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From Hidden Symmetry to Extra Dimensions:
5-dimensional models of electroweak symmetry breaking with brane-bulk interplay

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Alla mia famiglia e ad i miei amici (in particolare agli amici della "festa del 26", che sono la mia seconda casa), senza i quali niente di tutto questo avrebbe un senso. E a Silvia, che mi ha supportato e sopportato durante la stesura di questo lavoro.

Preface

All religions, arts and sciences are branches of the same tree. All these aspirations are directed toward ennobling man's life, lifting it from the sphere of mere physical existence and leading the individual towards freedom.

- A. Einstein

Particle physics is passing today through a peculiar stage in its development. A single theory is capable of explaining essentially the whole body of experimental results: it is the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge theory known as the "Standard Model" (SM), which is the fusion of the Glashow-Weinberg-Salam (GWS) theory [1, 2, 3] of Electroweak (EW) interactions and of Quantum Chromodynamics (QCD) [4, 5, 6, 7], the theory of strong interactions. However, the SM is not completely satisfactory from the theoretical point of view, for at least two distinct sets of reasons.

First, several of its ingredients are determined in a purely phenomenological way; for instance, while we have fundamental reasons to believe that subnuclear particles and their interactions should be well-described by a gauge theory, most other details of the SM are only fixed by experimental evidence, such as the choice of $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ as the correct gauge group; how many and which kind of matter (fermion) fields one has to include; the values of the particle masses; and so on.

In the second place, there is a list of fundamental questions that the SM leaves unanswered. Among these, three seems to me to have a particular significance:

- the theory does not include - at any level - the gravitational interaction, while a really complete, unified theory of the basic constituents of matter must of course describe gravity with the other three known fundamental forces. This point is however very difficult to address, due to the extreme weakness of the gravitational force in comparison to the other three interactions; this fact has so far prevented physicists from getting any experimental information about the behaviour of gravity at very short distances - that is, in the quantum regime. In fact, the standard naive estimate for the typical energy scale of quantum gravity yields an extremely high result, of the order of the Planck scale $M_P \simeq 10^{19}$ GeV, so high an energy that it seems unlikely to ever be explored in controlled experimental conditions;

- the mechanism that leads to the breaking of the EW symmetry at low energies is still not clearly understood; in the SM the symmetry breaking is mediated by a complex $SU(2)$ doublet of scalar particles, acquiring a VEV through an *ad hoc* chosen potential. This set-up requires the presence of a massive, neutral spin-0 excitation, in the physical spectrum, the *Higgs boson*. The existence of this particle is still unconfirmed by experiments; furthermore, fundamental scalars such as the Higgs are subject to the fine-tuning problem called the *hierarchy problem*, because their masses are unstable against radiative corrections:

- it was for some time believed that combined charge conjugation and parity (CP) invariance was a fundamental symmetry property of nature. Even though it is now clear from experiments that it is not the case, CP violation is only detected in the weak interactions. The absence of CP-violating terms in the strong sector is an experimental certainty, but it is not explained by any fundamental principle.

Every one of these issues - the first two in particular - has stimulated an enormous amount of work, giving birth to many hypothesis, new models and theoretical tools, all trying to help solving one or more of these puzzles. A huge variety of possible new phenomena was predicted, usually in the form of new particles and interactions; but experiments so far have given almost no hint about the presence of any new physics. On the contrary experimental data, at least in direct observations, have only been confirming the validity of the standard theory, with the only challenges to the SM predictions coming from indirect sources: astrophysics (there is by now almost universal agreement that the Universe should contain a huge amount of non-baryonic, non-relativistic dark matter, for which the SM provides no good candidate) and a number of experiments on neutrino oscillations, which proved that neutrinos cannot be massless (as it is by contrast hypothesized in the SM).

Collider experiments are about the only way we have for directly testing proposal for physics Beyond the Standard Model (BSM); so far, the energy region up to ~ 200 GeV has been explored. Reaching higher energies is very challenging from both a technical and an economical point of view; however, after a decade-long wait, the Large Hadron Collider (LHC) is going to begin operations, allowing us to reach up to and beyond the TeV region and, hopefully, to put to a test the many theories that have been proposed in the years and finally decide which one (if any) is the correct extension of the SM.

In the present work, I am going to focalize on the second one of the theoretical puzzles outlined above, the mystery of the electroweak breaking sector. In particular, I am going to explore in some detail one of its possible solutions, namely the existence of extra dimensions of space-time. I will now make a brief general introduction to the topics of EW symmetry breaking and extra dimensions, then give a more specific outline of the contents of the main body of this work.

Electroweak symmetry breaking

The unification of the electromagnetic and weak interactions in a single EW interaction by the GWS theory is one of the main elements of the SM. The GWS theory is a *spontaneously broken* non-abelian $SU(2)_L \otimes U(1)_Y$ gauge theory. A "spontaneous" breaking of symmetry - a phenomenon appearing in many different sectors of physics - occurs when the equations of motion are invariant under a symmetry, but some of their solutions are not. In the context of quantum field theory, this means, more specifically, that the vacuum state is not invariant under part of the symmetry group of the Lagrangian; from a phenomenological point of view, the symmetry is manifest in the interactions but not in the particle spectrum. In the SM theory, three linear combinations - the $W^{\pm}$ and Z bosons - of the four gauge fields associated with $SU(2)_L \otimes U(1)_Y$ symmetry get masses roughly of order $v \simeq 250$ GeV (which is referred to as the "EW breaking scale") - while a fourth, the photon, remains massless. This asymmetry in the spectrum leads to the completely different low-energy phenomenology observed in the two sectors of the theory (namely the electromagnetic and weak interactions), while at high energy the $SU(2)_L \otimes U(1)_Y$ symmetry is manifest.

The existence of the symmetry breaking is an experimental certainty, but the nature of the mechanism that brings it about is not. As I mentioned before, in the SM the breaking is realized through a complex $SU(2)$ scalar doublet; the doublet gets a nonzero VEV thanks to an *ad hoc* potential and gives masses to three out of four of the $SU(2)_L \otimes U(1)_Y$ gauge bosons via the Higgs mechanism. As a result, only one scalar degree of freedom remains in the theory, the Higgs boson. The main virtue of this explanation lies in its economy and simplicity: only one additional - and as yet unseen - particle is needed to obtain a consistent, calculable (at the perturbative level) and renormalizable theory. By contrast, consider the only known more economical alternative, the so-called EW chiral Lagrangian [8, 9]: this theory contains three scalars that transform nonlinearly under $SU(2)_L \otimes U(1)_Y$, and all of them are absorbed by gauge bosons to make them massive, so that there is no physical scalar degree of freedom left in the spectrum; however, the model is not renormalizable and this leads to a violation of perturbative unitarity - and consequently to a loss of validity of the description - around 1.7 TeV [10, 11].

Despite its simplicity and efficiency, the standard Higgs model does have some serious drawbacks, however. First of all, the mass of the Higgs boson is severely

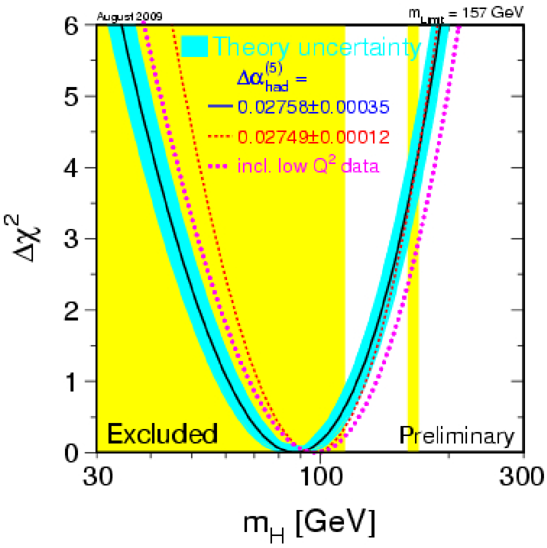


Figure 1: *The famous "Blueband Plot" [12] showing the constraints on the Higgs mass from a global fit of the SM parameters. The yellow region is excluded by direct experimental searches.*

constrained. A global fit of high-energy data would tend to favour a light Higgs, with mass $\lesssim$ 100 GeV (in the sense that the minimum of χ^2 for the global fit is below 100 GeV), but this is excluded by direct observation (see fig. 1). Also, there are strong theoretical motivations to keep the Higgs mass not too heavy; in particular, the Higgs mass must be less then about one TeV to avoid the onset of a strong interaction regime at that scale, resulting in a loss of calculability.

Another serious issue is the already mentioned hierarchy problem. Scalar particles, as the Higgs, are unstable against radiative corrections: their mass parameters are corrected by divergent terms which are quadratic in the cut-off. This is of no consequence if the cut-off is interpreted as a mathematical regulator with no physical meaning (this is possible in the SM since it is a renormalizable theory, potentially valid up to infinite energy). However, there is a very compelling argument in favour of the SM having a *physical* cut-off: as I already said, the model does not describe gravity; while this is not a problem at ordinary energies, since gravitational interactions are then completely negligible at the particle level, we know that gravity grows stronger with energy, eventually reaching a point where it can no longer be ignored. The conservative estimate for this to happen is around the *Planck mass*, $M_P \simeq 10^{19}$ GeV, the energy at which the classically-calculated Schwarzchild radius becomes comparable with the Compton wavelength of a particle. So, M_P acts as the "highest possible" effective cut-off for the SM. If one accepts the cut-off - whether it is really equal to M_P or not - to be a physical quantity, one can take the Wilsonian point of view and look at the renormalization of the Higgs boson mass parameter as the running from the "bare" value at the cut-off scale to the physical one at ordinary energies; it then seems very unnatural, in the sense that it requires an extremely fine tuning of the parameters, to get a mass of order 100 GeV from the cancellation of contributions at a much higher scale. This problem is not present for other particles in the theory: their mass parameters are symmetry-protected by gauge invariance, and only receive corrections which are logarithmic in the cut-off, and so are under control even when the cut-off is several orders of magnitude greater than the corresponding masses.

A simpler way of stating this problem is the metaphor of the "desert" in the energy landscape: it seems strange to have so much richness and diversity of phenomena appearing at various scales at low energy (think for instance about atomic interactions at the eV scale, nuclear at the MeV, QCD at the GeV, the spectrum of elementary particles extending up to around 100 GeV), and no new physics from the EW scale up to M_P, about 17 orders of magnitude further!

The attempts to avoid the hierarchy problem lead to many interesting possibilities. Among them, two general frameworks are particularly attractive. One is supersymmetry. In supersymmetric theories, there are the same number of fermionic and bosonic degrees of freedom, whose loops contribute to radiative corrections with opposite signs, leading to many cancellations and to the vanishing of quadratic divergences in mass corrections of the scalar fields. In other words, supersymmetry naturally protects the mass of the scalars in a similar way that gauge invariance protects that of the vectors in the SM.

The other interesting possibility is that the EW symmetry breaking is dynamical, driven by the presence of new strong interactions around the TeV scale. The precursor of this kind of theories is the so-called Technicolor (TC), which in its simplest, original form [13, 14, 15] postulates the presence of a QCD-like strongly interacting sector rescaled at a higher energy. The original proposal has been abandoned because it conflicts with electroweak precision tests; however, it represents only one possibility among an infinity of models sharing the same basic premise of strong interactions at a high energy scale, but with different implementations. In this kind of theories the Higgs may be absent or present as a composite state; in the latter case, the natural scale of its mass is set by the strong interactions and not by the UV cut-off of the theory. The main drawback of this kind of theories is the difficulty of executing calculations in a strong interaction regime, since most of the tools at our disposal only work in perturbation theory.

Extra Dimensions

An alternative to the above scenarios was proposed in the late '90s in the groundbreaking article by Arkani-Hamed, Dimopoulos and Dvali (ADD) [16]: the hierarchy problem may be solved if there is *no hierarchy at all*, and the "real" scale of quantum gravity is around one TeV, with M_P being only an "apparent" scale. This is possible if space-time is not really - as it appears to our observation - four dimensional, but rather has one or more extra spatial dimensions. The additional dimensions have to be compact to avoid deviation from Newton's law at macroscopic distances (the most recent experimental tests of the validity of Newton's law are detailed in [17]). The original set-up by ADD postulated a factorizable space-time geometry of the kind $\mathbb{R}^4 \times C$, C being a generic compact space. If the space C is n-dimensional with an average compactification radius of order R, the classical potential between two test masses m_1 and m_2 at a relative distance r is given, in the limiting cases $r \ll R$ and $r \gg R$, by:

$$\begin{aligned} V(r) &\sim \frac{m_1 m_2}{(M_P^*)^{n+2} r^{n+1}}, \quad r \ll R, \\ V(r) &\sim \frac{m_1 m_2}{(M_P^*)^{n+2} R^n r}, \quad r \gg R, \end{aligned} \tag{1}$$

as the particle feels the space-time to be effectively $(4+n)$-dimensional or 4-dimensional in the two cases. Here M_P^* is the Planck mass in the complete extended $(4+n)$-dimensional space-time. Matching the two expressions, it is evident that to an effectively 4-dimensional observator testing distances $r \gg R$, the Planck scale will appear to be

$$M_P^2 \simeq (M_P^*)^{n+2} V, \tag{2}$$

where $V \sim R^n$ is the volume of the compact space C. So, if V is large enough the real Planck scale M_P^* could be as low as one TeV, *de facto* eliminating the hierarchy between the EW and the gravity scale. If it were so, we would be testing quantum gravity effects already at LHC energies! It is easy to calculate the size of the extra dimensions as a function of the dimensionality of C, asking $M_P^* \sim 1$ TeV:

$$R \sim 10^{\frac{32}{n}-3} GeV^{-1} \sim 10^{\frac{32}{n}-19} m. \tag{3}$$

It is evident that the case $n = 1$ is already excluded by experiments, requiring a macroscopic extra dimension with radius $\sim 10^{13}$ m. However, for $n \geqslant 2$, I get $R \lesssim 10^{-3}$ m, which is at the limit of experimental precision; recent precision experiments [17] have found an upper limit $R < 44 \mu m$ at 95 % confidence level (effectively setting a lower limit on M_P^*: $M_P^* > 3.2$ TeV if $n = 2$). The SM fields, however, cannot freely propagate in extra dimensions of this size: their presence would have already been observed at collider experiments, which have probed far smaller distances. So it is necessary to additionally postulate some mechanism that localizes the non-gravitational fields to a 4-dimensional submanifold of space-time. In fact, ADD is really a class of models rather than a specific theory, since some of its predictions depend on the specific choice of this mechanism. Anyway, the basic difference in the behaviour of the fields (the SM fields stuck to a 4D submanifold but gravity propagating in the bulk of the extra-dimensional space-time) characteristic of the ADD scenario helps understanding intuitively the apparent weakness of gravity with respect to the other interactions: the strength of gravity is "diluted" by the fact that it is the only force free to propagate in the extra n dimensions.

One could argue that the ADD scenario does not really solve the hierarchy problem, but rather replaces it with a different question: admitting there are n extra compact dimensions, if the fundamental scale of gravity is M_P^*, one would naively expect them to have a size of order $(M_P^*)^{-1}$. But in the case of small n, the actual size needs to be several orders of magnitude bigger (if $M_P^* \sim 10^3$ GeV) to reproduce the apparent 4D value of the Planck mass M_P. What is the mechanism that stabilizes such an unnaturally large compactification scale?

In 1999, Randall and Sundrum (RS) [18] proposed an alternative scenario still based on extra dimensions that helped to address this issue. The model consists of a 5D theory with a non-factorizable metric of the form

$$ds^2 = e^{a(y)} dx^\mu dx^\nu - dy^2, \tag{4}$$

where y is the extra coordinate and $a(y) = -2ky$, with k the curvature parameter; y takes values on the interval $(0, L)$. Note that a metric of the form (4) with arbitrary a is the most general one can have in 5 dimensions while still retaining Lorentz invariance on the 4 noncompact directions. Space-time in the RS set-up is a slice of an AdS_5 space; the exponential warp factor triggers a red-shift along the 5^{th} dimension, effectively multiplying every parameter in the theory by a factor e^{-dky}, where $[Energy]^d$ is the parameter dimension in natural units ($\hbar = c = 1$), when considering phenomena at a given extra-dimensional coordinate y. Suppose then that only one fundamental mass scale exists, M_P, with the curvature parameter $k \sim M_P$. If the SM fields are localized near or at the $y = L$ border (also called the "IR brane" as opposed to the "UV brane" located at $y = 0$) of the AdS space, they would feel an effective cut-off scale $\tilde{M}_P = M_P\, e^{-2kL}$. Thanks to the exponential, it is not necessary to have a huge hierarchy between k and L^{-1} to bring $\tilde{M}_P$ down to ~ 1 TeV. Actually it is not necessary to confine all the SM fields near the IR brane, but just the Higgs.

RS models have inspired a huge literature; many alternative theories, all based on the assumption of a single warped extra dimension, have been proposed. To provide just a few notable examples, it is possible to use boundary conditions (BCs) in the extra dimension to break EW symmetry without a Higgs field [19, 20] or with a composite Higgs [21, 22], or to protect the Higgs mass by realizing the scalar field as a pseudo-goldstone boson of an approximate global symmetry [23, 24]. Furthermore, the RS set-up is useful to address not only the hierarchy problem, but other questions as well; most notably, the warp factor can help to explain the *flavour puzzle*, the fact that fermion masses span many orders of magnitude (equivalently, the fact that the Yukawa couplings in the symmetry breaking sector are all much less than unity), by localizing fermion fields in different points of the bulk of the extra dimension [25].

Finally, warped models have received special attention in view of the AdS/CFT correspondence [26]. According to this conjecture, 5D models on (a slice of) AdS space are "holographic duals" to 4D theories with (spontaneously broken) conformal invariance. The duality is strong/weak, so that when the 5D theory is in a perturbative regime the holographic dual is strongly interacting and vice versa. This fact provides an unique tool to make quantitative calculations in 4D strongly interacting theories, and creates a very interesting connection between extra-dimensional and TC-like theories.

Outline of the work

In the following chapters, I will investigate different topics in the context of alternative mechanisms of EW symmetry breaking in theories with a single extra dimension. As a first thing, in chapter 1 I will review the D-BESS model [27], an effective model of a TC-like strongly interacting sector. This model has the peculiar feature of having a suppressed contribution from the new physics to the EW precision observables (parametrized for instance through the ϵ parameters [28, 29, 30]), allowing for the presence of new vector resonance at a relatively low energy scale (about one TeV) without conflicting with experimental data. Then in chapter 2 I will show how this model can be generalized [31] to a "deconstructed" or "moose" model [32, 33] describing a whole tower of new vector states. Chapter 2 also contains my first original contribution, namely the calculation of the ϵ parameters in this generalized D-BESS model [34]. As I show in chapter 3, the moose extension of D-BESS can then be generalized to a 5-dimensional theory. The D-BESS model and its generalizations suffer the drawback of the unitarity constraint, which is as low as the one of the Higgsless SM [8, 9] (around 1.7 TeV). However, at least for a particular choice of the extra-dimensional background, a slice of AdS_5, in the 5-dimensional case it is possible to reintroduce an Higgs field, delaying unitarity violation to a scale $\gtrsim$ 10 TeV. In the "holographic" interpretation of AdS_5 models [35, 36], inspired by the AdS/CFT correspondence, this Higgs can be understood as a composite state and thus does not suffer from the hierarchy problem. The 5-dimensional GD-BESS on AdS_5 then provides a coherent description of the low energy phenomenology of a new strongly interacting sector up to energies significantly beyond the $\sim$ 2 TeV limit of the Higgsless SM, still showing a good compatibility with EW precision observables. In this version, the 5D GD-BESS is very similar to an RS1 model [18] with EW gauge fields propagating in the bulk and having brane-localized kinetic terms [37]. All of the material in chapter 3 is original, and is due to appear in a future work [38].

In the last chapter, I will review another study [39], to which I personally contributed, which is not directly related to D-BESS, but still addresses the problem of studying EW symmetry breaking in five dimensions. The study focuses on a model with two scalars, one propagating in the bulk of the extra dimension and the other confined to a 4-dimensional submanifold, and in particular on the effects of interaction terms between these two fields on the pattern of spontaneous symmetry breaking. The study is primarily technical in nature, and it does not claim to describe a realistic model. However, it is interesting to note that its most peculiar ingredient - namely the presence of a bulk Higgs field whose VEV gets distorted by brane-localized interactions - *can* be implemented in a potentially realistic theory. An interesting example of such a construction is the so called "Soft-Wall" SM [40], where the RS1 set-up is generalized by replacing the "hard" 5D cut-off at $y = \pi R$ (the brane) with a smooth boundary, which is provided by the VEV of a bulk scalar.

Chapter 1

The D-BESS model

As I have told in the preface, the presence of the hierarchy problem arguably makes the Higgs sector one of the most unsatisfactory elements of the SM, from a theoretical point of view. A very appealing option to avoid the problem is the possibility that EW symmetry is broken dynamically by a new strong interaction that manifests roughly at the TeV energy scale. The possibility was first analyzed by Weinberg [13] in 1975, while the first model to describe a concrete realization of this idea was the so-called "Technicolor" (TC) by Susskind [14] and Weinberg [15]. Even though this first model is excluded by experimental data (see for instance [41, 42]), the possibility of a dynamical origin for EW symmetry breaking remains open. With a slightly abuse of notation, and for the sake of simplicity, I will refer in the following to *any* theory sharing this basic premise as a TC-like theory.

Studying strongly interacting theories is in general difficult; furthermore, the physics community has been faced by a persistent absence of clear signals from the experiments regarding phenomena beyond the SM. For these reasons, no single candidate has emerged as the "simplest" or "most likely" TC model; in contrast, many very different possibilities have been explored during last years.

The prototype of strongly interacting theory, the only one which is well confirmed by experiments, is QCD. It is then natural to use our knowledge of the theory of quarks and gluons as a guide in the attempt to describe the phenomenology of a possible new strong interaction. The low-energy phenomenology of hadrons - which are interpreted as quark composites - is very different from what could be easily deduced by looking at the high-energy QCD Lagrangian; a good approximate description of hadron physics can be given by using an effective Lagrangian approach, in terms of chiral $SU(2) \otimes SU(2)$ or $SU(3) \otimes SU(3)$ Lagrangians. During the '80s, a class of theories was proposed which tried to give an analogous description of the low-energy phenomenology of the hypothetical strong TC sector. These models, called BESS (for Breaking Electroweak Symmetry Strongly) [43, 44, 45, 27], describe the physics of a set of new vector and axial vector massive particles which can be interpreted as composite states of the TC sector.

The BESS models, as TC models in general, are typically heavily constrained by EW precision measurements; in fact, even when the particles they describe have masses beyond the experimentally explored range, forbidding direct detection, the virtual effects due to the existence of these particles generally lead to experimentally unacceptable contributions to EW precision observables. Among these contributions, the most important are usually the *oblique* corrections, which can be parametrized in different ways, such as the S,T,U parameters by Peskin and Takeuchi [41, 42] or the ϵ_i ones by Altarelli, Barbieri and Jadach [28, 29, 30].

However, in this class at least one model exists, the D-BESS model [27], for which the contribution from new physics to the oblique corrections is suppressed. In this chapter, I will review this model; then, in chapters 2 and 3, I will show that it can be interpreted as a "moose" model [32] with only two sites, and thus it can be easily generalized to an N-site moose [31, 34]. In turn, the moose generalization of D-BESS (or GD-BESS) is the "deconstructed" (latticized only along the extra dimension) version 5-dimensional theory that maintains its most important feature, namely the description of new physics with a suppressed contribution to the EW precision parameters. As I will show, the resulting extra-dimensional model is closely related to a RS1 model [18] with gauge bosons propagating in the bulk of the extra dimension and localized kinetic terms on the IR brane (a similar model was also studied in [37]). The connection between the D-BESS and RS1 models is quite interesting, because the D-BESS model was originally formulated without any reference to the existence of extra dimensions; it is however not surprising in terms of the "holographic dual interpretation" [35, 36], based on the AdS/CFT correspondence conjecture [26], which considers the RS model as an effective description of a strongly interacting 4D theory.

1.1 A technical preamble

Before discussing the D-BESS model, I will briefly introduce two technical devices used in its construction, *nonlinear models* and *hidden symmetry.*

1.1.1 Nonlinear models

The study of nonlinear models was motivated by low-energy hadron physics, where light scalars are described as pseudo-goldstone bosons of spontaneously broken approximate global symmetries; for instance, pions can be described as goldstones arising from the breaking of the chiral $SU(2)\otimes SU(2)$ group to its diagonal subgroup $SU(2)_V$. The "nonlinear" label stems from the fact that, in these descriptions, the scalars transform nonlinearly under the global symmetry group G.

The comprehensive case of a theory invariant under the general action of a given continuous group G was studied in detail by Callan, Coleman, Wess and Zumino (CCWZ) [46, 47]. They found that any such theory is equivalent to one where the action of G on the fields is nonlinear but becomes linear when restricted to a subgroup H, and gave a standard representation of this action. Part of the degrees of freedom used in this description are the goldstone bosons of the spontaneous breaking $G \to H$. This is the part I will need for the construction of the D-BESS model and that I am going to illustrate in the following.

Let G be a generic (compact, simple) Lie group, H a proper subgroup, and $\mathfrak{g}$, $\mathfrak{h}$ their respective algebras (remember that since H is a subgroup of G, $\mathfrak{h}$ is a subalgebra of $\mathfrak{g}$). Choose V_i as generators of $\mathfrak{h}$, and complete them through another set A_j in such a way as to form a basis of $\mathfrak{g}$ orthonormal with respect to the Cartan-Killing inner product:

$$\mathrm{Tr}\big[V_i, V_j\big] = \frac{1}{2}\delta_{ij}, \quad \mathrm{Tr}\big[A_i, A_j\big] = \frac{1}{2}\delta_{ij}, \quad \mathrm{Tr}\big[V_i, A_j\big] = 0 \tag{1.1}$$

(in the above and in subsequent equations, the generators of G can be taken in any finite-dimensional representation). Now consider a G-valued field, $g_0(x)$. At least locally, it is always possible to decompose *uniquely* g as a product:

$$g_0(x) = \Sigma(x) h_0(x) \equiv e^{\xi_i(x) A_i} e^{u_i(x) V_i}, \tag{1.2}$$

that is, I can represent any $g \in G$ as a product of an element of H times an element of $G \backslash H$ chosen in such a way as to be the exponential of a generator orthogonal to H. Note that the field Σ is, in fact, a particular representative of a coset in G/H.

My goal is to build a G-invariant Lagrangian which uses the fields $\xi_i(x)$ as dynamical variables; to do this, the first step is to define a sensible action of G on them. This can be done in a standard way by using left multiplication:

$$(g, g_0) \mapsto g g_0 = g e^{\xi_i(x) A_i} e^{u_i(x) V_i} = e^{\xi'_i(x,\xi,g) A_i} e^{u'_i(x,\xi,g) V_i} \equiv \Sigma' h'_0 \tag{1.3}$$

by uniqueness of the decomposition (1.2). Eq. (1.3) gives an implicit transformation rule $(g, \xi) \mapsto \xi'$, which is the action I need. It can be shown that is properly defined (it satisfies the axioms for an action, $(gg', \xi) = (g, (g', \xi))$ for any g, $g' \in G$ and $(1, \xi) = \xi$). Clearly it is not linear in general; however, if I restrict the action to the subgroup H, that is if I act with $h \in H$, then

$$h g_0 = h e^{\xi_i A_i} h^{-1} h h_0 = e^{\xi_i h A_i h^{-1}} h h_0. \tag{1.4}$$

The transformation $A_i \to A'_i \equiv h A_i h^{-1}$ is an endomorphism of the Lie algebra $\mathfrak{g}$, that is A'_i is still an element of $\mathfrak{g}$. Since the A_i and V_i together form a basis of $\mathfrak{g}$, I can write:

$$A'_i = a_{ij} A_j + v_{ij} V_j. \tag{1.5}$$

The coefficients a_{ij} and b_{ij} can be determined by using the Cartan-orthogonality of the basis set (1.1). I get:

$$a_{ij} = 2\,\mathrm{Tr}\big[A'_i A_j\big],\; v_{ij} = 2\,\mathrm{Tr}\big[A'_i V_j\big]. \tag{1.6}$$

In fact, the coefficients v_{ij} are vanishing. Let me prove it:

$$\mathrm{Tr}\big[A'_i V_j\big] = \mathrm{Tr}\big[hA_i h^{-1} V_j\big] = \mathrm{Tr}\big[A_i h^{-1} V_j h\big]; \tag{1.7}$$

but since $\mathfrak{h}$ is a subalgebra of $\mathfrak{g}$ and $h \in H$, $h^{-1}V_j h$ is still $\in \mathfrak{h}$, then $h^{-1}V_j h = h_{ij}V_j$, so

$$\mathrm{Tr}\big[A'_i V_j\big] = \mathrm{Tr}\big[h_{ij} A_i V_j\big] = 0 \tag{1.8}$$

because the generators A_i and V_i are orthogonal. Then, by comparing eqs. (1.3) and (1.4), I can identify:

$$\xi'_i A_i = \xi_i a_{ij} A_j, \quad h'_0 = h h_0, \quad \Rightarrow \quad \xi'_i = \xi_j a_{ji}; \tag{1.9}$$

in conclusion, the action of G on the ξ_i becomes *linear* when restricted to H.

A general G-invariant Lagrangian for the fields ξ_i will have an infinite number of terms. However, these terms can be conveniently organized as an expansion on the number of derivatives. The first non-trivial term has two derivatives and can be written in terms of the $\mathfrak{g}$-valued Maurer-Cartan 1-form,

$$\alpha_\mu = \Sigma^{-1}\partial_\mu \Sigma \equiv a^i_\mu A_i + v^i_\mu V_i. \tag{1.10}$$

It is easy to show that under the action of G given in eq. (1.3), the coefficients a_μ and v_μ transform as

$$a'^i_\mu A_i = h'_0 a^i_\mu A_i {h'_0}^{-1}, \tag{1.11}$$
$$v'^i_\mu V_i = h'_0 v^i_\mu V_i {h'_0}^{-1} - (\partial_\mu h'_0){h'_0}^{-1}, \tag{1.12}$$

that is $a^i_\mu A_i$ transforms homogeneously, while $v^i_\mu V_i$ transforms as a gauge field. A simple two-derivatives invariant Lagrangian can then be written as:

$$\mathcal{L}^{(2)} = k^2\,\mathrm{Tr}\big[a^i_\mu a^{i\,\mu}\big]. \tag{1.13}$$

It can be shown that Lagrangian (1.13) is in fact the most general invariant Lagrangian with no more than two derivatives that can be written using only the ξ_i fields.

Lagrangian (1.13) exhibits spontaneous symmetry breakdown; in fact, it is G-invariant, and since the VEV of the scalar must be a constant, one can always choose $\langle\Sigma\rangle = \mathbf{1}$; by using transformation rule (1.3), then one can trivially prove that $\langle\Sigma\rangle$ is invariant under H but not under $G\backslash H$.

An especially simple and interesting situation is when G admits a parity-like automorphism:

$$P:\ V_i \to V_i, \quad A_i \to -A_i. \tag{1.14}$$

In this case, the transformation rule (1.3) can be written in a much more explicit form; applying P to both sides of the equality:

$$g\, e^{\xi_i A_i} = h_0^{-1} e^{\xi_i' A_i} h_0'$$

one gets

$$P(g) e^{-\xi_i A_i} = h_0^{-1} e^{-\xi_i' A_i} h_0'$$

and h_0' can be eliminated from the two equations, yielding

$$e^{2\xi_i' A_i} = g\, e^{2\xi_i A_i} P(g^{-1}). \tag{1.15}$$

A notable case in which this happens is the chiral $SU(N)_L \otimes SU(N)_R$ group, broken to the diagonal subgroup $SU(N)_V$: then if $T^a_{L,R}$ are the orthonormalized generators of the $SU(N)_{L,R}$ respectively, one can define $V^a = T^a_L + T^a_R$ and $A^a = T^a_L - T^a_R$; these are called "vector" and "axial vector" generators (note that while the vector generators close a subalgebra, the axial vector ones do not). The automorphism P corresponds to the operator exchanging $L \leftrightarrow R$.

Before going on to the next section, I will work out the $N = 2$ case fully, which is relevant to EW symmetry breaking and will be used throughout the rest of the chapter. The vector and axial vector generators are given by

$$V_a = \frac{1}{2}(\tau^a_L + \tau^a_R), \quad A_a = \frac{1}{2}(\tau^a_L - \tau^a_R), \quad a = 1, 2, 3 \tag{1.16}$$

where $\tau^a_{L,R}$ are Pauli matrices acting on the left/right sector. The field Σ becomes

$$\Sigma = e^{i\xi_a A_a} = e^{i\,\xi_a \frac{\tau^a_L}{2}} \otimes e^{-i\,\xi_a \frac{\tau^a_R}{2}}. \tag{1.17}$$

This means that Σ is the direct product of an $SU(2)$ matrix times its inverse; without loss of generality, I can simply drop one of the matrices and write Σ as an $SU(2)$-valued field. By using eq. (1.15) and remembering that the P operator exchanges left and right matrices, I can write the action of G on Σ as:

$$\Sigma \to g_L \Sigma g_R^\dagger. \tag{1.18}$$

The Lagrangian up to two derivatives (1.13) is also simplified in the $SU(2)\otimes SU(2)$ case: the Maurer-Cartan form $\Sigma^\dagger \partial_\mu \Sigma$ has no parallel component in this case and the Lagrangian simply becomes

$$\mathcal{L}^{(2)} = k^2 \mathrm{Tr}\big[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\big]. \tag{1.19}$$

Despite its simple form, the Lagrangian (1.19) is highly nontrivial; one can expand the exponentials

$$\Sigma = e^{i\,\xi_a \frac{\tau^a}{2}} \simeq 1 + i\,\xi_a \frac{\tau^a}{2} - \frac{1}{8}\xi_a\xi_a + \frac{i}{48}(\xi_a\tau_a)^3 + \dots \qquad (1.20)$$

and get an *infinite* number of interaction vertices. Keeping only the terms containing up to 4 fields, one gets

$$\mathcal{L}^{(2)} \simeq \frac{k^2}{2}\partial_\mu\xi_a\partial^\mu\xi^a + \frac{k^2}{6}\left(\partial_\mu\xi_a\partial^\mu\xi_b\xi_a\xi_b - \partial_\mu\xi_a\partial^\mu\xi_a\xi_b\xi_b\right);$$

and rescaling $\xi_a \to \frac{\pi_a}{k}$, one has properly normalized scalar kinetic terms for the three fields π_a, and a derivative four-π vertex suppressed by a double power of k (note that k has the dimension of an energy),

$$\mathcal{L}^{(2)} \simeq \frac{1}{2}\partial_\mu\pi_a\partial^\mu\pi^a + \frac{1}{6k^2}\left(\partial_\mu\pi_a\partial^\mu\pi_b\pi_a\pi_b - \partial_\mu\pi_a\partial^\mu\pi_a\pi_b\pi_b\right). \qquad (1.21)$$

It is easy to figure out that successive terms in the expansion will be suppressed by k^{n-2}, where n is the number of field in the vertex. The peculiar structure of this Lagrangian is typical of nonlinear models and helps to clarify two important facts:

1. This model is nonrenormalizable. It has an infinite number of couplings with a negative mass dimension (thus leading to divergences), each one requiring a different counterterm to cancel.
2. The infinite couplings of the model have a definite *hierarchy*. Higher order terms in the expansion are suppressed by greater and greater powers of k, which sets a characteristic scale of the model.

