Abstract

This work establishes a comparison between functions on derived loop spaces (Toën and Vezzosi, *Chern character, loop spaces and derived algebraic geometry*, in *Algebraic topology: the Abel symposium 2007*, Abel Symposia, vol. 4, eds N. Baas, E. M. Friedlander, B. Jahren and P. A. Østvær (Springer, 2009), ISBN:978-3-642-01199-3) and de Rham theory. If A is a smooth commutative k-algebra and k has characteristic 0, we show that two objects, $S^1 \otimes A$ and $\epsilon(A)$, determine one another, functorially in A. The object $S^1 \otimes A$ is the S^1-equivariant simplicial k-algebra obtained by tensoring A by the simplicial group $S^1 := B\mathbb{Z}$, while the object $\epsilon(A)$ is the de Rham algebra of A, endowed with the de Rham differential, and viewed as a ϵ-*dg-algebra* (see the main text). We define an equivalence φ between the homotopy theory of simplicial commutative S^1-equivariant k-algebras and the homotopy theory of ϵ-dg-algebras, and we show the existence of a functorial equivalence $\phi(S^1 \otimes A) \sim \epsilon(A)$. We deduce from this the comparison mentioned above, identifying the S^1-equivariant functions on the derived loop space LX of a smooth k-scheme X with the algebraic de Rham cohomology of X/k. As corollaries, we obtain *functorial* and *multiplicative* versions of decomposition theorems for Hochschild homology (in the spirit of Hochschild–Kostant–Rosenberg) for arbitrary semi-separated k-schemes. By construction, these decompositions are *moreover* compatible with the S^1-action on the Hochschild complex, on one hand, and with the de Rham differential, on the other hand.