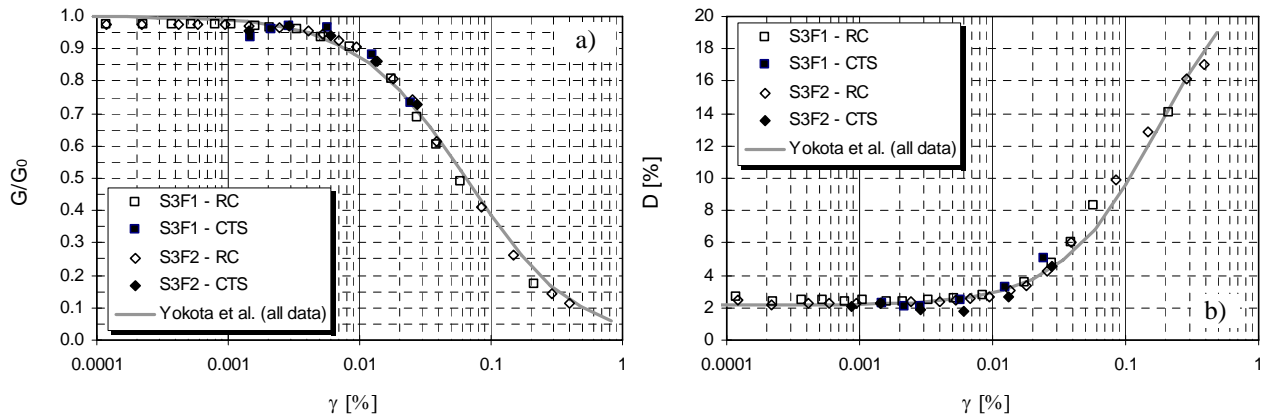
The confining effective stress dependent response of low-amplitude shear modulus at the end of primary consolidation obtained for the two samples is shown in Figure 2. The values of eq. (1) coefficients result:  $K = 326.29$  and  $n = 0.4472$ .



**Figure 2: Experimental values of  $G_0$  versus isotropic confining pressure from resonant column and cyclic torsional and calculated regression**

The experimental results of normalised shear modulus,  $G/G_0$  versus shear strain,  $\gamma$ , obtained for the two samples are plotted in Figure 3a. The model proposed by Yokota et al. (1981) (eq. (6)) was fitted to the experimental data and the values of  $\alpha$  and  $\beta$ , obtained for both the samples together are:  $\alpha = 18.639$  and  $\beta = 1.064$ . The best fit curve to the experimental data is also shown in Figure 3a.

The experimental values of the damping ratio,  $D$ , versus shear strain,  $\gamma$ , were determined using measurements of hysteretic loop area in CTS tests and by means of 'Amplitude Decay Method' in RC tests. The results for the two samples are plotted in Figure 3b. In the same Figure 3b the curve of eq. (9) fitting the experimental data is shown. The values of  $D_{max}$  and  $\lambda$  obtained for both the samples together are:  $D_{max} = 24.398$  and  $\lambda = 2.430$ . These relationships does not take into account modulus variations due to effects of the disturbance of specimens, of incorrect laboratory representation of grain-size distribution, of time and of the actual field stress conditions.



**Figure 3: Experimental data and regression curve of  $G/G_0$  ratio versus shear strain (a) and regression curve of damping ratio versus shear strain (b)**

Time effects on the small-strain shear modulus,  $G_0$ , were evaluated by means of the coefficients  $I_G$  and  $N_G$ . The experimental values of  $N_G$  range between 4.99 and 3.88 % in middle of the typical range suggested by Anderson and Stokoe II (1978) for clayey silt ( $1 < N_G < 14\%$ ). The mean value  $N_G = 4.43\%$  was assumed as design value.

To determine in situ shear waves velocity, a Down-Hole test was performed only to a depth of 20 m, by using an apparatus with two 3-D receivers and taking measurements of wave velocity at 1m intervals. The shear wave velocity profile obtained by means of the interval method is shown in Figure 1. As can be seen,  $V_S$  profile shows two parts: from 1m to 14m above g.l. increases almost linearly with depth, from  $V_S = 176$  m/sec to  $V_S = 345$  m/sec (regression line is:  $V_S = 12.8 z + 175.2$  e  $R^2 = 0.90$ ), below 14m and to 19m from g.l. it remains almost constant, with a mean value of 366 m/sec and a standard deviation of 3.7 m/sec.