The energy scale k is referred to as the *spontaneous symmetry breaking scale*; the nonlinear Lagrangian (1.19), despite being nonrenormalizable, gives an effective description of the physics of the goldstone bosons arising from the symmetry breaking $SU(2)_L \otimes SU(2)_R \to SU(2)_V$, which is valid *below* energies of the order of k.

An example: describing standard EW breaking nonlinearly

The model just analyzed can be used to provide the most economical description of the EW symmetry breaking sector of the SM [8, 9], sometimes referred to as the "Higgsless SM". It is well known that, whatever its explicit form, the EW breaking sector must possess the so-called *custodial* $SU(2) \otimes SU(2)$ global approximate symmetry [48, 49] to account for the experimental value of the ρ parameter. This is exactly the symmetry of our nonlinear model; furthermore, the breaking to $SU(2)_V$ gives just the three goldstones that are needed to provide mass for the $W^\pm$, Z bosons. The Higgs mechanism can be realized by gauging a $SU(2)_L \otimes U(1)_Y$

subgroup of the total symmetry, and changing the derivatives of Σ to covariant derivatives accordingly:

$$\partial_\mu \Sigma \to D_\mu \Sigma \equiv \partial_\mu \Sigma + ig\frac{\tau^a}{2} W_\mu^a \Sigma - ig' \Sigma \frac{\tau^3}{2} B_\mu . \tag{1.22}$$

The Lagrangian expanded up to quadratic order in the fields now reads

$$\begin{aligned}\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu \pi_a \partial^\mu \pi_a + \frac{g^2 k^2}{8} W_\mu^a W^{a\,\mu} + \frac{g'^2 k^2}{8} B_\mu B^\mu - \frac{g g' k^2}{4} W_\mu B^\mu \\ + \quad \text{gauge-goldstone mixing terms.}\end{aligned} \tag{1.23}$$

By setting $k = v \simeq 246$ GeV one recovers the standard gauge boson mass matrix. The drawback of this formulation is again the nonrenormalizability, which manifests itself as a violation of tree-level unitarity [10, 11]. The goldstone bosons can be completely eliminated by a gauge transformation (this is clear if one looks at the Lagrangian (1.19)), and the scattering of heavy gauge bosons proceeds just as in the SM in the unitary gauge, but without the possibility of exchanging the Higgs boson. The scattering of longitudinally polarized gauge bosons then diverges as E^2 with growing center-of-mass energy E; for instance the channel with the worst behaviour is $W_L^+ W_L^- \to W_L^+ W_L^-$ elastic scattering, whose leading term is [10, 11]

$$A_{WWWW} = \frac{G_F}{\sqrt{2}} 4E^2 (1 + \cos\theta) + O(E^0) \tag{1.24}$$

The growing with energy of the scattering amplitudes of longitudinal gauge bosons leads to unitarity violation at about 1.7 TeV.

This picture is in fact strongly connected to the standard one. The SM Higgs sector,

$$\begin{aligned}\mathcal{L}_H = D_\mu \Phi^\dagger D^\mu \Phi - V(\Phi^\dagger \Phi), \quad D_\mu \Phi = \partial_\mu \Phi + ig\frac{\tau^a}{2} W_\mu^a \Phi + i\frac{g'}{2} B_\mu \Phi, \\ V(\Phi^\dagger \Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2\end{aligned} \tag{1.25}$$

where Φ is the complex Higgs doublet, can be recast in a matrix form very similar to the nonlinear one described above. It is sufficient to define

$$\tilde{\Phi} = i\tau^2 \Phi^*, \quad M = \left(\tilde{\Phi}, \Phi\right); \tag{1.26}$$

I have

$$\frac{1}{2}\mathrm{Tr}\left[M^\dagger M\right] = \Phi^\dagger \Phi \tag{1.27}$$

and M transforms under $SU(2)_L \otimes U(1)_Y$ as

$$M(x) \to U_L(x) M(x) e^{i\alpha(x)\frac{\tau^3}{2}}, \; U_L \in SU(2), \; \alpha \in \mathbb{R}, \tag{1.28}$$

so that (1.25) becomes

$$\mathcal{L}_H = \mathrm{Tr}\big[D_\mu M^\dagger D^\mu M\big] - V\left(\frac{1}{2}\mathrm{Tr}\big[M^\dagger M\big]\right), \tag{1.29}$$

$$D_\mu M = \partial_\mu M + ig\frac{\tau^a}{2}W_\mu^a M - ig' M B_\mu \frac{\tau^3}{2}. \tag{1.30}$$

Since $M^\dagger M$ is proportional to the identity, it can be reparametrized as $\frac{\rho}{\sqrt{2}}U$, with ρ a real field and $U \in SU(2)$ (this reparametrization does not affect the physical observables). The Lagrangian (1.25) then separates as follows:

$$\mathcal{L}_H = \frac{1}{2}D_\mu\rho D^\mu\rho + \frac{\mu^2}{2}\rho^2 - \frac{\lambda}{4}\rho^4 + \rho^2\mathrm{Tr}\big[D_\mu U D^\mu U\big]. \tag{1.31}$$

Clearly, ρ is the field that acquires a VEV $v = \frac{\mu}{\sqrt{\lambda}}$. The fields in U can be eliminated by a gauge transformation. If one lets the mass of ρ to infinity with $\mu \to \infty$, and simultaneously also $\lambda \to \infty$ in such a way as to keep v constant, ρ decouples from the theory and the Lagrangian becomes identical to (1.19) [8, 9]. So, the meaning of the nonlinear description is clear: it gives the low-energy behaviour of the standard Higgs sector in the limit of a very large Higgs mass m_H (or at energies much lower than m_H).

1.1.2 Hidden Symmetry

The hidden symmetry approach consists basically of a clever way of introducing vector fields in the context of nonlinear models. In the late '70, it was noted [50, 51] that $G \to H$ nonlinear models can be reformulated as models with an enlarged symmetry $G \otimes H'$, where H' is a *gauge* symmetry group such that, locally, $H' \supseteq H$. In the simplest case one has $H' = H$ (still locally), the vector bosons associated with H' are not dynamical, and the standard nonlinear picture can be recovered via a gauge fixing. In short, the nonlinear model corresponds to the $G \otimes H'$-invariant model in a particular gauge (this is the sense in which the symmetry is "hidden" in the nonlinear model). A more interesting possibility lies in making the vector fields dynamical, by adding appropriate kinetic terms. These new vector degrees of freedom can be interpreted (as well as the scalars) as *composite states* of the underlying strong interaction, that is also responsible for the appearance of the kinetic terms.

Let me illustrate the approach concretely, again in the case of the $SU(2)_L \otimes SU(2)_R$ nonlinear model.
The symmetry gets enlarged to $\big[SU(2)_L \otimes SU(2)_R\big]_{global} \otimes [SU(2)_V]_{local}$; correspondingly, I have two $SU(2)$ dynamical variables, which I call L and R, with the following transformation laws:

$$L(x) \to h(x)L(x)g_L^\dagger, \quad R(x) \to h(x)R(x)g_R^\dagger, \quad L^\dagger L = R^\dagger R = \mathbf{1} \tag{1.32}$$

where

$$g_L \otimes g_R \in SU(2)_L \otimes SU(2)_R, \quad h(x) \in SU(2)_V.$$

Let me introduce the gauge fields V^a_μ of the hidden symmetry and the covariant derivatives of L and R:

$$D_\mu L(R) = \partial_\mu L(R) + i g_V V^a_\mu \frac{\tau^a}{2} L(R), \tag{1.33}$$

where V^a_μ, g_V are the gauge fields and coupling constant associated with the hidden symmetry. Let me also impose for simplicity a discrete symmetry [52],

$$P : L \leftrightarrow R, \tag{1.34}$$

I can write two invariants with two derivatives, namely

$$\mathcal{L}_V = \mathrm{Tr}\big[D_\mu L\, L^\dagger + D_\mu R\, R^\dagger\big]^2 \tag{1.35}$$

and

$$\mathcal{L}_A = \mathrm{Tr}\big[D_\mu L\, L^\dagger - D_\mu R\, R^\dagger\big]^2; \tag{1.36}$$

with the gauge choice $L = R^\dagger = \tilde{M}$, these reduce to

$$\mathcal{L}_A = \mathrm{Tr}\big[\partial_\mu \tilde{M}\, \tilde{M}^\dagger - \partial_\mu \tilde{M}^\dagger\, \tilde{M}\big]^2 = -\mathrm{Tr}\big[\partial_\mu\big[(\tilde{M}^\dagger)^2\big]\partial^\mu\big[\tilde{M}^2\big]\big];$$
$$\mathcal{L}_V = \mathrm{Tr}\big[2i\, g_V V^a_\mu \frac{\tau^a}{2} + \partial_\mu \tilde{M}\, \tilde{M}^\dagger + \partial_\mu \tilde{M}^\dagger\, \tilde{M}\big]^2.$$

The first invariant is equivalent, up to a multiplicative constant, to eq. (1.19); the second one, if there is no kinetic term for the vector field, gives an equation of motion that can easily be solved for V^a_μ:

$$V^a_\mu = i\, \mathrm{Tr}\big[\frac{\tau^a}{2g_V}(\partial_\mu \tilde{M}\, \tilde{M}^\dagger - \partial_\mu \tilde{M}^\dagger\, \tilde{M})\big]. \tag{1.37}$$

On shell, $\mathcal{L}_V$ becomes trivial, and the equivalence with the standard nonlinear model is fully demonstrated.

The picture of course changes if I allow the V^a_μ to be dynamical by adding standard kinetic terms for them. In this case, the invariant $\mathcal{L}_V$ is no longer trivial and the theory is no longer equivalent to the nonlinear $SU(2)_L \otimes SU(2)_R$. The symmetry of the vacuum is $(SU(2))^3_{diag.}$ (I need $g_L = g_R = h$ to keep the vacuum invariant), so this time I get six goldstone bosons from the spontaneous breaking. Since three of the broken generators are gauged, by the Higgs mechanism, I get an interacting theory of three massive vectors and three massless scalars; all of them can be regarded as low-energy effective, composite degrees of freedom of a more fundamental strong interacting sector.

1.2 D-BESS construction

I am now ready to review the D-BESS model [27]. The model is built on the ideas presented so far, with the additional goal, as I will show in the following, of making the contributions from the new physics to the EW precision observables as small as possible.

The set-up is as follows: the starting point is the spontaneously broken $SU(2)_L \otimes SU(2)_R \to SU(2)_V$ nonlinear model, as in the example of the previous section, but with a larger hidden symmetry group, a full $SU(2) \otimes SU(2)$. The total symmetry I require is $G \otimes H'$, with $G = \big(SU(2)_L \otimes SU(2)_R\big)_{global}$ and $H' = \big(SU(2)_L \otimes SU(2)_R\big)_{local}$; I want to break it down to $H = (SU(2))^4_{diag.}$, so I will have 9 goldstone bosons. The goldstones can be described using three independent $SU(2)$-valued fields, which I call L, R and M. They have the following transformation laws:

$$L(x) \to g_L L(x) h_L(x),\ \ R(x) \to g_R R(x) h_R(x),\ \ M(x) \to h_R^\dagger(x) M h_L(x), \tag{1.38}$$

where $g_L \otimes g_R \in G$ and $h_L \otimes h_R \in H'$. The vacuum corresponds to choosing $L = R = M = \mathbf{I}$. From eq. (1.38), it is then clear that the vacuum is invariant if and only if $g_L = g_R = h_L = h_R$, so that indeed $H = [SU(2)^4]_{diag.}$.

In addition to the $SU(2)$ fields, I also have two triplets of gauge vectors from the H' symmetry, W_L^a and W_R^a. Since H' gets spontaneously broken completely, the gauge fields become massive by absorbing 6 of the 9 goldstones present in the theory; in the end, the physical spectrum consists of six massive vectors and three massless scalars. The covariant derivatives for the L, R and M are, as implied by eq. (1.38):

$$\begin{aligned} &D_\mu L = \partial_\mu L - iL\mathbf{W}_{L\,\mu}, \quad D_\mu R = \partial_\mu R - iR\,\mathbf{W}_{R\,\mu} \\ &D_\mu M - i\mathbf{W}_{R\,\mu} M + iM\,\mathbf{W}_{L\,\mu}, \end{aligned} \tag{1.39}$$

where for simplicity I have used a different normalization for the gauge fields, absorbing the gauge coupling constants into them, and also introduced the shorthand notation:

$$\mathbf{W}_{L,R\,\mu} = W^a_{L,R\,\mu}\frac{\tau^a}{2}. \tag{1.40}$$

Finally I will require an additional symmetry:

$$P: \quad L \leftrightarrow R,\ M \leftrightarrow M^\dagger. \tag{1.41}$$

At this point it is useful to analyze the symmetry properties of this model. Aside from the gauge fields, the maximal *global* symmetry that can act naturally on the fields L, R, M without altering their character of $SU(2)$ matrices is $G_{max} = [SU(2) \otimes SU(2)]^3$, consisting of 6 independent $SU(2)$ matrices each one acting to the left or the right of one of the fields. I will show below that there exist special choices of the parameters, in the general Lagrangian of the theory, that enlarge the symmetry from $G \otimes H' \otimes P$ to $G_{max} \otimes P$. The D-BESS model corresponds to one of such choices; as I will show below, the enlarged symmetry is not merely a convenient choice from a mathematical point of view, but also has beneficial phenomenological consequences in suppressing the contributions to the EW precision parameters.

Let me write the general Lagrangian for the model. It is possible to write four invariants under $G \otimes H' \otimes P$ with up to two derivatives; they are [45]

$$\begin{aligned} I_1 &= \mathrm{Tr}\big[V_0 - V_1 - V_2\big]^2, \quad I_2 = \mathrm{Tr}\big[V_0 + V_2\big]^2 \\ I_3 &= \mathrm{Tr}\big[V_0 - V_2\big]^2, \quad I_4 = \mathrm{Tr}\big[V_1\big]^2, \end{aligned} \tag{1.42}$$

where

$$V_{0\,\mu} = L^\dagger D_\mu L, \quad V_{1\,\mu} = M^\dagger D_\mu M, \quad V_{2\,\mu} = M^\dagger R^\dagger D_\mu R\, M. \tag{1.43}$$

The strategy to build these invariants is to write down all the independent vectors containing one derivative that are

- invariant under G;
- covariant under H.

There are three such objects, namely the $V_{i\,\mu}$ of eq. (1.43), all of which transform as

$$V_{i\,\mu} \to h_L^\dagger V_{i\,\mu} h_L. \tag{1.44}$$

Any quadratic polynomial built with these objects and traced over will be trivially invariant under $G \otimes H'$; using three independent objects one can build 6 independent invariants, which reduce to the listed four when the P symmetry is also imposed.

The general Lagrangian can thus be written as

$$\mathcal{L} = \mathcal{L}_{kin} - \frac{v^2}{4} \sum_{i=1}^{4} a_i I_i, \tag{1.45}$$

where the a_i are free parameters, $\mathcal{L}_{kin}$ is the standard kinetic Lagrangian for the gauge fields,

$$\begin{aligned} \mathcal{L}_{kin} &= \frac{1}{2g''^2} \Big[\mathrm{Tr}\big[F_{\mu\nu}(\mathbf{L})\big]^2 + \mathrm{Tr}\big[F_{\mu\nu}(\mathbf{R})\big]^2\Big], \\ F_{\mu\nu}(\mathbf{L}) &\equiv \partial_\mu \mathbf{L}_\nu - \partial_\nu \mathbf{L}_\mu + i[\mathbf{L}_\mu, \mathbf{L}_\nu], \end{aligned} \tag{1.46}$$

and v represents, as in the example I have reviewed, a typical scale of the model. Note that the alternative normalization I have used requires an inverse squared coupling constant factor in front of every kinetic term. Also note that the P symmetry forces the two coupling constants of H, to be equal, namely g''.

Now let me examine the two limits in which the overall symmetry gets enhanced to the maximal one. The first corresponds to a special limit of the parameters, $a_1 = 0$, $a_2 = a_3$. With this choice, the Lagrangian becomes

$$\mathcal{L} = -\frac{v^2}{4}[2a_2 \operatorname{Tr}\big[V_0\big]^2 + a_4 \operatorname{Tr}\big[V_1\big]^2 + 2a_2 \operatorname{Tr}\big[V_2\big]^2], \tag{1.47}$$

and it is diagonal in the variables L, R, M (see eq. (1.43)). However, this is not sufficient to modify the overall symmetry, due to the local character of the group H' which leads to a non-trivial dependence on the gauge fields $\mathbf{W}_{L\mu}$ and $\mathbf{W}_{R\mu}$. However, if I consider the limit in which the gauge interaction is turned off, the Lagrangian reduces to

$$\mathcal{L} = \frac{v^2}{4}\left[2a_2\left[\operatorname{Tr}\big[\partial_\mu L^\dagger \partial^\mu L\big] + \operatorname{Tr}\big[\partial_\mu R^\dagger \partial^\mu R\big]\right] + a_4 \operatorname{Tr}\big[\partial_\mu M^\dagger \partial^\mu M\big]\right]. \tag{1.48}$$

The enlarged symmetry is manifest in this form. The theory now describes a set of nine massless scalars, interacting in triplets. This situation can be considered as a generalization of the vector symmetry described by Georgi in [53]. Though potentially interesting, this is not the case I wish to focus on, so I will not make any further consideration on it.

The other possibility, the one I am set to study, corresponds to the choice $a_4 = 0$, $a_2 = a_3 \equiv a/2$. In this case it is useful to rewrite I_1 as

$$I_1 = -\operatorname{Tr}\big[\partial_\mu U^\dagger \partial^\mu U\big], \tag{1.49}$$

where

$$U = LM^\dagger R^\dagger, \tag{1.50}$$

in terms of which the Lagrangian becomes

$$\begin{aligned}\mathcal{L} =& \frac{v^2}{4}\Big\{a_1 \operatorname{Tr}\big[\partial_\mu U^\dagger \partial^\mu U\big] + a\,\Big(\operatorname{Tr}\big[D_\mu L^\dagger D^\mu L\big]\\ &+\operatorname{Tr}\big[D_\mu R^\dagger D^\mu R\big]\Big)\Big\} + \mathcal{L}_{kin}.\end{aligned} \tag{1.51}$$

Again, the Lagrangian is a sum of three independent $SU(2) \otimes SU(2)$ invariants, so that the overall symmetry is $[SU(2) \otimes SU(2)]^3 \equiv G_{max}$, with an H' subgroup realized as a local symmetry. Note that this time there is no need to switch the gauge interactions off, and the spectrum still consists of three massless goldstones and two triplets of massive gauge bosons.

The simplest physical consequence of this parameter choice can be seen by looking at the mass matrix of the gauge bosons. The matrix can be obtained in general by substituting to the $SU(2)$ fields their VEV: $L,\ R,\ U \to \mathbf{1}$; it is equal to:

$$\mathcal{L}_{mass} = -\frac{v^2}{4}\left[a_2 \mathrm{Tr}\big[\mathbf{L}_\mu + \mathbf{R}_\mu\big]^2 + (a_3 + a_4)\mathrm{Tr}\big[\mathbf{L}_\mu - \mathbf{R}_\mu\big]^2\right]; \tag{1.52}$$

I can easily diagonalize this matrix by introducing vector and axial vector combinations of the fields:

$$\mathbf{V}_\mu = \frac{1}{2}(\mathbf{R}_\mu + \mathbf{L}_\mu), \quad \mathbf{A}_\mu = \frac{1}{2}(\mathbf{R}_\mu - \mathbf{L}_\mu), \tag{1.53}$$

so that

$$\mathcal{L}_{mass} = -v^2\left(a_2 \mathrm{Tr}\big[\mathbf{V}_\mu\big]^2 + (a_3 + a_4)\mathrm{Tr}\big[\mathbf{A}_\mu\big]^2\right). \tag{1.54}$$

So, in the limit $a_4 = 0,\ a_2 = a_3 \equiv a/2$, the modes $\mathbf{V}_\mu$ and $\mathbf{A}_\mu$ are completely degenerate or, equivalently, there is no mixing between $\mathbf{L}_\mu$ and $\mathbf{R}_\mu$. This is the reason why this model is called D(egenerate)-BESS. Moreover, the decoupling of the fields in Lagrangian (1.51) means that the would-be goldstones that get absorbed by the vectors to provide their mass are those in L and R, while those in U remain in the spectrum as massless particles. In fact, the goldstones in U are, in this limit, completely decoupled from the massive vectors. By eq. (1.52), the degenerate masses of the gauge bosons are given by

$$M^2_{L,R} = \frac{v^2}{4} a\, g''^2. \tag{1.55}$$

1.2.1 Adding weak interactions

The next step in the formulation of the model is to add the weak interactions. The most straightforward way to do so is to copy the strategy used for the nonlinear version of the standard EW breaking sector examined at the end of section 1.1.1, that is to promote an $SU(2)_L \otimes U(1)_Y$ subgroup of the global symmetry G to gauge symmetry. In this way, the last three surviving goldstone bosons, the ones in the field U, disappear from the physical spectrum to provide mass for the newly added $SU(2)_L \otimes U(1)_Y$ gauge bosons.

The Lagrangian of the model is obtained from eq. (1.51) via the minimal substitution

$$D_\mu L \to D_\mu L + i\,\tilde{\mathbf{W}}_\mu L, \quad D_\mu R \to D_\mu R + i\,\tilde{\mathbf{B}}_\mu R, \tag{1.56}$$

where $\tilde{\mathbf{W}}_\mu = \tilde{W}^a_\mu \frac{\tau^a}{2}$, $\tilde{\mathbf{B}}_\mu = \tilde{B}_\mu \frac{\tau^3}{2}$ and again the coupling constants are absorbed in the fields, and by adding canonical kinetic terms for the new gauge fields. The mass Lagrangian changes to

$$\begin{aligned}\mathcal{L}_{mass} = -\frac{v^2}{4}\Big[a_1 \operatorname{Tr}\big[\tilde{W}_\mu - \tilde{B}_\mu\big]^2 + a\,\big(\operatorname{Tr}\big[L_\mu - \tilde{W}_\mu\big]^2 \\ + \operatorname{Tr}\big[R_\mu - \tilde{B}_\mu\big]^2\big)\Big].\end{aligned} \tag{1.57}$$

The first term in eq. (1.57) reproduces the standard GWS gauge mass Lagrangian provided the identification $v^2 a_1 = 1/(\sqrt{2}G_F)$. It is then quite natural to make this choice and think about the model as a perturbation around the SM picture. So, in the following I will assume:

$$a_1 = 1, \quad v^2 = \frac{1}{\sqrt{2}G_F}. \tag{1.58}$$

One requires the lightest of the vector bosons to reproduce the SM fields. The remaining gauge fields will represent heavy vector and axial vector composites of the strong underlying sector. I have seen in eq. (1.55) that before switching on the weak interactions the heavy states have degenerate masses $M = v^2/4\,g''^2\,a$. If these "unperturbated" masses are heavy enough, the actual masses will be of the same order even after the gauging of $SU(2)_L \otimes U(1)_Y$. To make them heavy, I have two possible choices: I can take either g'' or $a \gg 1$. Since the first choice implies a strong interaction in the heavy sector, the second one is preferable. In fact, the combination $u = \sqrt{a}v$ sets the typical energy scale of the new sector; in this sense, it is evident that choosing $a \gg 1$, or $u \gg v$, corresponds to giving to the new states masses which are much higher than those of the SM fields.

It is interesting to examine the spectrum in some detail. Introducing the charged field combinations,

$$\tilde{W}^\pm = \frac{1}{\sqrt{2}}(\tilde{W}^1 \mp i\tilde{W}^2), \tag{1.59}$$

with similar relations for the other gauge fields, and switching back to the usual normalization:

$$\tilde{W}^a_\mu \to \tilde{g}\tilde{W}^a_\mu, \quad \tilde{B}_\mu \to \tilde{g}'\tilde{B}_\mu, \quad W^a_{L,R\,\mu} \to g'' W^a_{L,R\,\mu} \tag{1.60}$$

one finds

$$\begin{aligned}\mathcal{L}_{mass} =& \frac{v^2}{4}[(1+a)\tilde{g}^2\tilde{W}^+_\mu\tilde{W}^{-\,\mu} + a\,g''^2(W^+_{L\,\mu}W_L^{-\,\mu} + W^+_{R\,\mu}W_R^{-\,\mu}) \\ &- a\,\tilde{g}g''(\tilde{W}^+_\mu W_L^{-\,\mu} + \tilde{W}^-_\mu W_L^{+\,\mu})] \\ &+ \frac{v^2}{8}[(1+a)(\tilde{g}^2(\tilde{W}^3)^2 + \tilde{g}'^2\tilde{B}^2) + a\,g''^2((W^3_L)^2 + (W^3_R)^2) \\ &- 2\tilde{g}\tilde{g}'\tilde{W}^3_\mu\tilde{B}^\mu - 2a\,g''(\tilde{g}\tilde{W}^3_\mu W_L^{3\,\mu} + \tilde{g}'\tilde{B}_\mu W_R^{3\,\mu})]\end{aligned} \tag{1.61}$$

Let me look at the spectrum of the theory. The fields $W^\pm_R$ in the charged sector are decoupled from all the other states, and their mass is given by

$$M^2_{R^\pm} = \frac{v^2}{4}ag''^2 = \frac{u^2}{4}g''^2 \equiv M^2, \tag{1.62}$$

just like the degenerate masses of the heavy bosons in the limit of no weak interaction (as expected since they have no mixing with the $\tilde{W}^a$, $\tilde{B}$ fields).

By contrast, the fields $W_L^\pm$ mix with the $\tilde{W}^\pm$. The corresponding mass matrix could be diagonalized analytically, but it is more instructive to study it by using a perturbative expansion in $1/a$. The matrix can be written:

$$\mathcal{M}^2_{L^\pm} = \frac{u^2}{4}\begin{pmatrix} \tilde{g}^2 + \frac{1}{a}\tilde{g}^2 & -\tilde{g}g'' \\ -\tilde{g}g'' & g''^2 \end{pmatrix} \tag{1.63}$$

Aside the overall $\frac{u^2}{4}$ factor, $\mathcal{M}^2_{L^\pm}$ is decomposed in an order unity contribution plus a $O(1/a)$ perturbation. The eigenvalues are

$$m_W^2 = \frac{u^2}{4a}\frac{\tilde{g}^2 g''^2}{\tilde{g}^2 + g''^2} + O\left(\frac{u^2}{4a^2}\right), \tag{1.64}$$

$$M_L^2 = \frac{u^2}{4}(\tilde{g}^2 + g''^2) + \frac{u^2}{4a}\frac{\tilde{g}^4}{(\tilde{g}^2 + g''^2)} + O\left(\frac{u^2}{4a^2}\right); \tag{1.65}$$

The corresponding eigenvectors are, to leading order:

$$W^\pm = \frac{g''}{\sqrt{\tilde{g}^2 + g''^2}}\tilde{W}^\pm + \frac{\tilde{g}}{\sqrt{\tilde{g}^2 + g''^2}}W_L^\pm, \tag{1.66}$$

$$L^\pm = -\frac{\tilde{g}}{\sqrt{\tilde{g}^2 + g''^2}}\tilde{W}^\pm + \frac{g''}{\sqrt{\tilde{g}^2 + g''^2}}W_L^\pm, \tag{1.67}$$

and the next-to-leading corrections to the coefficients are of order $1/a$. The physical $W^\pm$ and $L^\pm$ particles are thus a combination of $\tilde{W}^\pm$ and $W_L^\pm$.

Looking at eq. (1.64), it is natural to define

$$\bar{g}^2 = \frac{\tilde{g}^2 g''^2}{\tilde{g}^2 + g''^2}, \tag{1.68}$$

so that the $W^\pm$ mass at leading order is given by

$$\tilde{m}_W^2 = \frac{\bar{g}^2 v^2}{4}; \tag{1.69}$$

note that this expression has exactly the same form of the SM tree level one.

In the neutral sector, the 4 fields $\tilde{W}^3$, $\tilde{B}$, W_L^3 and W_R^3 all mix together. The mass matrix is

$$\mathcal{M}^2_N = \frac{u^2}{4}\begin{pmatrix} \tilde{g}^2 + \frac{\tilde{g}^2}{a} & -\frac{\tilde{g}\tilde{g}'}{a} & -\tilde{g}g'' & 0 \\ -\frac{\tilde{g}\tilde{g}'}{a} & \tilde{g}'^2 + \frac{\tilde{g}'^2}{a} & 0 & -\tilde{g}'g'' \\ -\tilde{g}g'' & 0 & g''^2 & 0 \\ 0 & -\tilde{g}'g'' & 0 & g''^2 \end{pmatrix}; \tag{1.70}$$

It admits a massless eigenstate, which corresponds to the photon and is associated with the unbroken $U(1)$ symmetry of the vacuum:

$$A = \bar{g}\bar{s}_\theta \left(\frac{1}{\tilde{g}} \tilde{W}^3 + \frac{1}{\tilde{g}'} \tilde{B} + \frac{1}{g''} W_L^3 + \frac{1}{g''} W_R^3 \right), \tag{1.71}$$

where

$$\bar{s}_\theta = \frac{\bar{g}'}{\sqrt{\bar{g}^2 + \bar{g}'^2}}, \quad \text{with} \quad \bar{g}'^2 = \frac{\tilde{g}'^2 g''^2}{\tilde{g}'^2 + g''^2} \tag{1.72}$$

in analogy to (1.68). The remaining eigenvalues are

$$m_Z^2 = \frac{u^2}{4a}(\bar{g}^2 + \bar{g}'^2) + O\left(\frac{u^2}{4a^2}\right) \tag{1.73}$$

$$M_{L^3} = \frac{u^2}{4}\frac{g''^4}{g''^2 - \bar{g}^2} + \frac{u^2}{4a}\frac{\bar{g}^4}{g''^2 - \bar{g}^2} + O\left(\frac{u^2}{4a^2}\right) \tag{1.74}$$

$$M_{R^3} = \frac{u^2}{4}\frac{g''^4}{g''^2 - \bar{g}'^2} + \frac{u^2}{4a}\frac{\bar{g}'^4}{g''^2 - \bar{g}'^2} + O\left(\frac{u^2}{4a^2}\right); \tag{1.75}$$

then, calling $\mathbf{N}$ the matrix transforming the fields $\tilde{W}^3,\ \tilde{B},\ W_L^3,\ W_R^3$ that appear in the Lagrangian (1.61) in the mass eigenstates $A,\ Z,\ L^3,\ R^3$, I get

$$\mathbf{N} = \begin{pmatrix} \frac{\bar{g}'\bar{g}}{\tilde{g}\sqrt{\bar{g}'^2+\bar{g}^2}} & \frac{\bar{g}'\bar{g}}{\tilde{g}'\sqrt{\bar{g}'^2+\bar{g}^2}} & \frac{\bar{g}'\bar{g}}{g''\sqrt{\bar{g}'^2+\bar{g}^2}} & \frac{\bar{g}'\bar{g}}{g''\sqrt{\bar{g}'^2+\bar{g}^2}} \\ \frac{\bar{g}^2}{\tilde{g}\sqrt{\bar{g}'^2+\bar{g}^2}} & \frac{-\bar{g}'^2}{\tilde{g}'\sqrt{\bar{g}'^2+\bar{g}^2}} & \frac{\bar{g}^2}{g''\sqrt{\bar{g}'^2+\bar{g}^2}} & \frac{-\bar{g}'^2}{g''\sqrt{\bar{g}'^2+\bar{g}^2}} \\ -\frac{\bar{g}}{g''} & 0 & \frac{\bar{g}}{\tilde{g}} & 0 \\ 0 & -\frac{\bar{g}'}{g''} & 0 & \frac{\bar{g}'}{\tilde{g}'} \end{pmatrix}. \tag{1.76}$$

A preliminary study of the phenomenology of this model at the LHC or at a future linear collider has been done in [27]. Before going on, let me briefly consider fermions, which I have completely ignored up to this point. The simplest way to add them is to assume the usual representation assignments with respect to $SU(2)_L \times U(1)_Y$. Then, fermion mass terms can be generated by Yukawa couplings to the U field, as in the Higgsless SM. In this case fermion couplings to W and Z are the standard ones apart for the effect of the mixing with the additional vector bosons, and the fermions do not generate extra contributions to the EW precision parameters. Of course it is also possible to make a more general choice and add direct couplings of the fermions to the heavy gauge bosons. This extension [43, 44, 54] is certainly interesting, especially from the point of view of the continuum limit; however, for the sake of simplicity I will not consider it in this study.

The boson-fermion interactions will be then given by SM-like terms:

$$\begin{aligned}\mathcal{L}_{fermion} = &-\overline{\psi}\gamma^{\mu}\frac{(1-\gamma^5)}{2}\frac{\tau^a}{2}\psi\,\tilde{W}^a_{\mu} - \overline{\psi}\gamma^{\mu}\frac{(1-\gamma^5)}{2}\frac{B-L}{2}\psi\,\tilde{B}_{\mu}\\ &-\overline{\psi}\gamma^{\mu}\frac{(1+\gamma^5)}{2}\left(\frac{B-L}{2}+\frac{\tau^3}{2}\right)\psi\,\tilde{B}_{\mu}\end{aligned} \tag{1.77}$$

where ψ is a generic fermion doublet, and B, L are the baryon and lepton numbers respectively.

1.2.2 Low-energy limit

In this section, I am going to study the low energy limit of the model by eliminating the $\mathbf{W}_L$ and $\mathbf{W}_R$ fields using the solutions to their equations of motion for $M_{L,R} \to \infty$; in this limit, the kinetic terms of the new resonances are negligible, and the fields are not propagating degrees of freedom. The limit can be realized by sending $u \to \infty$; if a is also sent to infinity in such a way as to keep the scale v constant, the masses of the $W^{\pm}$ and Z fields remain finite.

