Abstract

This is the first of a series of papers devoted to lay the foundations of Algebraic Geometry in homotopical and higher categorical contexts. In this first part we investigate a notion of *higher topos*.

For this, we use S-categories (i.e. simplicially enriched categories) as models for certain kind of ∞-categories, and we develop the notions of *S-topologies*, *S-sites* and *stacks* over them. We prove in particular, that for an S-category T endowed with an S-topology, there exists a model category of stacks over T, generalizing the model category structure on simplicial presheaves over a Grothendieck site of Joyal and Jardine. We also prove some analogs of the relations between topologies and localizing subcategories of the categories of presheaves, by proving that there exists a one-to-one correspondence between S-topologies on an S-category T, and certain *left exact Bousfield localizations* of the model category of pre-stacks on T. Based on the above results, we study the notion of *model topos* introduced by Rezk, and we relate it to our model categories of stacks over S-sites.

In the second part of the paper, we present a parallel theory where S-categories, S-topologies and S-sites are replaced by *model categories*, *model topologies* and *model sites*. We prove that a canonical way to pass from the theory of stacks over model sites to the theory of stacks over S-sites is provided by the simplicial localization construction of Dwyer and Kan. As an

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example of application, we propose a definition of *étale K-theory of ring spectra*, extending the étale K-theory of commutative rings.