#### 4. COMPARISON BETWEEN ESTIMATED AND MEASURED PROFILES OF SHEAR WAVE VELOCITY

The value of the age factor for site,  $F_A$ , estimated by means of eq. (4) from the information about the deposit age  $t_c = 27,000$ -15,000 years (time since start of most recent significant change in stress history at the site), and the time to complete primary consolidation at site as a result of stress change,  $t_p = 50$  years, results between 2.5 and 2.7. The mean value,  $F_A = 2.6$ , was assumed as design value.

The estimate of the value of  $F_A$  with eq. (4) is very uncertain, because the estimates of both  $t_c$  and of  $t_p$  are very uncertain. However, an error in the estimating  $F_A$  over the term  $A_r = \Delta G$ , would not have a serious effect.

Therefore the difference between  $G_{lab}$  and  $G_{field}$  due to the long-term effect, from eqs. (1), (4), (5) and (8) results:

$$A_r = G_{field} - G_{lab} = F_A \cdot \frac{N_G}{100} \cdot G_{0lab} = 37.58 \cdot p_a \cdot \left( \frac{\sigma_0'}{p_a} \right)^{0.4472} \quad (8)$$

Finally, following the Anderson and Stokoe method, if shear modulus profile have not been determined in situ, the best estimate of the  $V_S$  profile from laboratory testing is presented in Figure 1. As can be seen,  $V_S$  profile estimated by means of the results of the laboratory tests taking into account the long-term time effects and  $V_S$  profile determined from Down-Hole test are match quite well. This is a better than good result, considering that: a) the gravel content increases in depth and produces an increase in soil stiffness, which can be noticed in  $V_S$  profile determined by the Down-Hole test but inevitably disregarded in  $V_S$  profile inferred from the results of laboratory tests; b) time  $t_c$ , since start of most recent significant change in stress history at the site, is not an only value but increase with depth; and finally c) to assess  $V_S$  profile from laboratory tests, only the results from two samples were available.

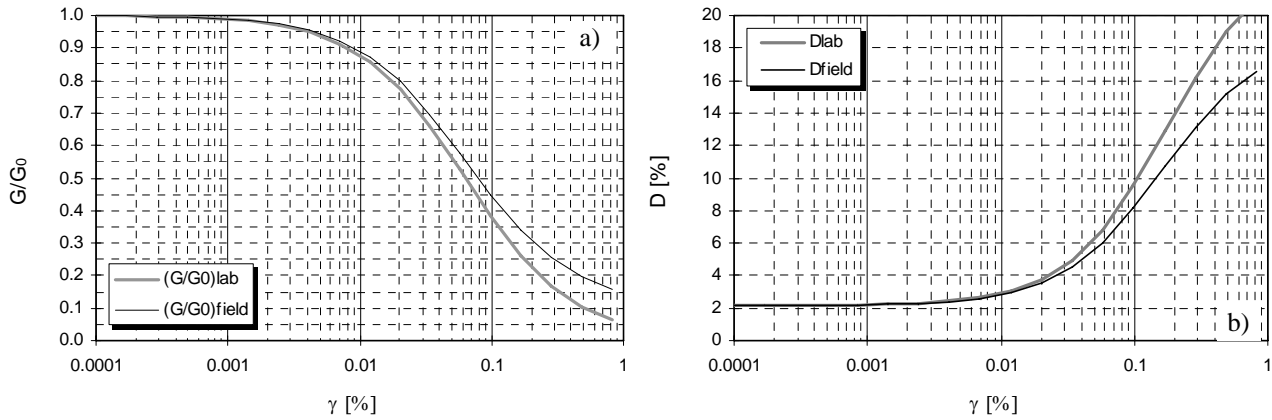
While in situ testing to determine small-strain shear modulus,  $G_0$  is always to be recommended, since the profile from in situ test are not always reliable because they depend a great deal on the uncertainties in measurements and interpretation methods and on the experience and technical ability of the company carrying out the work, comparison with the profile determined by means of the Anderson and Stokoe II method from laboratory test results is in any case useful.

#### 5. LOCAL SEISMIC RESPONSE ANALYSES

With the aim of analysing the influence of “long-term time” effects on seismic response, 1-D local seismic response analyses were performed on the previously described site, using the computer program PROSHAKE, a version for Windows of SHAKE [Schnabel et al., 1972].