The classical equations of motion for the $\mathbf{W}_{L,R}$ fields, as derived from Lagrangian (1.51) (adding the weak interactions) are:

$$\begin{aligned}\partial_{\mu}\mathbf{F}^{L\,\mu\nu} &= i[\mathbf{W}_{L\,\mu},\mathbf{F}^{L\,\nu\mu}] + \frac{u^2}{4}(\mathbf{W}^{\nu}_L - \tilde{\mathbf{W}}^{\nu})\\ \partial_{\mu}\mathbf{F}^{R\,\mu\nu} &= i[\mathbf{W}_{R\,\mu},\mathbf{F}^{R\,\nu\mu}] + \frac{u^2}{4}(\mathbf{W}^{\nu}_R - \tilde{\mathbf{B}}^{\nu}),\end{aligned} \tag{1.78}$$

where as usual $\mathbf{F}^{L,R}_{\mu\nu} = \partial_{\mu}\mathbf{W}_{L,R\,\nu} - \partial_{\nu}\mathbf{W}_{L,R\,\mu} + i[\mathbf{W}_{L,R\,\mu},\mathbf{W}_{L,R\,\nu}]$.

Let me solve eqs. (1.78) in the limit $u \to \infty$, $v = constant$. I get immediately

$$\mathbf{W}_{L\,\mu} = \tilde{\mathbf{W}}_{\mu}, \quad \mathbf{W}_{R\,\mu} = \tilde{\mathbf{B}}_{\mu}, \tag{1.79}$$

where the last equation means that only the third isospin component of $\mathbf{W}_R$ is different from zero. By substituting these equations in the total lagrangian (1.51) I get

$$\begin{aligned}\mathcal{L}_{eff} = &-\frac{v^2}{4}\mathrm{Tr}\big[\tilde{\mathbf{W}}_{\mu} - \tilde{\mathbf{B}}_{\mu}\big]^2\\ &+\frac{1}{2\tilde{g}^2}\mathrm{Tr}\big[F^{\mu\nu}(\tilde{\mathbf{W}})F_{\mu\nu}(\tilde{\mathbf{W}})\big] + \frac{1}{2\tilde{g}'^2}\mathrm{Tr}\big[F^{\mu\nu}(\tilde{\mathbf{B}})F_{\mu\nu}(\tilde{\mathbf{B}})\big]\\ &+\frac{1}{2g''^2}\mathrm{Tr}\big[F^{\mu\nu}(\tilde{\mathbf{W}})F_{\mu\nu}(\tilde{\mathbf{W}})\big] + \frac{1}{2g''^2}\mathrm{Tr}\big[F^{\mu\nu}(\tilde{\mathbf{B}})F_{\mu\nu}(\tilde{\mathbf{B}})\big]\end{aligned} \tag{1.80}$$

Eq. (1.80) shows that the effective contributions of the $\mathbf{W}_L$ and $\mathbf{W}_R$ particles add to the kinetic terms of the standard $\tilde{\mathbf{W}}$ and $\tilde{\mathbf{B}}$. By the following redefinition of the coupling constants

$$\frac{1}{g^2} = \frac{1}{\tilde{g}^2} + \frac{1}{g''^2}, \qquad \frac{1}{g'^2} = \frac{1}{\tilde{g}'^2} + \frac{1}{g''^2} \tag{1.81}$$

the effective Lagrangian becomes identical to the EW gauge part of the Lagrangian of the SM; the heavy degrees of freedom *decouple* from the theory in the limit $M_{(L,R)} \to \infty$. Note that the redefinition (1.81) is in fact identical to eqs. (1.68) and (1.72). In the decoupling limit for the heavy fields, I get $\bar{g} = g$ and $\bar{g}' = g'$.

Now, compare this case with the general one in which I have no enhanced symmetry, that is I keep arbitrary a_2, a_3 and a_4 in Lagrangian (1.45) and add the weak interactions by the minimal substitution (1.56). In the limit $u \to \infty$, it can be shown that the leading order solutions to the equations of motion for $\mathbf{W}_L$ and $\mathbf{W}_R$ are

$$\mathbf{W}_{L\,\mu} = \frac{1}{2}(1+z)\tilde{\mathbf{W}}_\mu + \frac{1}{2}(1-z)\tilde{\mathbf{B}}_\mu \tag{1.82}$$

$$\mathbf{W}_{R\,\mu} = \frac{1}{2}(1-z)\tilde{\mathbf{W}}_\mu + \frac{1}{2}(1+z)\tilde{\mathbf{B}}_\mu \tag{1.83}$$

where

$$z = \frac{a_3}{a_3 + a_4}. \tag{1.84}$$

The leading order effective Lagrangian becomes

$$\begin{aligned}
\mathcal{L}_{eff} = &-\frac{v^2}{4}\mathrm{Tr}\big[\tilde{\mathbf{W}}_\mu - \tilde{\mathbf{B}}_\mu\big]^2 \\
&+ \left(\frac{1}{2\tilde{g}^2} + \frac{1}{4g''^2}(1+z^2)\right)\mathrm{Tr}\big[F^{\mu\nu}(\tilde{\mathbf{W}})F_{\mu\nu}(\tilde{\mathbf{W}})\big] \\
&+ \left(\frac{1}{2\tilde{g}^2} + \frac{1}{4g''^2}(1+z^2)\right)\mathrm{Tr}\big[F^{\mu\nu}(\tilde{\mathbf{B}})F_{\mu\nu}(\tilde{\mathbf{B}})\big] \\
&+ \frac{1}{2g''^2}(1-z^2)\mathrm{Tr}\big[F^{\mu\nu}(\tilde{\mathbf{W}})F_{\mu\nu}(\tilde{\mathbf{B}})\big].
\end{aligned} \tag{1.85}$$

This is in general *not* equivalent to the SM Lagrangian. All the corrections to the standard case (that are, apart from a rescaling of the gauge coupling constants, contained in the mixing term $\mathrm{Tr}\big[F^{\mu\nu}(\tilde{\mathbf{W}})F_{\mu\nu}(\tilde{\mathbf{B}})\big]$), depend on the value of the parameter z. The choice $a_4 = 0$, $a_2 = a_3$, which characterizes the D-BESS model, implies $z = 1$, so that the corrections vanish. Note that it is not necessary to have $a_2 = a_3$ to get $z = 1$; however, the case $a_2 \neq a_3$ does not correspond to an increased symmetry of the Lagrangian, so it has no protection from radiative corrections, where deviations from the SM at leading order could reemerge. Similarly, the nonstandard term would vanish also for $z = -1$, but again this would not correspond to an enlargement of the symmetry. Finally, the limit which characterizes the D-BESS model is the only one in which one has a decoupling of the heavy fields in the limit of infinite mass.

Since the model at leading order in the low-energy limit $u \to \infty$ is indistinguishable from the SM, I have to go next-to-leading order to obtain the virtual effects of the new particles on EW precision observables, keeping also terms of order p^2/u^2. As it is well known, the dominant effect on precision observables comes from *oblique* corrections, or corrections to the gauge boson propagators; to study these contributions, I only need to calculate the bilinear effective Lagrangian. The solutions to eqs. (1.78) at next-to-leading order are:

$$\mathbf{W}_{L\,\mu} = \left(1 - \frac{\Box}{M^2}\right)\tilde{\mathbf{W}}_\mu + \Delta\mathbf{W}_{L\,\mu}, \tag{1.86}$$

$$\mathbf{W}_{R\,\mu} = \left(1 - \frac{\Box}{M^2}\right)\tilde{\mathbf{B}}_\mu + \Delta\mathbf{W}_{R\,\mu}, \tag{1.87}$$

where

$$\Delta\mathbf{W}_{L\,\mu} = \frac{1}{M^2}(\partial_\mu\partial^\nu\tilde{\mathbf{W}}^\mu - \partial_\mu[\tilde{\mathbf{W}}^\mu, \tilde{\mathbf{W}}^\nu] - [\tilde{\mathbf{W}}_\mu, F^{\mu\nu}(\tilde{\mathbf{W}})]), \tag{1.88}$$

$$\Delta\mathbf{W}_{R\,\mu} = \frac{1}{M^2}\partial_\mu\partial^\nu\tilde{\mathbf{B}}^\mu, \qquad M^2 = \frac{u^2 g''^2}{4}. \tag{1.89}$$

$\Delta\mathbf{W}_{L\,\mu}$ and $\Delta\mathbf{W}_{R\,\mu}$ contain linear terms proportional to the divergences of the fields and bilinear and trilinear terms that do not affect the self-energies of the EW gauge bosons, so I can neglect them in the derivation of my effective Lagrangian.

Substituting the solutions (1.86) in Lagrangian (1.51), writing down only the quadratic part and neglecting total divergences of the fields, I get:

$$\begin{aligned}\mathcal{L}^2_{eff} &= -\frac{1}{2}\tilde{W}^+_{\mu\nu}\tilde{W}^{-\,\mu\nu} - \frac{1}{4}\tilde{W}^3_{\mu\nu}\tilde{W}^{3\,\mu\nu} - \frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} \\ &+\frac{v^2g^2}{4}\tilde{W}^+_\mu\tilde{W}^{-\,\mu} + \frac{v^2g^2}{8}\tilde{W}^3_\mu\tilde{W}^{3\,\mu} + \frac{v^2g'^2}{8}\tilde{B}_\mu\tilde{B}^\mu - \frac{v^2gg'}{4}\tilde{W}^3_\mu\tilde{B}^\mu \\ &+\frac{1}{4g''^2}\frac{1}{M^2}\left(2g^2\tilde{W}^+_{\mu\nu}\Box\tilde{W}^{-\,\mu\nu} + g^2\tilde{W}^3_{\mu\nu}\Box\tilde{W}^{3\,\mu\nu} + g'^2\tilde{B}_{\mu\nu}\Box\tilde{B}^{\mu\nu}\right);\end{aligned} \tag{1.90}$$

I will now analyze the predictions of the effective Lagrangian (1.90) following ref. [55]. In the language of ref. [55], D-BESS is an "universal" theory, as long as the fermion only have standard couplings to the fields $\tilde{\mathbf{W}}$ and $\tilde{\mathbf{B}}$. These do not coincide with the standard $\mathbf{W}$ and $\mathbf{B}$, but rather are interpolating fields, made up of a superposition of the standard light and of the new heavy mass eigenstates. As long as their coupling to the fermions are of the standard form, eq. (1.77), the effects of new physics can be fully parametrized by the vacuum polarization amplitudes of these interpolating fields. Since the $\tilde{W}^3$ field mixes with $\tilde{B}$, I have in general four vacuum polarizations to consider, Π_{+-}, Π_{33}, Π_{BB}, Π_{3B}. Expanding up to the fourth power in momentum p, which is consistent with both the analysis of [55] and with Lagrangian (1.90), I can naively express these four self-energies in terms of 12 coefficients. However, these parameters are not all independent: three of them are fixed by EW input parameters (g, g', v):

$$\frac{1}{g^2} = \Pi'_{+-}(0), \quad \frac{1}{g'^2} = \Pi'_{BB}(0), \quad v^2 = -4\Pi_{+-}(0) \tag{1.91}$$

(derivatives of the Π's are taken with respect to p^2) and the masslessness of the photon implies two nontrivial relations among the coefficients of order zero. In the end I have to consider seven form factors, which can be defined as:

$$\begin{aligned}
\hat{S} = g^2\Pi'_{3B}(0), \quad \hat{T} = g^2 m_W^2(\Pi_{33}(0) - \Pi_{+-}(0)) \\
\hat{U} = -g^2(\Pi'_{33}(0) - \Pi'_{+-}(0)), \quad V = \frac{1}{2}g^2 m_W^2(\Pi''_{33}(0) - \Pi''_{+-}(0)) \\
X = \frac{1}{2}gg' m_W^2 \Pi''_{3B}(0), \quad Y = \frac{1}{2}g'^2 m_W^2 \Pi''_{BB}(0), \quad W = g^2 m_W^2 \Pi''_{33}(0).
\end{aligned} \tag{1.92}$$

From eq. (1.90), it is straightforward to calculate the correlators for the fields $\tilde{W}$ and $\tilde{B}$ and then to deduce the form factors defined in eq. (1.92). I get

$$\begin{aligned}
\Pi_{+-}(p^2) &= -\frac{1}{g^2}\left(g^2\frac{v^2}{4} - p^2 - p^4\frac{g^2}{M^2 g''^2}\right) \\
\Pi_{33}(p^2) &= -\frac{1}{g^2}\left(g^2\frac{v^2}{4} - p^2 - p^4\frac{g^2}{M^2 g''^2}\right) \\
\Pi_{BB}(p^2) &= -\frac{1}{g'^2}\left(g'^2\frac{v^2}{4} - p^2 - p^4\frac{g'^2}{M^2 g''^2}\right) \\
\Pi_{3B}(p^2) &= \frac{v^2}{4}.
\end{aligned} \tag{1.93}$$

It is straightforward to check that eq. (1.91) is identically satisfied; there is no need to rescale the fields or redefine the parameters g, g', v. Furthermore, as it follows from the equality of Π_{+-} and Π_{33} and the expression for Π_{3B} in eq. (1.93), I get $\hat{S} = \hat{T} = \hat{U} = V = X = 0$; there are only two nonvanishing form factors, namely W and Y:

$$W = \frac{g^2 m_W^2}{M^2 g''^2}, \quad Y = \frac{g'^2 m_W^2}{M^2 g''^2}. \tag{1.94}$$

For convenience, I also report the expressions for the contributions from new physics to the ϵ parameters [28, 29, 30], since these are better constrained by the data and widely used in the literature. The contributions can be deduced from the form factors [55]; I get,

$$\begin{aligned}\epsilon_1 &= \hat{T} - W + \frac{2Xs_\theta}{c_\theta} - \frac{Ys_\theta^2}{c_\theta^2},\\ \epsilon_2 &= \hat{U} - W + \frac{2Xs_\theta}{c_\theta} - V,\\ \epsilon_3 &= \hat{S} - W + \frac{X}{s_\theta c_\theta} - Y,\end{aligned} \tag{1.95}$$

where $\tan(\theta) = g'/g$. I find for the D-BESS model:

$$\epsilon_1 = -\frac{(c_\theta^4 + s_\theta^4)}{c_\theta^2}\overline{X}, \quad \epsilon_2 = -c_\theta^2\overline{X}, \quad \epsilon_3 = -\overline{X}, \tag{1.96}$$

with $\overline{X}$ given by

$$\overline{X} = \frac{m_Z^2}{M^2}\left(\frac{g}{g''}\right)^2. \tag{1.97}$$

Note that all the new physics contributions to the ϵ parameters are negative and of $O(m_Z^2/M^2)$. The decoupling property of the model is manifest in these expressions: in the limit $M \to \infty$ all the corrections to the ϵ parameters vanish. The factor m_Z^2/M^2 greatly suppress the contribution to the EW precision parameters; in the general $G \otimes H'$-invariant case, which does not show the decoupling, I would expect a factor of order unity in place of m_Z^2/M^2 in eq. (1.97). Furthermore, the $\overline{X}$ parameter contains another suppression factor, that is g^2/g''^2. The presence of this double suppression factor can have a crucial effect in allowing a relatively low-scale mass for the new resonances.

Chapter 2

Generalized D-BESS

2.1 Dimensional deconstruction

The D-BESS model that I reviewed in the previous chapter can be naturally generalized to a 5-dimensional theory. This extra-dimensional generalization is interesting at least for one reason: the resulting model is strikingly similar to a particular version of the RS1 model [37]. The D-BESS and the RS models were formulated following radically different approaches, so the connection between the two is nontrivial.

The key idea in generalizing the D-BESS construction [31, 34] is that of "dimensional deconstruction" which was introduced in a seminal paper by Arkani-Hamed, Cohen and Georgi (ACG) [32] and independently by Cheng, Hill and Pokorski [33]. The paper contained a compelling reflection on just what *is* an "extra dimension", along with the proposal of a simple model which, in a sense, *dynamically generates* a 5^{th} dimension.

What is the general behaviour of a theory in extra dimensions? We know, from the very direct point of view of everyday life, that the world appears to be 4 dimensional. However, it is possible that at short distance scales, shorter than those so far probed by collider experiments, the simplest description of natural phenomena may involve more than the ordinary three spacial and one temporal dimensions. The extra dimensions have to be radically different from the ones of our ordinary experience, and the simplest possibility is that they are *compact*, with a compactification scale R small enough to have rendered them invisible to experiments conducted so far. At large distances compared to the size of these compact dimensions, such a theory appears four dimensional, showing all the typical scaling properties: massless gauge forces fall off like the square of the distance, free energies of massless degrees of freedom scale like the fourth power of the temperature, and so forth. At energies roughly corresponding to the inverse compactification scale $\sim 1/R$, Kaluza-Klein (KK) excitations appear with a spectrum dictated by the nature of the compact space. At energies much higher than this scale, the extra dimensions become manifest: physics at distances much smaller than the compactification size does not feel the compactification, and the behaviour is that of a full-fledged higher dimensional theory.

There is one trouble with this picture: an extra-dimensional gauge theory has dimensionful couplings, so it is inherently nonrenormalizable. The theory thus requires a cut-off and, eventually, an UV completion. Furthermore, it may happen that the cut-off is not much higher than $1/R$ (that is, of the mass of the first or second KK excited state), so that the theory never reaches the stage in which the extra dimensions are really manifest. This is not a problem if one wishes to adopt the point of view of the effective theory, *i.e.* is not trying to write a complete, fundamental theory but just a phenomenological description valid in a limited energy range; however, the problem of giving a sensible UV completion is certainly a pending issue of extra-dimensional models, and may become phenomenologically relevant if one wishes to study the physics near the cut-off scale.

The proposal by ACG was, in a sense, to avoid the UV completion problem altogether by *dynamically building* an extra dimension. That is, one does not start with a 5-dimensional model; instead, a purely 4-dimensional model is built in such a way as to approximate a 5-dimensional one at *low* energies.

Let me illustrate the approach. Consider a $G^N \otimes G_s^{N-1}$ gauge theory in ordinary 4D spacetime, with $G = G_s = SU(2)$ and with a discrete symmetry imposed so that all G groups have a common coupling constant g and all G_s groups a different common coupling constant g_s. The theory also contains Weyl fermions, each transforming as a doublet under a $G \otimes G_s$ subgroup of the full gauge group, following a "nearest-neighbour" logic: that is, labelling the groups:

$$G \to G^i, \quad i = 1, \dots, N \qquad G_s \to G_s^i, \quad i = 1, \dots, N-1, \tag{2.1}$$

I have two kind of fermions, χ^i and ψ^i, that transform as

$$\chi^i \to g^i \chi^i \, g_s^{i\,\dagger}, \quad \psi^i \to g_s^i \psi^i \, g^{i+1\,\dagger}; \qquad i = 1, \dots, N-1, \tag{2.2}$$

with

$$g^i \in G^i, \quad g_s^i \in G_s^i.$$

The Lagrangian of the model, writing only superficially renormalizable terms, is thus

$$\begin{aligned}\mathcal{L} = \sum_{i=1}^{N-1} \Big[& \bar{\chi}^i \gamma^\mu D_\mu \chi^i + \bar{\psi}^i \gamma^\mu D_\mu \psi^i + \frac{1}{2g^2} \mathrm{Tr}\big[\mathbf{F}^i_{\mu\nu} \mathbf{F}^{i\,\mu\nu}\big] \\ & + \frac{1}{2g_s^2} \mathrm{Tr}\big[\mathbf{F}^{s,i}_{\mu\nu} \mathbf{F}^{s,i\,\mu\nu}\big] \Big] + \frac{1}{2g^2} \mathrm{Tr}\big[\mathbf{F}^N_{\mu\nu} \mathbf{F}^{N\,\mu\nu}\big],\end{aligned} \tag{2.3}$$

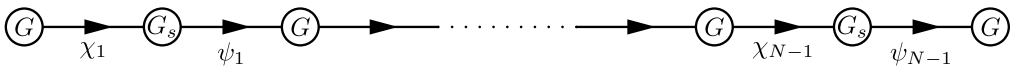


Figure 2.1: *The moose diagram describing model* (2.2)*. Circles represent gauge groups, and oriented straight lines represent fermions*

where $\mathbf{F}^{i(s,i)}_{\mu\nu}$ is the usual field strength for the gauge fields $\mathbf{A}^{i(s,i)} \equiv A^{a\,i(s,i)}\tau^a/2$ corresponding to the group $G^i(G^i_s)$, and the covariant derivatives of the fermions are defined as

$$D_\mu \chi^i = \partial_\mu \chi^i + i\, g \mathbf{A}^i_\mu \chi^i - i\, g_s \chi^i \mathbf{A}^{s,i}_\mu \tag{2.4}$$

$$D_\mu \psi^i = \partial_\mu \psi^i + i\, g_s \mathbf{A}^{s,i}_\mu \psi^i - i\, g \psi^i \mathbf{A}^{i+1}_\mu, \quad i = 1, \dots N-1, \tag{2.5}$$

in agreement with the transformation law (2.2).

This content of fields and symmetries can be conveniently summarized in terms of a so-called "moose" diagram. In such diagrams, circles are used to represent gauge groups, and oriented straight lines to represent fermions; a line moving away from a circle represents a set of Weyl fermions transforming as the fundamental representation of the corresponding gauge group, while a line moving towards a circle a set of Weyl fermions transforming as the conjugate of the fundamental. The theory described by eq. (2.2) corresponds to the diagram of fig. 2.1.

Note that by the Renormalization Group equations, the couplings g and g_s can be "dimensionally transmuted" to dimension-one parameters Λ and Λ_s; for example, for a pure $SU(2)$ theory with coupling $\tilde{g}$ I have, at the 1-loop level,

$$\alpha_{\tilde{g}} \equiv \frac{\tilde{g}^2}{4\pi} = \frac{3\pi}{11} \ln\left(\frac{\tilde{\Lambda}}{\mu}\right), \tag{2.6}$$

where μ is the renormalization scale. The dimensionful parameter $\tilde{\Lambda}$ sets the scale where the $SU(2)$ theory becomes strongly interacting.

The moose model (2.3) is asymptotically free, so that at high energy scales $E \gg \Lambda,\ \Lambda_s$, it is perturbative and describes a set of weakly interacting fermions and gauge bosons. But the phenomenology of the theory changes drastically at lower energies. I am interested in the limit $\Lambda_s \gg \Lambda$. Then, when $E \sim \Lambda_s$, the groups G are still in a perturbative regime, while the groups G_s become strongly interacting. By comparison with ordinary strong interactions I then expect the fermions to form condensates, with each pair of fermions connected to a given strong gauge group G^i_s acquiring a nonzero VEV:

$$\langle \chi^i \psi^i \rangle \sim 4\pi f_s^3 U^i, \quad i = 1, \dots N-1, \tag{2.7}$$

where $f_s = \Lambda_s/4\pi$ and U^i is a 2x2 unitary matrix. I also expect the emergence of a spectrum of hadron-like composite states with a typical mass scale of order Λ_s.

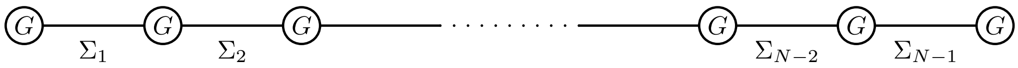


Figure 2.2: *The moose diagram describing model* (2.9)*. Circles represent gauge groups, and straight lines represent $SU(2)$ scalar multiplets.*

At energies *below* the scale Λ_s, the theory can be described as a nonlinear model with N gauge fields coupled to $N-1$ $SU(2)$-valued fields Σ^i, each transforming as

$$\Sigma^i \to g^i \Sigma^i {g^{i+1}}^\dagger. \tag{2.8}$$

This low-energy effective model can still be described by a moose diagram, where this time the solid lines represent $SU(2)$ fields transforming as in eq. (2.8) (fig. 2.2). The Lagrangian for the model can be written as

$$\mathcal{L}_{eff} = -\frac{1}{2g^2}\sum_{i=1}^{N} \mathrm{Tr}\big[\mathbf{F}^i_{\mu\nu})\big]^2 + f_s^2 \sum_{i=1}^{N-1} \mathrm{Tr}\big[D_\mu {\Sigma^i}^\dagger D^\mu \Sigma^i\big] + \ldots, \tag{2.9}$$

where the covariant derivative of the Σ^i field is defined by

$$D_\mu \Sigma^i = \partial_\mu \Sigma^i + i\mathbf{A}^i_\mu \Sigma^i - i\Sigma^i \mathbf{A}^{i+1}_\mu \tag{2.10}$$

and the dots stand for higher order terms, which are suppressed at low energy.

There are two important observations to be made on the low-energy moose Lagrangian. The first one is that eq. (2.9) describes, in fact, the discretized version of a 5D pure $SU(2)$ model, where the 5^{th} dimension has been latticized. Each value of the superscript on the fields corresponds to a different location along the extra dimension. The Σ^i fields are the so-called "link variables", always present in lattice gauge theory; they allow interactions between gauge fields located at neighbouring sites in the discretized dimension without spoiling the overall gauge invariance.

It is possible to establish a precise correspondence between the latticized and the full 5D theory [32, 33, 56]:

$$\frac{N}{gf_s} \to L, \quad \frac{N}{g^2} \to \frac{L}{g_5^2}, \tag{2.11}$$

where L is the length of the extra dimension and g_5 the 5-dimensional gauge coupling. The Σ^i fields can in turn be identified as Wilson lines connecting adjacent points along the extra dimension:

$$\Sigma^i = P\, exp\left(\int_{y_i}^{y_{i+1}} dy \mathbf{A}_5(y)\right), \tag{2.12}$$

where the y is the extra coordinate, the y_i are the vertices of the lattice and $\mathbf{A}_5$ is the 5^{th} component of the gauge field. The continuum description is recovered in the limit $N \to \infty$. Notice that, if the extra dimension is compact, that is, it has a finite L, this implies that f_s must also be sent to infinity. In the original construction, this is equivalent to send $\Lambda_s \to \infty$, *i.e.* to have G_s strongly interacting at an infinite scale.

The second relevant observation is that Lagrangian (2.9) has several aspects in common with an hidden symmetry construct, such as the D-BESS (see chapter 1 and in particular Lagrangian (1.51).

In fact, eq. (2.9) describes a collection of $SU(2)$-valued fields and $SU(2)$ gauge fields. Its vacuum state corresponds to $\Sigma^i \equiv \mathbf{I}$, $\forall i$, implying a spontaneous symmetry breaking:

$$G \equiv SU(2)^N \to H \equiv \left[SU(2)^N\right]_{diag.}; \tag{2.13}$$

this gives rise to $3N-3$ goldstone bosons that, since the symmetry that gets broken is local, are eaten by as many gauge bosons which become massive. The spectrum thus consists of $3N-3$ massive and 3 massless gauge bosons with no surviving scalar. The Lagrangian is not the most general that could be written with this field content and symmetry breaking pattern; it rather corresponds - and this also reminds D-BESS - to a very peculiar choice of the parameters, that makes the resulting theory identical to a discretized 5D $SU(2)$ gauge theory.

In view of this relation between moose models and models with hidden symmetries, a natural question arises: is it possible to generalize D-BESS in order to obtain a moose model sharing its most appealing characteristic, namely the doubly-suppressed contribution to the oblique parameters?

Answering to this question will be the main point of the next section.

2.2 GD-BESS: a moose model with "vanishing" S (or ϵ_3) parameter

Consider a moose model with the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\sum_{i=1}^{K}\frac{1}{g_i^2}\mathrm{Tr}\big[\mathbf{F}^i_{\mu\nu})\big]^2 + \sum_{i=1}^{K+1} f_i^2\mathrm{Tr}\big[D_\mu\Sigma^{i\dagger}D^\mu\Sigma^i\big], \tag{2.14}$$

where

$$D_\mu\Sigma^i = \partial_\mu\Sigma^i + i\mathbf{A}^{i-1}_\mu\Sigma^i - i\Sigma^i\mathbf{A}^i_\mu, \quad \mathbf{A}^0 = \mathbf{A}^{K+1} = 0. \tag{2.15}$$

Eq. (2.14) is a generalization of eq. (2.9); I have dropped the symmetry requirement that forced all the g_i and all the f_i - I will call these last parameters "link coupling constants" - to be equal, and added two more "link scalars", namely Σ^0

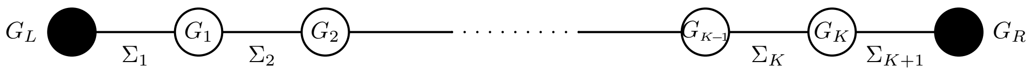


Figure 2.3: *The moose diagram describing model* (2.14). *Circles represent gauge groups, and straight lines represent* $SU(2)$ *scalar multiplets. Black circles at the ends of the moose represent global symmetries.*

and Σ^{K+1}. In addition to its $SU(2)^K$ gauge invariance, the new Lagrangian has a $SU(2)_L \otimes SU(2)_R$ global symmetry: it is possible to multiply Σ^0 on the left and Σ^{K+1} on the right by two independent constant $SU(2)$ matrices without altering the Lagrangian. The theory can be represented by the moose in fig. 2.3.

Once again, the vacuum state corresponds to $\Sigma^i \equiv \mathbf{I}$. This time, the symmetry breaking is $SU(2)^{K+2} \to \left[SU(2)^{K+2}\right]_{diag.}$[1]; $3K+3$ goldstone bosons arise from the symmetry breaking, of which only $3K$ disappear to provide masses for the gauge fields. The mass matrix for the vector fields can be easily obtained by setting all the Σ^i equal to their VEV, $\Sigma^i \to \mathbf{I}$, and by rescaling $\mathbf{A}^i \to g_i \mathbf{A}^i$. I get

$$\mathcal{L}_{mass} = \sum_{i=1}^{K+1} f_i^2 \mathrm{Tr}\left[g_{i-1}\mathbf{A}^{i-1} - g_i \mathbf{A}^i\right]^2 \equiv \sum_{i,j=1}^{K} (\mathbf{M}_{(2)})_{ij} \mathbf{A}^i_\mu \mathbf{A}^{j\,\mu} \tag{2.16}$$

with

$$(\mathbf{M}_{(2)})_{ij} = g_i^2 (f_i^2 + f_{i+1}^2)\delta_{ij} - g_i g_{i+1} f_{i+1}^2 \delta_{i,j-1} - g_j g_{j+1} f_{j+1}^2 \delta_{i,j+1}. \tag{2.17}$$

By construction, $\mathbf{M}_{(2)}$ is a symmetric matrix and can be diagonalized by an orthogonal transformation S. I assume that the parameters g_i, f_i are chosen in such a way that $\mathbf{M}_{(2)}$ is positive definite, otherwise I will certainly get an unphysical picture. I will call $\tilde{\mathbf{A}}^i$ the mass eigenstates and M_i^2 the corresponding eigenvalues, so that

$$\mathbf{A}^i = \sum_{n=1}^{K} S_{in} \tilde{\mathbf{A}}^n, S^T_{in} (\mathbf{M}_{(2)})_{nm} S_{mj} = M_i^2 \delta_{ij}. \tag{2.18}$$

2.2.1 ϵ_3 at leading order

Historically, one of the biggest difficulties with TC models has been to get a sufficiently small contribution to the S parameter [41, 42], or the related ϵ_3 [28, 29, 30]. In the case of the D-BESS model, however, I have shown (sec. 1.2.2) that the contribution is suppressed and vanishes in the limit of infinite mass for the new

[1] in general; as I am going to show, for special choices of the parameters, the symmetry of the vacuum may be enlarged

resonances, due to decoupling. To get a proper generalization of D-BESS in the present case of a linear moose, I will now calculate the leading correction to ϵ_3 by the new physics described by Lagrangian (2.14) by making use of the dispersive relation introduced in refs. [41, 42]. I have

$$\epsilon_3 = -\frac{g^2}{4\pi}\int_0^\infty \frac{ds}{s^2} Im[\Pi_{VV}(s) - \Pi_{AA}(s)], \tag{2.19}$$

where g is the weak isospin gauge coupling, and $\Pi_{VV(AA)}$ is the current-current correlator

$$\begin{aligned} \int d^4x\, e^{-iq\cdot x}\langle \mathbf{J}^\mu_{V(A)} \mathbf{J}^\nu_{V(A)}\rangle &= i g^{\mu\nu}\Pi_{VV(AA)}(q^2) \\ &+ (q^\mu q^\nu - \text{proportional terms}), \end{aligned} \tag{2.20}$$

where $\mathbf{J}^\mu_{V(A)}$ is the vector (axial) Noether current associated with the global $SU(2)_L \otimes SU(2)_R$ symmetry acting at the ends of the moose; I have

$$\mathbf{J}^\mu_{V(A)} = J^{a\,\mu}_{V(A)} \tau_{V(A)}, \tag{2.21}$$

with $\tau_{V(A)}$ being the vector (axial) generators of $SU(2)_L \otimes SU(2)_R$.

I can calculate the correlator $\Pi_{VV(AA)}$, and then ϵ_3, by defining vector (axial) "decay constants" for the mass eigenstates $\tilde{\mathbf{A}}^i$ from the matrix elements of the vector (axial) current between the vacuum and the one vector boson state, as

$$\langle 0|J^{a\,\mu}_{V(A)}|\tilde{A}^{b\,i}(p,\epsilon)\rangle = g_{i\,V(A)}\delta^{ab}\epsilon^\mu, \tag{2.22}$$

where $|\tilde{A}^{b\,i}(p,\epsilon)\rangle$ is the b isospin component of the i^{th} vector mass eigenstate with polarization ϵ. Then I can write

$$Im(\Pi_{VV(AA)}(s)) = -\pi \sum_i g^2_{iV(A)}\delta(s - M_i^2), \tag{2.23}$$

so that

$$\epsilon_3 = \frac{g^2}{4}\sum_i \left(\frac{g^2_{i\,V}}{M_i^4} - \frac{g^2_{i\,A}}{M_i^4}\right). \tag{2.24}$$

The decay constants (2.22) can in turn be easily calculated by looking at how the fields Σ_i transform under vector and axial $SU(2)_L \otimes SU(2)_R$ transformations. I get

$$\begin{aligned} \text{vector}: \Sigma^1 \to T\,\Sigma^1, &\quad \Sigma^{K+1} \to \Sigma^{K+1}T^\dagger, \\ \text{axial}: \Sigma^1 \to T\,\Sigma^1, &\quad \Sigma^{K+1} \to \Sigma^{K+1}T, \end{aligned} \tag{2.25}$$

where T is a generic $SU(2)$ matrix and the fields Σ^i with $i = 2, \ldots K$ are invariant. Then the vector and axial vector currents are

$$J^a_{V(A)\,\mu} = f_1^2 g_1 A^{a\,1}_\mu \pm f^2_{K+1} g_K A^{a\,K}_\mu, \tag{2.26}$$

and the decay constants

$$g_{i\,V(A)} = f_1^2 g_1 S_{1i} \pm f_{k+1}^2 g_K S_{Ki}, \tag{2.27}$$

with the S_{ij} defined in eq. (2.18).

Substituting eq. (2.27) in eq. (2.24) I find

$$\epsilon_3 = g^2 g_1 g_K f_1^2 f_{K+1}^2 \sum_i \frac{S_{1i} S_{Ki}}{M_i^4} = g^2 g_1 g_K f_1^2 f_{K+1}^2 (\mathbf{M}_{(2)}^{-2})_{1K}. \tag{2.28}$$

The above expression can be rewritten [31]:

$$\epsilon_3 = g^2 \sum_{i=1}^{K} \frac{(1-y_i) y_i}{g_i^2}, \tag{2.29}$$

where I introduced the notation:

$$y_i = \sum_{j=1}^{i} \frac{f^2}{f_j^2}, \quad \text{with} \quad \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2}. \tag{2.30}$$

Eqs. (2.30) implies $0 \leqslant y_i \leqslant 1$; then from eq. (2.29) I get immediately:

$$\epsilon_3 \geqslant 0, \tag{2.31}$$

that is ϵ_3 is a semipositive definite expression. Moreover, if all the coupling constants are of the same order, and the same holds for the link coupling constants:

$$g_i \sim g_c, \quad f_i \sim f_c, \quad \forall i, \tag{2.32}$$

I can easily estimate the typical size for ϵ_3 as

$$\epsilon_3 \sim \frac{g^2}{g_c^2} \tag{2.33}$$

It is important to bear in mind that, in all the calculations in this section, I have consistently neglected the weak interactions: the Lagrangian (2.14) only describes the new physics contribution. To have a potentially realistic theory, I must add four new vector bosons by gauging an $SU(2)_L \otimes U(1)_Y$ subgroup of the global $SU(2)_L \otimes SU(2)_R$. These will mix with the fields $\mathbf{A}^i$ and four new eigenstates will appear, eliminating the three remaining goldstone bosons from the spectrum (one of them is massless). The $SU(2)_L \otimes U(1)_Y$ gauge fields will also contribute to the vector and axial currents, changing the expression (2.28) for ϵ_3. However, as long as all the new physics eigenstates are heavy, with masses $M_i \gg m_Z$, I expect the corrections to (2.28) to be suppressed by a factor of order $m_Z^2/\bar{M}^2$, where $\bar{M}$ is a characteristic mass scale of the heavy sector, roughly equal to the smallest of the masses M_i. The result (2.28) can be regarded as the leading order in an expansion on the weak interactions.

2.2.2 Cutting a link

The calculations of the preceding section led to an explicit leading-order formula for ϵ_3. We have seen that ϵ_3 is semipositive definite. Is it possible to saturate the bound and get ϵ_3 to vanish at this order? The answer to this question is positive. In fact, if any of the f_i, with $i = 2, \cdots, K$, vanishes, then the mass matrix $\mathbf{M}_{(2)}$ is block-diagonal (this can be easily checked by looking at its definition (2.17)); it follows then that $\mathbf{M}_{(2)}^{-2}$ is also block-diagonal, implying $(\mathbf{M}_{(2)}^{-2})_{1K} = 0$ and $\epsilon_3 = 0$ by eq. (2.28). I will refer to this situation as "cutting a link" [31].

The same result can be also derived from the explicit expression (2.29). In fact, if I send $f_m \to 0$, I have by eq. (2.30) also $f^2 \to 0$, $f_i \to \delta_{i,m}$ and $y_i = \sum_{j=1}^{i} \delta_{j,m} = \theta_{i,m}$, where I defined the discrete step function

$$\theta_{i,j} = \begin{cases} 1, & for\, i \geq j, \\ 0, & for\, i < j. \end{cases} \tag{2.34}$$

Then I obtain

$$\epsilon_3 = g^2 \sum_{i=1}^{K} \frac{(1-\theta_{i,m})\theta_{i,m}}{g_i^2} = 0. \tag{2.35}$$

However, this choice has a drawback: cutting a link corresponds to losing one of scalar multiplets Σ^i, which is instead necessary to give masses to the gauge bosons of the standard $SU(2)_L \times U(1)_Y$. This problem can be solved by adding to the Lagrangian a new term,

$$f_0^2 Tr[\partial_\mu U^\dagger \partial^\mu U], \tag{2.36}$$

where U is an $SU(2)$-valued field transforming under an $SU(2)_{\tilde{L}} \otimes SU(2)_{\tilde{R}}$ (prior to the gauging of the weak interactions) as

$$U \to \tilde{g}_L U \tilde{g}_R^\dagger, \quad \tilde{g}_{L(R)} \in SU(2)_{\tilde{L}(\tilde{R})}, \tag{2.37}$$

and f_0 is a new parameter related to the Fermi scale. This additional term does not contribute to ϵ_3 to leading order, because the U field does not couple to the gauge fields A_μ^i, $i = 1, \cdots, K$ so that the gauge boson mass matrix $\mathbf{M}_{(2)}$ - still before the switching on of the weak interactions - is not changed by the addition of the U kinetic term.

It is interesting to note that in the full model without a cut link, a similar field U' transforming under $SU(2)_L \otimes SU(2)_R$ can be realized as a product of all the Σ^i:

$$U' = \prod_{i=1}^{K+1} \Sigma^i; \tag{2.38}$$

this definition is analogous to eq. (1.50) in the D-BESS model.

Notice also, that the fact that cutting a link leads to a vanishing of ϵ_3, can be understood in terms of an enhancement of the global symmetry. The symmetry group is in fact enlarged from

$$SU(2)_L \otimes SU(2)_R \otimes \big(SU(2)\big)^K \tag{2.39}$$

to

$$SU(2)_L \otimes SU(2)_R \otimes \big(SU(2)\big)^K \otimes SU(2)_{\tilde{L}} \otimes SU(2)_{\tilde{R}}, \tag{2.40}$$

(2.41)

since the global symmetry $SU(2)_{\tilde{L}} \otimes SU(2)_{\tilde{R}}$ under which the kinetic term for the field U is invariant does not coincide with the symmetry $SU(2)_L \otimes SU(2)_R$ acting upon the scalar fields Σ_1 and Σ_{K+1}. Note that this is no longer true when I gauge the weak interactions; then I need to identify $SU(2)_{\tilde{L}(\tilde{R})}$ with $SU(2)_{L(R)}$, otherwise the U field is decoupled from the rest of the Lagrangian and cannot supply the goldstone modes needed to give mass to the $W^\pm$ and the Z.

The enhanced global symmetry acts as a custodial symmetry [57], and this explains why the ϵ_3 parameter is vanishing at leading order; when the symmetry is broken by adding the weak interactions, nonzero contributions of order $m_Z^2/\bar{M}^2$ are expected.

The new Lagrangian for the model, with the m link cut and the field U, is given by

$$\begin{aligned}\mathcal{L} =& f_0^2 \mathrm{Tr}\big[\partial_\mu U^\dagger \partial^\mu U\big] + \sum_{i=1}^{m-1} f_i^2 \mathrm{Tr}\big[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i\big] \\ &+ \sum_{i=m+1}^{K+1} f_i^2 \mathrm{Tr}\big[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i\big] - \frac{1}{2g_i^2}\sum_{i=1}^{K} \mathrm{Tr}\big[(F^i_{\mu\nu})^2\big].\end{aligned} \tag{2.42}$$

The model corresponding to the Lagrangian (2.42) is shown in Fig. 2.4. Before the weak gauging I have three disconnected chains (this is the reason why the symmetry gets enhanced). It is worth underlining that the difference with respect to the linear moose model (2.14) lies the fact that a link is cut (a link coupling constant f_m is vanishing) and that the invariant term containing the corresponding scalar field Σ_m is substituted by the invariant involving the field U coupling the two ends of the chain. Cutting a link implies that, in the unitary gauge, the gauge fields A^i_μ become massive by eating the Σ_i fields, while the goldstone bosons which will give masses to the standard gauge bosons once the gauge group $SU(2)_L \times U(1)_Y$ is switched on are contained in the U field.

In the end, I have succeeded in showing that it is possible to build a moose model with an extra custodial symmetry that forces all the ϵ_i parameters to be zero at leading order in the weak interactions (note that the parameters ϵ_1 and ϵ_2 are vanishing too, because of the presence of the usual $SU(2)_{L+R}$ custodial symmetry [48]).

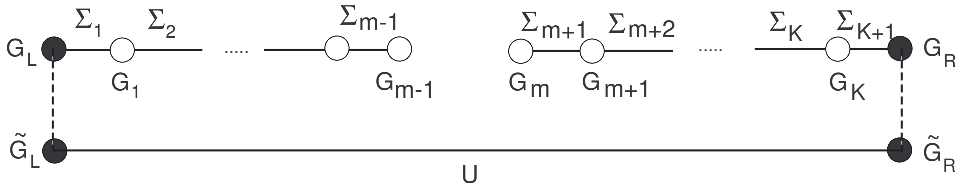


Figure 2.4: *Graphic representation of the linear moose model with the m link cut described by the Lagrangian (2.42). The dashed lines represent the identification of the global symmetry groups after weak gauging.*

2.2.3 The ϵ parameters at next-to-leading order

I am now going to switch on the gauge interactions in order to calculate the next-to-leading order ($\sim m_Z^2/\bar{M}^2$) corrections to the ϵ parameters. As I saw in section 1.2.1, the first step is to promote a $SU(2)_L \otimes U(1)_Y$ subgroup of $SU(2)_L \otimes SU(2)_R$ to a gauge symmetry, by generalizing the covariant derivatives of Σ^1 and Σ^K via the minimal substitution

$$\begin{aligned} D_\mu \Sigma_1 &= \partial_\mu \Sigma_1 + i\tilde{\mathbf{W}}_\mu \Sigma_1 - i\Sigma_1 \mathbf{A}^1_\mu, \\ D_\mu \Sigma_{K+1} &= \partial_\mu \Sigma_{K+1} + i\mathbf{A}^K_\mu \Sigma_{K+1} - i\Sigma_{K+1}\tilde{\mathbf{B}}_\mu, \end{aligned} \tag{2.43}$$

where $\tilde{\mathbf{W}} \equiv \tilde{W}^a \frac{\tau^a}{2}$ and $\tilde{\mathbf{B}} \equiv \tilde{B}^3 \frac{\tau^3}{2}$ are the $SU(2)_L \otimes U(1)_Y$ gauge fields, promoting the derivatives of U to covariant derivatives:

$$D_\mu U = \partial_\mu U + i\tilde{W}^a_\mu \frac{\tau^a}{2} U - i\tilde{B} U \frac{\tau^3}{2} \tag{2.44}$$

and adding standard kinetic terms for $\tilde{\mathbf{W}}$ and $\tilde{\mathbf{B}}$:

$$-\frac{1}{2\tilde{g}^2} \mathrm{Tr}\left[\mathbf{F}^{\tilde{W}}_{\mu\nu}\right]^2 - \frac{1}{2\tilde{g}'2} \mathrm{Tr}\left[\mathbf{F}^{\tilde{B}}_{\mu\nu}\right]^2, \tag{2.45}$$

where $\tilde{g}$, $\tilde{g}'$ are the $SU(2)_L \otimes U(1)_Y$ coupling constants and

$$\begin{aligned} \mathbf{F}^{\tilde{W}}_{\mu\nu} &= \partial_\mu \tilde{\mathbf{W}}_\nu - \partial_\nu \tilde{\mathbf{W}}_\mu + i[\tilde{\mathbf{W}}_\mu, \tilde{\mathbf{W}}_\nu], \\ \mathbf{F}^{\tilde{B}}_{\mu\nu} &= \partial_\mu \tilde{\mathbf{B}}_\nu - \partial_\nu \tilde{\mathbf{B}}_\mu. \end{aligned} \tag{2.46}$$

Up to now, I have not required any particular symmetry for the coupling constants g_i and the link couplings f_i. Henceforth, however, I will impose a reflection invariance with respect to the ends of the moose, both for simplicity and to obtain a more direct generalization of the D-BESS. I get the following relations among the couplings

$$f_i = f_{K+2-i}, \quad g_i = g_{K+1-i}. \tag{2.47}$$

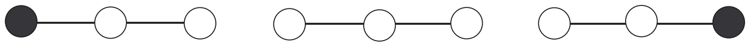


Figure 2.5: *For $K = 2N + 1$, putting one of the f_i to zero in a reflection invariant model one is left with a string containing more vector fields than scalars.*

Figure 2.6: *For $K = 2N$, cutting the central link I am left with two strings, each of them ending with a gauge field.*

Then, If K is odd, $K = 2N + 1$, I have an even number of scalar fields, and when I put one of the link couplings, say f_m to zero, this implies that also f_{K+2-m}, which is connected to f_m by the reflection symmetry must vanish. This leads to an unphysical situation, for I do not have enough scalars to give mass to all vector multiplets. This situation is illustrated in Fig. 2.5: the original string is broken in three pieces with the central one containing more vector fields than scalar ones. As a consequence there are massless vector fields in the spectrum of the theory. The matrix $\mathbf{M}_{(2)}$ is singular and eq. (2.28) is not applicable as it stands.

The situation is different for K even, $K = 2N$, since in this case I can cut the central link, remaining in the condition depicted in Fig. 2.6: I am left with two disconnected strings, each of them with a gauge field at one end point. With the field U, I have just enough scalars to give mass to every vector field, and $\mathbf{M}_{(2)}$ is still nonsingular. Another interesting point is that, due to the reflection invariance, the two blocks of the mass matrix (which, remember, is block diagonal) are equal. Therefore there is - prior to adding the weak interactions - complete degeneracy between vector and axial vector resonances. The limit $N = 1$ in this case corresponds exactly to the D-BESS model, with the identifications $g_1 = g''$ and $f_1^2 = u^2/4$. For these reasons, in the following I will impose $K = 2N$, with the reflection symmetry and the cut on the central link.

Summing up, the Lagrangian of the bosonic sector of the generalized D-BESS (which I will call GD-BESS for short in the following [34]) model is

$$\begin{aligned}\mathcal{L} = & \sum_{i=1, i\neq N+1}^{2N+1} f_i^2 \mathrm{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] + f_0^2 \mathrm{Tr}[D_\mu U^\dagger D^\mu U] \\ & - \frac{1}{2\tilde{g}^2}\mathrm{Tr}[(\mathbf{F}_{\mu\nu}^{\tilde{W}})^2] - \frac{1}{2\tilde{g}'^2}\mathrm{Tr}[(\mathbf{F}_{\mu\nu}^{\tilde{B}})^2] - \frac{1}{2g_i^2}\sum_{i=1}^{2N}\mathrm{Tr}[(\mathbf{F}_{\mu\nu}^{i})^2]\end{aligned} \tag{2.48}$$

I am ready to go on and calculate the ϵ parameters. As a first thing, I substitute in eq. (2.48) their VEV to all Σ^i, $\Sigma^i \to \mathbf{I}\ \forall i$, and separate the contribution of the kinetic terms from that of "link" terms. I get:

$$\mathcal{L}_{kin} = -\frac{1}{4g_i^2}\sum_{i=0}^{2N+1}(A^{i,a}_{\mu\nu} - \epsilon^{abc}A^{i,b}_{\mu}A^{i,c}_{\nu})^2, \tag{2.49}$$

$$\mathcal{L}_{link} = \sum_{\substack{i=1 \\ (i\neq N+1)}}^{2N+1}\frac{f_i^2}{2}(A^{i-1,a}_{\mu} - A^{i,a}_{\mu})^2 + \frac{f_0^2}{2}(\tilde{W}^a_\mu - \tilde{B}^a_\mu)^2, \tag{2.50}$$

where I made the identifications:

$$\begin{gathered} A^{0,a}_\mu = \tilde{W}^a_\mu, \quad A^{2N+1,3}_\mu = \tilde{B}_\mu, \quad A^{2N+1,1}_\mu = A^{2N+1,2}_\mu = 0 \\ g_0 = \tilde{g}, \quad g_{2N+1} = \tilde{g}' \end{gathered} \tag{2.51}$$

and defined:

$$A^{i,a}_{\mu\nu} = \partial_\mu A^{i,a}_\nu - \partial_\nu A^{i,a}_\mu, \quad i = 0, \cdots, 2N+1. \tag{2.52}$$

From the Lagrangian (2.48), I can derive the classical equations of motion for the $\mathbf{A}^i_\mu$ fields:

$$\begin{aligned} \partial_\mu \mathbf{F}^{i\,\mu\nu} =& \, i[\mathbf{A}^i_\mu, \mathbf{F}^{i\,\nu\mu}] + [f_i^2(\mathbf{A}^{i-1\,\nu} - \mathbf{A}^{i\,\nu}) \\ & - f_{i+1}^2(\mathbf{A}^{i\,\nu} - \mathbf{A}^{i+1\,\nu})], \qquad\qquad i = 1, \cdots, 2N, \end{aligned} \tag{2.53}$$

where again I have identified

$$\mathbf{A}^0_\mu = \tilde{\mathbf{W}}_\mu, \quad \mathbf{A}^{2N+1}_\mu = \tilde{\mathbf{B}}_\mu. \tag{2.54}$$

I need to solve these equations of motion in the low-energy limit $p^2 \ll M_i^2, \ \forall i$, where p represents the typical momentum scale of the processes I wish to describe. The mass spectrum cannot be determined analytically in the general case, so to implement the approximation I need an estimate for the mass scale $\bar{M}$, that I can obtain by looking at the mass matrix (2.17). In fact, every term in (2.17) is a sum of contributions proportional to $f_i^2 g_j^2$ for some i, j; then, I can consider the limit:

$$p^2 \ll f_i^2 g_j^2, \quad i = 1, \cdots, N, N+2, \cdots, 2N+1; \ j = 1, \cdots, 2N. \tag{2.55}$$

I will get a more explicit estimate for the mass scale $\bar{M}$ in the following, and check the consistence of this approximation. Also remember that in the D-BESS case, which corresponds to the $N = 1$ limit of the model I am examining, the leading term in the masses of the heavy vector states was $g''u/2$, that is $g_1 f_1/2$ in the formalism of the present model.

If I now rewrite eq. (2.53) as

$$\partial_\mu \mathbf{F}^{i\,\mu\nu} + i[\mathbf{A}^i_\mu, \mathbf{F}^{i\,\mu\nu}] = [f_i^2(\mathbf{A}^{i-1\,\nu} - \mathbf{A}^{i\,\nu}) - f_{i+1}^2(\mathbf{A}^{i\,\nu} - \mathbf{A}^{i+1\,\nu})], \tag{2.56}$$

I can see that all the quantities on the left-hand side are of higher order with respect to those on the right-hand side (remember that, since I am using a noncanonical normalization, there is a factor g_i implicit in every gauge field $\mathbf{A}^i$). To leading order, the equations reduce to

$$f_i^2(\mathbf{A}_\mu^{i-1} - \mathbf{A}_\mu^i) - f_{i+1}^2(\mathbf{A}_\mu^i - \mathbf{A}_\mu^{i+1}) = 0, \tag{2.57}$$

which implies

$$\mathbf{A}_\mu^N = \mathbf{A}_\mu^{N-1} = \ldots = \mathbf{A}_\mu^1 = \tilde{\mathbf{W}}_\mu \tag{2.58}$$

and

$$\mathbf{A}_\mu^{N+1} = \mathbf{A}_\mu^{N+2} = \ldots = \mathbf{A}_\mu^{2N} = \tilde{\mathbf{B}}_\mu. \tag{2.59}$$

Substituting this leading order expressions in the Lagrangians (2.49), (2.50), writing down only the bilinear terms, since I only need to calculate the gauge boson propagators, I get

$$\begin{aligned}\mathcal{L}_{eff}^2 = -\frac{1}{2}(\frac{1}{\tilde{g}^2} + \frac{1}{\overline{G}^2})\tilde{W}_{\mu\nu}^+\tilde{W}^{-\,\mu\nu}\\ -\frac{1}{4}(\frac{1}{\tilde{g}^2} + \frac{1}{\overline{G}^2})\tilde{W}_{\mu\nu}^3\tilde{W}^{3\,\mu\nu} - \frac{1}{4}(\frac{1}{\tilde{g}'^2} + \frac{1}{\overline{G}^2})\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu}\\ + f_0^2\tilde{W}_\mu^+\tilde{W}^{-\,\mu} + \frac{f_0^2}{2}\tilde{W}_\mu^3\tilde{W}^{3\,\mu} + \frac{f_0^2}{2}\tilde{B}_\mu\tilde{B}^\mu - f_0^2\tilde{W}_\mu^3\tilde{B}^\mu,\end{aligned} \tag{2.60}$$

where

$$\frac{1}{\overline{G}^2} = \sum_{k=1}^{N}\frac{1}{g_k^2} = \sum_{k=N+1}^{2N}\frac{1}{g_k^2}. \tag{2.61}$$

This expression exactly reproduces the SM electroweak gauge Lagrangian, by identifying

$$\frac{1}{g^2} = (\frac{1}{\tilde{g}^2} + \frac{1}{\overline{G}^2}), \quad \frac{1}{g'^2} = (\frac{1}{\tilde{g}'^2} + \frac{1}{\overline{G}^2}), \quad f_0^2 = \frac{v^2}{4} \equiv \frac{(\sqrt{2}\,G_F)^{-1}}{4}. \tag{2.62}$$

Note that I get

$$f_0^2 g^2 \simeq M_W^2, \quad f_0^2(g^2 + g^{'2}) \simeq M_Z^2 \tag{2.63}$$

in the limit $\bar{M} \to \infty$. The situation is directly analogue to that of D-BESS: as the masses M_i of the new resonances go to infinity, the standard picture is recovered. The model is decoupling! This means that the model is indeed a generalization of D-BESS, so any deviation from the SM at low energy will be suppressed at least by a factor $\frac{p^2}{\bar{M}^2}$.

Let me go now to the next-to-leading order by solving the equations of motion for the $\mathbf{A}^i$ iteratively. I substitute the leading order solutions (2.58) and (2.59) in the left-hand side of eq. (2.56), and get

$$\mathbf{A}_\nu^i = \tilde{\mathbf{W}}_\nu - c_i\mathbf{K}_\nu, \quad i = 1, \ldots, N; \tag{2.64}$$

$$\mathbf{A}_\nu^i = \tilde{\mathbf{B}}_\nu - c_i\mathbf{H}_\nu, \quad i = N+1, \ldots, 2N; \tag{2.65}$$

where I have introduced:

$$\mathbf{K}_\nu = \partial^\mu \mathbf{F}^{\tilde{W}}_{\mu\nu} + i[\tilde{\mathbf{W}}^\mu, \mathbf{F}^{\tilde{W}}_{\mu\nu}], \quad \mathbf{H}_\nu = \partial^\mu \mathbf{F}^{\tilde{B}}_{\mu\nu},$$
$$c_i = \sum_{j=1}^{i} \frac{1}{f_j^2} \sum_{k=j}^{N} \frac{1}{g_k^2} = c_{N+i} = \sum_{j=N+i}^{2N+1} \frac{1}{f_j^2} \sum_{k=N+1}^{j} \frac{1}{g_k^2}, \quad i = 1, \ldots, N. \tag{2.66}$$

Notice that the coefficients c_i are positive definite and of order $O(\frac{1}{M^2})$; furthermore, the reflection symmetry implies $c_i = c_{2N+1-i}$.

If I now insert the solutions (2.64), (2.65) in Lagrangians (2.49), (2.50) I get, at the bilinear level,

$$\begin{aligned}\mathcal{L}^2_{eff} &= -\frac{1}{2g^2}\tilde{W}^+_{\mu\nu}\tilde{W}^{-\,\mu\nu} - \frac{1}{4g^2}\tilde{W}^3_{\mu\nu}\tilde{W}^{3\,\mu\nu} - \frac{1}{4g'^2}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} \\ &+\frac{v^2}{4}\tilde{W}^+_\mu\tilde{W}^{-\,\mu} + \frac{v^2}{8}\tilde{W}^3_\mu\tilde{W}^{3\,\mu} + \frac{v^2}{8}\tilde{B}_\mu\tilde{B}^\mu - \frac{v^2}{4}\tilde{W}^3_\mu\tilde{B}^\mu \\ &+\frac{1}{4\overline{G}^2}\frac{1}{\bar{M}^2}\left(2\tilde{W}^+_{\mu\nu}\Box\tilde{W}^{-\,\mu\nu} + \tilde{W}^3_{\mu\nu}\Box\tilde{W}^{3\,\mu\nu} + \tilde{B}_{\mu\nu}\Box\tilde{B}^{\mu\nu}\right);\end{aligned} \tag{2.67}$$

where

$$\frac{1}{\bar{M}^2} = C\overline{G}^2, \quad C \equiv \sum_{i=1}^{N} \frac{c_i}{g_i^2} \equiv \sum_{i=N+1}^{2N} \frac{c_i}{g_i^2}. \tag{2.68}$$

Eq. (2.67) is identical to eq. (1.90) which describes the low-energy limit of the D-BESS model, with the replacements:

$$M \to \bar{M}, \quad \frac{1}{g''^2} \to \frac{1}{\overline{G}^2}. \tag{2.69}$$

This means that I can repeat the calculation at the end of section 1.2.2 *verbatim*, and that the ϵ parameters are given, also in the general case, by eq. (1.96), of course with the replacements (2.69) [34].

Let me comment on the mass scale $\bar{M}$. Eq. (2.68) gives an explicit, if complicated, expression for it. Recalling the definition of the c_i in (2.66), I can see that it is indeed of order $f_i g_j$, so the approximation (2.55) indeed makes sense. In the D-BESS limit $N = 1,\ g_1 \to g'',\ f_1^2 \to u^2/4$, it is easy to check that $\bar{M}$ is exactly - as it should - equal to M, the degenerate mass of the heavy states before the gauging of weak interactions: by using eq. (2.61), I have in fact in this case $\frac{1}{g_1^2} = \frac{1}{g''^2}$, and $\bar{M}^2 = f_1^2 g_1^2 = g''^2 u^2/4 = M$. In the next simplest case, $N = 2$ and $g_1 = g_2 \equiv g_c$, $f_1 = f_2 \equiv f_c$, I get

$$\bar{M}^2 = \frac{2}{5} f_c^2 g_c^2; \tag{2.70}$$

by diagonalizing $\mathbf{M}_{(2)}$, I find that the squared mass of the lightest eigenstate in this case is, up to corrections of $O(m_Z^2/\bar{M}^2)$:

$$M_1^2 \simeq 0.38 f_c^2 g_c^2. \tag{2.71}$$

Then I see that $\bar{M}$ is roughly equal to the mass of the lightest eigenstate. This result will be explicitly confirmed, in the next chapter, in the continuum limit, at least for some special choices of the 5D metric.

2.2.4 SM input parameters

There is another way of deriving the expression (1.96) for the ϵ parameters, which is also useful to get explicit expressions for the standard input parameters of the EW sector, α, G_F and m_Z in terms of the parameters of the model. The starting point is still the low-energy effective Lagrangian (2.67). Going back to the usual normalization $\tilde{W}^a \to g\tilde{W}^a$, $\tilde{B} \to \tilde{g}'\tilde{B}$, introducing $\tilde{A}$ and $\tilde{Z}$ fields from $\tilde{W}^3$ and $\tilde{B}$ in the usual way:

$$\begin{pmatrix} \tilde{W}^3 \\ \tilde{B} \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \tilde{Z} \\ \tilde{A} \end{pmatrix}, \tag{2.72}$$

where $s_\theta = g'/\sqrt{g^2 + g'^2}$, and substituting in eq. (2.67), I get:

$$\begin{aligned} \mathcal{L}^2_{eff} = & -\frac{1}{2}\tilde{W}^+_{\mu\nu}\tilde{W}^{-\,\mu\nu} - \frac{1}{4}\tilde{Z}_{\mu\nu}\tilde{Z}^{\mu\nu} - \frac{1}{4}\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} \\ & +\tilde{M}^2_W\tilde{W}^+_\mu\tilde{W}^{-\,\mu} + \frac{\tilde{M}^2_Z}{2}\tilde{Z}_\mu\tilde{Z}^\mu + \frac{1}{2\bar{M}^2}[z_W\tilde{W}^+_{\mu\nu}\Box\tilde{W}^{-\,\mu\nu} \\ & +\frac{z_\gamma}{2}\tilde{A}_{\mu\nu}\Box\tilde{A}^{\mu\nu} + \frac{z_Z}{2}\tilde{Z}_{\mu\nu}\Box\tilde{Z}^{\mu\nu} + z_{Z\gamma}\tilde{A}_{\mu\nu}\Box\tilde{Z}^{\mu\nu}] \end{aligned} \tag{2.73}$$

where

$$\begin{aligned} & z_W = \frac{g^2}{\overline{G}^2},\ z_\gamma = \frac{2e^2}{\overline{G}^2}, \quad \text{with} \quad e = gs_\theta = g'c_\theta, \\ & z_Z = \frac{g^2(c^4_\theta + s^4_\theta)}{c^2_\theta\overline{G}^2},\ z_{Z\gamma} = \frac{gg'c_{2\theta}}{\overline{G}^2},\ \tilde{M}^2_W = \frac{v^2g^2}{4},\ \tilde{M}^2_Z = \frac{v^2(g^2 + g'^2)}{4}. \end{aligned} \tag{2.74}$$

If I now rescale the fields according to

$$\begin{aligned} \tilde{W}^\pm_\mu &= \left(1 + \frac{z_W}{2}(\frac{\Box}{\bar{M}^2} - \frac{\tilde{M}^2_W}{\bar{M}^2})\right)W^\pm_\mu, \\ \tilde{Z}_\mu &= \left(1 + \frac{z_Z}{2}(\frac{\Box}{\bar{M}^2} - \frac{\tilde{M}^2_Z}{\bar{M}^2})\right)Z_\mu, \\ \tilde{A}_\mu &= \left(1 + \frac{z_\gamma}{2}\frac{\Box}{\bar{M}^2}\right)A_\mu + z_{Z\gamma}\frac{\Box}{\bar{M}^2}Z_\mu, \end{aligned} \tag{2.75}$$

I can get rid of the anomalous "$\Box$" terms in the quadratic part of the Lagrangian. Then I obtain

$$\begin{aligned} \mathcal{L}^2_{eff} = & -\frac{1}{2}W^+_{\mu\nu}W^{-\,\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} \\ & + m^2_W W^+_\mu W^{-\,\mu} + \frac{m^2_Z}{2}Z_\mu Z^\mu, \end{aligned} \tag{2.76}$$

where:

$$m_W^2 = \tilde{M}_W^2 \left(1 - z_W \frac{\tilde{M}_W^2}{\bar{M}^2}\right), \quad m_Z^2 = \tilde{M}_Z^2 \left(1 - z_Z \frac{\tilde{M}_Z^2}{\bar{M}^2}\right). \tag{2.77}$$

The effective Lagrangian in (2.76) has the same form as the SM one; as a bonus, I have explicit expressions (of course to order $m_Z^2/\bar{M}^2$) for the masses of the W and Z boson. However, the rescaling (2.75) of the fields will affect the couplings to the fermions, which I can assume to be of the standard form (1.77) as in the D-BESS model. If I shift to the $\tilde{W}^{\pm}$, $\tilde{A}$, $\tilde{Z}$ basis in eq. (1.77), and rescale the fields according to (2.75), then the effective expression for the fermion currents interactions are the following:

$$\begin{aligned}
\mathcal{L}^{int}_{charg.} &= -\frac{g}{\sqrt{2}}\bar{\psi}_u\gamma^\mu(1-\frac{\gamma^5}{2})\left(1+\frac{z_W}{2}(\frac{\Box}{\bar{M}^2}-\frac{\tilde{M}_W^2}{\bar{M}^2})\right)\psi_d W^+_\mu + (h.c.)\\
\mathcal{L}^{int}_{neut.} &= -\frac{g}{c_\theta}\left(1+\frac{z_Z}{2}(\frac{\Box_Z}{\bar{M}^2}-\frac{\tilde{M}_Z^2}{\bar{M}^2})\right)\bar{\psi}\,\gamma^\mu\Big[\frac{\tau^3}{2}\frac{(1-\gamma^5)}{2}\\
&\qquad - Qs_\theta^2\left(1+\frac{c_\theta}{s_\theta}z_{Z\gamma}\frac{\Box_Z}{\bar{M}^2}\right)\Big]\psi\, Z_\mu\\
\mathcal{L}^{int}_{e.m.} &= \quad g s_\theta\,\bar{\psi}\,\gamma^\mu Q\psi\left(1-\frac{z_\gamma}{2}\frac{\Box}{\bar{M}^2}\right)A_\mu,
\end{aligned} \tag{2.78}$$

where I use the convention that $\Box_Z$ does only operate on Z and $Q = \frac{\tau^3}{2} + \frac{B-L}{2}$.