Two different analyses were performed. In the first, the low amplitude shear modulus profile deduced from  $V_S$  in situ measurements and the normalized shear modulus curve  $G(\gamma)/G_0$  from the results of the laboratory tests were assumed. In the second analysis, low-strain shear modulus profile and normalized shear modulus curve  $G(\gamma)/G_0$  assumed in the numerical modeling were deduced from the results of the laboratory tests taking into account long-term time effects. It can be observed that an only  $G(\gamma)/G_0$  curve was adopted since it was supposed  $F_A \cdot N_G$  independent from the mean effective confining pressure and consequently independent from depth.

In Figure 4 the curve  $G(\gamma)/G_0$  from laboratory tests and the curve  $G(\gamma)/G_0$  obtained for the site taking into account long-term time effects are represented. It can be observed that the two curves are markedly different only for high shear strain levels; therefore the difference in local seismic response obtained when long-term time effects are taken into account or ignored, can be relevant only for strong seismic input motion.



**Figure 4: Comparison between  $G/G_0$  curves (a) and damping ratio curves (b) without and with the “long-term time” effects**

The following analyses were carried out to verify and quantify the aforesaid hypothesis. In this light, the analyses of the local seismic response at the examined site were performed both with the conventional procedure and considering the long-term time effects by assuming two different input motions.

### 5.1 Input data

Stratigraphic data and geotechnical properties and parameters for the site are taken from the results of in situ and laboratory tests, according to the criteria described in the previous paragraphs. Bedrock was imposed at a depth of 24.2m from g.l. with a shear wave velocity  $V_s=1000\text{m/s}$ ; for modulus reduction and damping were assumed the curves for rock included in sample data files for the original SHAKE program [Schnabel et al., 1970].

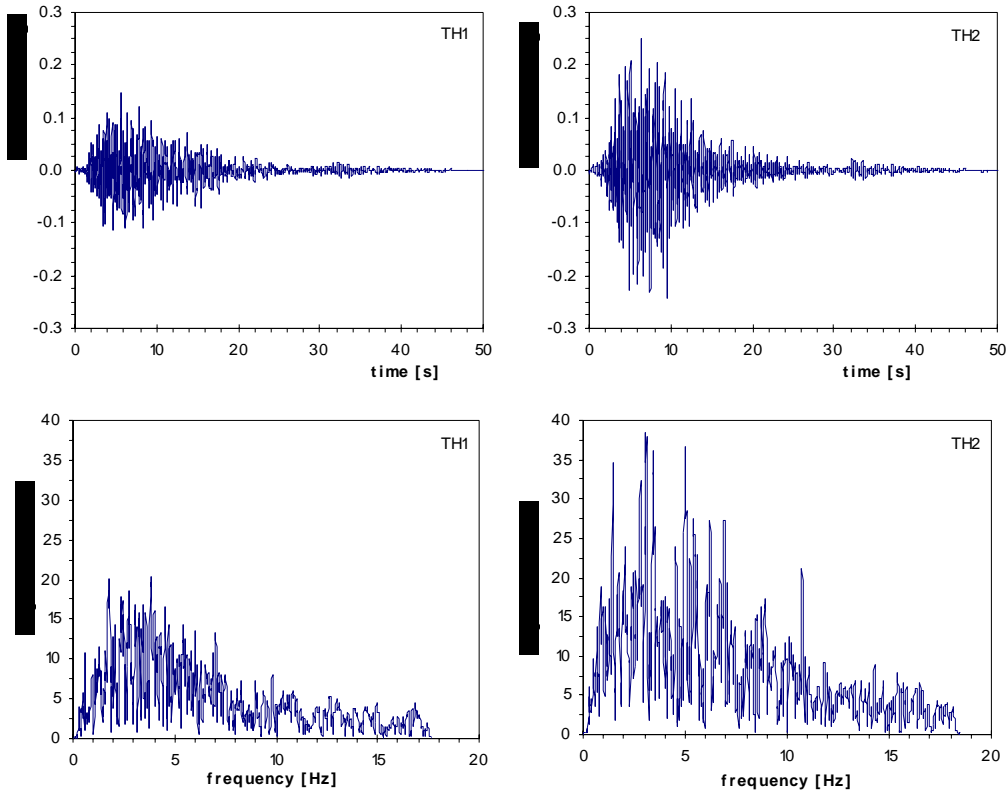
Two seismic input were assumed in the analyses. As no accelerometric recording exist for the examined site, two artificial accelerograms representative of the earthquakes expected in the area with a return period of 475 and 974 years, and local Magnitude 5.5 and 6 respectively, were adopted [Pergalani, 2005]. They were generated with a stochastic-probabilistic method, which was considered the most appropriate in defining input seismic motion acceleration for seismic microzonation [Marcellini et al., 2001].