I see that the photon-fermion interaction at zero momentum correctly predicts $e = gs_\theta$ as the physical value of the electric charge. The Fermi constant G_F can be measured from the μ decay, still at zero momentum. I have:

$$\begin{aligned}
\frac{G_F}{\sqrt{2}} &= \frac{e^2}{8s_\theta^2}\left(1 - z_W\frac{m_W^2}{\bar{M}^2}\right)\frac{1}{m_W^2}\left(1 + z_W\frac{m_W^2}{\bar{M}^2}\right)\\
&= \frac{e^2}{8s_\theta^2 c_\theta^2 m_Z^2}\left(1 - z_Z\frac{m_Z^2}{\bar{M}^2}\right),
\end{aligned} \tag{2.79}$$

where I have substituted the physical masses m_W and m_Z to $\tilde{M}_W$ and $\tilde{M}_Z$ since they only differ by terms of $O(m_Z^2/\bar{M}^2)$, which are negligible in a term which is already of the same order. From eq. (2.79) I can define the effective Weinberg angle (see [29]):

$$\frac{G_F}{\sqrt{2}} \equiv \frac{e^2}{8s_{\theta_0}^2 c_{\theta_0}^2 m_Z^2} \;\Rightarrow\; s_{\theta_0}^2 c_{\theta_0}^2 = s_\theta^2 c_\theta^2\left(1 + z_Z\frac{m_Z^2}{\bar{M}^2}\right), \tag{2.80}$$

that is

$$s_{\theta_0}^2 = s_\theta^2\left(1 + \frac{c_\theta^2}{c_{2\theta}} z_Z \frac{m_Z^2}{\bar{M}^2}\right). \tag{2.81}$$

Finally, I can calculate the ϵ parameters. They can be defined in terms of the three observables $\Delta\rho$, Δk and Δr_W [29], which parametrize the effective neutral current and the m_W/m_Z mass ratio:

$$\left(1-\frac{m_W^2}{m_Z^2}\right)\frac{m_W^2}{m_Z^2}=\frac{s_{\theta_0}^2 c_{\theta_0}^2}{1-\Delta r_W}; \tag{2.82}$$

$$\begin{aligned}&\mathcal{L}^{int}_{neut.} = \\ &-\frac{e}{s_{\theta_0}c_{\theta_0}}\sqrt{1+\Delta\rho}\;\overline{\psi}\left((\frac{T_L^3}{2}-Qs_{\theta_0}^2(1+\Delta k))\gamma^\mu-\frac{T_L^3}{2}\gamma^\mu\gamma^5\right)\psi\, Z_\mu;\end{aligned} \tag{2.83}$$

the combinations yielding the ϵ parameters are:

$$\epsilon_1 = \Delta\rho \tag{2.84}$$

$$\epsilon_2 = c_{\theta_0}^2\Delta\rho+\frac{s_{\theta_0}^2}{c_{2\theta_0}}\Delta r_W-2s_{\theta_0}^2\Delta k \tag{2.85}$$

$$\epsilon_3 = c_{\theta_0}^2\Delta\rho+c_{2\theta_0}\Delta k. \tag{2.86}$$

Comparing eqs. (2.82) and (2.81), and eqs. (2.83) and (2.78), I get

$$\Delta r_W = z_\gamma+\frac{c_{2\theta}}{s_\theta^2}z_W\left(1+\frac{m_W^2}{\bar{M}^2}\right)-\frac{c_\theta^2}{s_\theta^2}z_Z\left(1+\frac{m_W^2}{\bar{M}^2}\right) \tag{2.87}$$

$$\Delta\rho = -z_Z\frac{m_Z^2}{\bar{M}^2} \tag{2.88}$$

$$\Delta k = \frac{c_\theta^2}{c_{2\theta}}\left(z_\gamma-z_Z\left(1+\frac{m_W^2}{\bar{M}^2}\right)\right)+\frac{c_\theta}{s_\theta}z_{Z\gamma}\left(1+\frac{m_W^2}{\bar{M}^2}\right) \tag{2.89}$$

and the ϵ parameters can be easily derived in perfect agreement with eq. (1.96).

Chapter 3

D-BESS in 5 dimensions

Let me study the continuum limit of the generalized D-BESS model. As I discussed in the preface, on the one hand the GD-BESS model was formulated as an effective description at low-energy of an hypothetical strongly interacting sector, responsible for EW symmetry breaking; on the other hand, in recent years, by the AdS/CFT correspondence [26, 58], physicists have come to think of models in five dimensions as "holographic" duals of strongly interacting 4D ones [35, 36]. Giving a 5D description of GD-BESS is then a logical step. While studying this 5-dimensional extension, furthermore, I have clarified a not-so-obvious fact: there is at least a particular limit, where the 5-dimensional D-BESS becomes very similar to an realization of the RS1 model [18], specifically the one proposed in ref. [37]. This limit will be explored in section 3.4.2.

3.1 The cut link in 5 dimensions

As it was shown in section 2.1, a linear moose model (such as that of eq. (2.3) and fig. 2.1) is the discretized version of a SU(2) 5-dimensional gauge theory. In the example of that section, I considered a very simple model with constant couplings. The generalized D-BESS model has a number of features in comparison to that very basic example. First of all, it allows for different values of the link and coupling constants at different sites on the moose, which in the continuum limit correspond to different values of the coordinate of the extra dimension. Thus eq. (2.11), which gives the correspondence between the 4D and the 5D parameters, needs to be generalized. Then, as explained in the previous section, I need to gauge part of the global symmetry acting at the two ends of the moose to implement the weak interactions. Finally, to suppress the contribution of the heavy vector fields to the ϵ parameters, I need to "cut a link" and to add to the Lagrangian an apparently nonlocal field U which connects the gauge fields of the $SU(2)_L \otimes U(1)_Y$ local symmetry.

To be able to properly describe this generalization, I need a representation for the 5D metric. Since the deconstructed model possesses ordinary 4D Lorentz invariance, the extra-dimensional metric must be compatible with this symmetry. Such a metric can in general be written in the form:

$$ds^2 = b(y)\eta_{\mu\nu}\, dx^\mu dx^\nu + dy^2, \tag{3.1}$$

where η is the standard Lorentz metric with the $(-,+,+,+)$ signature choice, y the variable corresponding to the extra dimension and $b(y)$ is a generic positive definite function, usually known as the "warp factor". I normalize $b(y)$ by requesting that $b(0) = 1$. For definiteness, I will consider a finite extra dimension, with $y \in (0, \pi R)$. By the standard convention of 5D theories, the endpoints of the interval will also be called *branes*. With this choice, eq. (2.11) becomes in the GD-BESS case:

$$\frac{g_i^2}{N} \to \frac{g_5^2}{\pi R}, \quad f_i^2 \to b(y)\frac{N}{\pi R g_5^2}, \tag{3.2}$$

where g_5 is a 5D gauge coupling, with mass dimension $-1/2$. As can be seen, a general choice for the g_i implies that g_5 is "running" , with an explicit dependence on the extra variable. In the following, I will not consider this possibility, but rather restrict for simplicity to a constant coupling (as it is standard in the literature), so, from the 4D side, I will have $g_i \equiv g_c$.

The trickiest part of the generalization, however, is to interpret the cutting of the link. To understand this properly, I can start by noticing that the cut link prevents any direct contact between the two sides of the moose; the fields on the left only couple to those on the right through the field U. In this sense, the moose is split by the cut in two separate pieces, linked by U. So is, in the continuum limit, the extra dimension. Due to the reflection symmetry (see section 2.2.3 and eq. (2.47)), the two pieces are identical to each another, at a site-by-site level, from every point of view: field content, coupling constants g_i, link couplings f_i. The right way to look at this set up is to *understand the sites connected by the reflection symmetry as describing the same point along the extra dimension*: for example, I can see the fields $\mathbf{A}^i$ and $\mathbf{A}^{2N-i}$ not as values of the same 5-dimensional $SU(2)$ gauge field at two different points along the extra dimension, but as *components* of a single $SU(2) \otimes SU(2)$ gauge field at the *same* extra-dimensional location. This is consistent because by eq. (3.2) the warp factor - and thus the 5-dimensional metric - at a given site only depends on the value of the link coupling constant f_i, which is equal at points identified by the reflection symmetry, that can then describe the same point y_i on the 5^{th} dimension. The situation is depicted graphically in figure 3.1: it is equivalent to "flipping" one of the pieces of the moose and superposing it to the other one. In this way, I do not obtain an 5D $SU(2)$ gauge theory, but an $SU(2)_L \otimes SU(2)_R$ one, with the left part of the moose describing the $SU(2)_L$ gauge theory and the right part $SU(2)_R$ and the coupling constants of the two sectors of the gauge group identified by a discrete symmetry. The field U no longer appears

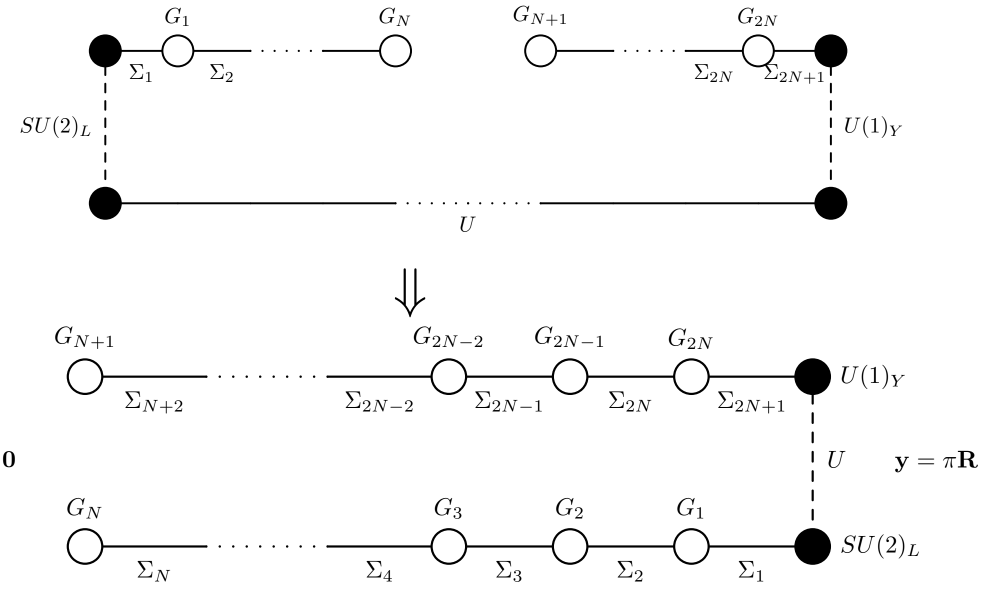


Figure 3.1: *Interpretation of the cut link in the continuum limit of the GD-BESS model. The first half of the moose is "flipped" and superimposed to the second half. In this way, the N^{th} and the $N+1^{th}$ sites are identified with the $y = 0$ brane, while the 1^{st} and the $2N+1^{th}$ with the $y = \pi R$ one.*

as nonlocal, but rather as confined at one end of the extra-dimensional segment.

The last point to consider is the presence of different gauge fields - the ones corresponding to $SU(2)_L \otimes U(1)_Y$ - at the two ends of the moose, which are identified with one of the endpoints of the 5D interval (which for definiteness I will take to be $y = \pi R$). This can be accounted for by considering *localized kinetic terms* at $y = \pi R$ for the 5D gauge fields; the fields $\tilde{W}$ and $\tilde{B}$ can then be simply identified with the values of the $SU(2)_L$ and of the third component of the $SU(2)_R$ 5D gauge fields respectively. Notice that the "flipped" GD-BESS moose has $N+1$ sites: N for the $SU(2)_L \otimes SU(2)_R$ gauge fields and a last one for the fields corresponding to $SU(2)_L \otimes U(1)_Y$. By convention, I will map this last site to the $y = \pi R$ end of the extra dimension; the other endpoint, $y = 0$, will correspond to the gauge fields living next to the cut link, $\mathbf{A}^N$ and $\mathbf{A}^{N+1}$.

Putting all this together, the 5D limit of GD-BESS is described by the action

$$
\begin{aligned}
S = \int d^4x \int_0^{\pi R} \sqrt{-g}\, dy \Bigg[-\frac{1}{4g_5^2} L^a_{MN} L^{a\,MN} - \frac{1}{4g_5^2} R^a_{MN} R^{a\,MN} \\
+ \delta(y) \left(-\frac{1}{4\tilde{g}^2} L^a_{\mu\nu} L^{a\,\mu\nu} - \frac{1}{4\tilde{g}'^2} R^3_{\mu\nu} R^{3\,\mu\nu} - \frac{\tilde{v}^2}{4} (D_\mu U)^\dagger D^\mu U + \mathit{fermions} \right) \Bigg],
\end{aligned}
\tag{3.3}
$$

where:

- with the usual convention, the greek indices run from 0 to 3, while capital latin ones take the values $(0, 1, 2, 3, 5)$, with "5" labelling the extra direction
- g is the determinant of the metric tensor g_{MN}, defined by

$$ds^2 = g_{MN} dx^M dx^N \equiv b(y)\, \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{3.4}$$

- L^a_{MN} and R^a_{MN} are the $SU(2)_L \otimes SU(2)_R$ gauge field strengths:

$$L(R)^a_{MN} = \partial_M W^a_{L(R)\;N} - \partial_N W^a_{L(R)\;M} + i\epsilon^{abc} W^b_{L(R)\;M} W^c_{L(R)\;N}; \tag{3.5}$$

  the fields $W^a_{L(R)}$ represent the continuum limit of the $A^{a\,i}$
- $\tilde{g}$, $\tilde{g}'$, g_5 are three in general different gauge couplings. $\tilde{g}$ and $\tilde{g}'$ are the direct analogous of their deconstructed counterparts. g_5 is the bulk coupling, it has mass dimension $-\frac{1}{2}$, and it is the 5D limit of the g_i, as can be seen by eq. (3.2)[1]
- the brane scalar U is an $SU(2)$-valued field, with its covariant derivative defined by:

$$D_\mu U = \partial_\mu U + i W^a_{L\,\mu} \frac{\tau^a}{2} U - i W^3_{R\,\mu} U \frac{\tau^3}{2}, \tag{3.6}$$

  in exact analogy with eq. (2.44). Note that the field U is analogous to the one that describes the standard Higgs sector in the limit of an infinite Higgs mass, see eq. (1.31). It can be conveniently parametrized in terms of three real scalars,

$$U = \exp(\frac{i\pi^a \tau^a}{2\tilde{v}}); \tag{3.7}$$

- the fermionic terms, which I take to be confined on the brane for simplicity, have the usual SM form

It is important to notice that the action (3.3) does not define the physics of the model uniquely: I still have the freedom of choosing *boundary conditions* (BCs) for the fields. In fact, in a 4D theory it is commonly understood that the fields should vanish in the limit $x \to \infty$. By contrast, in a theory living on a compact extra dimension, this is not the only possibility. The choice of the BCs in general may depend on the geometry of the extra dimension (for instance, if I compactify the extra dimension on a circle, I will get periodic BCs) and it is part of the definition of the model. In the case I am considering, the extra dimension is an interval. The choice of the BCs depends on how big is the part of the gauge symmetry I wish to preserve on the boundaries. A possible choice is to set the fields to zero

[1] The left-right symmetry inherited from GD-BESS implies an equality between the couplings the $SU(2)_L$ and $SU(2)_R$ sectors of the bulk gauge group, which I have chosen to maintain in the text for simplicity. However, it is straightforward to generalize to the case in which the model has two distinct couplings $g_{5\,L}$ and $g_{5\,R}$. The conclusions of this chapter are not altered in any significant way.

(similarly to the 4D case) on one or both the endpoints, for instance $y = 0$; in this case the gauge symmetry will be broken at $y = 0$, and a gauge transformation, to leave the action invariant, will have to be chosen in such a way as to reduce to the identity as $y \to 0$. In this case, one speaks of *Dirichlet* BCs. Otherwise, I can choose to leave the symmetry unbroken; in this case, the values of the fields on the borders are completely unconstrained. One can also use a combination of these two possibilities; a well-known example in the literature where this happens is the Higgsless model [19, 20], which uses a complicated set of BCs in order to achieve a specific symmetry breaking pattern on the borders of the extra dimension.

This BC ambiguity is absent in deconstructed models: the BCs get implicitly specified by the way in which the discretization of the 5^{th} dimension is realized. This means that the GD-BESS model that I have studied in the previous chapter already has a specific set of "built-in" BCs. These can be understood by looking at the residual gauge symmetry at the ends of the moose. It is apparent that, after the "flipping" depicted in fig. 3.1, at the N^{th} and $(N+1)^{th}$ sites, corresponding at $y = 0$ in the continuum limit, I have the full $SU(2)_L \otimes SU(2)_R$ gauge invariance. By contrast, at the 0^{th} and $(2N+1)^{th}$, corresponding to $y = \pi R$, the gauge symmetry is broken down to $SU(2)_L \otimes U(1)_Y$. To do this, I have to impose Dirichlet BCs on two of the $SU(2)_R$ gauge fields at $y = \pi R$, while all the other fields, and all the fields at $y = 0$ are unconstrained. The complete gauge symmetry breaking pattern is thus as follows: I have an $SU(2)_L \otimes SU(2)_R$ gauge invariance in the bulk, unbroken on the $y = 0$ brane and broken by a combination of Dirichlet BCs and scalar VEV (of the U field) to $U(1)_{e.m.}$ on the $y = \pi R$ brane.

In the remainder of this chapter, I will study the model defined by the action (3.3). First of all, I will perform a general analysis of the full 5D theory by the standard technique of the Kaluza-Klein (KK) expansion. Then I will look at the low-energy limit and derive expression for the ϵ parameters; the results will confirm that this is indeed the 5D limit of GD-BESS. Finally, I will make some remarks on the phenomenology of the model in correspondence with two interesting choices for the geometry of the 5^{th} dimension, that of a flat dimension ($b(y) \equiv 1$) and that of a slice of AdS_5 ($b(y) = e^{-2ky}$).

3.2 KK expansion

Since I wish to keep the metric generic for the moment, a convenient strategy is to expand the gauge fields $W^a_{L(R)\,M}$ (and the goldstones π^a) directly in terms of mass eigenstates [59, 60]. So I define:

$$
\begin{aligned}
W^a_{L\,\mu}(x,y) &= \sum_{j=0}^{\infty} f^a_{L\,j}(y)\, V^{(j)}_\mu(x), \qquad & W^a_{L\,5}(x,y) &= \sum_{j=0}^{\infty} g^a_{L\,j}(y)\, G^{(j)}(x),\\
W^a_{R\,\mu}(x,y) &= \sum_{j=0}^{\infty} f^a_{R\,j}(y)\, V^{(j)}_\mu(x), \qquad & W^a_{R\,5}(x,y) &= \sum_{j=0}^{\infty} g^a_{R\,j}(y)\, G^{(j)}(x),
\end{aligned}
\tag{3.8}
$$

$$
\pi^i(x) = \sum_{j=0}^{\infty} c^i_j\, G^{(j)}(x).
$$

The expansion (3.8) is written in full generality; it allows for a maximal mixing of the gauge fields. *A priori*, this means that fields with different isospin index could be mixed. The index "(j)" labels all the mass eigenstates. This choice is in fact more general than is needed; eventually, I will choose BCs for the model in such a way that 3 decoupled towers of eigenstates exist, so that many of the above wavefunctions (or constant coefficients in the case of the brane scalars) are vanishing. I have written eq. (3.8) in a general form to emphasize the fact that the decoupling only occurs *a posteriori*, following from two requests that I make on the expansion: first, the wavefunctions must form complete sets, and second, upon substituting the expansion and performing the integration over the extra-dimensional variable y, a *diagonal* bilinear Lagrangian must result, *i.e.* the fields defined in eq. (3.8) have to be mass eigenstates.

Since the proof of the diagonalization is somewhat technical, I will proceed in reverse order, first defining the three sectors of the model, together with the conditions that the corresponding wavefunctions have to satisfy, then show how the three sectors are derived by the request of diagonalizing the KK expanded Lagrangian. The three sectors are:

- A *left charged* sector coming from the expansion of the $(W_L^{1,2})_M$ fields and the brane scalars $\pi^{1,2}$. The explicit form of the expansion is

$$
\begin{aligned}
W^{1,2}_{L\,\mu}(x,y) &= \sum_{n=0}^{\infty} f^{1,2}_{L\,n}(y)\, W^{1,2\,(n)}_{L\,\mu}(x),\\
W^{1,2}_{L\,5}(x,y) &= \sum_{n=0}^{\infty} g^{1,2}_{L\,n}(y)\, G^{1,2\,(n)}_{L}(x),\\
\pi^{1,2}(x) &= \sum_{n=0}^{\infty} c^{1,2}_n\, G^{(n)}(x).
\end{aligned}
\tag{3.9}
$$

  The wavefunctions of the vector fields satisfy the equation of motion:

$$
\hat{D} f^{1,2}_{L\,n} = -m^2_{L\,n} f^{1,2}_{L\,n}, \tag{3.10}
$$

  where I defined the differential operator:

$$
\hat{D} \equiv \partial_y(b(y)\partial_y(\cdot)), \tag{3.11}
$$

and the set of BCs:

$$\partial_y f^{1,2}_{L\,n} = 0 \qquad \text{at } y = 0, \tag{3.12}$$

$$\left(\frac{\tilde{g}^2}{g_5^2}\partial_y - b(\pi R)^{-1}\, m_n^2 + \frac{\tilde{g}^2\tilde{v}^2}{4}\right) f^{1,2}_{L\,n} = 0 \qquad \text{at } y = \pi R. \tag{3.13}$$

The scalar profiles are fixed by the conditions:

$$g^{1,2}_{L\,n} = \frac{1}{m_{L\,n}}\partial_y f^{1,2}_{L\,n}, \qquad c^{1,2}_n = \frac{\tilde{v}}{2m_{L\,n}} f^{1,2}_{L\,n}|_{\pi R}. \tag{3.14}$$

Note that in this sector no massless solution is allowed; in fact, eq. (3.10) together with the Neumann BC at $y = 0$ (3.12) imply that a massless mode must have a *constant* profile, and a constant, massless solution cannot satisfy the BC at $y = \pi R$ (3.13). Also note that eq. (3.10) and the BCs (3.12) and (3.13) are diagonal in the isospin index, so I have $f^1_{L\,n} = f^2_{L\,n}$.

Some caution must be used in writing down the completeness and orthogonality relations for the $f^{1,2}_{L\,n}$ mode functions. The differential operator $\hat{D}$ (3.11) is in fact not hermitian with respect to the ordinary scalar product when evaluated on functions obeying BCs of the kind (3.13), due to the presence of terms explicitly containing the eigenvalues $m_{L\,n}$ which are induced by πR-localized terms in the action. To obtain the correct completeness and orthogonality properties of this function set, a generalized scalar product must be used which takes into account such terms. This is given by

$$\left(f^{1,2}_{L\,n}, f^{1,2}_{L\,m}\right)_{\tilde{g}} = L_m^2\delta_{mn}, \quad (f,h)_{\tilde{g}} = \frac{1}{g_5^2}\int_0^{\pi R} dy\; fh + \frac{1}{\tilde{g}^2}\; fh|_{\pi R}\,, \tag{3.15}$$

where L_m sets the normalization. Since the scalar product $(\cdot,\cdot)_{\tilde{g}}$ is dimensionless, I will set: $L_m \equiv 1$. This will ensure that the kinetic terms of the bosons of this sector are canonically normalized. From this definition I deduce the completeness relation:

$$\begin{aligned}&\frac{1}{g_5^2}\sum_k f^{1,2}_{L\,k}(y) f^{1,2}_{L\,k}(z) + \frac{1}{\tilde{g}^2}\delta(z-\pi R)\sum_k f^{1,2}_{L\,k}(y) f^{1,2}_{L\,k}(\pi R)\\ &= \delta(y-z);\end{aligned} \tag{3.16}$$

- A *right charged* sector coming from the expansion of $(W^{1,2}_R)_M$. The explicit form of the expansion this time is

$$\begin{aligned}W^{1,2}_{R\,\mu}(x,y) &= \sum_{n=0}^{\infty} f^{1,2}_{R\,n}(y)\, W^{1,2\,(n)}_{R\,\mu}(x),\\ W^{1,2}_{R\,5}(x,y) &= \sum_{n=0}^{\infty} g^{1,2}_{R\,n}(y)\, G^{1,2\,(n)}_{R}(x).\end{aligned} \tag{3.17}$$

The wavefunctions of the vector fields satisfy a similar equation of motion:

$$\hat{D} f^{1,2}_{Rn} = -m^2_{Rn} f^{1,2}_{Rn}, \tag{3.18}$$

and the set of BCs:

$$\partial_y f^{1,2}_{Rn} = 0 \qquad \text{at } y = 0, \tag{3.19}$$

$$f^{1,2}_{Rn} = 0 \qquad \text{at } y = \pi R. \tag{3.20}$$

The scalar profiles are given by:

$$g^{1,2}_{Rn} = \frac{1}{m_{Rn}} \partial_y f^{1,2}_{Rn}. \tag{3.21}$$

Again, in this sector there is no massless solution, for the constant profile of a massless mode is incompatible with the BC (3.20). Also, the equation of motion and the BCs are again diagonal in the isospin index, so $f^1_{Rn} = f^2_{Rn}$. The right charged sector obeys the usual L^2 orthogonality property:

$$\left(f^{1,2}_{Rn}, f^{1,2}_{Rm}\right) \equiv \frac{1}{g_5^2} \int_0^{\pi R} dy\ f^{1,2}_{Rn} f^{1,2}_{Rm} = R^2_m \delta_{mn}, \tag{3.22}$$

where the factor $1/g_5^2$ has been inserted to compensate for the mass dimension of the integral, so that I can normalize: $R_m \equiv 1$, again ensuring that the kinetic terms will have the canonical normalization.

- Finally, a *neutral* sector coming from the expansion of $(W^3_L)_M$, $(W^3_R)_M$ and π^3. The expansion has the form

$$\begin{aligned}
W^3_{L\,\mu}(x,y) &= \sum_{n=0}^{\infty} f^3_{L\,n}(y)\, N^{(n)}_\mu(x), \quad W^3_{L\,5}(x,y) = \sum_{n=0}^{\infty} g^3_{L\,n}(y)\, G^{(n)}_N(x), \\
W^3_{R\,\mu}(x,y) &= \sum_{n=0}^{\infty} f^3_{R\,n}(y)\, N^{(n)}_\mu(x), \quad W^3_{R\,5}(x,y) = \sum_{n=0}^{\infty} g^3_{R\,n}(y)\, G^{(n)}_N(x), \\
\pi^3(x) &= \sum_{j=0}^{\infty} c^3_j\, G^{(j)}(x);
\end{aligned} \tag{3.23}$$

the equation of motion and the BCs for the vector profiles are given by:

$$\hat{D} f^3_{L,R\,n} = -m^2_{N\,n} f^3_{L,R\,n}, \tag{3.24}$$

$$\partial_y f^3_{L,R\,n} = 0 \qquad \text{at } y = 0, \tag{3.25}$$

$$\begin{cases} \left(\frac{\tilde{g}^2}{g_5^2}\partial_y - b(\pi R)^{-1}\ m^2_n + \frac{\tilde{g}^2 \tilde{v}^2}{4}\right)\ f^3_{L\,n} - \frac{\tilde{g}^2 \tilde{v}^2}{4}\ f^3_{R\,n} = 0 \\ \left(\frac{\tilde{g}'^2}{g_5^2}\partial_y - b(\pi R)^{-1}\ m^2_n + \frac{\tilde{g}'^2 \tilde{v}^2}{4}\right)\ f^3_{R\,n} - \frac{\tilde{g}'^2 \tilde{v}^2}{4}\ f^3_{L\,n} = 0 \end{cases} \qquad \text{at } y = \pi R, \tag{3.26}$$

and the scalar profiles satisfy

$$\begin{aligned} g^3_{L,R\,n} &= \frac{1}{m_{N\,n}} f^3_{L,R\,n}, && \text{if } m_{N\,n} \neq 0 \\ g^3_{L,R\,n} &= 0 && \text{if } m_{N\,n} = 0 \\ c^3_n &= \frac{\tilde{v}}{2m_n} \left(f^3_{L\,n} - f^3_{R\,n}\right)\big|_{\pi R}. \end{aligned} \tag{3.27}$$

In contrast to the charged ones, the neutral sector admits a single massless solution; I have $m_{N\,0} = 0$. Eqs. (3.24) and (3.25) imply for a massless mode that both $f^3_{L\,n}$ and $f^3_{R\,n}$ must be constant; then, using also eq. (3.26) I get:

$$f^3_{L\,0} = f^3_{R\,0} \equiv f_0, \tag{3.28}$$

where f_0 is a constant. The massless mode has to be identified with the photon $\Rightarrow N^{(0)}_\mu \equiv A_\mu$; since it is the only massless mode in the spectrum I have that the symmetry of the vacuum is, correctly, just $U(1)_{e.m.}$. The "charged" and "neutral" labels I have given to the three sectors refer to their transformation properties with respect to this unbroken symmetry.

As in the case of the left charged sector, the BC at $y = \pi R$ in this case explicitly contains the mass of the n^{th} mode, so that again the basis wavefunctions $f^3_{L\,n}$ and $f^3_{R\,n}$ have nonstandard orthogonality properties. The correct relations are:

$$\left(f^3_{L\,n}, f^3_{L\,m}\right)_{\tilde{g}} = (N^L_m)^2 \delta_{mn}, \quad \left(f^3_{R\,n}, f^3_{R\,m}\right)_{\tilde{g}'} = (N^R_m)^2 \delta_{mn}, \tag{3.29}$$

where $(\cdot,\cdot)_{\tilde{g}'}$ is defined in a way analogous to $(\cdot,\cdot)_{\tilde{g}}$ (eq. (3.15)). Completeness relations similar to that in eq. (3.16) also hold. Note that it is not possible to set both N^L_n and N^R_n to 1. In fact, since they obey the same differential equation (3.24) and the same BC at $y = 0$ (3.25), $f^3_{L\,n}$ and $f^3_{R\,n}$ are proportional to each other:

$$f^3_{L\,n} = K_n f^3_{R\,n}, \tag{3.30}$$

and the constants K_n are fixed by the BC at $y = \pi R$ (3.26). To get also in this case canonically normalized kinetic terms I have to set:

$$(N^L_n)^2 + (N^R_m)^2 = 1; \tag{3.31}$$

the ratio $(N^L_n)/(N^R_n)$ will be fixed by the value of K_n and by eq. (3.29). In particular, for the massless mode it is easy to get

$$\frac{1}{f_0^2} = \frac{2\pi R}{g_5^2} + \frac{1}{\tilde{g}^2} + \frac{1}{\tilde{g}'^2}. \tag{3.32}$$

3.2.1 Derivation of the conditions for the KK expansion

I will now show how eqs. from (3.9) to (3.27) can be derived from the request that the effective 4D Lagrangian is diagonal. Throughout the following calculation, I will only need to work with the bilinear gauge part of the action (3.3). Expanding the gauge fields as in eq. (3.8) *without assuming anything a priori* on the form of the functions f^a_j and g^a_j and the constants c^a_j and carrying out the integration with respect to the extra dimension, I get

$$\begin{aligned}\mathcal{L}^{(2)} = &-\frac{1}{4}V^{(j)}_{\mu\nu}V^{(k)\,\mu\nu}A_{jk} - \frac{1}{2}V^{(j)}_{\mu}V^{(k)\,\mu}B_{jk}\\ &-\frac{1}{2}\partial_\mu G^{(j)}\partial^\mu G^{(k)}C_{jk} + V^{(j)}_\mu\partial^\mu G^{(k)}D_{jk},\end{aligned} \tag{3.33}$$

where I defined the matrices:

$$\begin{aligned}A_{jk} &= \frac{1}{g_5^2}\int_0^{\pi R} dy\,\left(f^a_{L\,j}f^a_{L\,k} + f^a_{R\,j}f^a_{R\,k}\right) + \frac{1}{\tilde{g}^2}f^a_{L\,j}f^a_{L\,k}\big|_{\pi R}\\ &\quad + \frac{1}{\tilde{g}'^2}f^3_{R\,j}f^3_{R\,k}\big|_{\pi R};\\ B_{jk} &= \frac{1}{g_5^2}\int_0^{\pi R} dy\,b(y)\,\left(\partial_y f^a_{L\,j}\partial_y f^a_{L\,k} + \partial_y f^a_{R\,j}\partial_y f^a_{R\,k}\right)\\ &\quad + \frac{\tilde{v}^2}{4}b(\pi R)\left(f^a_{L\,j}f^a_{L\,k} + f^3_{R\,j}f^3_{R\,k} - 2f^3_{L\,j}f^3_{R\,k}\right)\big|_{\pi R};\\ C_{jk} &= \frac{1}{g_5^2}\int_0^{\pi R} dy\,b(y)\,\left(g^a_{L\,j}g^a_{L\,k} + g^a_{R\,j}g^a_{R\,k}\right) + c^a_j c^a_k;\\ D_{jk} &= \frac{1}{g_5^2}\int_0^{\pi R} dy\,b(y)\partial_y\,\left(f^a_{L\,j}g^a_{L\,k} + f^a_{R\,j}g^a_{R\,k}\right)\\ &\quad - \frac{\tilde{v}}{2}\left(f^a_{L\,j}c^a_k - f^3_{R\,j}c^3_k\right)\big|_{\pi R}.\end{aligned} \tag{3.34}$$

In the expanded Lagrangian (3.33), it is possible to recognize vector and scalar kinetic-like terms, vector mass-like terms and vector / would-be goldstone mixings. However, all those terms are in general not diagonal with respect to the KK number. This is of course a direct consequence of the general nature of the expansion (3.8). However, if the expanded theory is to be consistent, it must be possible to obtain the actual physical degrees of freedom - with explicitly diagonal mass and kinetic terms - by defining appropriate linear combinations of the modes $V^{(j)}_\mu$ and $G^{(j)}$. I then introduce a still general basis change in field space:

$$V^{(j)}_\mu = R_{jk}\,\tilde{V}^{(k)}_\mu; \quad G^{(j)} = S_{jk}\,\tilde{G}^{(k)}, \tag{3.35}$$

and *require* the Lagrangian (3.33) to be diagonal in terms of the new degrees of freedom $\tilde{V}^{(j)}_\mu$ and $\tilde{G}^{(j)}$. This means that the matrices R^TAR, R^TBR, S^TCS and R^TDS (all the fields are real, so I can choose the matrices R and S to be orthogonal) have to be diagonal. Since in general it is not possible to diagonalize four independent matrices using just two rotations, I will need to impose a set of *consistency conditions* on the wavefunctions $f^a_{L,R\,j}$ and $g^a_{L,R\,j}$ and the constants c^a_j, that will determine the wave functions uniquely.