The acceleration time histories, named TH1 and TH2 respectively, were scaled to 0.15g and 0.25g respectively and were assumed as input motions on outcropping rock. Table 1 summarized the main parameters of the two input motions; the corresponding acceleration time histories and Fourier spectra are represented in Figure 5.

**Table 1: Main parameters of the two input motions assumed in the local seismic response analyses**

	TH1	TH2
PGA [g]	0.15	0.25
Arias intensity [cm/s]	38.28	114.70
Predominant period [sec]	0.26	0.33
Bracketed duration [sec]	14.25	15.12
Trifunac duration [sec]	13.99	11.02





**Figure 5: Acceleration time history and Fourier spectra of the input motions**

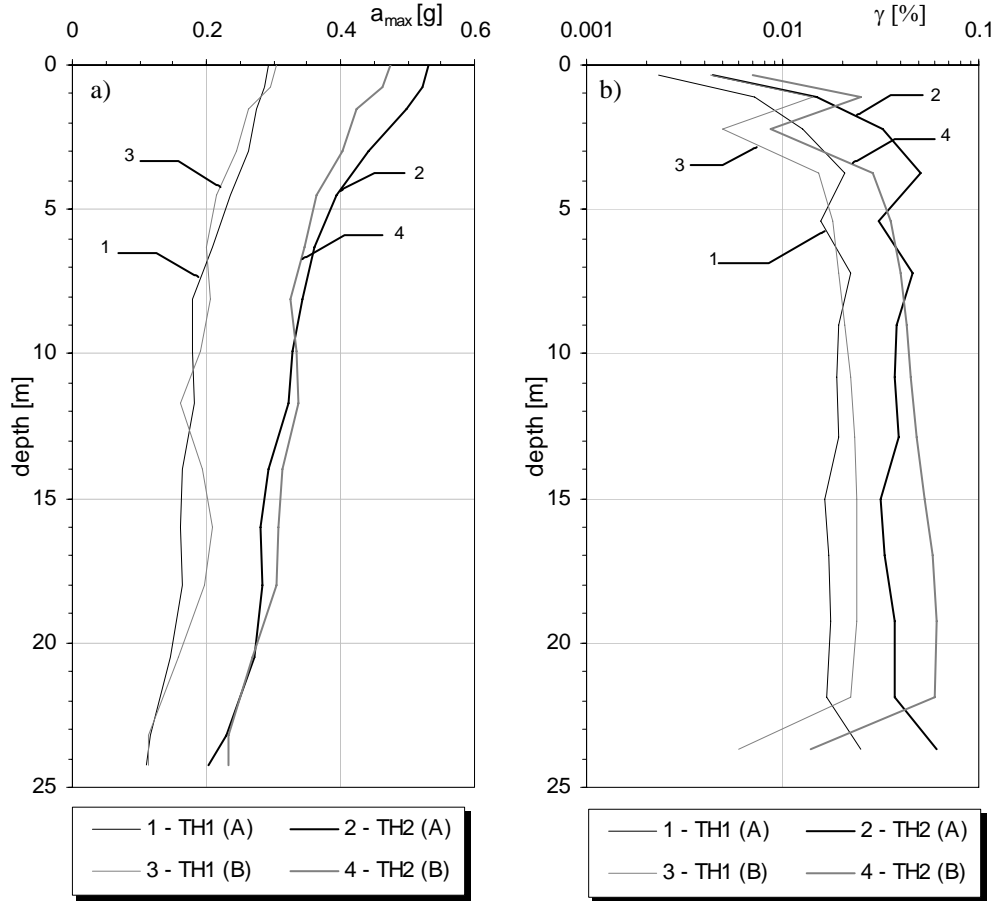
## 5.2 Results

The long-term time effects modify the shape of the  $G/G_0$  curve measured by laboratory testing, and these modifications are very dependent on the seismic input. For the site examined, the design earthquake having a return period of 475 years produces low-amplitude shear strains in the sub-soil, and the results of the local seismic response analysis, for the same  $V_S$  profile, are not influenced by the correction of the  $G/G_0$  curve.