Let me define:

$$f^a_{L,R\,j} = R_{jk}\,\tilde{f}^a_{L,R\,k}; \quad g^a_{L,R\,j} = S_{jk}\,\tilde{g}^a_{L,R\,k}, \quad c^a_j = S_{jk}\,\tilde{c}^a_k; \tag{3.36}$$

the conditions that I need to impose on the KK modes are then:

$$\begin{aligned}
&\frac{1}{g_5^2}\int_0^{\pi R} dy\left(\tilde{f}^a_{L\,j}\tilde{f}^a_{L\,k} + \tilde{f}^a_{R\,j}\tilde{f}^a_{R\,k}\right) + \frac{1}{\tilde{g}^2}\tilde{f}^a_{L\,j}\tilde{f}^a_{L\,k}|_{\pi R} \\
&+ \frac{1}{\tilde{g}'^2}\tilde{f}^3_{R\,j}\tilde{f}^3_{R\,k}|_{\pi R} = a_j\delta_{jk};
\end{aligned} \tag{3.37a}$$

$$\begin{aligned}
&\frac{1}{g_5^2}\int_0^{\pi R} dy\, b(y)\left(\partial_y\tilde{f}^a_{L\,j}\partial_y\tilde{f}^a_{L\,k} + \partial_y\tilde{f}^a_{R\,j}\partial_y\tilde{f}^a_{R\,k}\right) \\
&+ \frac{\tilde{v}^2}{4}b(\pi R)\left(\tilde{f}^a_{L\,j}\tilde{f}^a_{L\,k} + \tilde{f}^3_{R\,j}\tilde{f}^3_{R\,k} - 2\tilde{f}^3_{L\,j}\tilde{f}^3_{R\,k}\right)|_{\pi R} = b_j\delta_{jk};
\end{aligned} \tag{3.37b}$$

$$\frac{1}{g_5^2}\int_0^{\pi R} dy\, b(y)\left(\tilde{g}^a_{L\,j}\tilde{g}^a_{L\,k} + \tilde{g}^a_{R\,j}\tilde{g}^a_{R\,k}\right) + \tilde{c}^a_j\tilde{c}^a_k = c_j\delta_{jk}; \tag{3.37c}$$

$$\begin{aligned}
&\frac{1}{g_5^2}\int_0^{\pi R} dy\, b(y)\left(\partial_y\tilde{f}^a_{L\,j}\tilde{g}^a_{L\,k} + \partial_y\tilde{f}^a_{R\,j}\tilde{g}^a_{R\,k}\right) \\
&\qquad - \frac{\tilde{v}}{2}\left(\tilde{f}^a_{L\,j}\tilde{c}^a_k - \tilde{f}^3_{R\,j}\tilde{c}^3_k\right)\Big|_{\pi R} = d_j\delta_{jk}.
\end{aligned} \tag{3.37d}$$

I want to reduce the set of eqs. (3.37) to a more explicit form. As a first thing, consider the integral appearing in the left-hand side of eq. (3.37b). It can be rewritten

$$\begin{aligned}
&\int_0^{\pi R} b(y)\,\partial_y\tilde{f}^a_{L\,j}\partial_y\tilde{f}^a_{L\,k}\,dy + \ (L\to R) \\
= -&\int_0^{\pi R}\tilde{f}^a_{L\,j}\partial_y(b(y)\,\partial_y\tilde{f}^a_{L\,k})\,dy + b(\pi R)\,\tilde{f}^a_{L\,j}\partial_y\tilde{f}^a_{L\,k}\big|_0^{\pi R} + \ (L\to R).
\end{aligned} \tag{3.38}$$

If the eigenfunctions $\tilde{f}^a_{L,R\,j}$ satisfy the equation of motion:

$$\hat{D}f^a_{L,R\,j} = -m_j^2 f^a_{L,R\,j}, \tag{3.39}$$

where I leave the eigenvalue m_j for now unspecified, then the integral in eq. (3.38) can be further simplified to

$$-m_k^2\int_0^{\pi R}\tilde{f}^a_{L\,j}\tilde{f}^a_{L\,k}\,dy + b(\pi R)\,\tilde{f}^a_{L\,j}\partial_y\tilde{f}^a_{L\,k}\big|_0^{\pi R} + \ (L\to R). \tag{3.40}$$

Now notice from eqs. (3.37) that the left and right wavefunctions only mix through their 3^{rd} isospin components. So the conditions (3.37) receive three separate contributions, one from left wavefunctions with isospin $a = 1, 2$, another from $a = 1, 2$ right wavefunctions and the last one from mixed left/right $a = 3$ modes. The simplest, most natural choice is to diagonalize the three contributions independently. In this way, I will get three different sets of BCs, that is three decoupled towers of mass eigenstates. While this may not be the most general solution to eqs. (3.37), it is consistent with the symmetry breaking pattern. The general expansion (3.8) can then be recast into a more explicit form:

$$\begin{aligned}
W^{1,2}_{L\,\mu}(x,y) &= \sum_{n=0}^{\infty} f^{1,2}_{L\,n}(y)\, W^{1,2\,(n)}_{L\,\mu}(x), & W^{1,2}_{L\,5}(x,y) &= \sum_{n=0}^{\infty} g^{1,2}_{L\,n}(y)\, G^{1,2\,(n)}_{L}(x),\\
W^{1,2}_{R\,\mu}(x,y) &= \sum_{n=0}^{\infty} f^{1,2}_{R\,n}(y)\, W^{1,2\,(n)}_{R\,\mu}(x), & W^{1,2}_{R\,5}(x,y) &= \sum_{n=0}^{\infty} g^{1,2}_{R\,n}(y)\, G^{1,2\,(n)}_{R}(x),\\
W^{3}_{L\,\mu}(x,y) &= \sum_{n=0}^{\infty} f^{3}_{L\,n}(y)\, N^{(n)}_{\mu}(x), & W^{3}_{L\,5}(x,y) &= \sum_{n=0}^{\infty} g^{3}_{L\,n}(y)\, G^{(n)}_{N}(x),\\
W^{3}_{R\,\mu}(x,y) &= \sum_{n=0}^{\infty} f^{3}_{R\,n}(y)\, N^{(n)}_{\mu}(x), & W^{3}_{R\,5}(x,y) &= \sum_{n=0}^{\infty} g^{3}_{R\,n}(y)\, G^{(n)}_{N}(x),\\
\pi^{1,2}(x) &= \sum_{n=0}^{\infty} c^{1,2}_{n}\, G^{(n)}(x), & \pi^{3}(x) &= \sum_{n=0}^{\infty} c^{3}_{n}\, G^{(n)}(x).
\end{aligned} \tag{3.41}$$

As a consequence of this redefinition, the equation of motion (3.39) can also be more explicitly rewritten as three separate equations:

$$\hat{D} f^{1,2}_{L\,n} = -m^2_{L\,n} f^{1,2}_{L\,n}, \tag{3.42}$$
$$\hat{D} f^{1,2}_{R\,n} = -m^2_{R\,n} f^{1,2}_{R\,n}, \tag{3.43}$$
$$\hat{D} f^{3}_{L,R\,n} = -m^2_{N\,n} f^{3}_{L,R\,n}, \tag{3.44}$$

to emphasize the fact that to each sector corresponds a different set of eigenvalues. These three equations reproduce precisely eq. (3.10), (3.18) and (3.24).

To go on, assume that the wavefunctions obey orthogonality conditions:

$$(f^a_{L\,m}, f^a_{L\,n})_{\tilde{g}} = \delta_{mn}, \quad \frac{1}{g_5^2}(f^{1,2}_{R\,m}, f^{1,2}_{R\,n})_{L^2} = \delta_{mn}, \quad (f^3_{R\,m}, f^3_{R\,n})_{\tilde{g}'} = \delta_{mn}, \tag{3.45}$$

where the $(\cdot,\cdot)_{\tilde{g}}$ scalar product was defined in eq. (3.15). With this assumption, the left-hand side of eq. (3.37a) becomes diagonal, and the equation itself is satisfied by choosing $a_n \equiv 1$. Furthermore, eq. (3.37b) splits into three independent conditions:

$$\begin{aligned} b^L_n \delta_{mn} =& - m^2_{L\,n}\, \delta_{mn} + \left(\frac{m^2_{L\,n}}{\tilde{g}^2} + \frac{\tilde{v}^2}{4} b(\pi R)\right) \tilde{f}^{1,2}_{L\,m} \tilde{f}^{1,2}_{L\,n}|_{\pi R} \\ &+ b(\pi R) \left(\tilde{f}^{1,2}_{L\,m} \partial_y \tilde{f}^{1,2}_{L\,n}\right)|^0_{\pi R}, \end{aligned} \tag{3.46}$$

$$\begin{aligned} b^R_n \delta_{mn} =& - m^2_{R\,n}\, \delta_{mn} + \left(\frac{m^2_{R\,n}}{\tilde{g}^2} + \frac{\tilde{v}^2}{4} b(\pi R)\right) \tilde{f}^{1,2}_{R\,m} \tilde{f}^{1,2}_{R\,n}|_{\pi R} \\ &+ b(\pi R) \left(\tilde{f}^{1,2}_{R\,m} \partial_y \tilde{f}^{1,2}_{R\,n}\right)|^0_{\pi R}, \end{aligned} \tag{3.47}$$

$$\begin{aligned} b^N_n \delta_{mn} =& - m^2_{N\,n}\, \delta_{mn} + \frac{m^2_{N\,n}}{\tilde{g}^2} \left(\tilde{f}^3_{L\,m} \tilde{f}^3_{L\,n} + \tilde{f}^3_{R\,m} \tilde{f}^3_{R\,n}\right)|_{\pi R} \\ &+ \frac{\tilde{v}^2}{4} b(\pi R) \left(\tilde{f}^3_{L\,m} \tilde{f}^3_{L\,n} + \tilde{f}^3_{L\,m} \tilde{f}^3_{L\,n} - 2 \tilde{f}^3_{L\,m} \tilde{f}^3_{L\,n}\right)|_{\pi R} \\ &+ b(\pi R) \left(\tilde{f}^3_{L\,m} \partial_y \tilde{f}^3_{L\,n} + \tilde{f}^3_{R\,m} \tilde{f}^3_{R\,n}\right)|^0_{\pi R}, \end{aligned} \tag{3.48}$$

which are identically satisfied as soon as the $f^a_{L,Rn}$ obey the BCs (3.12), (3.13), (3.19), (3.20), (3.25) and (3.26). Notice that eq. (3.10), (3.18), (3.24) together with the above mentioned BCs guarantee the orthogonality of the wavefunctions that I assumed in eq. (3.45), so I have a self-consistent solution of eqs. (3.37a) and (3.37b). To complete the diagonalization and finally get an expanded bilinear Lagrangian, I just need to solve the last two equations in the set (3.37). This can be obtained by imposing the conditions (3.14), (3.21) and (3.27) respectively on the scalar profiles of the three sectors.

3.2.2 The expanded Lagrangian

After the expansion, the gauge Lagrangian is reduced to the form:

$$\begin{aligned} \mathcal{L}^{(2)}_{gauge} =& - \frac{1}{2} W^{+\,(n)}_{L\,\mu\nu} W_L^{-\,(n)\,\mu\nu} - \frac{1}{2} W^{+\,(n)}_{R\,\mu\nu} W_R^{-\,(n)\,\mu\nu} - \frac{1}{4} N^{(n)}_{\mu\nu} N^{(n)\,\mu\nu} \\ & - \left|\partial_\mu G_L^{+\,(n)} - m_{L\,n} W^{+\,(n)}_{L\,\mu}\right|^2 - \left|\partial_\mu G_R^{+\,(n)} - m_{R\,n} W^{+\,(n)}_{L\,\mu}\right|^2 \\ & - \frac{1}{2}\left(\partial_\mu G_N^{(n)} - m_{N\,n} N^{(n)}_\mu\right)^2 \\ + \Big\{ & i\, g^L_{klm} \left[N^{(m)}_{\mu\nu} W_L^{+\,(k)\,\mu} W_L^{-\,(l)\,\nu} + N^{(m)}_\mu (W_L^{-\,(l)\,\mu\nu}) W^{+\,(k)}_{L\,\nu} - h.c.\right] \\ & + g^{2\,LL}_{klmn} \Big[W_L^{+\,(k)\,\mu} W_L^{-\,(l)\,\nu} W_L^{+\,(m)\,\rho} W_L^{-\,(n)\,\sigma} (\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma})\Big] \\ + & \; g^{2\,LN}_{klmn} \Big[W_L^{+\,(k)\,\mu} W_L^{-\,(l)\,\nu} N^{(m)\,\rho} N^{(n)\,\sigma} (\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma})\Big] + (L \leftrightarrow R)\Big\} \end{aligned} \tag{3.49}$$

taking into account contributions from both brane and bulk terms. The bilinear part of the Lagrangian is, as announced, diagonal. The trilinear and quadrilinear coupling constants g^{L}_{klm}, $g^{2\,LL}_{klmn}$, $g^{2\,LN}_{klmn}$ are defined in terms of the gauge profiles:

$$g^{L}_{klm} = \frac{1}{g_5^2}\int_0^{\pi R} dy f^1_{L\,k} f^1_{L\,l} f^3_{L\,m} + \frac{1}{\tilde{g}^2} f^1_{L\,k} f^1_{L\,l} f^3_{L\,m}|_{\pi R}, \tag{3.50}$$

$$g^{2\,LL}_{klmn} = \frac{1}{g_5^2}\int_0^{\pi R} dy f^1_{L\,k} f^1_{L\,l} f^1_{L\,m} f^1_{L\,n} + \frac{1}{\tilde{g}^2} f^1_{L\,k} f^1_{L\,l} f^1_{L\,m} f^1_{L\,n}|_{\pi R}, \tag{3.51}$$

$$g^{2\,LN}_{klmn} = \frac{1}{g_5^2}\int_0^{\pi R} dy f^1_{L\,k} f^1_{L\,l} f^3_{L\,m} f^3_{L\,n} + \frac{1}{\tilde{g}^2} f^1_{L\,k} f^1_{L\,l} f^3_{L\,m} f^3_{L\,n}|_{\pi R} \tag{3.52}$$

(remember that $f^1_{L(R)\,n} \equiv f^2_{L(R)\,n}$); similar definitions hold for the coupling constants g^{R}_{klm}, $g^{2\,RR}_{klmn}$, $g^{2\,RN}_{klmn}$ of the right sector, but without any contribution from boundary terms due to eq. (3.20).

An important observation can be made concerning the couplings $g^{L(R)}_{kl0}$. These give the coupling of $N^{(0)}$, which I identified with the photon, with the charged fields; as a consequence, they should all be equal to the electric charge, for any value of k, l. By the definition (3.50) and eq. (3.28), I immediately get:

$$g^{L}_{kl0} = g^{R}_{kl0} \equiv f_0 \delta_{kl}, \tag{3.53}$$

thanks to the fact that the wavefunctions $f^1_{L\,k}$ and $f^1_{R\,k}$ form an orthonormal basis. Then I conclude that

$$f_0 = e. \tag{3.54}$$

Now, using the fact that $K_0 = 1$ (see eq. (3.30) and eq. (3.26)), and the normalization conditions of the neutral sector (3.29), (3.31), I can derive an expression for the electric charge as a function of the model parameters:

$$\frac{1}{e^2} = \frac{2\pi R}{g_5^2} + \frac{1}{\tilde{g}^2} + \frac{1}{\tilde{g}'^2}. \tag{3.55}$$

The actual profiles and masses can of course only be obtained by specifying the warp factor $b(y)$. However, it is possible to put, in general, the various equation of motion and BCs (eqs. from (3.9) to (3.27)) in a more compact form. In fact, equations of motion (3.10), (3.18) and (3.24) all have the same general form, $\hat{D}f = -m^2 f$. This equation is a second order ODE, so it admits two independent solutions. Following ref. [60], I can introduce two convenient linear combinations $C(y, m_n)$ and $S(y, m_n)$ ("warped sine and cosine") such that

$$C(0, m) = 1, \quad \partial_y C(0, m) = 0; \qquad S(0, m) = 0, \quad \partial_y S(0, m) = m \tag{3.56}$$

with $m \neq 0$ (I have already seen that there is a single massless mode and that its profile is constant). In the limit of a flat extra dimension, these functions reduce to the ordinary sine and cosine.

Thanks to the Neumann BCs on the $y = 0$ brane (3.12), (3.19), (3.25), the vector profiles $f^a_{L,R\,n}$ are all proportional to $C(y, m_n)$. The eigenvalues, that is the physical masses of the vector fields $m_{L\,n}$, $m_{R\,n}$ and $m_{N\,n}$, are then fixed by the BCs on the IR brane (3.13), (3.20) and (3.26). For the three sectors I can easily derive three eigenvalue equations:

Left charged:

$$\tfrac{\tilde{g}^2}{g_5^2} C'(\pi R, m_{L\,n}) - (b(\pi R)\; m_{L\,n}^2 - \tfrac{\tilde{g}^2 \tilde{v}^2}{4})\, C(\pi R, m_{L\,n}) = 0 \tag{3.57}$$

Right charged:

$$C(\pi R, m_{R\,n}) = 0 \tag{3.58}$$

Neutral:

$$\begin{aligned}
&\Big(\tfrac{\tilde{g}^2}{g_5^2} C'(\pi R, m_{N\,n}) - (b(\pi R)\; m_{N\,n}^2 - \tfrac{\tilde{g}^2 \tilde{v}^2}{4})\, C(\pi R, m_{N\,n})\Big) \cdot \\
&\Big(\tfrac{\tilde{g}'^2}{g_5^2} C'(\pi R, m_{N\,n}) - (b(\pi R)\; m_{N\,n}^2 - \tfrac{\tilde{g}'^2 \tilde{v}^2}{4})\, C(\pi R, m_{N\,n})\Big) \\
&= \tfrac{\tilde{g}^2 \tilde{g}'^2 \tilde{v}^4}{16} C(\pi R, m_{N\,n})^2
\end{aligned} \tag{3.59}$$

In section 3.4 I will make extensive use of these equations for specific choices of the warp factor and of the parameters of the models to obtain explicit examples of the KK spectrum.

3.3 Low energy limit and EW precision observables

I can obtain a convenient low-energy approximation of the theory by using the so-called *holographic* approach [58, 35, 36, 61, 62, 63, 64], which consists in integrating out the bulk degrees of freedom in the functional integral. For the purposes of the present calculation it is sufficient to take into account just the tree-level effects of the heavy resonances, so the integration can be done by simply eliminating the bulk fields from the Lagrangian via their classical equations of motion; moreover, bulk gauge self-interactions can be neglected.

The equations to be solved are:

$$\hat{D}\, W^a_{L(R)\,\mu}(p, y) = (p^2 \delta_{\mu\nu} - p_\mu p_\nu) W^{a\,\nu}_{L(R)}(p, y) \tag{3.60}$$

where $\hat{D}$ defined in eq. (3.11)) and I have Fourier transformed with respect to the first four coordinates. As previously discussed, on the $y = 0$ brane, I do not want to make any assumptions on the value of the fields; so I leave their variations arbitrary, and since there are no localized terms on the brane, this leads to Neumann boundary conditions for all of the fields:

$$\begin{cases} \partial_y W^a_{L\,\mu} = 0 \\ \partial_y W^a_{R\,\mu} = 0 \end{cases} \qquad y = 0. \tag{3.61}$$

At the other end of the AdS segment, I account for the presence of localized terms by imposing four fields to be equal to generic *source fields*, while the other two (those corresponding to the right charged sector) are vanishing:

$$\begin{cases} W^a_{L\,\mu} = \tilde{W}^a_\mu \\ W^3_{R\,\mu} = \tilde{B}_\mu \\ W^{1,2}_{R\,\mu} = 0 \end{cases} \qquad y = \pi R. \tag{3.62}$$

The first step in solving the equations is to split the fields in their *longitudinal* (aligned with p_μ) and *transversal* parts. The operator $(p^2\delta_{\mu\nu} - p_\mu p_\nu)$ is vanishing when acting on the longitudinal part, while it is simply equivalent to p^2 when acting on the transversal one. In this way, each equation can be split into two simpler ones:

$$\begin{cases} \hat{D}\, W^{a,\,tr}_{L/R\,\mu} = p^2\, W^{a,\,tr}_{L/R\,\mu} \\ \hat{D}\, W^{a,\,long}_{L/R\,\mu} = 0 \end{cases} \tag{3.63}$$

Taking into account the boundary conditions (3.61), (3.62), and defining $|p^2| \equiv \omega^2$, eqs. (3.63) are simply solved; the solutions are given by

$$\begin{cases} W^{a,\,tr}_{L\,\mu} = (\tilde{C}(y,\omega) - \dfrac{\tilde{C}'(0,\omega)}{\tilde{S}'(0,\omega)}\tilde{S}(y,\omega))\tilde{W}^{a,\,tr}_\mu \\ W^{a,\,long}_{L\,\mu} = \tilde{W}^{a,\,long}_\mu \\ W^{3,\,tr}_{R\,\mu} = (\tilde{C}(y,\omega) - \dfrac{\tilde{C}'(0,\omega)}{\tilde{S}'(0,\omega)}\tilde{S}(y,\omega))\tilde{B}^{tr}_\mu \\ W^{3,\,long}_{R\,\mu} = \tilde{B}^{long}_\mu \\ W^{1,2}_{R\,\mu} = 0 \end{cases} \tag{3.64}$$

with

$$\begin{aligned} \hat{D}\,(\tilde{S},\tilde{C}) &= -\,\omega^2(\tilde{S},\tilde{C}); \\ \tilde{S}(\pi R,\omega) = 0,\ &\tilde{S}'(\pi R,\omega) = \omega; \\ \tilde{C}(\pi R,\omega) = 1,\ &\tilde{C}'(\pi R,\omega) = 0. \end{aligned} \tag{3.65}$$

As it can be seen, the first two components of the right sector drop out from the low-energy effective Lagrangian altogether. This corresponds to the fact that in general the right charged sector, in contrast to the left charged and neutral ones, has no superposition with the IR brane, which is where the fermions are confined; as a consequence, they do not give any contribution to four fermion processes and to EW precision observables.

Before substituting the solutions, note that the bulk Lagrangian can be reduced - through an integration by parts - to a surface term plus a term proportional to the equations of motion,

$$\mathcal{L}^{(2)}_{bulk} = -\frac{1}{2g_5^2}\left(\partial_y (W^{a,\,tr}_{L\,\mu} b(y) W^{a,\,tr\,\mu}_{L}) - W^{a,\,tr}_{L\,\mu}((\hat{D}-p^2)\delta^\mu_\nu + p^\mu p_\nu) W^{a,\,tr\,\nu}_{L}\right) \\ + (L \to R). \tag{3.66}$$

After the substitution, most of the terms vanish due to the BCs; I am only left with

$$\mathcal{L}^{(2)}_{bulk} = -\frac{1}{2g_5^2} W^a_{L\,\mu} b(y)\; \partial_y W^{a\,\mu}_{L}|_{\pi R} + (L \to R,\ a \to 3); \tag{3.67}$$

taking into account the definition of the $\tilde{C}$ and $\tilde{S}$ functions (3.65), eq. (3.67) reduces to

$$\mathcal{L}^{(2)}_{bulk} = \frac{\omega\; b(\pi R)}{2g_5^2}\left.\frac{\tilde{C}'}{\tilde{S}'}\right|_0 \left(\tilde{W}^{a,\,tr}_\mu\;\tilde{W}^{a,\,tr\,\mu} + \tilde{B}^{tr}_\mu\;\tilde{B}^{tr\,\mu}\right). \tag{3.68}$$

Eq. (3.68) has a complicated dependence on ω hidden in the functions $\tilde{S}'|_0$, $\tilde{C}'|_0$. In order to extract the low-energy behaviour of the theory, let me expand in ω. This can be done in general, without needing to specify $b(y)$. In fact, using eq. (3.65), it is not difficult to show that the functions $\tilde{C}(y,\omega)$ and $\tilde{S}(y,\omega)$ obey the integral equations:

$$\tilde{C}(y,\omega) = 1 - \omega^2 \int_y^{\pi R} dy'\, b^{-1}(y') \int_{y'}^{\pi R} dy''\, \tilde{C}(y'',\omega) \tag{3.69}$$

$$\tilde{S}(y,\omega) = \omega \int_y^{\pi R} dy'\, b^{-1}(y') - \omega^2 \int_y^{\pi R} dy'\, b^{-1}(y') \int_{y'}^{\pi R} dy''\, \tilde{S}(y'',\omega), \tag{3.70}$$

from which I can derive a low-energy expansion (small ω):

$$\tilde{C}(y,\omega) = 1 - \omega^2 \int_y^{\pi R} dy'\, y'\, b^{-1}(y') \\ + \omega^4 \int_y^{\pi R} dy'\, b^{-1}(y') \int_{y'}^{\pi R} dy'' \int_{y''}^{\pi R} dy'''\, y''' b^{-1}(y''') + \ldots \tag{3.71}$$

$$\tilde{S}(y,\omega) = \omega \int_y^{\pi R} dy'\, b^{-1}(y') \\ - \omega^3 \int_y^{\pi R} dy'\, b^{-1}(y') \int_{y'}^{\pi R} dy'' \int_{y''}^{\pi R} dy'''\, b^{-1}(y''') + \ldots \tag{3.72}$$

I will substitute expansions (3.71), (3.72) in eq. (3.68), keeping terms up to $O(\omega^4)$, which - as I will soon show - will reproduce the Standard Model (SM) plus corrections of order $m_Z^2/\bar{M}^2$, where $\bar{M}$ is given by:

$$\frac{1}{\bar{M}^2} = \frac{1}{\pi R}\int_0^{\pi R} dy \int_y^{\pi R} dz\ z\ b^{-1}(z) \tag{3.73}$$

and, as I will show later, is of the order of the mass of the lightest resonance that I have integrated out. After the substitution, the bulk Lagrangian becomes

$$\begin{aligned}\mathcal{L}^{(2)}_{bulk} = & -\frac{\pi R}{2g_5^2}\Big(\tilde{W}^a_\mu(p^2\eta^{\mu\nu} - p^\mu p^\nu)(1-\frac{p^2}{\bar{M}^2})\tilde{W}^a_\nu \\ & + \tilde{B}_\mu(p^2\eta^{\mu\nu} - p^\mu p^\nu)(1-\frac{p^2}{\bar{M}^2})\tilde{B}_\nu\Big);\end{aligned} \tag{3.74}$$

Notice that the parameter $\bar{M}$ can be related to the integrals introduced in [65],

$$\frac{1}{\bar{M}^2} = I_2(\pi R) - I_1(\pi R) \tag{3.75}$$

where

$$I_1(y) = \frac{1}{\pi R}\int_0^y \int_0^z dz'\ z' b^{-1}(z'), \quad I_2(y) = \int_0^y dz'\ z' b^{-1}(z') \tag{3.76}$$

Finally, the effective Lagrangian is obtained by adding the above contribution to the one coming from the brane. Switching back to the coordinate space, the final expression is

$$\begin{aligned}\mathcal{L}^{(2)}_{eff} = & -\frac{1}{4g^2}\tilde{W}^a_{\mu\nu}\tilde{W}^{a\ \mu\nu} - \frac{1}{4g'^2}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} \\ & -\frac{v^2}{8}\left(\tilde{W}^a_\mu\tilde{W}^{a\ \mu} + \tilde{B}_\mu\tilde{B}^\mu - 2\tilde{W}^3_\mu\tilde{B}^\mu\right) \\ & +\frac{\pi R}{4g_5^2}\left(\tilde{W}^a_{\mu\nu}\frac{\Box}{\bar{M}^2}\tilde{W}^{a\ \mu\nu} + \tilde{B}_{\mu\nu}\frac{\Box}{\bar{M}^2}\tilde{B}^{\mu\nu}\right) \\ & + \textit{bosonic self-interactions} + \textit{fermion terms}\end{aligned} \tag{3.77}$$

where I have introduced the effective couplings

$$\frac{1}{g^2} = \frac{1}{\tilde{g}^2} + \frac{1}{\bar{g}_5^2}, \qquad \frac{1}{g'^2} = \frac{1}{\tilde{g}'^2} + \frac{1}{\bar{g}_5^2}. \tag{3.78}$$

and

$$v = \tilde{v}\, b(\pi R), \quad \bar{g}_5^2 = g_5^2/\pi R. \tag{3.79}$$

Lagrangian (3.77) is identical to (2.67); the only difference lies in the definition of the mass parameter $\bar{M}$. But eqs. (2.66) and (2.68), together with the condition $g_i \to g_c$ (which I have to impose to avoid an y-dependent 5D coupling), imply

$$\left.\frac{1}{\bar{M}^2}\right|_{decon.} = \sum_{i=1}^{N}\frac{1}{g_c^2}\sum_{j=N+i}^{2N+1}\frac{j-N}{f_j^2 g_c^2} = \sum_{i=1}^{N}\frac{1}{g_c^2}\sum_{j=i}^{N+1}\frac{j}{f_j^2 g_c^2}, \tag{3.80}$$

which, by eq. (3.2), is the discretization of eq. (3.73). This is the final piece of evidence that the 5D model I am studying in this chapter is the correct 5D limit of GD-BESS.

Starting from eq. (3.77), I can retrace all the steps that bring from (2.67) to the calculation of the ϵ parameters and of the standard input parameters α, G_F and m_Z in terms of the model parameters. For convenience I rewrite the results:

$$\alpha \equiv \frac{e^2}{4\pi} = \frac{g^2 s_\theta^2}{4\pi}, \tag{3.81}$$

$$m_Z^2 = \tilde{M}_Z^2 \left(1 - z_Z \frac{\tilde{M}_Z^2}{\bar{M}^2}\right), \quad \text{with} \quad \tilde{M}_Z^2 = \frac{v^2(g^2 + g'^2)}{4}, \tag{3.82}$$

$$\frac{G_F}{\sqrt{2}} \equiv \frac{e^2}{8 s_{\theta_0}^2 c_{\theta_0}^2 m_Z^2}, \quad \text{with} \quad s_{\theta_0}^2 c_{\theta_0}^2 = s_\theta^2 c_\theta^2 \left(1 + z_Z \frac{m_Z^2}{\bar{M}^2}\right), \tag{3.83}$$

$$\epsilon_1 = -\frac{(c_\theta^4 + s_\theta^4)}{c_\theta^2}\overline{X}, \quad \epsilon_2 = -c_\theta^2 \overline{X}, \quad \epsilon_3 = -\overline{X}, \tag{3.84}$$

with

$$\overline{X} = \frac{m_Z^2}{\bar{M}^2}\left(\frac{g}{\bar{g}_5}\right)^2. \tag{3.85}$$

with $\tan(\theta) = g'/g$.

Notice that by eqs. (3.78) and (3.81) I get

$$e^2 = \frac{1}{\frac{1}{\bar{g}^2} + \frac{1}{\bar{g}_5^2} + \frac{1}{\bar{g}'^2} + \frac{1}{\bar{g}_5^2}}, \tag{3.86}$$

which is in perfect agreement with eq. (3.55).

In section 3.4, I will study the constraints on the model parameter space by EW precision parameter for two choices of the warp factor, $b(y) \equiv 1$ (flat extra dimension) and $b(y) = e^{-2ky}$ (a slice of AdS_5).

3.3.1 Notes on unitarity and the Higgs field

As any gauge theory in 5 space-time dimensions, the 5D D-BESS model has couplings with negative mass dimension and is therefore not renormalizable. In the KK expanded 4D theory emerging from the compactification of the extra dimension, the nonrenormalizability manifests as a partial wave unitarity violation at tree level at an energy scale proportional to the inverse square of the gauge coupling [66]. While a detailed study of the unitarity properties of the model was beyond the scope of the present work, it is still possible (and interesting) to give an estimate based on naive dimensional analysis. In flat space, the naive estimate for a gauge theory with dimensional coupling constant g_5 gives a cut-off $\Lambda = (16\pi^2)/g_5^2$ [67].

In a warped space, the cut-off is dependent on the location along the fifth dimension: starting from Λ at the $y = 0$ brane, it is redshifted along the interval (as is every other energy scale in the theory), getting down to $\Lambda' = \Lambda\sqrt{b(\pi R)}$ upon reaching the $y = \pi R$ brane. To get an estimate for the Kaluza-Klein 4D effective theory, we will use the most restrictive cut-off:

$$\Lambda' = \frac{16\pi^2}{g_5^2}\sqrt{b(\pi R)}. \tag{3.87}$$

In addition to the one coming from the negative mass dimension bulk coupling g_5 (or equivalently from the infinite tower of KK excitations), the 5D D-BESS has another, more stringent unitarity bound: the one coming from the U field on the $y = \pi R$ brane. In this model, in fact, the longitudinal components of the electroweak gauge bosons are only coupled to the U field. As a consequence the corresponding scattering amplitudes violate partial wave unitarity at the same energy scale as in the Higgsless SM [8] that I reviewed at the end of section 1.1.1, that is $\Lambda_{cut-off} \simeq 1.7$ TeV. The violation of unitarity is not postponed to higher scales as in the 5 dimension Higgsless model [19, 20]. This situation exactly mirrors the one of the 4D D-BESS and the deconstructed GD-BESS models.

However, this problem can be easily cured by generalizing the U field to a matrix containing an additional real scalar excitation ρ, mimicking in the matrix formulation of the standard Higgs sector:

$$U \to M \equiv \frac{\rho}{\sqrt{2}}U. \tag{3.88}$$

Just as in the case of the Higgsless SM, the exchange of the new scalar degree of freedom ρ cancels the growing with energy terms in the scattering of the longitudinal EW gauge bosons, delaying unitarity violation. A similar process of unitarization via the addition of scalar fields was also studied in the context of the D-BESS model in ref. [68].

Once I have added the extra scalar field ρ, I can give it a potential:

$$V(\rho) = -\frac{\mu^2}{2}\rho^2 + \frac{\lambda}{4}\rho^4, \tag{3.89}$$

whereupon the field ρ acquires a VEV $\tilde{v} = \frac{\mu}{\sqrt{\lambda}}$ and a mass $m_H = \sqrt{2b(\pi R)}\mu$. I then expand as usual:

$$\rho = h + \tilde{v}; \tag{3.90}$$

h is an SM-like Higgs. With this expansion, the Lagrangian is equal to that of eq. (3.3) plus kinetic, mass and interaction terms for h. The interactions between h

and the gauge bosons help unitarizing the scattering of the longitudinally polarized vectors, and that the unitarity violation is postponed to the scale typical of a 5D theory, Λ'.

Notice that I could just have added the extra scalar from the beginning, substituting the term containing U with a standard complex scalar doublet. I did not do so because I first wanted to study the continuum limit of the GD-BESS model. In the GD-BESS case, the presence of a physical scalar seemed undesirable since it seemed to reintroduce the hierarchy problem. In the continuum limit, however, at least for a particular choice of the extra-dimensional background, the slice of AdS_5 that I will analyze in section 3.4.2, the Higgs can be interpreted as a composite state - just as the KK excitations of the gauge bosons - by the AdS/CFT correspondence [26, 58, 35, 36], sidestepping the hierarchy problem.

3.4 Phenomenology

In this last section, I am going to do a brief phenomenological study of the continuum GD-BESS in correspondence of two particular choices for the warp factor $b(y)$: the *flat limit*, $b(y) \equiv 1$ and the *RS limit*, $b(y) = e^{-2ky}$. In both cases, I will report spectrum examples, limits from electroweak precision parameters and naive unitarity cut-off.