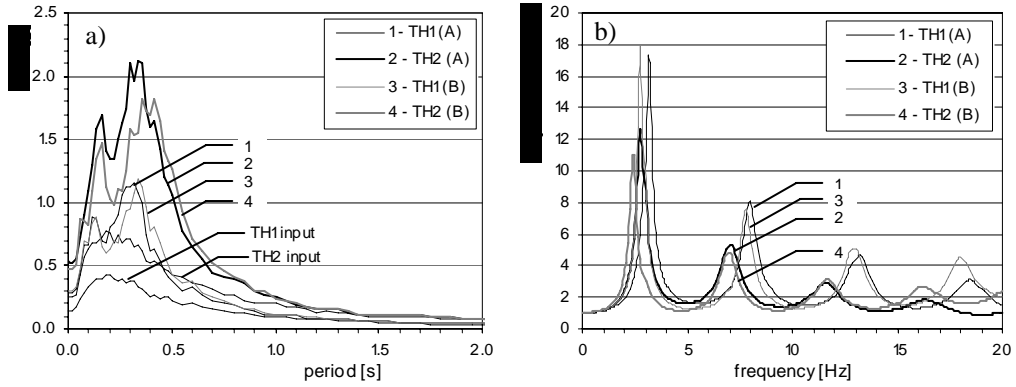
Therefore, for both the design earthquakes TH1 and TH2, analyses for the two following cases have been carried out: A)  $V_S$  profile measured by field testing and  $G/G_0$  curve not corrected and B)  $V_S$  profile estimated by laboratory testing and  $G/G_0$  curve corrected for the long-term time effects.

The results of the 1-D local seismic response analyses were examined in time and frequencies domain and are summarized in Figures 6 and 7. For the design earthquake having a return period of 974 years a clear difference in terms of maximum shear strain profile can be observed (curves TH2-A and TH2-B of Figure 6b), with a much less marked difference in terms of maximum acceleration profile (curves TH2-A and TH2-B of Figure 6a). Pseudo-acceleration response spectra for 5% of damping are presented in Figure 7a. As can be observed the differences in spectra acceleration for both the designed input motions and both the considered  $V_S$  profiles (cases A and B) are negligible.

Spectral amplification functions between soil surface and bedrock are shown in Figure 7b. It can be noted that for the stronger design earthquake (i.e. for higher shear strain amplitude) the natural frequencies of the deposits move to smaller values and amplification factor decreases.



**Figure 6: Peak ground acceleration (a) and effective shear strain (b) profiles obtained from local seismic response analyses**



**Figure 7: Pseudo-acceleration response spectra for 5% damping (a) and spectral amplification function (b)**

## 6. CONCLUSIONS

The uncertainties of the results of the local seismic response analyses can be ascribed to three different sources of error: 1. the seismic input, 2. the geotechnical input, and 3. the method of the analysis. In this paper, we try to quantify the effect of the two first sources of error for a specified site.

When carrying out local seismic response analyses, and in general for every geotechnical problem, in situ testing and laboratory testing are both necessary and complementary for defining the geotechnical input. In particular, the stratigraphy and the  $V_s$  profile by in situ testing and the shape of the  $G/G_0$  curve by laboratory testing on

undisturbed specimens must be determined. The uncertainties are caused by sometimes dubious reliability of in situ and laboratory testing results, due to the inherent variability of soil properties, to insufficient sample size, to the impossibility of reproducing the field conditions in the laboratory, to human errors in formulating the geotechnical model and in interpreting the experimental results.

Some geotechnical data can be estimated or measured by in situ testing only (e.g. the stratigraphy), or by laboratory testing only (e.g. the shape of the  $G/G_0$  curve), while other geotechnical data can be estimated or measured by means of both testing methods (e.g. the  $G_0$  profile). In this latter case generally the results obtained by field testing are more precise but less accurate than results obtained by laboratory testing. It is therefore advisable to compare geotechnical input data ( $G_0$  profiles) estimated by different methods to use in the analysis the more precise profile from field testing, after having checked that it is also sufficiently accurate.

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