3.4.1 Flat extra dimension

In this case, I have $b(y) \equiv 1$. This immediately implies (using eq. (3.73))

$$\bar{M} = \frac{\sqrt{3}}{\pi R}. \tag{3.91}$$

To get an interesting phenomenology at an accessible scale, I need $\bar{M} \sim$ TeV. The basic parameters of the model are πR, the gauge couplings g_5, $\tilde{g}$ and $\tilde{g}'$, the VEV of the scalar field $\tilde{v}$ (which is $\equiv v$ since $b = 1$) and its self-coupling constant λ. The latter is only used in the determination of the Higgs mass m_H; three out of four of the remaining parameters can be expressed in terms of the three measured quantities that are customarily chosen as input parameters for the SM, α, G_F and m_Z. Using eqs. (3.81) and (3.83) I can easily derive:

$$g^2 = \frac{4\pi\alpha}{s^2_{\theta_0}} \left(1 + \frac{4\pi\alpha(c^4_\theta + s^4_\theta)}{\bar{g}^2_5 c^2_\theta c_{2\theta}} \frac{m^2_Z}{\bar{M}^2}\right), \tag{3.92}$$

$$g'^2 = \frac{4\pi\alpha}{c^2_{\theta_0}} \left(1 - \frac{4\pi\alpha(c^4_\theta + s^4_\theta)}{\bar{g}^2_5 s^2_\theta c_{2\theta}} \frac{m^2_Z}{\bar{M}^2}\right), \tag{3.93}$$

$$v^2 = \frac{4}{g^2 + g'^2} m^2_Z \left(1 - \frac{4\pi\alpha(c^4_\theta + s^4_\theta)}{\bar{g}^2_5 c^4_\theta} \frac{m^2_Z}{\bar{M}^2}\right); \tag{3.94}$$

then, using definitions (3.78), I obtain also $\tilde{g}$ and $\tilde{g}'$. The free parameters of the model are then just πR and g_5, or equivalently πR and $\bar{g}_5$. The order of magnitude of πR is fixed by eq. (3.91) together with the request $\bar{M} \sim$ TeV, while $\bar{g}_5$ is constrained by eq. (3.78). In fact, since I need $\tilde{g}^2$ and $\tilde{g}'^2$ to be positive, eq. (3.78) implies $\bar{g}_5 > g,\ g'$. Numerically this means

$$g_5 \gtrsim 0.65, \tag{3.95}$$

because $g \simeq \sqrt{\frac{4\pi\alpha}{s^2_{\theta_0}}} = 0.65$ and $g' \simeq \sqrt{\frac{4\pi\alpha}{c^2_{\theta_0}}} = 0.36$ up to corrections of $O(m_Z^2/\bar{M}^2)$.

I am ready to go on and calculate the spectrum. In the flat limit, the C and S functions (eq. (3.56)) reduce to ordinary trigonometric functions:

$$C(y,m) = \cos(my), \qquad S(y,m) = \sin(my). \tag{3.96}$$

However, even in this very simple case only the eigenvalue equation for the right charged sector (3.58) can be analytically solved. I get

$$m_{R\,n} = \frac{2n-1}{2R}, \quad n = 1, 2, \ldots \tag{3.97}$$

The equations (3.57), (3.59) defining eigenvalues for the other two sectors have to be solved numerically. Some general remarks can be made at a qualitative level, however.

Eq. (3.57) can be recast in the form

$$m_{L\,n}\tan(m_{L\,n}\pi R) = -\frac{g_5^2}{\tilde{g}^2}(m^2_{L\,n} - \frac{\tilde{g}^2\tilde{v}^2}{4}); \tag{3.98}$$

the eigenvalues of the left charged sector are then determined by the intersection of the curves: the trigonometric curve $\tan(m\pi R)$ and the parabola $-\frac{g_5^2}{\tilde{g}^2}(m^2 - \frac{\tilde{g}^2\tilde{v}^2}{4})$. The $-\frac{\tilde{g}^2\tilde{v}^2}{4}$ term - originating from the $y = \pi R$ brane mass term in the action (3.3) - raises the vertex of the parabola, allowing for an intersection of the curves near $m = 0$, and a corresponding very light eigenstate $m_{L\,0}$, which can be identified with m_W. For bigger values of m, the parabola goes down as $-m^2$, and the intersection are nearer and nearer the asymptotes of $\tan(m\pi R)$ (which correspond to the zeroes of $\cos(m\pi R)$, and thus to the eigenvalues of the right charged sector, (3.97)), that are evenly spaced with a pace $1/R$. The situation is illustrated in fig. 3.2.

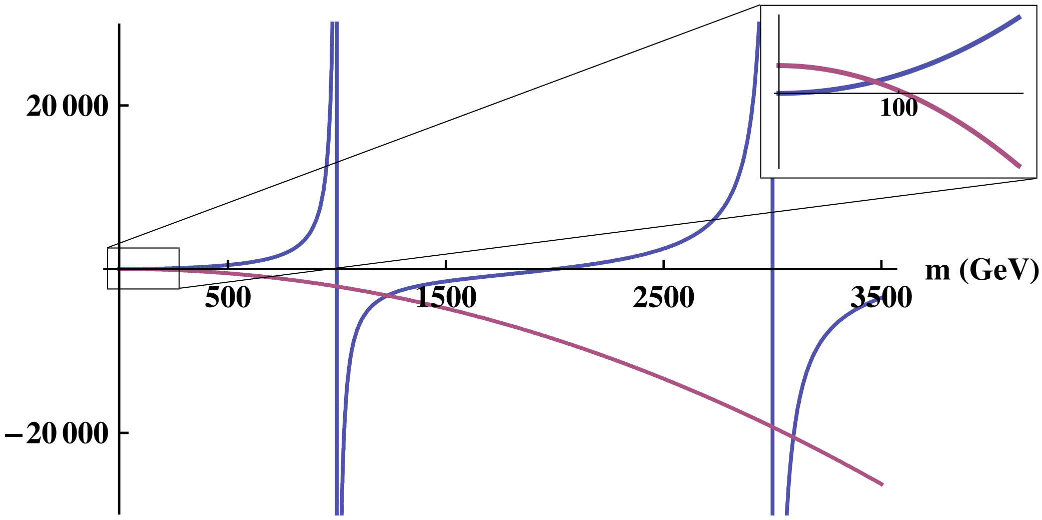


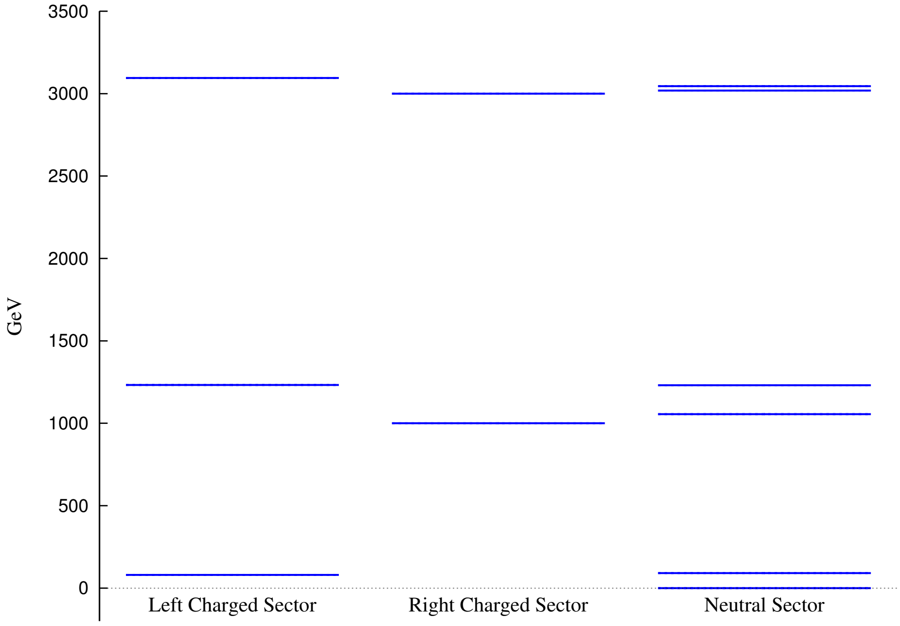
Figure 3.2: *Qualitative analysis of the eigenvalue equation* (3.98) *for the left charged sector. The intersections corresponding to the first and second KK excitations can be clearly seen. On the top right, on a bigger scale, the intersection corresponding to the zero mode, W.*

The neutral sector has a more complicated eigenvalue equation (3.59). However, it can be easily checked that, as soon as $m \gg m_Z$, the right-hand side of the equation is negligible so that it can be approximated:

$$\begin{aligned}&\left(\frac{\tilde{g}^2}{g_5^2} m \sin(\pi R m_{N\,n}) + (m_{N\,n}^2 - \frac{\tilde{g}^2 \tilde{v}^2}{4}) \cos(\pi R m_{N\,n})\right) \cdot \\ &\left(\frac{\tilde{g}'^2}{g_5^2} m \sin(\pi R m_{N\,n}) + (m_{N\,n}^2 - \frac{\tilde{g}'^2 \tilde{v}^2}{4}) \cos(\pi R m_{N\,n})\right) = 0.\end{aligned} \tag{3.99}$$

The eigenvalue equation can then be approximately factorized into two independent ones; the first one is identical to eq. (3.98), the second one is similar with the replacement $g \to g'$. The tower of the neutral eigenstates is then composed by two subtowers, one of which almost identical to the one of the left sector. In fig. 3.3, I show the lightest part of the spectrum in an explicit example corresponding to a particular choice of the parameters.

Besides the spectrum, it is very important to check the model against EW precision observables. In fig. 3.4, I show the allowed region at 95% C.L. in parameter space $(M_1, \bar{g}_5)$, based on the new physics contribution to the ϵ parameters. $M_1 \equiv m_{R\,1} = 1/2R$ is the mass of the lightest KK excitation, that is of the first eigenstate of the right charged sector. It is important to note that M_1 and $\bar{M}$ are of the same order (their rate is $2\sqrt{3}/\pi$, see eqs. (3.91) and (3.97)). This can seem surprising, since the right sector is not involved in the calculation of the EW precision observables. The point is simply that, when the brane kinetic terms are big enough (that is when



	0 mode(s)	1^{st} **KK excit.(s)**	2^{nd} **KK excit.(s)**
Left charg.	80.4	1230	3095
Right charg.	-	1000	3000
Neutral	$0, 91.2$	$1060, 1230$	$3020, 3100$

Figure 3.3: *Light spectrum (zero modes and first two KK excitations) for the model in the flat limit, with the following parameter choice:* $\pi R = 1.57 \cdot 10^{-3}$ GeV^{-1}*,* $\bar{g}_5 = 1$*. All masses are in GeV. The corresponding naive unitarity cut-off is* $\sim 10^5$ GeV*, so every shown state is well within the unitarity limit.*

the ratio $\tilde{g}^2/g_5^2$ is $\lesssim M_1$), the first KK excitations of the left charged and neutral sector are also of order M_1; in fact, in this limit, the contribution of the kinetic term (the one proportional to $m^2_{L,N\,n}$) is dominant in the BCs (3.13), (3.26), which thus approximate the Dirichlet BC characteristic of the right charged sector. This is exactly the limit which is interesting from the phenomenological point of view, because it corresponds to the situation in which the $g^2/\bar{g}_5^2 < 1$ and the contribution to the ϵ parameters is suppressed.

The contour is obtained by considering the following experimental values for the ϵ parameters:

$$\begin{aligned} \epsilon_1 &= (+5.4 \pm 1.0)10^{-3} \\ \epsilon_2 &= (-8.9 \pm 1.2)10^{-3} \\ \epsilon_3 &= (+5.34 \pm 0.94)10^{-3} \end{aligned} \tag{3.100}$$

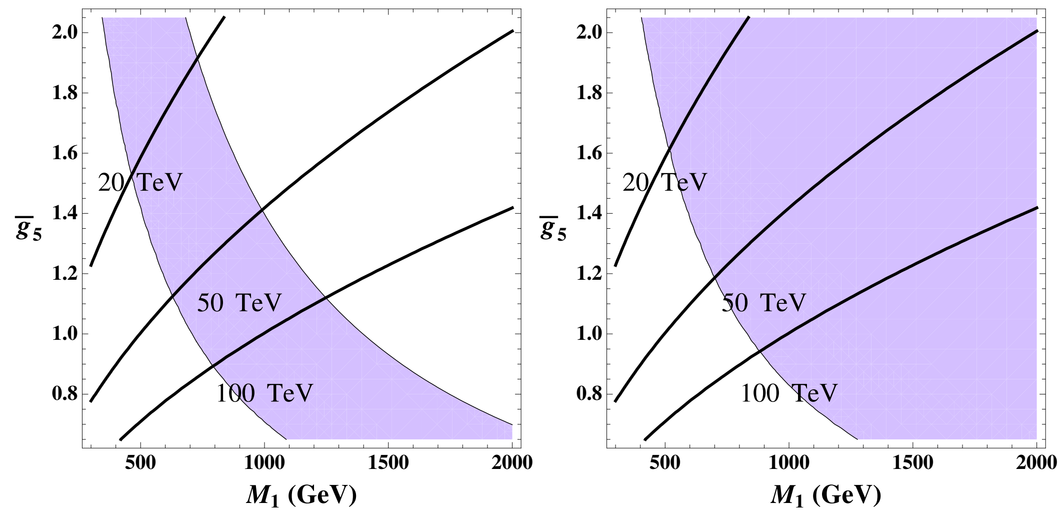


Figure 3.4: *Allowed regions in the* $(M_1, \bar{g}_5)$ *parameter space for a flat extra dimension, for two values of the Higgs mass:* $m_H = 1$ TeV *(on the left) and* $m_H = 300$ GeV *(on the right), based on electroweak precision constraints, and unitarity constraints from naive dimensional analysis (contours correspond to the model UV cut-off from unitarity).*

with correlation matrix

$$\begin{pmatrix} 1 & 0.60 & 0.86 \\ 0.60 & 1 & 0.40 \\ 0.86 & 0.40 & 1 \end{pmatrix}; \tag{3.101}$$

(taken from [69]), and adding to the present model contribution the one from radiative corrections in the SM. To fix the SM contribution, I must know the top mass m_t and assign an Higgs mass m_H. I have set $m_t = 171.2$ (*Review of Particle Physics*, 2008 edition) and repeated the fit for two different test values of the Higgs mass, $m_H = 1$ TeV and $m_H = 300$ GeV. I get:

$$\epsilon_1 = 3.2\ 10^{-3},\ \epsilon_2 = -6.5\ 10^{-3}, \quad \epsilon_3 = 6.7\ 10^{-3}, \quad \text{for } m_H = 1 \text{ TeV}; \tag{3.102}$$

$$\epsilon_1 = 4.7\ 10^{-3},\ \epsilon_2 = -7.0\ 10^{-3}, \quad \epsilon_3 = 6.1\ 10^{-3}, \quad \text{for } m_H = 300 \text{ GeV}; \tag{3.103}$$

(these SM contributions are obtained as a linear interpolation from the values listed in [70]).

Fig. 3.4 also reports contours that correspond to several values of the naive unitarity cut-off, (3.87). As it can be seen, the model is potentially compatible with EW precision data, even for a relatively small mass scale for the new heavy vector states (remember that, thanks to the decoupling, in the limit $M_1 \to \infty$ the SM picture is recovered, that is the region on the far right in fig. 3.4 gives the constraints in the SM case). The main drawback of the model in this limit is that since it has a single extra dimension, which is compact, small and flat, it does not help solving the hierarchy problem: the Higgs mass must still be adjusted through a fine-tuning exactly as in the SM.

3.4.2 The model on a slice of AdS_5

Probably, the most interesting case from the phenomenological point of view is that of an exponentially warped extra-dimension, a slice of AdS_5 space. This case corresponds to choosing $b(y) = e^{-2ky}$. The interest of this limit lies both in the possibility of solving the hierarchy problem thanks to an exponential suppression of mass scales on the $y = \pi R$ brane (or IR brane, which is where the Higgs is located) and in the AdS/CFT correspondence [26, 58, 35, 36], according to which a model on AdS_5 can be viewed as the dual description of a strongly interacting model on four dimensions. In particular, in AdS/CFT fields localized near the IR brane are interpreted as duals to composite states of the strong sector; in this interpretation the Higgs field is no longer a fundamental field, but only an effective low-energy degree of freedom, just like the KK excitations of the gauge fields.

With this choice I am in a sense come full circle, since I started my theoretical exploration by considering the D-BESS model, which gives a 4D low-energy effective description of a strongly interacting sector; I generalized that model first to a moose one, then to a 5-dimensional one; finally, thanks to the AdS/CFT correspondence, I can read the generalized 5D model again as an effective description of a strongly interacting theory.

The choice $b(y) = e^{-2ky}$ implies (again by eq. (3.73))

$$\frac{1}{\bar{M}^2} = \frac{1}{4k^2}\left(\frac{e^{2k\pi R}(2k^2(\pi R)^2 - 2k\pi R + 1) - 1}{k\pi R}\right); \tag{3.104}$$

the model has now an extra parameter, the curvature k, in addition to the usual πR, $\bar{g}_5$, $\tilde{g}$, $\tilde{g}'$, $\tilde{v}$ and λ. Eqs. (3.92), (3.93) and (3.94) still hold (with the new definition of $\bar{M}$ (3.104)); then, after fixing the standard EW input parameters, I am left with three free quantities, πR and $\bar{g}_5$ and k. Note that $\bar{g}_5$ is still constrained by eq. (3.95). Then, if I want this model to be a potential solution to the hierarchy problem, as the RS1 model [18], I need to fix the curvature parameter k to be around the Planck scale, $M_P \simeq 10^{19}$ GeV. Then, to have $\bar{M}$ around one TeV, I need $k\pi R \simeq 35$.

Let me look at the spectrum again. In this case, the C and S functions (eq. (3.56)) are given by:

$$\begin{aligned} S(y,m) &= \frac{e^{ky}}{2k}\pi\, m\left(J_1\left(\tfrac{m}{k}\right)Y_1\left(\tfrac{e^{ky}m}{k}\right) - J_1\left(\tfrac{e^{ky}m}{k}\right)Y_1\left(\tfrac{m}{k}\right)\right) \\ C(y,m) &= \frac{e^{ky}}{2k}\pi\, m\left(J_1\left(\tfrac{e^{ky}m}{k}\right)Y_0\left(\tfrac{m}{k}\right) - J_0\left(\tfrac{m}{k}\right)Y_1\left(\tfrac{e^{ky}m}{k}\right)\right), \end{aligned} \tag{3.105}$$

where J_i and Y_i are Bessel function of the first and of the second kind respectively. In this case, not even the condition for the right charged eigenstates can be solved analytically. However, using standard properties of the Bessel functions it is possible to give an estimate for the first eigenvalue,

$$M_1 \simeq k\, e^{-k\pi R} \frac{2\sqrt{2}}{\sqrt{4k\pi R - 3}}, \tag{3.106}$$

and for the characteristic spacing between two adjacent states, which is approximately constant and equal to $\Delta M = \pi k e^{-k\pi R}$.

The qualitative analysis made for the flat case generalizes almost verbatim to the AdS case. In particular, I have again that $m_{L\,1} \simeq m_{N\,1} \simeq M_1$. The main difference is the typical distance between two adjacent eigenstates, which is given by ΔM rather than simply by $1/R$. In fig. 3.5, I show an example of spectrum corresponding to a particular choice of the model parameters. It is interesting to compare this situation to the one of flat case; even though the masses of the first KK level in each sector are roughly the same, the appearance of the second KK level is delayed to a much higher scale.

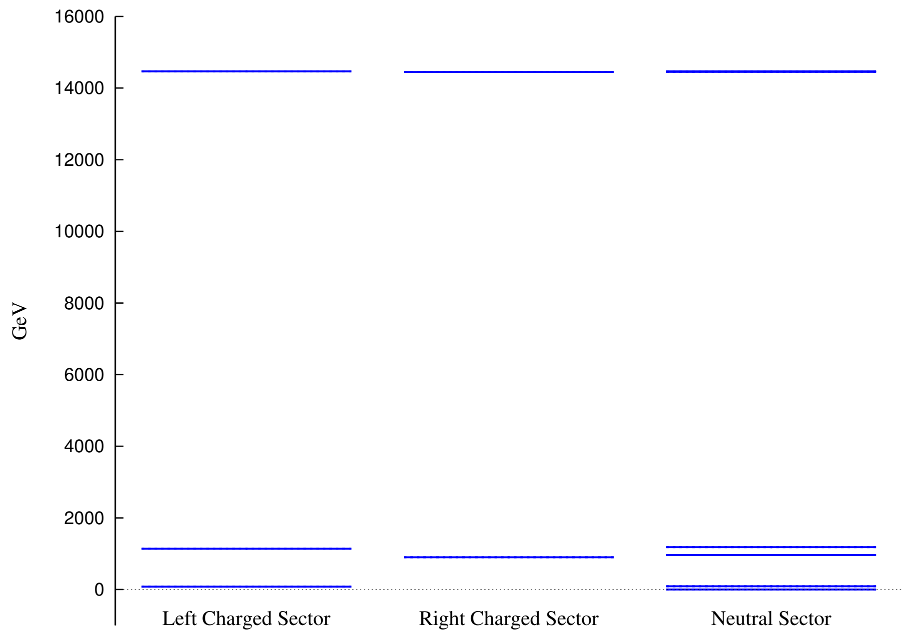
Also in this case, I have checked the model against EW precision data using the ϵ parameters. In fig. 3.6, I show the allowed region at 95% C.L. in parameter space $(M_1, \bar{g}_5)$; experimental data and SM radiative correction are the same of the flat case. Fig. 3.6 also reports contours that correspond to different values of the naive unitarity cut-off, (3.87). Notice that in this case, the UV cut-off due to unitarity is generally much lower than it was in the flat case. Nevertheless, the model is again potentially compatible with EW precision data, even when the new heavy vector states have masses around one TeV. The unitarity cut-off scale, which is quite low, calls for an UV extension of the model at an energy scale which is not much higher than the one potentially reached by the LHC; still the scenario described by the model seems interesting and deserves an accurate study.

The physical content of the 5D GD-BESS on an AdS background is very similar to the one of the RS1-like model described in [37]. In that reference, the authors studied a $SU(2)_L \otimes U(1)_Y$ 5D gauge theory in AdS background, with localized kinetic terms on the IR brane. The main difference between this set-up and the one I have outlined in this chapter is that I have considered a larger $SU(2)_L \otimes SU(2)_R$ bulk gauge symmetry. Notice, however, that if I add fermions in the simplest way, that is by localizing them on the IR brane (mimicking what I did for the D-BESS and GD-BESS models), then the extra gauge fields (that correspond to what I called the "right charged sector") are almost impossible to detect experimentally, since they cannot interact with the fermions (by eq. (3.20) they have no superposition with the IR brane). In fact, as can be seen by the effective Lagrangian calculation of section 3.3, they do not contribute to the ϵ parameters either. In conclusion, even if the bulk gauge group is different, the phenomenology of the two models is almost identical (the situation change, however, if fermions are allowed to propagate in the bulk).

This is a very interesting conclusion for this study: working with a completely bottom-up approach, starting from an effective 4D theory - the D-BESS model - and generalizing, I have arrived at a 5D model that quite closely reproduces a particular version of RS1.

3.5 Summary

In this chapter I have examined the continuum limit of the GD-BESS model, and contribution from the new physics it describes to the ϵ parameters. I have found that this 5D model has a set-up similar to a RS1 with the gauge fields propagating in the bulk of the extra-dimension. The contribution to the ϵ parameters is of



	0 mode(s)	1^{st} **KK excit.(s)**	2^{nd} **KK excit.(s)**
Left charg.	80.4	1180	14500
Right charg.	-	900	14450
Neutral	$0, 91.2$	$962, 1180$	$14450, 14500$

Figure 3.5: *Light spectrum (zero modes and first two KK excitations) for the model in the RS limit, with the following parameter choice:* $k = 5.9 \cdot 10^{18}$ GeV, $\pi R = 5.9 \cdot 10^{-18}$ GeV^{-1}, $k\pi R = 35$, $\bar{g}_5 = 1$. *All masses are in GeV. The corresponding naive unitarity cut-off is* $16.8 \cdot 10^3$ GeV. *In comparison with the flat case, it can be seen that the second KK level is pushed to a much higher energy scale, near but still within the unitarity limit.*

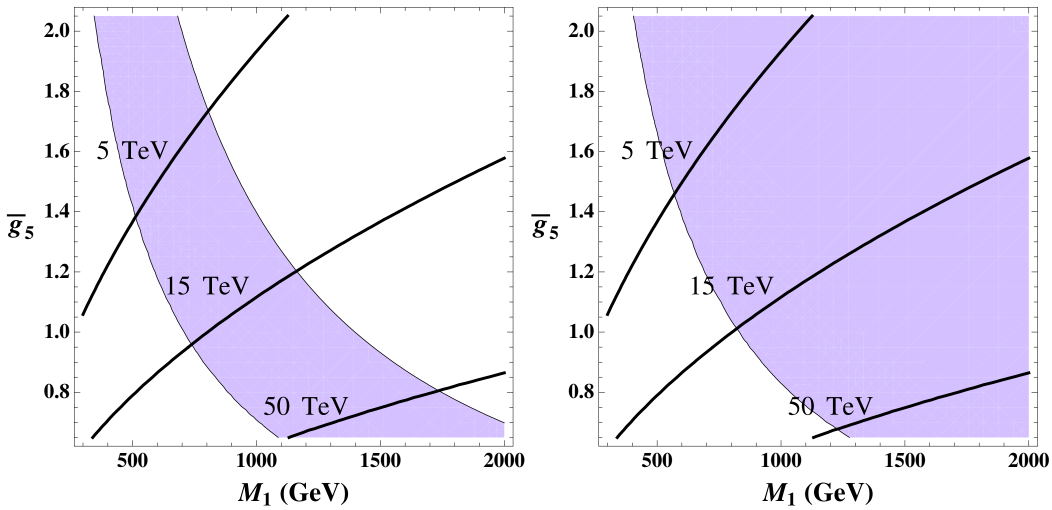


Figure 3.6: *Allowed regions in the* $(M_1, \bar{g}_5)$ *parameter space for the model in the RS limit* ($b(y) = e^{-2ky}$*, with* $k\pi R$ *fixed at* 35*), for two values of the Higgs mass:* $m_H = 1$ TeV *(on the left) and* $m_H = 300$ GeV *(on the right), based on electroweak precision constraints, and unitarity constraints from naive dimensional analysis (contours correspond to the model UV cut-off from unitarity).*

order $\frac{g^2}{\bar{g}_5^2}\frac{m_Z^2}{M_1^2}$, where M_1 is the mass of the first KK excitation; this is similar to what was found in the deconstructed case [34] and has an additional suppression factor $\frac{g^2}{\bar{g}_5^2}$ with respect to what is naively expected in a RS1 model (see for instance []). To have $\frac{g^2}{\bar{g}_5^2}$ significantly lower than unity one needs to have quite strongly interacting new physics; however, there is a significant portion of the parameter space in which the predicted values of the ϵ parameters are sufficiently near their experimental value, the new physics has a scale low enough (around a TeV) to be potentially detectable at the LHC and the naive unitarity cut-off, signalling the breakdown of perturbation theory, is $\geqslant 10$ TeV.

Chapter 4

Modified spontaneous symmetry breaking pattern by brane-bulk interactions

In this last chapter, I will leave behind the analysis of the D-BESS model and of its generalizations and study a rather different problem, though still related to EW symmetry breaking in extra dimensions. I will in fact present a detailed study of the vacuum configuration of a 5-dimensional model, where a scalar field living in the bulk gets, in some regions of the model parameter space, a nontrivial vacuum profile, explicitly dependent on the extra coordinate.

Vacuum solutions with a non trivial behaviour in the extra coordinate have been investigated in the literature, in particular to understand chirality properties or fermion masses and mixings, [71, 72, 73], or just to study the existence and stability of non trivial scalar configurations in simple $\lambda\phi^4$ theories on the circle or the orbifold [74, 75, 76, 77, 78], extending the pioneering paper on field localization in extra dimensions by Rubakov and Shaposhnikov [79].

In this study, I want to focus on the modification of the naive vacuum configuration when delta-like interactions are present between brane and bulk fields. Brane terms are always generated by radiative corrections, even in the absence of tree level brane couplings [72]. The coefficients of these operators are free parameters of any 5D model, and a generic study should in principle always consider them. The effects of such terms are often significant; for instance, the presence of brane kinetic terms is instrumental in getting sufficiently suppressed contribution to the EW precision parameters when light (of order TeV) KK excitations are considered in RS1-like models [37], as it is the case also of the 5D GD-BESS. The effect of brane kinetic terms has been investigated for scalar, fermion and gauge theories in [72, 80, 37, 81, 82, 83, 84]; by contrast, here I will focus on interaction terms.

For simplicity I will illustrate these effects in a simple two-Higgs doublet model in five dimensions, assuming one Higgs in the bulk and the second one on the brane [39]. Models of this type have been considered mainly from the phenomenological point of view as the simplest extensions of the Standard Model (SM) in five dimensions without supersymmetry [85, 86, 87, 88, 89].

In the analysis of these models usually one assumes the existence of a constant vacuum solution for the bulk field, which does not depend on the extra coordinate, without discussing whether the two-Higgs potential admits such a solution. In general (see for instance [90]) a constant solution does not exist, unless a particular relation among the quadrilinear couplings of the bulk and brane Higgs potential is satisfied. In the following, I provide analytic expressions for vacuum solutions and build explicit examples with non trivial profiles corresponding to configurations which are absolute minima of the energy density.

4.1 Delta-like interactions between brane and bulk fields

In this section, I review the $SU(2)_L \times U(1)_Y$-invariant two-Higgs model in five dimensions, with the field Φ_1 propagating in the bulk and the field Φ_2 localized on the brane at $y = 0$. I will use this simple model to illustrate the mechanism I wish to study, namely the existence of nontrivial vacuum configurations. The action of the model is:

$$S = \int_a^b dy \int d^4x \left\{ \mathcal{L}^{(5)} + \mathcal{L}^{(4)} \right\} , \tag{4.1}$$

$$\mathcal{L}^{(5)} = \partial_M \Phi_1^\dagger \partial^M \Phi_1 - V^{(5)}(\Phi_1) , \tag{4.2}$$

$$\mathcal{L}^{(4)} = \delta(y) \Big[\partial_\mu \Phi_2^\dagger \partial^\mu \Phi_2 - V^{(4)}(\Phi_1, \Phi_2) \Big] , \tag{4.3}$$

where $M = \mu, 5$ and $a < 0 < b$. Note that Φ_1 has energy dimension $3/2$, whereas Φ_2 has dimension 1. There could be some other fields, but, for the following discussion, only Φ_1 and Φ_2 are relevant. In order to identify the vacuum state, I need to solve the equations of motion

$$(-\partial_y^2 + \Box)\Phi_1 = \frac{\delta V^{(5)}}{\delta \Phi_1} + \delta(y) \frac{\delta V^{(4)}}{\delta \Phi_1} , \tag{4.4}$$

$$\delta(y) \Box \Phi_2 = \delta(y) \frac{\delta V^{(4)}}{\delta \Phi_2} , \tag{4.5}$$

$$\sum_{\Phi_\alpha = \Phi_1, \Phi_1^\dagger} \int d^4x \Big[\Big(\frac{\delta \mathcal{L}^{(5)}}{\delta \partial_y \Phi_\alpha} \delta \Phi_1 \Big)_{y=b} - \Big(\frac{\delta \mathcal{L}^{(5)}}{\delta \partial_y \Phi_\alpha} \delta \Phi_1 \Big)_{y=a} \Big] = 0 \, . \tag{4.6}$$

The last term comes from the boundary conditions, and could also give rise to contributions that can be recast in terms of $\delta(y-a)$ and $\delta(y-b)$ functions and are thus similar to those that I will consider next. This said, and for simplicity, I will choose periodic boundary conditions so that eq. (4.6) is automatically satisfied. If this was not the case, one should repeat for this boundary term the same analysis I will follow below for the $\delta(y)$ term.

The vacuum manifold corresponds to those solutions of the above equations of motion with minimum energy. Customarily, one considers constant solutions, i.e., $\Phi_1 = v_1, \Phi_2 = v_2$, so that the vacuum manifold corresponds to the minima of the potential, and, in particular, $\delta V^{(5)}/\delta\Phi_1 = 0$ and $\delta V^{(4)}/\delta\Phi_i = 0$ with $i = 1, 2$.

However, I will show in the following that the presence of delta-like interactions between brane and bulk fields modifies the vacuum manifold in such a way that static field configurations are not allowed any more. I will show that this effect is non-perturbative and that even an infinitesimal value of such a coupling could avoid the presence of the naively expected pattern of spontaneous symmetry breaking on the brane.

In order to illustrate these effects I will concentrate on a model used in the literature, although my considerations are applicable to more general solutions of the kind described above (and probably involving other kind of fields like fermions, or more complicated interaction terms, as long as brane-bulk interactions are present).

4.1.1 An example within 5D extensions of the Standard Model

I consider a minimal 5D extension of the SM with two scalar fields. For the moment it is irrelevant whether the compactification is done on the $[-\pi R, \pi R]$ circle with periodic boundary conditions or in an orbifold S^1/Z_2, of length πR, since I am only interested in vacuum configurations. Of course, for oscillations around the vacuum the orbifold would lead to fields with definite y-parity.

In this simple model the $SU(2)_L$ and $U(1)_Y$ gauge fields and the Higgs field Φ_1 propagate in the bulk while the Higgs field Φ_2 lives on the brane at $y = 0$. The Lagrangian of the gauge Higgs sector is given by (see [89] for a review)

$$\int_{-\pi R}^{\pi R} dy \int dx\, \mathcal{L}(x,y) = \int_{-\pi R}^{\pi R} dy \int dx \Big\{ -\frac{1}{4} B_{MN}B^{MN} - \frac{1}{4} F^a_{MN}F^{aMN} + \mathcal{L}_{GF}(x,y) + (D_M\Phi_1)^\dagger (D^M\Phi_1) + \delta(y)(D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) - V(\Phi_1,\Phi_2)\Big\}, \quad (4.7)$$

where B_{MN}, F^a_{MN} are the $U(1)_Y$ and $SU(2)_L$ field strengths and a is the $SU(2)_L$ index. The covariant derivative is defined as $D_M = \partial_M - ig_5 A^a_M \tau^a/2 - ig_5' B_M/2$.

For simplicity, I will consider a Higgs potential symmetric under the discrete symmetry $\Phi_2 \to -\Phi_2$, which is given by

$$V(\Phi_1,\Phi_2) = \mu_1^2\,(\Phi_1^\dagger\Phi_1) \,+\, \lambda_1\,(\Phi_1^\dagger\Phi_1)^2 \,+\, \delta(y)\,\Big[\,\frac{1}{2}\,\mu_2^2\,(\Phi_2^\dagger\Phi_2) \,+\, \frac{1}{2}\,\lambda_2\,(\Phi_2^\dagger\Phi_2)^2$$
$$+\,\frac{1}{2}\,\lambda_3\,(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \,+\, \frac{1}{2}\,\lambda_4\,(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \,+\, \lambda_5\,(\Phi_1^\dagger\Phi_2)^2 \,+\, \text{h.c.}\Big], \tag{4.8}$$

where the dimensionalities of the couplings are: 1 for μ_1 and μ_2, -1 for $\lambda_1, \lambda_3, \lambda_4$ and λ_5, whereas λ_2 is dimensionless.

The vacuum state manifold corresponds to configurations which are both energy minima and solutions of the following equations of motion:

$$(-\partial_y^2 + \Box)\Phi_1 = \mu_1^2\Phi_1 + 2\lambda_1\,(\Phi_1^\dagger\Phi_1)\Phi_1$$
$$+\,\delta(y)\,\Big[\lambda_3\,\Phi_1(\Phi_2^\dagger\Phi_2) + \lambda_4\,\Phi_2(\Phi_2^\dagger\Phi_1) \,+ 2\,\lambda_5\,(\Phi_1^\dagger\Phi_2)\Phi_2\Big]\,, \tag{4.9}$$

$$\delta(y)\Box\Phi_2 = \delta(y)\Big[\mu_2^2\Phi_2 + 2\lambda_2\,(\Phi_2^\dagger\Phi_2)\Phi_2 + \lambda_3\,(\Phi_1^\dagger\Phi_1)\Phi_2$$
$$+\,\lambda_4\,(\Phi_2^\dagger\Phi_1)\Phi_1 +\, 2\lambda_5\,(\Phi_1^\dagger\Phi_2)\Phi_1\Big]\,. \tag{4.10}$$

However, one could naively think, and it is sometimes assumed [85, 86, 87, 88, 89], that the extrema of the potential correspond to constant configurations $\Phi_1 = (0, v_1/\sqrt{4\pi R})$, $\Phi_2 = (0, v_2/\sqrt{2})$. Let me note, however, that if I substitute such constant solutions into the equations of motion above, I find

$$0 = v_1\left(\mu_1^2 + 2\lambda_1\frac{v_1^2}{4\pi R}\right)\,, \tag{4.11}$$

$$0 = v_1\,v_2^2\,\left(\lambda_3 + \lambda_4 + 2\lambda_5\right)\,, \tag{4.12}$$

$$0 = v_2\left(\mu_2^2 + \lambda_2 v_2^2 + \frac{v_1^2}{4\pi R}(\lambda_3 + \lambda_4 + 2\lambda_5)\right)\,. \tag{4.13}$$

If the trivial solutions $v_1 = v_2 = 0$ correspond to a minimum, I get a trivial vacuum configuration and no spontaneous symmetry breaking. When implementing a spontaneous symmetry breaking one customarily builds the Lagrangian in such a way that $\mu_1^2 < 0$, $\mu_2^2 < 0$ and thus $v_1 \neq 0$ and $v_2 \neq 0$ correspond to the minimum. But, due to eq.(4.12), this can only happen if $\lambda_3 + \lambda_4 + 2\lambda_5 = 0$. This may come as a surprise since these constants parametrize the interaction of brane and bulk fields and are, in principle, independent. Thus, *even the tiniest value of an interaction with* $\lambda_3 + \lambda_4 + 2\lambda_5 \neq 0$ *destroys the simplest ansatz of a translationally invariant vacuum state in the* y *direction.*

If one requires $\lambda_3 + \lambda_4 + 2\lambda_5 = 0$ [90], the minimum of the potential corresponds to the "constant ansatz". In this way, the Higgs fields are expanded in the standard form

$$\Phi_1(x,y) = \begin{pmatrix} \frac{i}{\sqrt{2}}(\omega^1 - i\omega^2) \\ \frac{1}{\sqrt{2}}(\frac{v_1}{\sqrt{2\pi R}} + h_1 - i\omega^3) \end{pmatrix},\ \Phi_2(x) = \begin{pmatrix} \frac{i}{\sqrt{2}}(\pi^1 - i\pi^2) \\ \frac{1}{\sqrt{2}}(v_2 + h_2 - i\pi^3) \end{pmatrix}, \tag{4.14}$$

where $v_1 \equiv \sqrt{-2\pi R\mu_1^2/\lambda_1}$ and $v_2 \equiv \sqrt{-\mu_2^2/\lambda_2}$ are the VEVs of the scalar fields and $v^2 = v_1^2 + v_2^2 = (\sqrt{2}G_F)^{-1}$. Let me remark that I assume $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 > -2\sqrt{2\pi R\lambda_1\lambda_2}$, otherwise the potential will not be bounded from below.

In the following, I will study the vacuum configuration of this model in the general case $\lambda_3 + \lambda_4 + 2\lambda_5 \neq 0$. I will find that, by including such a term, the spatial invariance in the 5^{th} dimension y is broken and nontrivial vacuum configurations are obtained from solutions of eqs. (4.9) and (4.10). For certain choices of parameters, the assumption that the vacuum state is independent of y might nonetheless be a good approximation, although the VEV of Φ_1 could be rather different from what could be guessed naively.

4.2 Static solutions of the equations of motion

Following the previous discussion, in this section I will first search for solutions of the equations of motion that could play the role of the true vacuum. Then, in the next section, I will study whether these solutions have a lower energy than the trivial vacuum so that they can trigger a spontaneous symmetry breaking. In particular, I will look here for solutions that maintain 4D Poincarè invariance (*i.e.* they do not depend on the 4D space-time coordinates x), but still have a dependence on y. I can then recast the static vacuum solutions as

$$\langle\Phi_1(x,y)\rangle = \begin{pmatrix} 0 \\ \varphi_1(y) \end{pmatrix}, \quad \langle\Phi_2(x)\rangle = \begin{pmatrix} 0 \\ \varphi_2 \end{pmatrix}, \tag{4.15}$$

where $\varphi_1(y)$ is a real-valued field, and φ_2 a real constant.

For the sake of simplicity, and because I just want to illustrate the effects due to the presence of a $\delta(y)$ term, I will study the limit $\lambda_4 = \lambda_5 = 0, \lambda_3 \neq 0$. Therefore, the equations of motion, eqs.(4.9) and (4.10) for non-trivial vacuum solutions in this model, are reduced to

$$\partial_y^2\varphi_1(y) - \varphi_1(y)\left[\mu_1^2 + 2\lambda_1\varphi_1(y)^2 + \delta(y)\,\lambda_3\varphi_2^2\right] = 0\,, \tag{4.16}$$

$$\delta(y)\,\varphi_2\left[\mu_2^2 + 2\lambda_2\,\varphi_2^2 + \lambda_3\,\varphi_1(y)^2\right] = 0\,. \tag{4.17}$$

The above solutions have an associated energy density per unit volume:

$$\begin{aligned}\mathcal{H} = \int_{-\pi R}^{\pi R} dy\, &\left[(\partial_y\varphi_1(y))^2 + \mu_1^2\varphi_1(y)^2 + \lambda_1\varphi_1(y)^4\right.\\ &\left.+\,\delta(y)\,\left(\mu_2^2\varphi_2^2 + \lambda_2\varphi_2^4 + \lambda_3\varphi_1(y)^2\varphi_2^2\right)\right].\end{aligned} \tag{4.18}$$

As is usually done, I account for the presence of the δ-function by solving the δ-less equation

$$\partial_y^2\varphi_1(y) - \varphi_1(y)\left[\mu_1^2 + 2\lambda_1\varphi_1(y)^2\right] = 0 \tag{4.19}$$

in the bulk regions $y < 0$ and $y > 0$ separately, and then connecting the two partial solutions by using the following boundary conditions:

- continuity in $y = 0$:

$$\varphi_1(0^-) = \varphi_1(0^+) \equiv \varphi_1(0); \tag{4.20}$$

- discontinuity of the first derivative in $y = 0$ with a gap $\lambda_3 \varphi_2^2 \varphi_1(0)$:

$$\varphi_1'(0^+) - \varphi_1'(0^-) = \lambda_3 \varphi_2^2 \varphi_1(0), \tag{4.21}$$

  where by eq.(4.17) I should have

$$\varphi_2^2 = -\frac{\mu_2^2}{2\lambda_2} - \frac{\varphi_1(0)^2 \lambda_3}{2\lambda_2}, \quad \text{with} \quad \varphi_2^2 > 0. \tag{4.22}$$

4.2.1 Solutions in the bulk

Let me analyze the most general solution of eq.(4.19) obeying 4D Poincaré invariance. Following [91, 75] I first multiply both sides by $\partial_y \varphi_1(y)$ and then integrate in y, to get

$$\frac{1}{2}(\partial_y \varphi_1(y))^2 - \frac{1}{2}\mu_1^2 \varphi_1(y)^2 - \frac{\lambda_1}{2}\varphi_1(y)^4 = e_0, \tag{4.23}$$

where e_0 is a constant quantity. Thus, integrating again I find

$$y - y_0 = \pm \int_{\varphi_1(y_0)}^{\varphi_1(y)} \frac{dt}{\sqrt{\mu_1^2 t^2 + \lambda_1 t^4 + 2e_0}}. \tag{4.24}$$

This integral can be solved analytically in terms of Jacobi elliptic functions [92, 93]. Such methods are well known, and thus I only provide the necessary steps to understand my notation. In particular, the exact solution depends on the nature of the roots of the polynomial

$$P_4(t) \equiv \lambda_1 t^4 + \mu_1^2 t^2 + 2e_0. \tag{4.25}$$

These are given by

$$t^2 = \frac{-\mu_1^2 \pm \sqrt{\mu_1^4 - 8e_0\lambda_1}}{2\lambda_1} \equiv \frac{-\mu_1^2}{2\lambda_1}\left(1 \mp \beta^2\right), \tag{4.26}$$

with

$$\beta^2 = \sqrt{1-\alpha}, \quad \alpha = \frac{8e_0\lambda_1}{\mu_1^4}. \tag{4.27}$$

Hence, depending on the values of α, I have the following cases:

A) $\alpha < 0$; $P_4(t)$ has two real and two complex solutions. I can therefore make use of the definition of the Jacobi elliptic $\mathrm{cn}(x, k^2)$ function:

$$\int_1^x \frac{dt}{\sqrt{(1-t^2)(1-k^2+k^2t^2)}} = \mathrm{cn}^{-1}(x, k^2), \tag{4.28}$$

to rewrite eq.(4.24) as follows:

$$y - y_0 = \pm\frac{a}{\sqrt{N}} \int_{\frac{\varphi_1}{a}(y_0)}^{\frac{\varphi_1}{a}(y)} \frac{dt}{\sqrt{(1-t^2)(1-k^2+k^2t^2)}}. \tag{4.29}$$

This is achieved by rescaling $t \to at$, so that

$$P_4(t) \to \lambda_1 a^4 t^4 + a^2\mu_1^2 t^2 + 2e_0 \equiv N(1-t^2)(1-k^2+k^2t^2), \tag{4.30}$$

where

$$\begin{aligned} k^2 &= \frac{1}{2}\left(1+\frac{1}{\beta^2}\right),\ a^2 = \frac{-\mu_1^2}{2\lambda_1}(1+\beta^2) > 0, \\ N &= \frac{-\mu_1^4}{2\lambda_1}\beta^2(1+\beta^2) < 0. \end{aligned} \tag{4.31}$$

In this way I finally get what I will call the "A type" solution

$$\varphi_1^A(y) = \pm\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1+\beta^2}\ \mathrm{nc}\left(|\mu_1|\beta(y-y_0), \frac{1}{2}(1-\frac{1}{\beta^2})\right), \tag{4.32}$$

where I used the relation $\mathrm{cn}(ix, k^2) = \dfrac{1}{\mathrm{cn}(x, 1-k^2)} \equiv \mathrm{nc}(x, 1-k^2)$.

B) $0 \leq \alpha \leq 1$, that is, $0 \leq \beta \leq 1$; in this case, $P_4(t)$ has four real solutions.

Again, I rescale $t \to at$; then I can match $P_4(t)$ to

$$P_4(at) \to N(1-t^2)(1-k^2t^2), \tag{4.33}$$

which leads to a Jacobi elliptic $\mathrm{sn}(x, k^2)$ solution

$$\int_0^x \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}} = \mathrm{sn}^{-1}(x), \tag{4.34}$$

with

$$\begin{aligned} k^2 &= \frac{1-\beta^2}{1+\beta^2}, \qquad a = \frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1-\beta^2}, \\ N &= 2e_0 = \frac{\mu_1^4}{4\lambda_1}(1-\beta^4) > 0\,, \end{aligned} \tag{4.35}$$

thus leading to what I will call "B1 type" solution

$$\varphi_1^{B1}(y) = \pm \frac{|\mu_1|}{\sqrt{2\lambda_1}} \sqrt{1-\beta^2}\ \mathrm{sn}\left(\frac{|\mu_1|}{\sqrt{2}} \sqrt{1+\beta^2}(y-y_0), \frac{1-\beta^2}{1+\beta^2} \right), \qquad (4.36)$$

which is an oscillating function of y that satisfies $\varphi_1^{B1}(y_0) = 0$.

But I can also recast $P_4(t)$ as

$$P_4(at) \to N(1-t^2)(t^2-1+k^2), \qquad (4.37)$$

which now leads to a Jacobi elliptic $\mathrm{dn}(x, k^2)$ solution

$$\int_1^x \frac{dt}{\sqrt{(1-t^2)(t^2-1+k^2)}} = \mathrm{dn}^{-1}(x), \qquad (4.38)$$

by identifying,

$$\begin{aligned} k^2 &= \frac{2\beta^2}{1+\beta^2}, \qquad a = \frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1+\beta^2}, \\ N &= \frac{-\mu_1^4}{4\lambda_1}(1+\beta^2)^2 < 0. \end{aligned} \qquad (4.39)$$

This is what I will call a "B2 type" solution, which does not oscillate. It satisfies $\varphi_1^{B2}(y_0)/a = 1$, and can be written as

$$\varphi_1^{B2}(y) = \pm \frac{|\mu_1|}{\sqrt{2\lambda_1}} \sqrt{1+\beta^2}\ \mathrm{dc}\left(\frac{|\mu_1|}{\sqrt{2}} \sqrt{1+\beta^2}(y-y_0), \frac{1-\beta^2}{1+\beta^2} \right), \qquad (4.40)$$

where I have used the relation $\mathrm{dn}(ix, k^2) = \mathrm{dc}(x, 1-k^2)$

C) $\alpha > 1$. In this case β^2 is pure imaginary and $P_4(t)$ has no real solutions. I can rewrite eq. (4.24) as:

$$y - y_0 = \pm \int_{\tilde{\varphi_1}(y_0)}^{\tilde{\varphi_1}(y)} \frac{d\tilde{t}}{\sqrt{(\tilde{t}^2-(1+\sqrt{1-\alpha}))(\tilde{t}^2-(1-\sqrt{1-\alpha}))}}, \qquad (4.41)$$

where I have made the rescaling:

$$t \to \tilde{t} = \frac{\sqrt{2\lambda_1}}{|\mu_1|} t. \qquad (4.42)$$

This integral is not equal to the inverse of a Jacobi elliptic function, as those of the previous cases. However, although in a somewhat more tedious way, it can be solved by using the standard techniques for elliptic integrals [92, 93].

The general solution is:

$$\varphi_1^C(y) = \frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{\frac{1}{2k^2-1}} \times \tag{4.43}$$
$$\frac{\mathrm{dn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{\frac{1}{2k^2-1}}\,(y-y_0),k^2\right) \pm \sqrt{1-k^2}\,\mathrm{sc}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{\frac{1}{2k^2-1}}\,(y-y_0),k^2\right)}{\mathrm{dn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{\frac{1}{2k^2-1}}\,(y-y_0),k^2\right) \mp \sqrt{1-k^2}\,\mathrm{sc}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{\frac{1}{2k^2-1}}\,(y-y_0),k^2\right)},$$

with

$$k^2 = \frac{1}{2}\left(1+\frac{1}{\sqrt{\alpha}}\right). \tag{4.44}$$

Let me now build the complete solutions of eq.(4.19) by imposing suitable boundary conditions in $y=0$ and $y=\pi R$.

4.2.2 Matching conditions

From integration, I initially have four free constants, two on the left side of the brane $y<0$, that I call y_{0L} and β_L (y_{0L},α_L in the case of type C solutions), and two more on the right side, $y>0$, called y_{0R} and β_R (again, y_{0R},α_R for solutions of type C). This fixes the shape of the function in the intervals, but, since the fields and their derivatives always appear squared in the action, there is an overall sign ambiguity, as it happens in the naive case with $\lambda_3=\lambda_4=\lambda_5=0$ where the vacuum solution for one Higgs in the bulk is given by either v_1 or $-v_1$.

Nevertheless, I am just looking for static minima of the action, which is symmetric under $y\leftrightarrow -y$. Hence the vacuum states must be even or odd under $y\leftrightarrow -y$, which implies $\beta_L=\beta_R\equiv\beta$. Also note that solutions which are antisymmetric under $y\leftrightarrow -y$ satisfy trivially the boundary condition (4.21); however if I require the continuity of $\varphi_1(y)$ in $y=\pi R$, its derivative has at least two nodes (one in the $(0,\pi R)$ region and the other in the $(-\pi R,0)$ one), so it cannot correspond to a global minimum of the energy (as I have explicitly checked numerically). In conclusion I am only interested in *even* solutions and therefore $y_{0L}=-y_{0R}\equiv y_0$.

Summarizing, apart from the overall sign arbitrariness, I am left with two constants β, y_0 that parametrize the space of possible candidates for vacuum configurations.

Furthermore my solutions should be of class C^1 except in $y=0$, and possibly in $y=\pm\pi R$ where I could impose some additional boundary conditions. At $y=0$ the left and right solutions should match each other according to eqs.(4.20), (4.21) and (4.22). The first one is automatically satisfied for even or odd functions, as in the present case. If non trivial solutions do exist, then I must have $\mu_2^2<0$, so (4.22) tells me that, for $\lambda_3>0$, $\varphi_1(0)$ is bounded by $\varphi_1(0)^2<-\mu_2^2/\lambda_3$. However, eq.(4.21) gives a relation between the two parameters β, y_0, that has to be solved numerically. All in all, there is just one free parameter left. This one can be fixed if I impose an additional boundary condition on $y = \pm\pi R$. As I will show, the boundary condition could be as simple as requiring continuity of the first derivative in $\pm\pi R$, but other choices are possible. Similarly to the terms in eq.(4.6), one could even think of another delta-like interaction term localized in a mirror brane in $y = \pm\pi R$.

In summary, by imposing the $y = 0$ boundary conditions in eqs.(4.20), (4.21) and (4.22), together with an additional boundary condition on $y = \pm\pi R$, one has sufficient constraints to fix, up to a global sign, the complete vacuum configuration in terms of the bulk solutions A, B1, B2, C detailed in the previous section.

In general I found that a given choice of parameters does not allow the existence of all kind of solutions. Of course, the trivial solution $v_1 = v_2 = 0$ is always present, but it will not correspond to the true vacuum if one of the solutions described above has a lower energy. This will lead to a spontaneous symmetry breaking with a pattern that is not translationally invariant in the y variable. For some choice of parameters it can also happen that non trivial solutions cannot be found, so that there is no spontaneous breaking of symmetry.

In the next section I will show, with explicit examples, that for certain choices of the parameters, the non-trivial configurations do exist and have lower energy densities than the trivial $\Phi_1 = \Phi_2 = 0$ one. These solutions lead to a spontaneous symmetry breaking with a pattern which is nonstandard, since the vacuum is not translationally invariant in the extra coordinate, and the VEV of the bulk scalar field is not related to the Lagrangian parameters in the usual manner.

4.3 Examples of non-trivial vacuum configurations

I have shown how the solutions are basically fixed by the boundary conditions, once one knows the Lagrangian parameters. Let me now remark that in the model I have considered in Section 4.1 there are five independent parameters $\lambda_1, \lambda_2, \lambda_3$ and μ_1, μ_2. Note that, in the realistic case for the usual two-Higgs doublet one customarily chooses the parameters with the constraint $v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$, which fixes one of the Lagrangian parameters in terms of the others, and provides the standard mass for the electroweak gauge bosons once the covariant $SU(2)_L \times U(1)_Y$ derivatives are considered.

Since I want to illustrate how the symmetry breaking pattern can be modified with non-trivial brane interactions, I will impose a similar constraint. However, since $\varphi_1(y)$ is not a constant, I have to look back to the kinetic terms of the scalar fields in the Lagrangian in eq. (4.7).

Recalling that $D_M = \partial_M - ig_5 A_M^a \tau^a/2 - ig_5' B_M/2$, I see that the KK zero modes of the gauge fields will obtain their masses from the vacuum configuration $\varphi_1(y)$ of the scalar field Φ_1 and the VEV φ_2 of Φ_2 through the combination

$$v^2 \equiv 4\int_0^{\pi R} \varphi_1(y)^2\, dy + 2\,\varphi_2^2 = 4\int_0^{\pi R} \varphi_1(y)^2\, dy - \frac{\mu_2^2 + \lambda_3\,\varphi_1(0)^2}{\lambda_2}\,, \qquad (4.45)$$

where I have used eq.(4.22).

Of course, in the $\lambda_3 \to 0$ limit, I recover the usual relation $v^2 = v_1^2 + v_2^2$, but, in the general case, since the VEV φ_1 depends explicitly on y, an integration over the compactified fifth dimension is required. Once again, imposing that for the true vacuum $v^2 = (246 \text{ GeV})^2$, with v defined in eq.(4.45), fixes one of the Lagrangian parameters in terms of the others.

Note that eq. (4.45) is really an approximation: in fact, if the scalar ϕ_1 gets a nontrivial vacuum profile, then the gauge vectors relative to the broken generators will also have nontrivial wavefunctions. As a consequence, the relation between the gauge boson mass and v will also be modified with respect to the standard case by a quantity of the order of the relative variation of the ϕ_1 along the extra dimension. For the precise relation one needs the explicit calculation of the profile of gauge modes, which lies beyond the scope of this work.

I will show that, depending on the boundary conditions on $y = \pm\pi R$, I can still find solutions for which the "constant ansatz" may be a good approximation, although the vacuum expectation value of the Φ_1 field on the $y = 0$ brane might be rather different from v_1. In addition, there are solutions which change sizably in the extra dimension and should not be approximated by a constant value. Both cases will be illustrated with the following examples.

4.3.1 Quasi-constant vacuum in the extra dimension

Let me impose, as a boundary condition, the continuity of the first derivative of $\varphi_1(y)$ in $y = \pi R$. The periodicity, moreover, identifies the point πR with the point $-\pi R$; so, what I in fact require is: $\varphi_1'(\pi R) = \varphi_1'(-\pi R)$. But φ_1' is an odd function, then $\varphi_1'(-\pi R) = -\varphi_1'(\pi R)$ also comes true. So I conclude that $\varphi_1'(\pi R) = 0$, that is, πR is a maximum or a minimum for $\varphi_1(y)$.

Let me now make the following choice of parameters:

$$\begin{aligned} &\pi R = (1\text{TeV})^{-1}, \quad |\mu_1| = 165 \text{ GeV}, \quad \lambda_1 = 0.5 \times 2\pi R, \\ &\lambda_2 = 1, \quad \lambda_3 = 0.85 \times 2\pi R. \end{aligned} \qquad (4.46)$$

Since I require $v = 246\,\text{GeV}$, in eq.(4.45), apart from a global sign, there is only one continuous solution of eq. (4.21), that turns out to be of the B1-type, and can be written as follows:

$$\varphi_1^{B1}(y) = +\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1-\beta^2}\ \mathrm{sn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{1+\beta^2}(y-y_0), \frac{1-\beta^2}{1+\beta^2}\right), \quad y>0, \quad (4.47)$$

$$\varphi_1^{B1}(y) = -\frac{|\mu_1|}{\sqrt{2\lambda_1}}\sqrt{1-\beta^2}\ \mathrm{sn}\left(\frac{|\mu_1|}{\sqrt{2}}\sqrt{1+\beta^2}(y+y_0), \frac{1-\beta^2}{1+\beta^2}\right), \quad y<0, \quad (4.48)$$

with $|\mu_2| \simeq 220$ GeV, $\beta \simeq 0.79$ and $y_0 \simeq 0.012$ GeV^{-1}. Here, for definiteness, I have taken the sign in front of the $y > 0$ solution to be positive, but of course, there is another solution with the opposite sign and the same energy.

The energy density can be calculated using eq. (4.18); I find that it is equal to $-(179\text{ GeV})^4$, which is less than the $(0\text{ GeV})^4$ associated with the trivial static solution, thus confirming the fact that I am in presence of spontaneous symmetry breakdown. Actually, since there are no other solutions, the one I have found, shown in fig. 4.1 corresponds to a global minimum and can be identified with the true vacuum. As it can be seen from the figure, a constant solution in this case would be an adequate approximation, since the difference between $\varphi_1(0)$ and $\varphi_1(\pi R)$ is less than 1%. However, I should note that the VEV of the Φ_1 field on the $y = 0$ brane is $\varphi_1(0) \simeq 139$ GeV, very different from the corresponding $v_1 \simeq 233$ GeV which would be obtained with the parameter choice (4.46) and $\lambda_3 = 0$. This is a 63% decrease that can modify the spectrum of the KK excitations with respect to the one of the naive ansatz even if the vacuum configuration is almost constant.

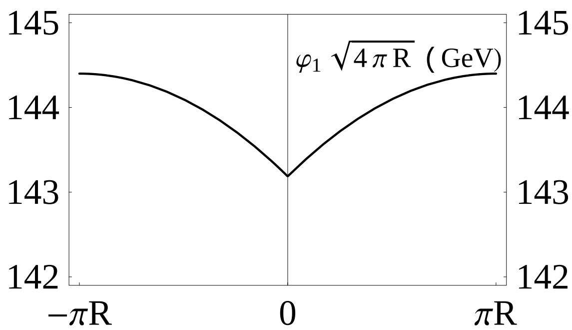


Figure 4.1: *Vacuum configuration for the choice of parameters of sect. 4.3.1. Note that, by taking it as constant, (as it is for* $\lambda_3 = 0$*), might be a good approximation, since its variation from* $y = 0$ *to* $y = \pi R$ *is less than* 1%*. However,* $\varphi_1(0) \simeq 233\,\mathrm{GeV}$ *when* $\lambda_3 = 0$*, instead of* $\varphi_1(0) \simeq 143\,\mathrm{GeV}$ *here.*

4.3.2 Sizable violation of translational invariance in the extra dimension

Let me allow for a discontinuity of the first derivative in $y = \pi R$ assuming for $\varphi_1'(\pi R)$ a given value different from 0, and make this different choice of parameters:

$$\pi R = (1\text{TeV})^{-1}, \quad |\mu_1| = 60 \text{ GeV}, \quad \lambda_1 = 0.5 \times 2\pi R, \\ \lambda_2 = 2, \quad \lambda_3 = 10 \times 2\pi R. \tag{4.49}$$

Again, the minimum corresponds to a B1 type solution as in eqs. (4.47) and (4.48), but with $|\mu_2| \simeq 349$ GeV, $\beta \simeq 0.1$ and $y_0 \simeq 0.15\,\text{GeV}^{-1}$. The energy density in this case is $\simeq -(245\,\text{GeV})^4$, again indicating a spontaneous symmetry breaking. Incidentally, in this case there is also another solution, of type A, but it has a positive energy density and thus it does not correspond to a vacuum state.

In fig. (4.2), I show the vacuum configuration for the choice of parameters of eq. (4.49). I can note that, in this case, the constant approximation would not be appropriate, since the difference between $\varphi_1(0)$ and $\varphi_1(\pi R)$ is more than 20%. Moreover, $\varphi_1(0) \simeq 19$ GeV while $v_1 \simeq 85$ GeV, so the corresponding difference is even greater than that of the previous case.

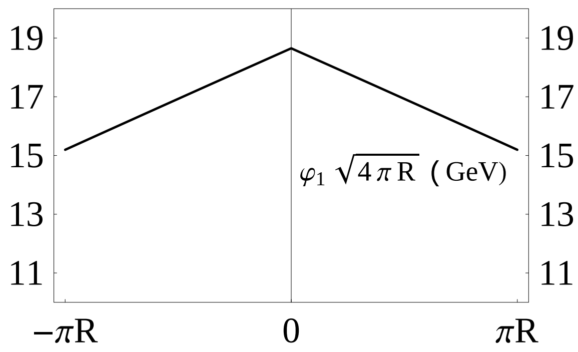


Figure 4.2: *Vacuum configuration for the choice of parameters of sect. 4.3.2. I see that a constant φ_1 is not a good approximation: the variation from $y = 0$ to $y = \pi R$ is about 22%. The variation of $\varphi_1(0)$ with respect to the non-interacting case is even greater; I would have $\varphi_1(0) \simeq 85\,\text{GeV}$ for $\lambda_3 = 0$ (with the other parameters kept constant), while $\varphi_1(0) \simeq 19\,\text{GeV}$ here.*

4.4 Summary

In this chapter, I have shown how the explicit breaking of translational invariance on the extra dimension induced by delta-like interactions between scalar bulk and brane fields translates into the vacuum configuration. This effect modifies the naively expected pattern of spontaneous symmetry breakdown in extra dimensional extensions of the Standard Model containing such terms. In particular I have found that, if a general form for the scalar potential is considered, constant non trivial solutions of the equation of motion for the scalar fields on the bulk cannot be found. I am thus forced to consider a vacuum configuration for the scalar bulk field that depends on the extra coordinate y.

I have used a simple two-Higgs model to illustrate these effects, and, in particular, I have derived the shape of the vacuum configuration in two examples: in the first one, the y dependence is weak, so that a constant configuration may still be a good approximation; however, the value of the VEV on the brane of the scalar bulk field is significantly shifted with respect to the case with no brane-bulk interactions, and this could cause a modification of the Kaluza-Klein spectrum of the bulk fields after the spontaneous symmetry breaking. In the second example, the y dependence is much stronger, and a constant solution would only be a poor approximation to the actual vacuum configuration.

Conclusions and future perspectives

The exact nature of the mechanism that leads to the breakdown of electroweak symmetry at low energies is one of the deepest open questions in particle physics. Now that the Large Hadron Collider is at last becoming operative, we may be able to get an answer; while waiting for the first experimental data, it is worthwhile to explore the potential EW breaking scenarios from a theoretical point of view.

In the Standard Model, the mechanism of EW symmetry implies the presence a fundamental scalar particle, the Higgs boson, with a mass around 100 GeV. However, this mechanism is affected by a serious fine-tuning problem, the hierarchy problem, because the mass of the Higgs boson is not protected against radiative corrections and would naturally be expected to be as large as the physical UV cut-off of the SM, which could be as high as $M_P \simeq 10^{19}$ GeV.

Two possible solutions to the hierarchy problem are the technicolor theories [13, 14, 15] (that postulate the presence of new strong interactions around the TeV scale) and extra-dimensional theories [16, 18, 94]. These seemingly unrelated classes of theories have in fact a profound connection through the AdS/CFT correspondence [26].

In this work, I have started by examining the D-BESS model [27], a low-energy effective TC theory. Generic TC models usually have difficulties in satisfying the constraints coming from EW precision measurements [41, 42]. The D-BESS model, however, possesses an $(SU(2) \otimes SU(2))^2$ custodial symmetry [57] that leads to a suppressed contribution from the new physics to the EW precision observables (parametrized for instance through the ϵ parameters [28, 29, 30]), making it possible to have new vector bosons at a relatively low energy scale (around a TeV). This new vector states are interpreted as composites of a strongly interacting sector.

I have then studied the generalization of the D-BESS model (GD-BESS) [31, 34], first to a deconstructed or "moose" model [32], then to a 5-dimensional theory. The D-BESS model and both of its generalizations suffer the drawback of the unitarity constraint, which is as low as that of the Higgsless SM [8, 9], that is around 1.7 TeV.

However, (at least for a particular choice of the extra-dimensional background) in the 5-dimensional model it is possible to reintroduce an Higgs field, delaying unitarity violation to a scale $\gtrsim$ 10 TeV. In the "holographic" interpretation of AdS_5 models [35, 36], inspired by the AdS/CFT correspondence, this Higgs can be understood as a composite state and thus does not suffer from the hierarchy problem. The 5-dimensional GD-BESS on AdS_5 then provides a coherent description of the low energy phenomenology of a new strongly interacting sector up to energies significantly beyond the $\sim$ 2 TeV limit of the Higgsless SM, still showing a good compatibility with EW precision observables. In this version, the 5D GD-BESS is very similar to an RS1 model [18] with EW gauge fields propagating in the bulk and having brane-localized kinetic terms [37]. The calculation of the ϵ parameters in GD-BESS (section 2.2.3) and its 5-dimensional generalization (chapter 3) are original contributions to this work, [34, 38].

While the 5D generalization of (G)D-BESS, especially on an AdS background, is certainly interesting and worth studying, it cannot be considered a fully realistic model, because of its treatment of the fermions. The obtained results for the EW precision parameters only hold if fermions are confined to the $y = \pi R$ brane. However, in this case one generically expects the emergence of four-fermion operators which induce unacceptable flavour violations [95, 96, 97, 98, 99, 100]. The well-known cure to this problem is letting fermions propagate in the bulk, so an obvious follow-up to the work done in this thesis is studying the behaviour of 5D GD-BESS with bulk fermions.

Another interesting future development is a detailed investigation on the possibility of having a heavy Higgs boson, with a mass of order 300 GeV or more. In the SM, global fits indicate that the Higgs mass cannot be higher than about 160 GeV; by contrast, in GD-BESS the constraints do not seem so stringent (see figs. 3.4 and 3.6) and an heavy Higgs mass could be allowed.

In the last chapter of this work, I described another study, not directly related to D-BESS, but still concerning EW symmetry breaking in five dimensions. This is another original contribution, published in [39]. The study focused on a model with two scalars, one propagating in the bulk of the extra dimension and the other confined to a brane, and in particular on the effects of interaction terms between the brane and the bulk fields on the pattern of spontaneous symmetry breaking. While the study was primarily technical in nature, without any claim to describe a fully realistic model, its most peculiar ingredient - namely the presence of a bulk Higgs field whose VEV gets distorted by brane-localized interactions - can be implemented in a potentially realistic model. An interesting example is the so called "Soft-Wall" SM [40], where the RS1 set-up is generalized by replacing the "hard" 5D cut-off at $y = \pi R$ (the brane) with a smooth boundary, which is provided by the VEV of a bulk scalar.

In conclusion, while the SM Higgs mechanism provides the most efficient and economical explanation of the spontaneous electroweak symmetry breaking, it is still not verified by experiments and it is not completely satisfactory from a theoretical point of view; theories in extra dimensions provide a fascinating alternative to the standard picture, that implies the existence of an interesting phenomenology that could be observed at LHC.

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